

Ground States Structure of Ruthenium Isotopes with Neutron $N = 60, 62$

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Abstract

In this paper, Ruthenium Isotopes with neutron $N = 60, 62$ have been studied the ground state bands using Matlab computer code interacting boson model (IBM-1). We apply IBM-1 formula for O(6) symmetry in Ru isotopes with neutron $N = 60, 62$. The theoretical energy levels up to spin-parity 12^+ have been obtained for $^{104,106}\text{Ru}$ isotopes. The yrast states, gamma band, beta band, and B(E2) values are calculated for those nuclei. The experimental and calculated $R_{4/2}$ values indicate that the even-even $^{104-106}\text{Ru}$ isotopes have O(6) dynamic symmetry. The calculated results are compared to the experimental data and are found in good harmony with each other. The plots of the potential energy surface of both nuclei are O(6) characters.

Keywords

Ruthenium Isotopes, Interacting Boson Model-1, Potential Energy, Energy Level

1. Introduction

Recently, Ruthenium isotope has been a focus of the nuclear structure of many theoretical and experimental investigations. The low-lying even nuclei had been successfully explained nuclear collective characters using the interacting boson model-1 (IBM-1) [1]. In the first beginning the collective states can be described by a system of identical bosons N_b . These are S-boson $L = 0$ and d-boson $L = 2$. There is no discrepancy between neutron and proton in IBM-1. There are three dynamical symmetries indicated by U(5), SU(3) and O(6) analogous to spherical vibrator, deformed rotor, and γ -soft respectively. The microscopic a harmonic

vibrator approach (MAVA) used in investigating the lower level collective states in Ruthenium isotopes [2].

The Ruthenium isotopes have atomic number $Z = 44$. It belongs near to closed shell Sn (magic number $Z = 50$). The external forms of even $^{104-106}\text{Ru}$ isotopes have $g_{9/2}^{-6}$ (6 proton holes) and $g_{9/2}^{10,12}$ (10 and 12 neutron particles) close to magic number 50. This configuration has been investigated the ground state structure from spherical to deformed symmetry. The edifice of yrast levels and electromagnetic strength of Ru isotopes studied by many scientists [3] [4] [5] [6] [7].

Recently, the properties of the yrast level were studied in Pd isotopes with even neutron $N = 54 - 64$ [8]. The electromagnetic reduced transition strength of Cd isotopes with $N = 66 - 74$ were investigated [9]. The B(E2) value of yrast band of even $^{102-112}\text{Pd}$ and $^{96-102}\text{Ru}$ isotopes [10] [11] [12] were investigated by interacting boson model (IBM-1). The low-lying level of ^{184}W and ^{184}Os nuclei were investigated [13] [14].

The present aim particularly focuses on the structure of the ground state band and the potential energy surfaces to find the dynamical symmetry of even $^{104-106}\text{Ru}$ isotopes by the application of IBM.

2. Method of Calculation

The Interacting Boson Model (IBM) gives occupation to truncated model space for nuclei with N number of nucleons. It provides a quantitative description of identical particles with forming pairs of angular momentum 0 and 2.

The Hamiltonian of IBM-1 [15]:

$$H = \sum_{i=1}^N \varepsilon_i + \sum_{i<j}^N V_{ij} \quad (1)$$

Here ε is energy of boson and V_{ij} is the potential energy of boson between i and j .

Hamiltonian is from multi-pole form [16]

$$H = \varepsilon \hat{n}_d + a_0 (\hat{P} \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) + a_2 (\hat{Q} \cdot \hat{Q}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) + a_4 (\hat{T}_4 \cdot \hat{T}_4) \quad (2)$$

Here

$$\begin{aligned} \hat{n}_d &= (d^\dagger \cdot \tilde{d}), \quad \hat{P} = \frac{1}{2}(\tilde{d} \cdot \tilde{d}) - \frac{1}{2}(\tilde{s} \cdot \tilde{s}) \\ \hat{L} &= \sqrt{10} [d^\dagger \times \tilde{d}]^{(1)} \\ \hat{Q} &= [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} - \frac{1}{2} \sqrt{7} [d^\dagger \times \tilde{d}]^{(2)} \\ \hat{T}_3 &= [d^\dagger \times \tilde{d}]^{(3)}, \quad \hat{T}_4 = [d^\dagger \times \tilde{d}]^{(4)} \end{aligned}$$

Here P is the pairing operator for s and d bosons, Q is quadrupole operator, \hat{n}_d is number of d boson, L is operator of angular momentum, and T_3 octuplet operators and T_4 is hexadecapole operators.

The Hamiltonian starting with U(6) and finishing with group O(2) as given in Equation (2) is bringing to a lower state of three limits, γ -soft O(6), the vibration U(5) and the rotational SU(3) nuclei [17]. We know that in the SU(3) limits, the effective parameter is the quadrupole a_2 , in the O(6) limit the effective parameter is the pairing a_0 , in U(5) limits, the effective parameter is ε .

The Hamiltonian and eigen-values for the three limits [18]:

U(5):

$$\left. \begin{aligned} \hat{H}_{U(5)} &= \varepsilon \hat{n}_d + a_1 (\hat{L} \cdot \hat{L}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) + a_4 (\hat{T}_4 \cdot \hat{T}_4) \\ E(n_d, \nu, L) &= \varepsilon n_d + K_1 n_d (n_d + 4) + K_4 \nu (\nu + 3) + K_5 L (L + 1) \end{aligned} \right\} \quad (3)$$

with

$$\begin{aligned} K_1 &= 1/12 a_1 \\ K_4 &= -1/10 a_1 + 1/7 a_3 - 3/70 a_4 \\ K_5 &= -1/14 a_3 + 1/14 a_4 \end{aligned}$$

O(6):

$$\left. \begin{aligned} \hat{H}_{O(6)} &= a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \\ E(\sigma, \tau, L) &= K_3 [N_b (N_b + 4) - \sigma (\sigma + 4)] + K_4 \tau (\tau + 3) + K_5 L (L + 1) \end{aligned} \right\} \quad (4)$$

with

$$\begin{aligned} K_3 &= 1/4 a_0 \\ K_4 &= 1/2 a_3 \\ K_5 &= -1/10 a_3 + a_1 \end{aligned}$$

SU(3):

$$\left. \begin{aligned} \hat{H}_{SU(3)} &= a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \\ E(\lambda, \mu, L) &= K_2 (\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda \mu) + K_5 L (L + 1) \end{aligned} \right\} \quad (5)$$

with

$$\begin{aligned} K_2 &= 1/2 a_2 \\ K_5 &= a_1 - 3/8 a_2 \end{aligned}$$

K_1 , K_2 , K_3 , K_4 , and K_5 are other forms of strength parameters.

Then applying particular limit of symmetry (O(6), SU(3), U(5)) to determine the frame of a set of nuclei is more advantageous than full Hamiltonian of IBM-1. It comprise multi-free parameters those make it simple to fit the structure of a nuclei. A flow chart of method of calculation is given in **Figure 1**.

3. Results and Discussion

The obtained results have discussed for yrast energy level, γ -band, β -band, effective charge used to reproduce B(E2) values, transition probabilities B(E2), maxing ratio and contour plots of the potential energy surfaces using IBM-1.

The γ -unstable limit has applied for $^{104,106}\text{Ru}$ nuclei using data of experimental energy ratios ($E_2: E_4: E_6: E_8 = 1:2.5:4.5:6.5$). In the framework of IBM-1, the even

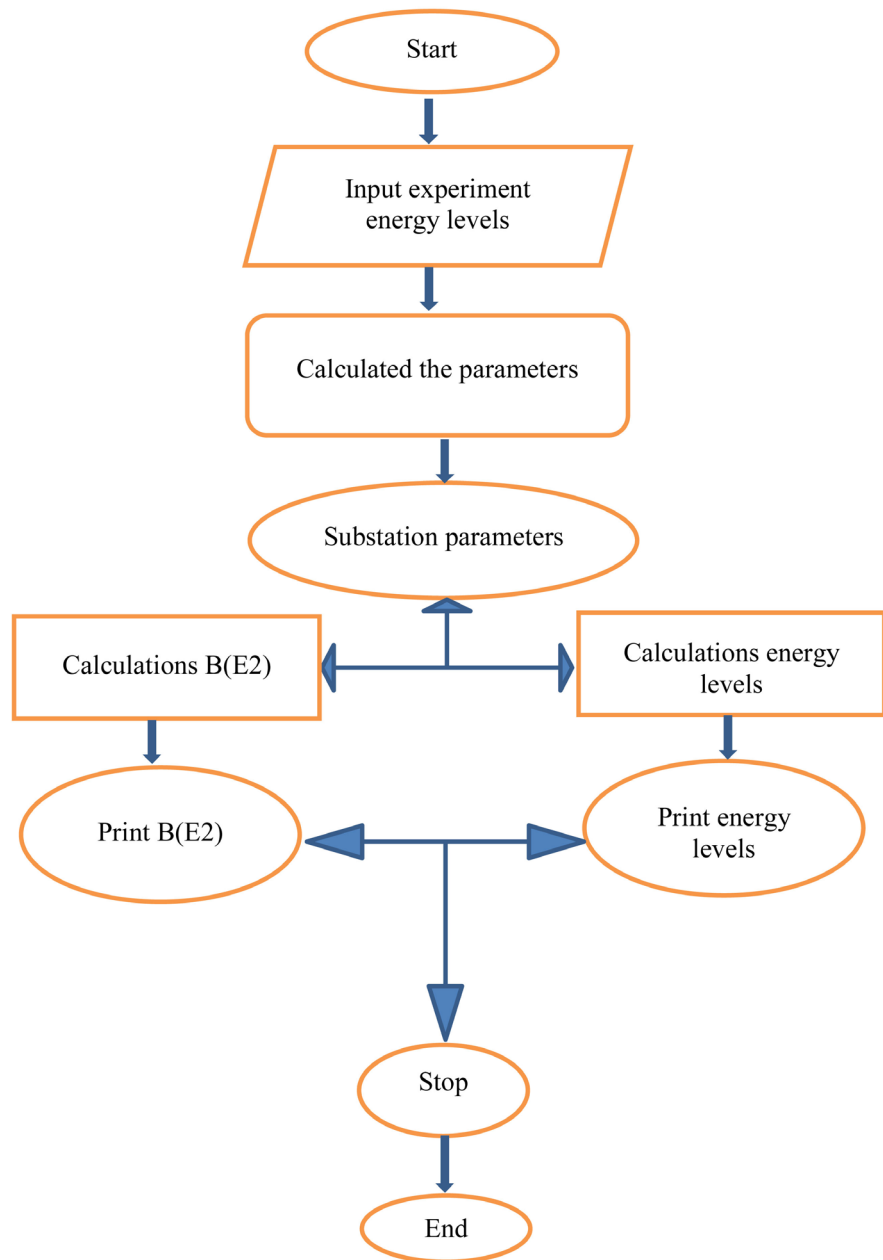


Figure 1. Flowchart of method of calculation.

$^{104-106}\text{Ru}$ nuclei have three protons boson hole and five and six neutrons boson particle respectively. Therefore total bosons numbers of ^{104}Ru and ^{106}Ru nuclei are 8 and 9, respectively. The IBM-1 models carry out with no difference between the bosons of proton and neutron. The energy ratio $R = E_{4_1^+}/E_{2_1^+}$ gives the information of the symmetry shapes of a nucleus. The symbol $E_{2_1^+}$ and $E_{4_1^+}$ is at the energy level 2_1^+ and 4_1^+ respectively. It is known that the $R = E_{4_1^+}/E_{2_1^+} \approx 2$ is for U(5), $R = E_{4_1^+}/E_{2_1^+} \approx 2.5$ is for O(6) and $R = E_{4_1^+}/E_{2_1^+} \approx 3.33$ for SU(3) [19] [20]. The experimental $R_{4/2}$ of ^{104}Ru and ^{106}Ru isotopes is 2.48 and 2.60, respectively. **Figure 2** shows, $R_{4/2}$ values of ^{104}Ru and ^{106}Ru isotopes are O(6) symmetry.

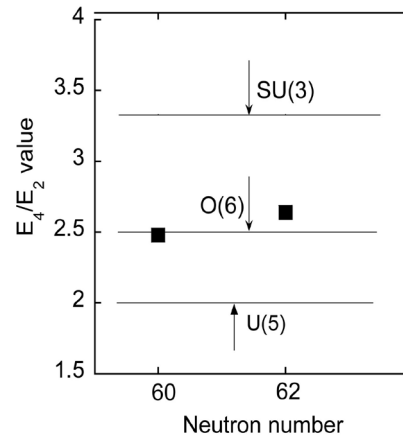


Figure 2. The arrows indicate the line of $E(4_1^+)/E(2_1^+)$ values of the U(5), O(6) and SU(3) limits. The $E(4_1^+)/E(2_1^+)$ values of experimental data²² of the $^{104,106}\text{Ru}$ isotopes are presented as function of neutrons.

The best fit was taken up to 12^+ of Ru isotopes with neutron $N = 60, 62$. The parameters were determined the experimental eigen values ($E(n_b, \nu, L)$) from the Equation (4), where n_b , ν and L are quantum numbers. The parameters in the present data are shown in **Table 1**.

The calculated energy levels as well as experimental data are presented in **Table 2**. According to the weight of fitting the Ru-104 and Ru-106 nuclei are good candidates of O(6) symmetry. The calculation of γ -bands and β -bands are compared with experimental data and presented to **Table 3** and **Table 4**. From the tables, the IBM calculations and experimental results are in good agreements [21].

The reduced electric transition probabilities give the more information on the structure of nuclei. The E2 transition operator must be a Hermitian tensor of rank two; consequently, the number of bosons must conserve.

$$T^{E2} = \alpha_2 [d^\dagger s + s^\dagger d]^{(2)} + \beta_2 [d^\dagger d]^{(2)} \quad (6)$$

Here T^{E2} is the operator of reduced matrix elements of the E2. (s^\dagger, d^\dagger) are creation and (s, d) are annihilation operators for s and d bosons. α_2 indicated the effective quadrupole charge and β_2 is dimensionless coefficient, $\beta_2 = \chi\alpha_2$

$$B(E2, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_f | T^{E2} | J_i \rangle \right|^2 \quad (7)$$

The parameters, α_2 and β_2 of Equation (6), were adjusted to reproduce the experimental $B(E2, 2_1^+ \rightarrow 0_1^+)$. The effective charge (e_b) in present calculation is shown in **Table 5**. The values of e_b were estimated to reproduce experimentally $B(E2, 2_1^+ \rightarrow 0_1^+)$. The values $\beta_2 = 0$ for $^{104,106}\text{Ru}$ isotopes because these nuclei have the O(6) property. The calculated values of B(E2) transitions with experimental data are presented in **Table 6** for Ru isotopes with neutron $N = 60, 62$ in this study [21]. The calculated data of IBM-1 is good agreements with the available experimental results.

Table 1. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, excepted N and CHQ.

A	N	ϵ	a_0	a_1	a_2	a_3	a_4	$CHQ(\chi)$
^{104}Ru	8	0.000	0.1098	0.0180	0.000	0.1770	0.000	0.000
^{106}Ru	9	0.000	0.0990	0.0102	0.000	0.1513	0.000	0.000

Table 2. g -band (in MeV) for even $^{104-106}\text{Ru}$ nuclei.

J^π	^{104}Ru		^{106}Ru	
	IBM	Exp.	IBM	Exp.
0^+	0.000	0.000	0.000	0.000
2^+	0.3558	0.3580	0.2726	0.2700
4^+	0.8910	0.8884	0.6570	0.7147*
6^+	1.6056	1.5564	1.1532	1.2958*
8^+	2.4996	2.3204	1.7612	1.9734*
10^+	3.5730	3.1119	2.4810	2.7050
12^+	4.8258	-	3.3126	3.4500*

Table 3. γ -band (in MeV) for even $^{104-106}\text{Ru}$ nuclei.

J^π	^{104}Ru		^{106}Ru	
	IBM	Exp.	IBM	Exp.
2^+	0.8868	0.8931	0.7256	0.7923
3^+	1.5966	1.2424	1.2610	1.0915*
4^+	1.5990	1.5026	1.3002	1.3068*
5^+	2.4870	1.8723*	1.9082	1.6411*
6^+	2.4906	2.1966*	1.9670	1.9078*
7^+	3.5568	-	2.6672	2.2841*
8^+	3.4878	-	2.7456	2.9600*
9^+	4.8060	-	3.6360	-

Table 4. β -band (in MeV) for even $^{104-106}\text{Ru}$ nuclei.

J^π	^{104}Ru		^{106}Ru	
	IBM	Exp.	IBM	Exp.
0^+	0.9882	0.9882	0.9900	0.9906
2^+	1.3440	1.5154	1.2626	1.3922
4^+	1.8792	2.0808	1.6470	-
6^+	2.5938	-	2.1432	-
8^+	3.5616	-	2.7512	-
10^+	4.8120	-	3.5380	-

Table 5. Effective charge used to reproduce B(E2) values for even $^{104-106}\text{Ru}$ nuclei.

A	N	e_B (eb)
^{104}Ru	8	0.0935
^{106}Ru	9	0.0916

Table 6. Experimental and the IBM-1 values of B(E2) for even $^{104-106}\text{Ru}$ nuclei (in e^2b^2).

$I_i \rightarrow I_f$	IBM-1	EXP.	IBM-1	Exp.
	^{104}Ru		^{106}Ru	
$2_1^+ \rightarrow 0_1^+$	0.1679	0.1682	0.1966	0.1966
$4_1^+ \rightarrow 2_1^+$	0.2273	0.2149	0.2689	-
$4_2^+ \rightarrow 2_2^+$	0.1282	-	0.1541	-
$6_1^+ \rightarrow 4_1^+$	0.2448	-	0.2941	-
$6_2^+ \rightarrow 4_2^+$	0.1626	-	0.2000	-
$8_1^+ \rightarrow 6_1^+$	0.2384	-	0.2934	-
$10_1^+ \rightarrow 8_1^+$	0.2152	-	0.2747	-
$10_2^+ \rightarrow 8_2^+$	0.0941	-	0.0000	-
$2_2^+ \rightarrow 2_1^+$	0.2273	0.1957	0.2689	-
$4_2^+ \rightarrow 4_1^+$	0.1166	-	0.1401	-
$6_2^+ \rightarrow 6_1^+$	0.0759	-	0.0933	-
$8_2^+ \rightarrow 8_1^+$	0.0000	-	0.0659	-

The application of potential energy surface (PES) gives the information to find microscopic and geometric shapes such as spherical, prolate, oblate and γ independent (γ soft). It gives us about symmetry, the shape of nuclei, the minimum deepness and the change of the shape. The PES of the IBM Hamiltonian was drawn by the Skyrme mean with

$$|N, \beta, \gamma\rangle = \frac{1}{\sqrt{N!}} (b_c^+)^N |0\rangle, \quad (8)$$

$$b_c^+ = (1 + \beta^2)^{-1/2} \left\{ S^+ + \beta \left[\cos \gamma (d_0^+) + \sqrt{1/2} \sin \gamma (d_2^+ + d_{-2}^+) \right] \right\}, \quad (9)$$

The energy surface $E(N, \beta, \gamma)$ for O(6) limits as a function of β and γ , has been calculated [1] [22]. Here, β were indicated the total deformation of a nucleus. **Figure 3** shows the contour plots in the γ - β plane resulting from $E(N, \beta, \gamma)$ for ^{104}Ru and ^{106}Ru isotopes. The potential surfaces are approximately independent of gamma only. In this figure, the color lines show the values of the potential energy surface in MeV. The mapped IBM energy surfaces of ^{104}Ru and ^{106}Ru are O(6) characters.

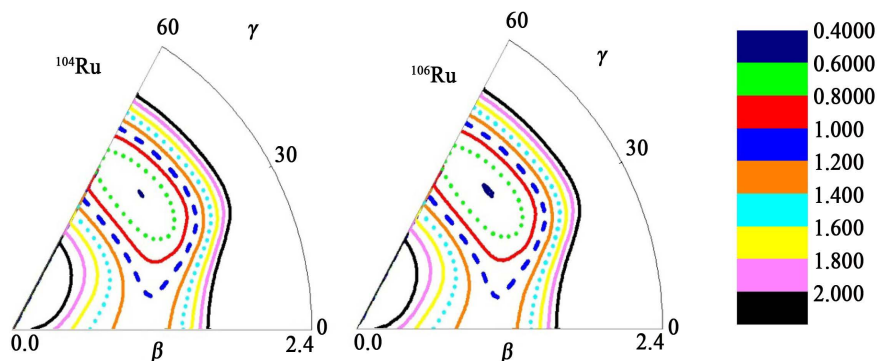


Figure 3. Contour map of potential energy surfaces for Ru isotopes with $N = 60, 62$.

4. Conclusion

The yrast band, gamma band and beta band, electromagnetic transition and potential energy surface of ^{104}Ru and ^{106}Ru isotopes calculated in terms of $O(6)$ limit of interacting boson model-1. The energy levels up to 12^+ of $^{104,106}\text{Ru}$ nucleus found by the best fitted of the parameters in the Hamiltonian of the IBM-1. The analyses of the IBM-1 results for the ground state band suggest a satisfactory agreement with the experimental data. The nobility and contribution of this work included that the framework of interacting boson approximations shows the Ru with neutron numbers 60 and 62 considered gamma soft $O(6)$ symmetry.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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