

Analytical and Numerical Study of the Hydro-Mechanical Behavior of a Cantilever Retaining Wall in Upward Seepage Conditions

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How to cite this paper: Kuma, M.M., Leonard, N., Bertrand, P.J., Arnaud, K.N., Gilbert, T., Elvis, A. and Abong, P.C. (2024) Analytical and Numerical Study of the Hydro-Mechanical Behavior of a Cantilever Retaining Wall in Upward Seepage Conditions. *World Journal of Engineering and Technology*, 12, 914-937.

<https://doi.org/10.4236/wjet.2024.124056>

Received: February 5, 2024

Accepted: October 18, 2024

Published: October 21, 2024

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Abstract

Poor design of ground water evacuation mechanisms is often blocked and leads to the rise of ground water behind the wall. As a result, free water behind the wall that is not quickly evacuated, increases the lateral pressure and thus favors overturning failure. The resolution of the overturning problem in cantilever retaining walls caused by hydro-mechanical interaction was studied. An analytical and numerical method was used to study this type of wall-floor interaction. Then Coulomb's design criterion against overturning to develop a mathematical model that compute analytical factor of safety against overturning in different water conditions and heel lengths was used. The modeling and simulation of this system in the Cast3m software which took into account a wide variety of floor and wall properties were performed. The numerical factor of safety against rollover was obtained, and the graphs for the factor of safety versus heel length and immersion depth for both methods were plotted. From ($0 \leq H_w \leq H/3$), water effect is not dangerous to wall stability against overturning and from ($H/3 < H_w \leq H$), water effect is very dangerous to wall stability against overturning. For analytical and numerical methods, the heel can be pre-dimensioned against overturning as: L_c : $[0.27H \ 0.38H]$, $[0.29H \ 0.43H]$ for $0 \leq H_w \leq H/3$; $[0.33H \ 0.45H]$, $[0.39H \ 0.53H]$ for $H/3 < H_w \leq 2H/3$; $[0.5H \ 0.6H]$, $[0.50H \ 0.67H]$ for $2H/3 < H_w \leq H$. The numerical method guaranteeing more safety than the analytical method, Cantilever retaining walls can thus be pre-dimensioned considering Clayey-Sand soil in hydro-mechanical conditions.

Keywords

Cantilever, Retaining Wall, Overturning, Hydro-Mechanical, Soil-Structure, Interaction

1. Introduction

Retaining wall constructions have been widely applied over the years in construction areas where large excavations are costly. The topography of different geographical areas warrants the need for earth-retaining structures. In recent decades, the use of earth retaining walls has widely expanded in road construction projects so that a variety of these structures are used in modern transportation systems [1]. Due to complex topography, and excessive rainfall, moisture accumulation at the back of retaining walls is hazardous to the stability of the retaining wall because they modify the modes of interactions which can lead to failure. Backfill materials such as sand, and pouzzolane can at once ease the free drainage of this water while reducing the risk of wall failure caused by water. This work consists to study the soil structure interactions in the presence of excessive moisture content backfilled with sand and pouzzolane. Since moisture is completely inevitable in the design of retaining walls, the influence on active earth pressures, passive earth pressures, and ground pressures is worth studying. In 2019, Majid *et al.* did a series of 1-g shaking table tests using variable-amplitude harmonic excitations was performed on 0.8 m high MSE/soil nail hybrid retaining (MSE/SN) wall models to investigate the seismic behavior of this innovative retaining earth structure. It was found that the deformation mode and the horizontal displacements of the MSE/SN walls were highly dependent on the length of the nails, such that $L/H = 0.7$ can be defined as the critical ratio in seismic conditions for MSE/SN walls which have been reinforced with strips having a constant length. Irrespective of the different nail lengths, the pattern of the observed failure mechanism included a moving block which was delineated by a two-part failure plane consisting of a concave curve and an inclined line with a certain point of intersection. Also, a consistent range of the normalized horizontal displacements (D_x/H), about 0.55% - 1.10%, corresponding to the formation of local shear bands, and a range of $D_x/H = 5.0\% - 5.6\%$, corresponding to the development of active wedge failure, were determined. In 2020, Fu-quan Chen *et al.* did a study on Passive earth pressure of narrow cohesionless backfill against inclined rigid retaining walls under translation mode. Their results showed an increase in the passive earth pressure and the number of slip surfaces when the backfill space decreased, in which the passive earth pressure was nonlinearly distributed. Frydman and Keissar carried out a series of centrifugal model tests on rigid retaining walls with sand backfill to observe the changes in earth pressures behind the wall from at-rest conditions to active conditions [2]. It is observed that the coefficient of active lateral earth pressure decreases with the depth and is smaller than the value calculated with the Rankine theory. Centrifugal

tests were carried out to investigate on the arching effects on unyielding retaining walls with narrow backfill width, while the lateral earth pressure acting on a retaining wall with narrow backfills is clearly smaller than the estimation based on the Rankine theory or Coulomb's theory [3]. Ilyas Saribas *et al.* did a study on the effects of the use of two different types of recycled aggregates with known characteristics as backfill materials in newly built cantilever-reinforced concrete retaining walls on the seismic performance of the walls [4]-[7]. The physical properties of the recycled aggregates used as backfill materials were determined using aggregate tests. Subsequently, analytical studies for the reinforced concrete retaining walls containing recycled aggregates in the amounts of 25%, 50%, 75%, and 100% were performed under seismic and static loads and the results were compared with those obtained for the retaining walls containing 100% natural aggregate as the backfill material. The experimental and analytical studies showed that the internal friction angles and effective ground acceleration coefficients significantly affected the overturning moment and total active pressure values of the retaining walls. The results led to the conclusion that recycled aggregates can be partially or completely used as the backfill material in retaining walls.

This work has relevance in regions of excessive rainfall and groundwater movement. A retaining wall designed without the consideration of water movement can lead to an increase in active pressures, which is unsafe for the wall's stability. Excessive moisture can lead to the overturning of the wall, slip circle failures, and sliding. A static structure that moves due to excessive load endangers the stability of the region it retains. In this paper, modeling the 2D cantilever retaining wall in seepage conditions was performed, and the stability against overturning was checked with varying heel length and water depth. A correlation between factor of safety against overturning and heel length was performed.

2. Materials and Methods

2.1. Methods

In this part of this article, the objective was to present the methodology of this research work. The general objective of this work has been to analytically and numerically study the hydro-mechanical behavior of SSI in Cantilever retaining walls in case of overturning failure. The analytical methodology and the numerical methodology were presented. The Mathematical equations which form the criterion for design and numerical model which forms the basis for simulating the hydro-mechanical SSI interaction in Cast3m. The physical, mechanical and geometrical parameters of soil and concrete which will help us to simulate this multi-physical system were presented. This protocol of research helped us to study Concrete Cantilever retaining walls of 3 m height, retaining Sandy Clayey soils in the presence of rising water table (upward seepage). In this part of the work, the parameters were exploited, the heel length, rising water table, resulting Active Earth pressure and Settlement were presented.

The Analytical and Numerical methods were chosen because it helped us

manipulate various parameters to obtain varied structure-soil behavior which is difficult to obtain from an experimental work. The Analytical gives us discrete solutions and the Numerical gives us numerical solutions that approach the reality. The Cast3m software and MS. Excel were employed to attain this research objective.

2.2. Analytical Method

The analytical method applied here was used to compute values of active Earth pressures, Horizontal thrusts, overturning moments, and resistant moment against overturning with the goal of obtaining Factors of safety for heel length variation and water height rise. Microsoft Excel spreadsheet was used to compute these values.

2.2.1. Materials

In order to achieve the objective of modelling the hydro-mechanical behavior of the retaining wall analytically. The following materials were used:

- 1) Concrete parameters (Density, geometrical parameters of wall);
- 2) Clayey-Sand soil parameters (Saturated density, humid density, submerged density, Angle of internal friction of soil);
- 3) Water (density);
- 4) Excel spread sheet (for analytical computation).

2.2.2. Analytical Protocol

A table of values for different variations of the heel by considering the geometrical parameters of the wall was developed. The physical properties of the materials were defined, the densities, angle of internal frictions were defined. The parameters used for computing the active earth pressure coefficient were defined. The Coulomb's method was chosen to compute active earth pressure coefficient because the wall is not frictionless. The overturning moments, resistant moments, Factors of safety for varied heel lengths and water altitude behind the wall were calculated. This method is presented below:

▪ *Wall, soil and Coulomb's parameters*

The height of the retaining wall was considered fixed at 3 m. This height is preferable because it is the height that suits this kind of retaining wall contrary to other walls (mass retaining walls, MSE) which can go to heights of over 3 m. **Tables 1-3** respectively show the geometrical parameters and computational value, the physical properties of soil and retaining wall and the coulomb's parameters for commutating active earth pressure coefficient.

Table 1. Geometrical parameters and computational value.

Total height of retaining wall (m)	HT	3
Length of front heel (m)	B1	0.3
Thickness of stem (m)	B2	0.3

Continued

Length of heel (m)	LC	0.3	0.5	1	1.5	2	2.5
Height of submerged soil (m)	H_w	0	1	2	3		
Total length of base (m)	BT			BI + B2 + LC			
Thickness of foundation (heel) (m)	H3				0.3		

Table 2. Physical properties of soil and retaining wall.

Physical properties of materials	Values
Density of Concrete (ρ_c : kg·m ⁻³)	2500
Density of water (ρ_w : kg·m ⁻³)	1000
Unsubmerged density of soil (ρ_{us} : kg·m ⁻³)	1700
Saturated density of soil (ρ_{sat} : kg·m ⁻³)	2031.25
g. Acceleration due to gravity m·s ⁻²	9.81

Table 3. Coulomb’s parameters for commutating active earth pressure coefficient.

Wall backfill angle (β)	90°
Angle of inclination of soil wedge (α)	0°
Soil angle of internal friction (ϕ')	30°
Wall-soil friction angle (δ)	2/3 $\phi' = 20^\circ$

The above parameters enabled us to calculate the Coulomb Active Earth pressure coefficient K_a using equation.

▪ **Resisting Moment (M_{RES})**

The different weights, lever arm and moments that tend to stabilize the wall, were calculated using the MS Excel spread sheet.

For Water level = 0 m.

$$M_{RES}^i = Wi \times Li \tag{2.1}$$

For water level ($H_w \neq 0$ m).

Total Resisting moment (M_{RES}) is given by:

$$M_{RES} = \sum_{i=1}^3 Wi \times Li \tag{2.2}$$

▪ **Overturning Moment (M_{ov})**

The overturning moment was computed following the table as shown below. In Case of Water presence and absence of water. The Vertical pressures were calculated, and the horizontal pressures were calculated using the Active Earth pressure coefficients. The Horizontal thrust was then calculated together with their lever arm from the Point C at the base of the wall about which the Overturning moments were calculated.

The Active Earth Pressure force is given by:

$$P_a = \frac{1}{2} K_a \times \rho_{us} \times g \times H^2 \quad (2.3)$$

When there is no rising ground water: $H_w = 0$:

$$\text{Active Earth Pressure Force (N)} \quad P_{a1} = \frac{1}{2} K_a \times \rho_{us} \times g \times HT^2 \quad (2.4)$$

$$\text{Lever Arm (m)} \quad L4 = \frac{HT}{3} + H3 \quad (2.5)$$

When there is rising ground water: $H_w \neq 0$. **Table 4** show the active earth pressure computation.

Table 4. Active earth pressure computation for $H_w \neq 0$.

S/N	Active Earth Pressure Force (N)	Lever Arm (m)
Unsubmerged Soil	P_{a11} $HT - H_w = H1$	$L5$ ($H2 = H_w$)
Submerged soil (2)	P_{a11}	$L6$
Submerged soil (3)	P_{a2}	$L7$
Water Pressure	P_{a3}	$L7$

$$P_{a11} = \frac{1}{2} K_a \times \rho_{us} \times g \times (HT - H_w)^2 \quad (2.20)$$

$$L5 = \frac{HT - H_w}{3} + H_w + H3 \quad (2.6)$$

$$P_{a2} = (\rho_{sat} - \rho_{us}) \times g \times K_a \times H_w^2 \times \frac{1}{2} \quad (2.7)$$

$$L6 = (0.5 \times H_w) + H3 \quad (2.8)$$

$$L7 = (H_w/3) + H3 \quad (2.9)$$

$$P_{a3} = \frac{1}{2} \rho_w \times g \times H_w^2 \quad (2.10)$$

$$M_{ov} = (P_{a11} \times L5) + (P_{a11} \times L6) + (P_{a2} \times L7) + (P_{a3} \times L7) \quad (2.11)$$

▪ **Analytical factor of Safety (FOS_A)**

For a retaining wall to be considered safe against overturning, the Factor of safety is a ratio between the Moment tending to resist overturning and the moment tending to cause overturning. This Value is said to be greater than or equal to 2 to 3.

$$2 \leq FOS_A = \frac{M_{RES}}{M_{ov}} \leq 3 \quad (2.12)$$

From this methodology of the analytical work, the values of safety factor against overturning for different heel lengths and water levels were calculated. Overturning moments, Resisting moments with varied heel lengths and water level.

2.3. Numerical Method

The numerical method was also used to simulate the same system. As peculiar as this method is, it helped us to numerically obtain values of stresses in the soil, settlements which enabled us to calculate Overturning moment from resulting stresses and Resisting moments from the resulting weights of that favor the stability of the wall. However, this method takes into consideration several constraints which the analytical model does not. This method used three models, concrete, soil, and interface, to simulate the hydro-mechanical interaction. Below is a presentation of the materials and method of the numerical method.

2.3.1. Materials

The materials used for this method are given below:

- Cast3m software for simulation;
- Soil numerical model;
- Concrete Numerical model;
- Interface numerical model for Concrete and Soil, Soil and Soil;

▪ *System of coherent measuring units use in CAST3M*

CAST3M does not have any particular system of measuring units. It is to the user to provide the data in a coherent system checking the fundamental law of dynamics: $F = M \cdot a$. Where F : force, M : mass and a : acceleration. Once the measuring units used in the data are defined, all the results will be expressed in these same units. There is an exception to this rule concerning the measurement of the angles which must always be expressed in degrees. On the other hand, the temperatures and the thermal dilation coefficient must be expressed in coherent units. Presented below is a methodology of how the software functions. The first part is the definition of the problem. The second part consists of resolving the problem.

▪ *Soil Numerical Model*

The model chosen for soil in the numerical approach is the CamClay Model while the analytical model takes into consideration, the soil densities, in saturated and unsaturated conditions, as well as the internal angle of friction. These parameters were used to represent the hydro-mechanical conditions of the soil in the case of rising groundwater. The soil retained by the wall and the foundation soil are of the same type but vary in their cohesions, pre-consolidation pressures, void ratios, Poisson's coefficients. The type of soil used for this work is Clayey Sand. Soil is an elasto-plastique Material which typically follows the **CamClay criteria**. Parameters are defined in **Table 5** and **Table 6** above.

Table 5. Submerged retained soil parameters.

YUN2	NU2	RHO2 = ρ_{sat}	FI2	E02	M2	COHE2	P02	G12
180×10^6 MPa	0.4	2031.25	28°	0.6	1.1	0.15×10^6 MPa	0.4×10^6 MPa	58.07×10^6 MPa

Table 6. Unsubmerged retained soil and foundation soil parameters.

YUN3	NU3	RHO3	FI3	E03	M3	COHE3	P03	G13
211×10^6 MPa	0.35	1700	30°	0.6	1.2	0.20×10^6 MPa	0.4×10^6 MPa	72.26×10^6 MPa

2.3.2. Numerical Protocol

▪ *Calculation Option*

A 2D model to simplify this work was considered. The Cast3m software by default considers a default thickness of 1 m. Calculation was 2-dimensional with lines segment of 2 nodes and a triangular finite surface. The density of the finite element was 0.1 which implies that for a unit length, the segment is broken into ten other line segments.

▪ *Geometry*

The geometry of the model was programmed using the Cast3m software (**Figure 1**).

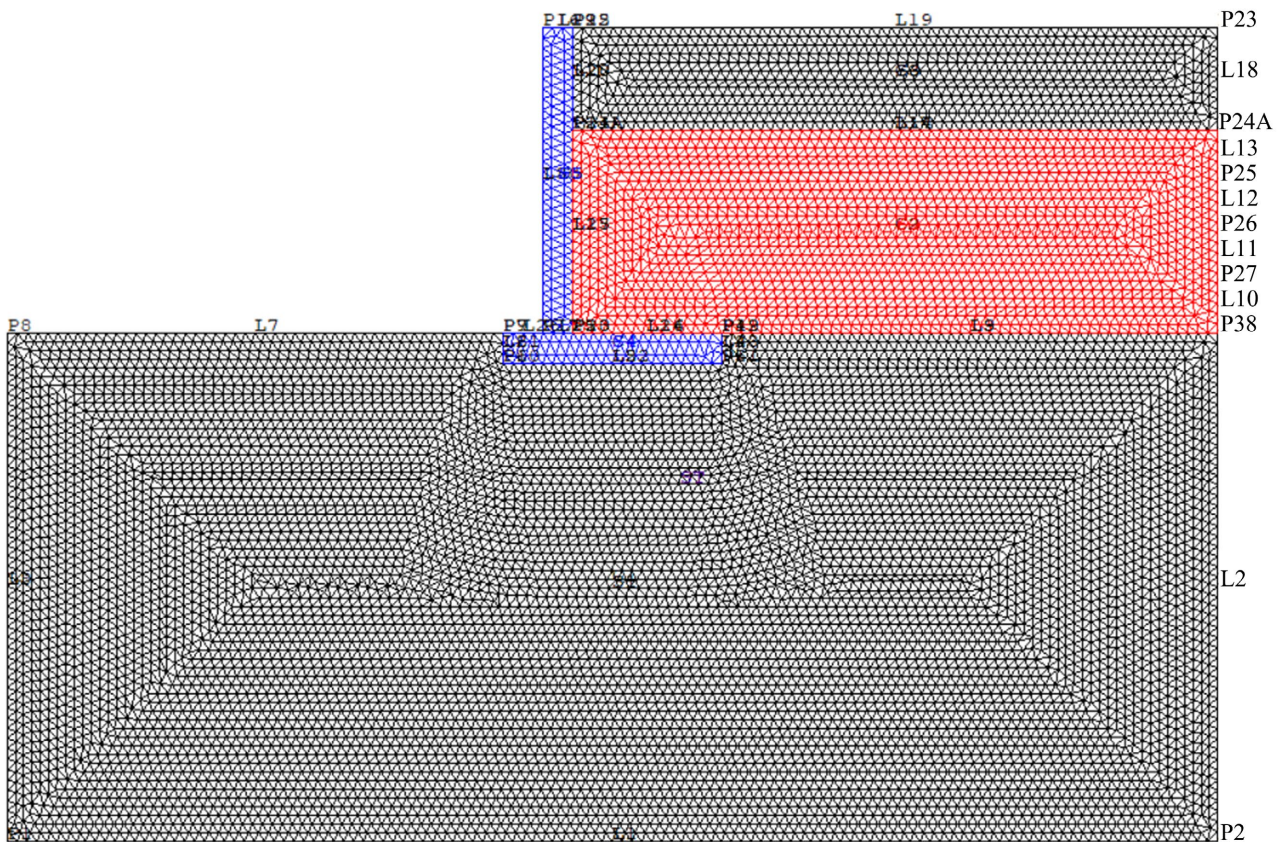


Figure 1. Discretized geometrical model (Cast3m software).

▪ *Materials modes of behavior and characteristics*

The materials mode of behavior under loadings, as already defined above in materials was defined. These material properties were varied in case of water influence in soil. The case of submerged soil.

▪ *Boundary Conditions*

The Cast3m software calculates the stiffness matrix of the Cantilever Retaining wall and the Soil in order to obtain a solution. Since the work is done in 2 dimensions, with three degrees of freedom: translation in ox axis, oy axis and rotation. **Figure 2** show the boundary condition of the model.

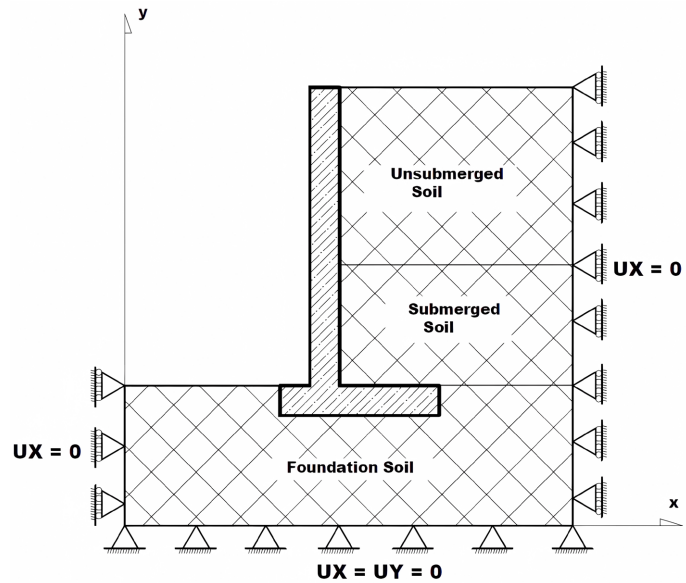


Figure 2. Boundary condition of the numerical model.

▪ **Loading**

The Cast3m software makes provision or use to load the model in two kinds of load application: by imposing a force or by imposing a displacement. In this work, loading by force was suitable for the numerical model because it permitted us to know the exact load to apply on the wall and soil in conformity with the loads computed in the numerical work. This method of load application permitted us to produce the corresponding displacements and stresses. After the loads are applied, the software then computes the solution in two ways: non-linear and linear calculation [8]. The interest relates only to the maximum values of displacements, constraints and forces. A linear solution method in which the model’s own weight is applied in a single iteration was chosen. The software to calculate the self-weight of each constituent element of model and distribute it evenly to all finite elements of that element was applied. **Figure 3** and **Figure 4** respectively show the self-weight of cantilever wall (Cast3m) and self-weight of soil (Cast3m).

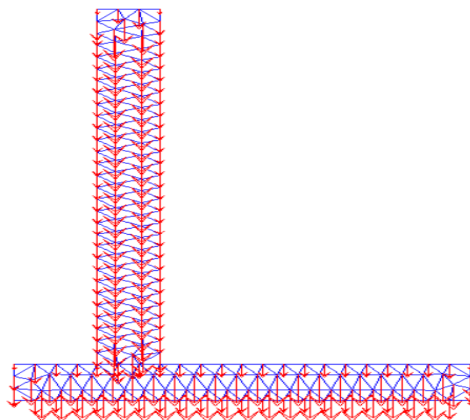


Figure 3. Self-weight of cantilever wall (Cast3m).

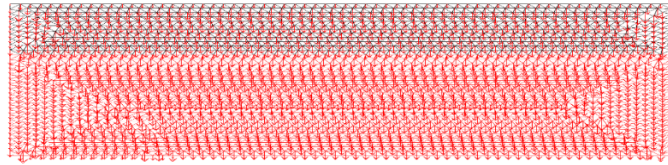


Figure 4. Self-weight of soil (Cast3m).

▪ *Data Extraction and Exploitation*

As specified in the objectives, the graphs of active Earth pressure, safety factor with heel length was plotted. The digital method involves the judicious choice of points from which data will be extracted. The horizontal stress from three main points was extracted as: **P20**, **P21A**, and **P22** at the base of the wall, at the level of the water table and at the summit, respectively. In order to see the effect of increasing the heel length on the wall displacement, the horizontal displacement at the summit of the wall on point P16 was extracted. **Figure 5** shows how data were extracted.

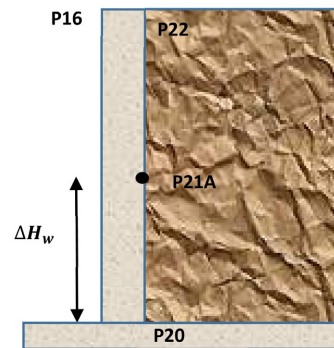


Figure 5. Points of data extraction on numerical model.

In order to calculate the overturning moment base on the numerical result. The horizontal stresses from the different points given above were extracted, then the principle of vertical stress increase as depth increases was applied. Two cases are possible in case of vertical stresses. The principle is explained in **Figure 6** below.

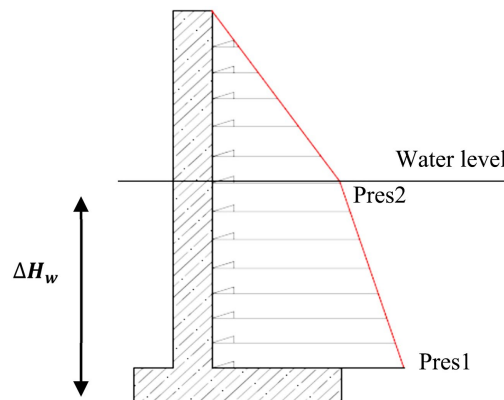


Figure 6. Possible horizontal stress configurations.

The pressures: Pres1 and Pres2 were used to calculate the horizontal thrust in both submerged and unsubmerged cases, just as the figure above. From the software, the overturning moments, resisting moments, factor of safety were calculated.

▪ **Displacements and stresses and deformation**

The peculiarity with the Cast3m software was that it would enable us to get results of different stresses in the soil and the retaining wall for different water conditions. Most importantly, the stress evolution from the base of the heel to the top of the backfill soil was observed. The analysis with pictures was performed, the displacements and deformation of the wall for different water conditions. This will help us analyse the von Mises stresses which is calculated for a particular failure criterion.

3. Results and Interpretation

The results were obtained from the numerical and analytical models. The results of influence of water level in the soil on the retaining wall stability and the influence of heel length on the factor of safety against overturning were presented in this part. Influence of water level on the factor of safety of the retaining wall stability was presented. The analytical solution and the numerical solution were compared.

3.1. Displacement Fields and Stress Field Analysis

Cast3m software permitted us to have stress fields and displacement fields of different hydro-mechanical conditions. For these analyses, water depths of 0, 1, 2 and 3 m which simultaneously corresponds to 0, $H/3$, $2H/3$, and H were chosen.

▪ **Horizontal displacements**

Yielding walls are walls that have the tendency to move forward under lateral loads. Below is presented horizontal displacement fields for varying water depths (Figures 7-10).

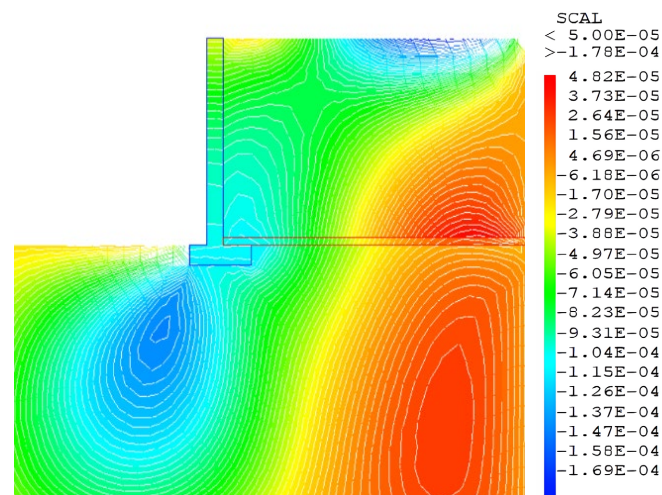


Figure 7. Horizontal displacement for $H_w = 0$ (Unsaturated soil conditions).

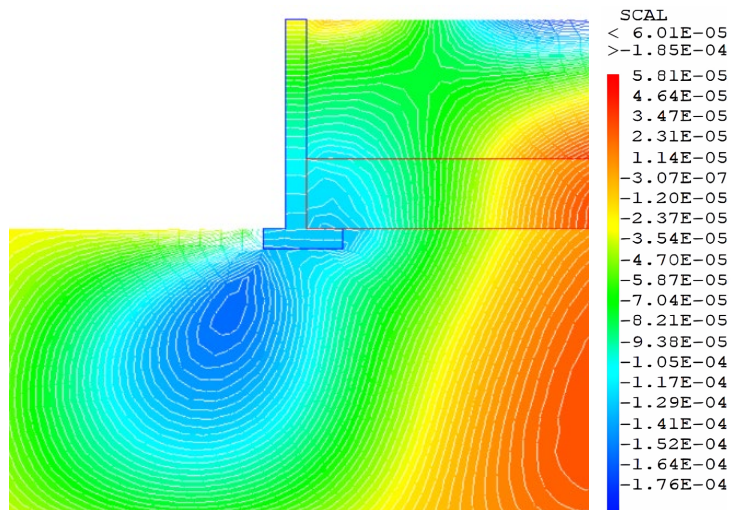


Figure 8. Horizontal displacement for $H_w = H/3$ (Partially saturated soil conditions).

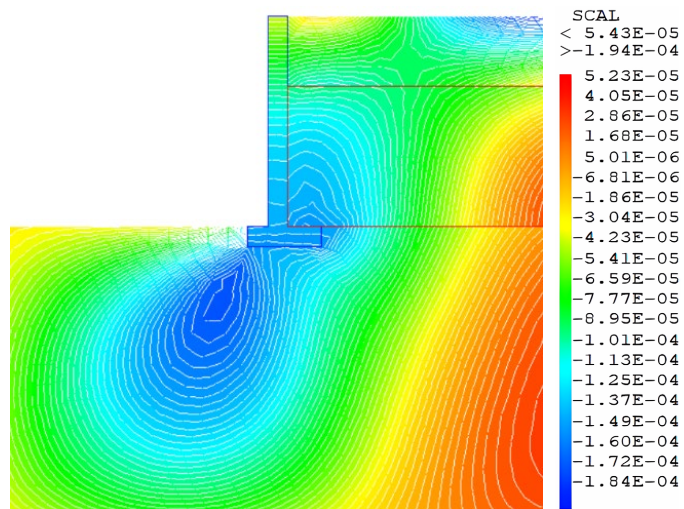


Figure 9. Horizontal displacement for $H_w = 2H/3$ (Partially saturated soil conditions).

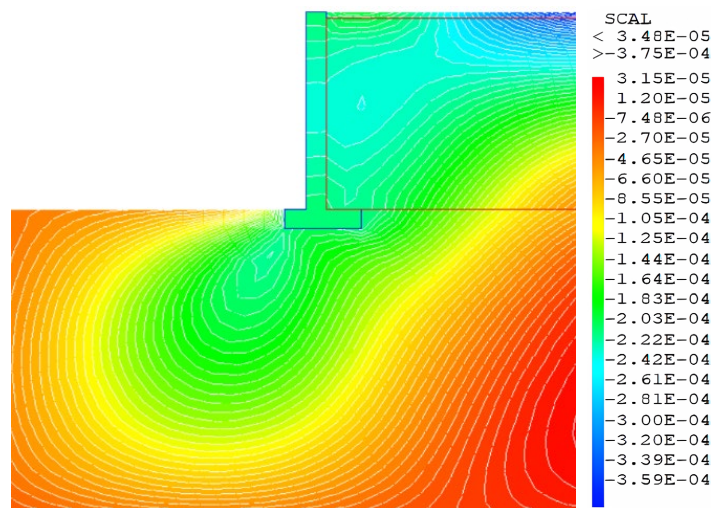


Figure 10. Horizontal displacement for $H_w = H$ (Fully saturated soil conditions).

- The combined figures of horizontal displacement indicate that the soil behind the wall and beneath the toe tend to move forward.
- A displacement field bulb around the toe which increases as the submerged depth increases was observed. This movement is responsible for the face tilt of the wall—Overturning.

From the results, extract horizontal displacement values at top of the wall was collected. **Figure 11** shows the influence of water depth on the horizontal displacement of the wall.

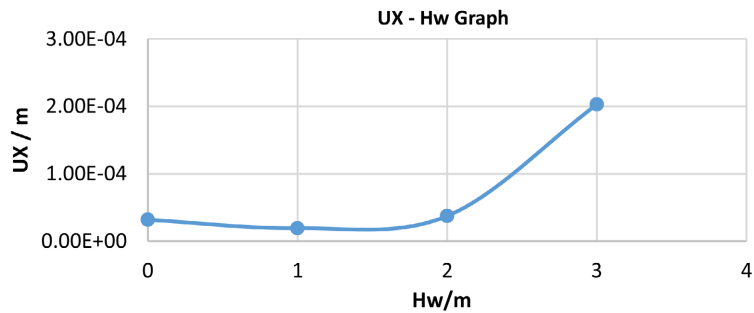


Figure 11. Horizontal displacement—Submerged soil depth Graph.

- The influence of water depth on the horizontal displacement of the wall was observed. This result increase in displacement is due to increase in earth pressure.
- Water level depth greatly affects wall displacement from $2H/3$ to H .

Between a submerged depth of 0 to $2H/3$ the displacement is constant, but experiences a great increase from $2H/3$.

▪ **Horizontal Stress field**

The software also enables us to analyze horizontal stresses that act in the soil and in the retaining wall. **Figures 12-15** below show the horizontal stress fields for different submerged conditions.

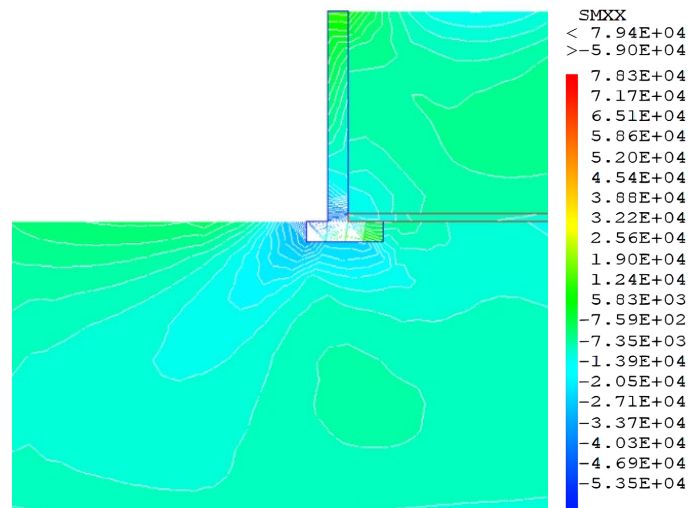


Figure 12. Horizontal stress for $H_w = 0$ (Unsaturated soil conditions).

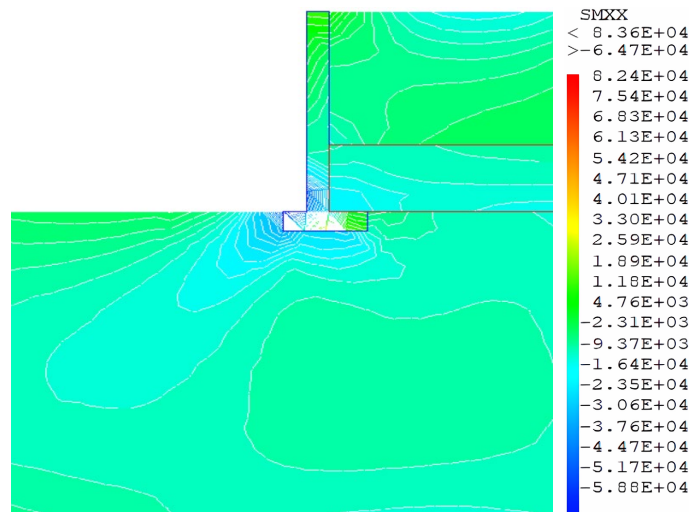


Figure 13. Horizontal stress for $H_w = H/3$ (Partially saturated soil conditions).

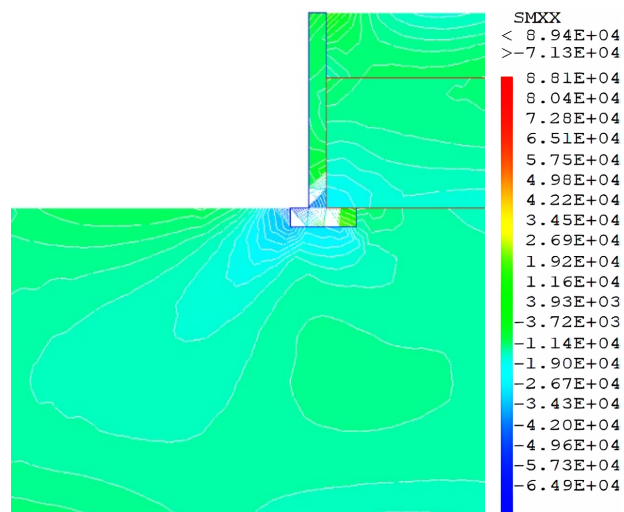


Figure 14. Horizontal stress for $H_w = 2H/3$ (Partially saturated soil conditions).

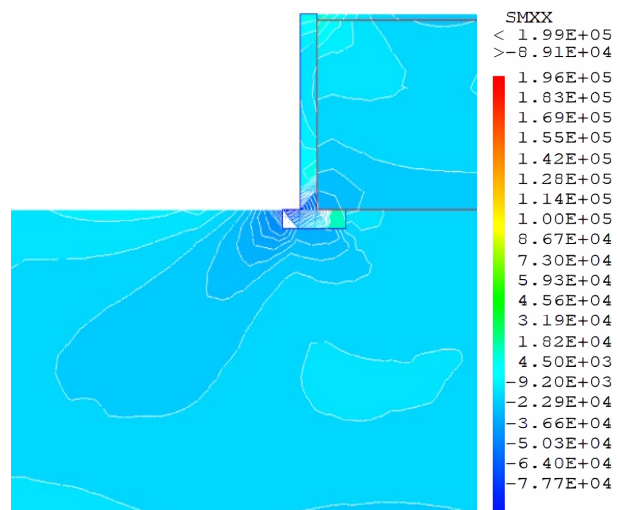


Figure 15. Horizontal stress for $H_w = H$ (Fully saturated soil conditions).

- From the Horizontal stress fields for different submerged depths, horizontal pressure behind the wall increases as the submerged depth increases. This pressure is most intense at the base of the wall.
- At fully submerged depth, the whole regions of soil around the retaining wall tend to exert horizontal pressure.

3.2. Influence of Water Level on the Overturning Moment

The water level height was varied from 0 to 3 m. The overturning moment was computed analytically and numerically. The results are presented below in **Figure 16**.

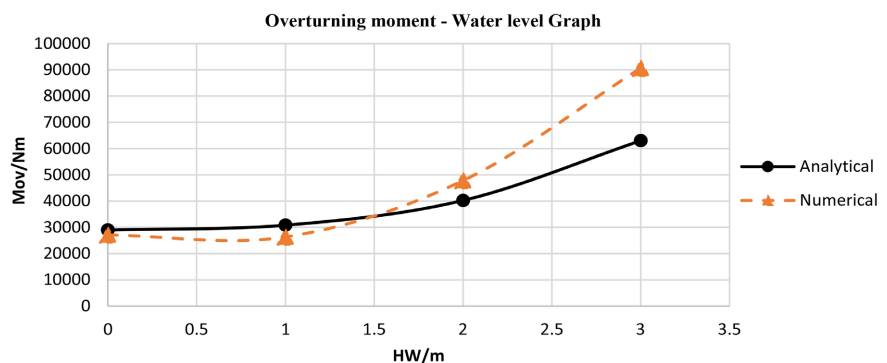


Figure 16. Overturning moment—water level depth.

- From **Figure 16**, the rise of water table behind the retaining wall, increases the overturning moment. This means that the stability of the wall against overturning is decreased with increase in water level.
- The Numerical solution shows that there is a more adverse effect of overturning than for the analytical solution. This difference should be due to the fact that the parameters taken into consideration for numerical solution are more compared to the analytical solution. The Numerical solution models and simulates the structural interaction between both materials and considers the deformability of the soil.

3.3. Influence of Water Level on the Factor of Safety

Here, graphs of FOS against water level (H_w) were plotted to see the effect of increasing water level on safety against overturning. The following results were obtained in **Figure 17** below.

- **Figure 17** shows the influence of water level depth for definite heel lengths. The figure shows that, the longer the heel, the more the factor of safety against overturning. However, increase in water level decreases this factor of safety—the effect being more for smaller heel lengths;
- From the figure, a stationary regime where there is very little or no influence of water depth on factor of safety against overturning was observed. Based on this finding, water level behind the retaining wall has effect only from a third of the wall height.

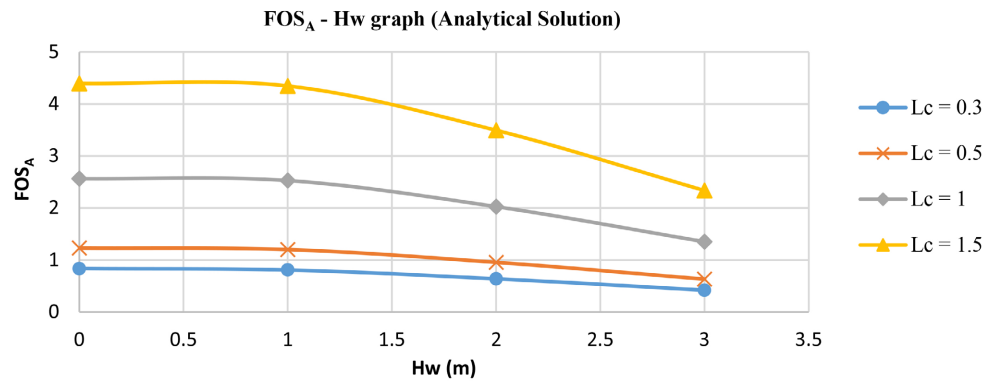


Figure 17. Factor of safety—water level depth from analytical solution.

$$H_w \leq \frac{H}{3} \tag{3.1}$$

No influence of water on overturning

$$\frac{H}{3} < H_w \leq H \tag{3.2}$$

water begins to influence the stability against overturning.

Figure 18 below shows the factor of safety for different heel lengths and water depth.

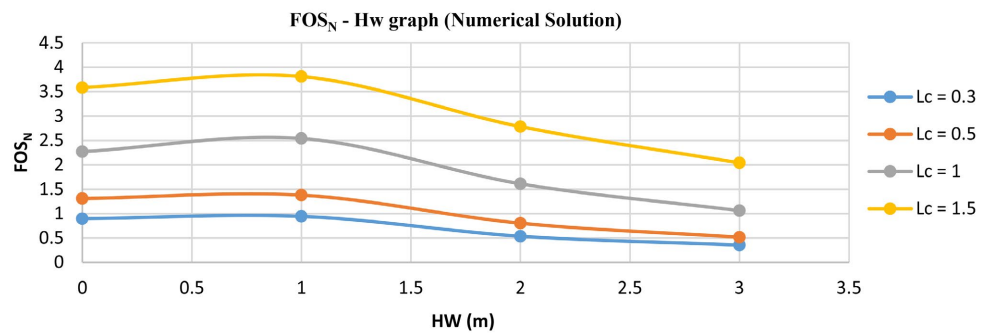


Figure 18. Factor of safety—water level depth graph-Numerical solution.

- Figure 18 above shows the influence of water level depth on factor of safety against overturning for different heel lengths, for the numerical work. This influence is similar to those obtained from the analytical solution – two regimes of influence are seen: a third of the wall height were water depth is not a threat to stability against overturning and above a third where water depth is a threat to stability against overturning.
- The Equations (3.1) and (3.2) are true for the results obtained numerically

3.4. Influence of Heel Length on the Factor of Safety against Overturning

At different water levels behind the soil, the length of the heel was varied, computed the factor of safety for both the numerical and analytical work (Figure 19).

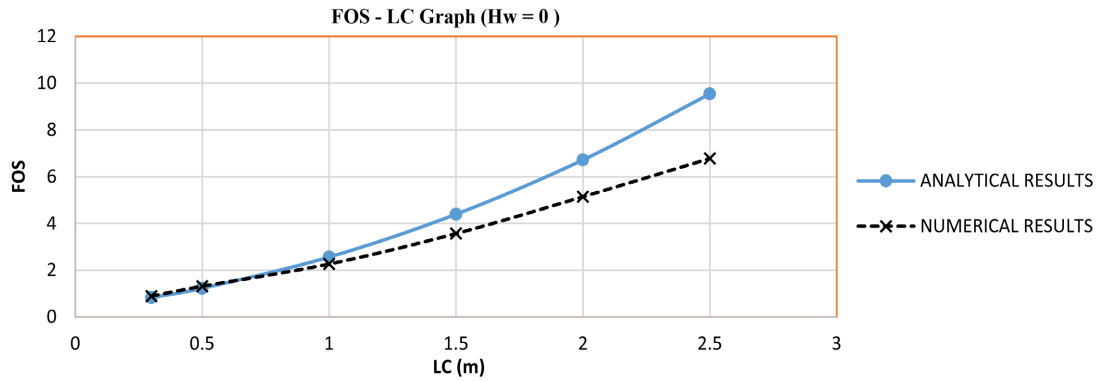


Figure 19. Factor of safety—Heel length Graph for $H_w = 0$.

- Figure 19 above shows the effect of increasing the heel length and its influence on the factor of safety against overturning for the analytical and the numerical solution. In the absence of water, increasing the heel length increasing the stability of the wall against overturning – this is true for both methods. However, the numerical solution gives a lesser factor of safety for a give heel length as compared to factor of safety for analytical solution.
- The difference between the numerical and analytical solution is due to the fact that, more soil parameters are used, and most importantly, the soil is compressible.

Figure 20 below show different overturning moments, resisting moments, factors of safety taken, for different heel length for both the numerical and analytical solutions for water depth of 1 m.

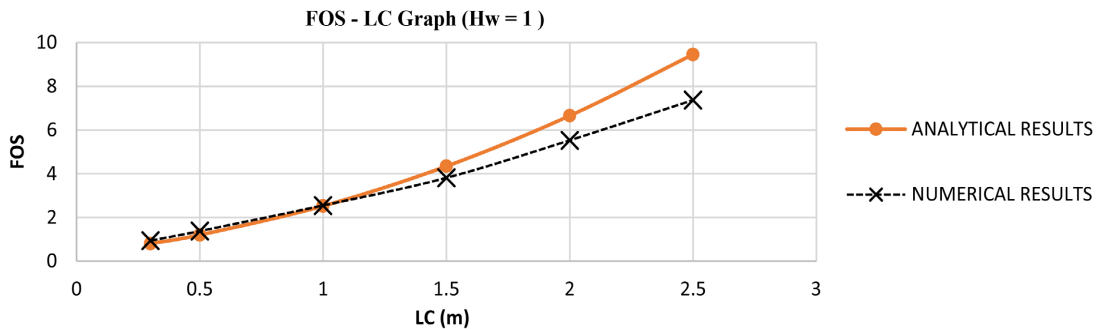


Figure 20. Factor of safety—Heel length Graph for $H_w = 1$.

- Figure 20 shows the effect of increase heel length on Factor of safety against overturning for a water depth of 1 m. The graphs show that there was an increase in factor of safety against overturning.
- The rate of increase was lesser for the numerical analysis than the analytical analysis. This was the same phenomenon for the case where water was absent in the soil.

Figure 21 below show the values for resistant moments and overturning moments for different heel lengths for a water depth of 2 m.

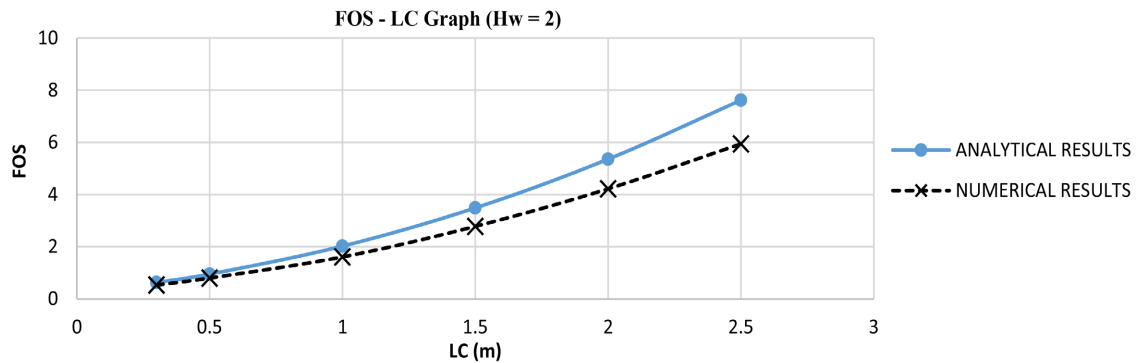


Figure 21. Factor of Safety—Heel length Graph for $H_w = 2$.

- **Figure 21** show the effect of increase heel length on Factor of safety against overturning for a water depth of 2 m. The graphs show that there was an increase in factor of safety against overturning.
- The rate of increase was lesser for the numerical analysis than the analytical analysis. This was the same phenomenon for the case where water was absent in the soil.

Figure 22 below shows different Overturning moments, resisting moments, factors of safety taken, for different heel length for both the numerical and analytical solutions for water depth of 1 m.

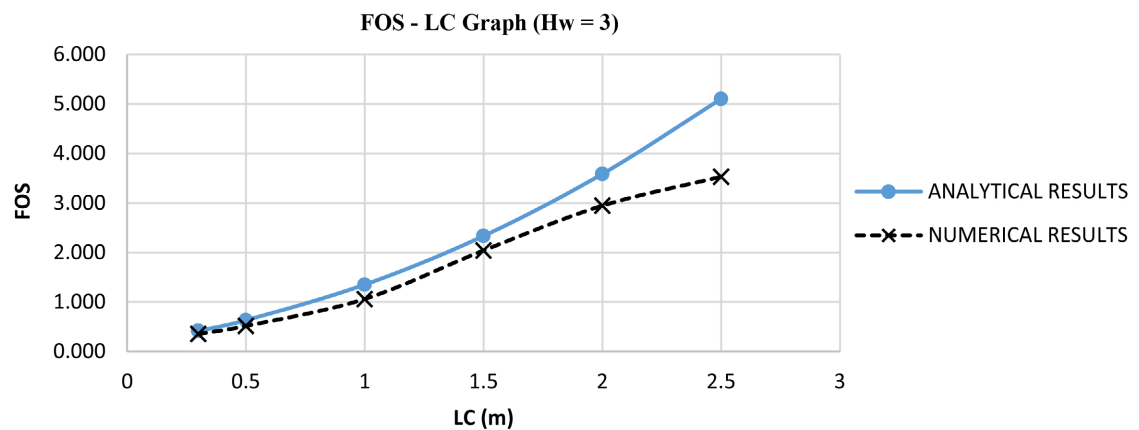


Figure 22. Factor of Safety—Heel length Graph for $H_w = 3$.

- **Figure 22** above shows that, for a fully submerged soil, the increase of heel length also leads to an increase in factor of safety against overturning. This is true for both the numerical and analytical solutions.
- The numerically and analytical solutions for a fully submerged soil were closely related than for the rest of the submerged cases and unsubmerged case.

In order to better appreciate the influence of heel length on the global variation of water depth, **Figure 23** combines the solutions obtained analytically and numerically.

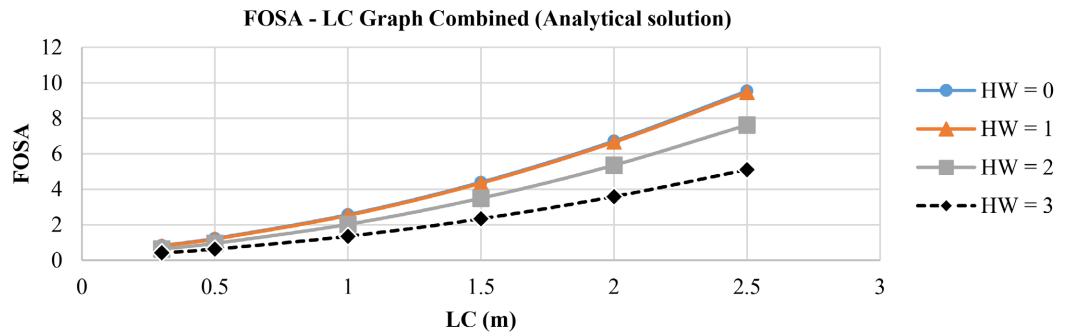


Figure 23. Combined factor of safety—Heel length graph for analytical solution.

- Figure 23 shows the combined graphs of heel length influence on factor of safety against overturning for different submerged water levels for the analytical solution. The results obtained showed that there is a general increase on the factor of safety against overturning for different submerged water levels. However, this increase decreases with increase of submerged depth.
- For $H_w = 0$ and $H_w = 1$ m, results were similar. Indicating that, for water levels ≤ 1 m, increasing heel length, increases the factor of safety at the same rate (Figure 24).

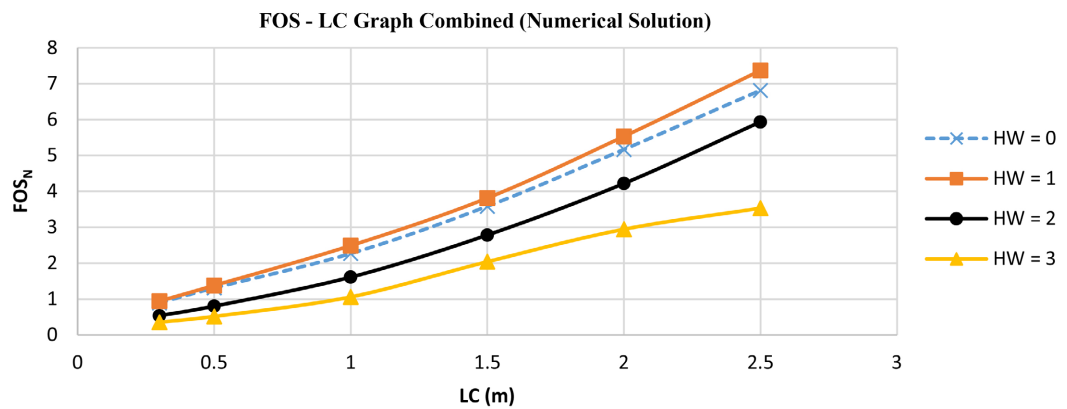


Figure 24. Combined factor of safety—Heel length graph for numerical solution.

- Figure 24 shows the combined graphs of heel length influence on factor of safety against overturning for different submerged water levels for the numerical solution. The results obtained showed that there is a general increase on the factor of safety against overturning for different submerged water levels. However, this increase decreases with increase of submerged depth.
- For $H_w = 0$ and $H_w = 1$ m, results were similar. Indicating that, for water levels ≤ 1 m, increasing heel length, increases the factor of safety at the same rate.

3.5. Pre-Dimensioning Recommendations against Overturning in Clayey-Sand Soils

3.5.1. Analytical Pre-Dimensioning

Based on the results obtained, empirical norms for heel lengths which can be

adopted when designing or pre-dimensioning retaining walls to be able to counter overturning in unsaturated, partially saturated and fully saturated conditions with little or no drainage system provided. The lower boundary values are chosen for a factor of safety of 2, against overturning while the upper boundary values are chosen for a factor of safety of 3, against overturning (Figures 25-27).

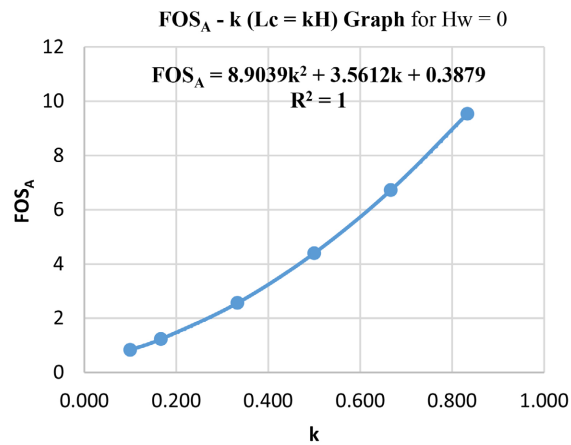


Figure 25. FOS_A - k Graph for H_w = 0.

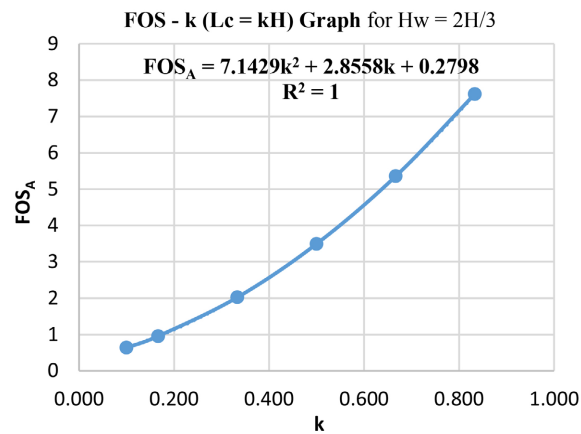


Figure 26. FOS_A - k graph for H_w = 2H/3.

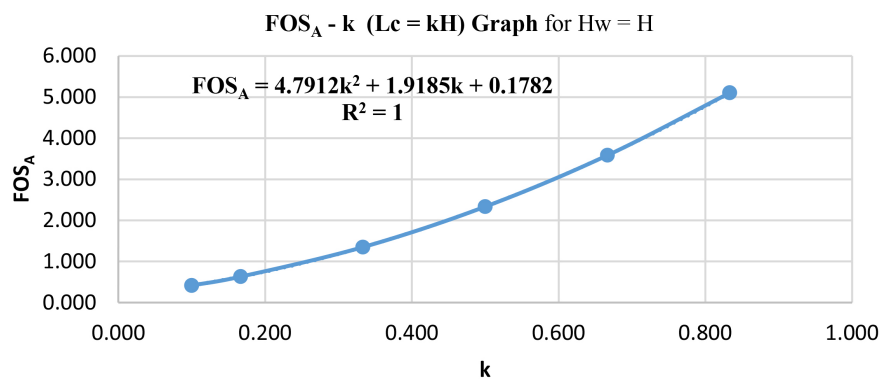


Figure 27. FOS_A - k graph for H_w = H.

3.5.2. Numerical Pre-Dimensioning

Figures 28-30 show the numerical pre-dimensioning.

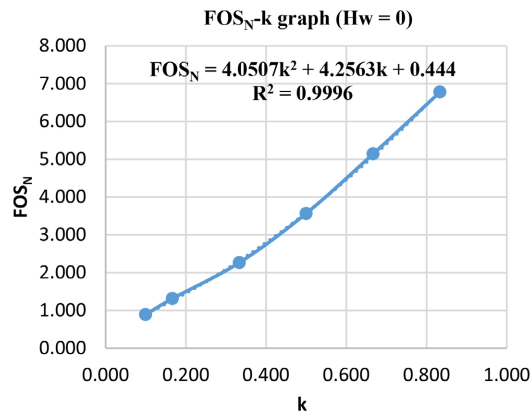


Figure 28. FOS_N - k graph for H_w = 0.

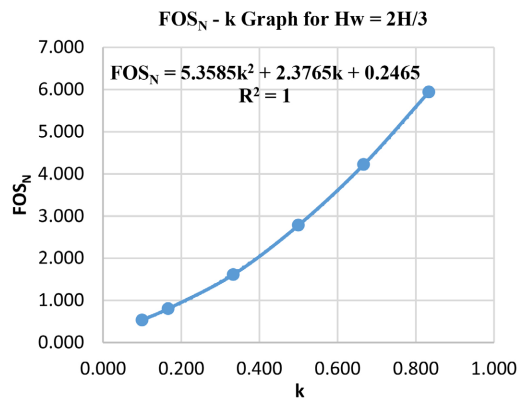


Figure 29. FOS_N - k graph for H_w = 2H/3.

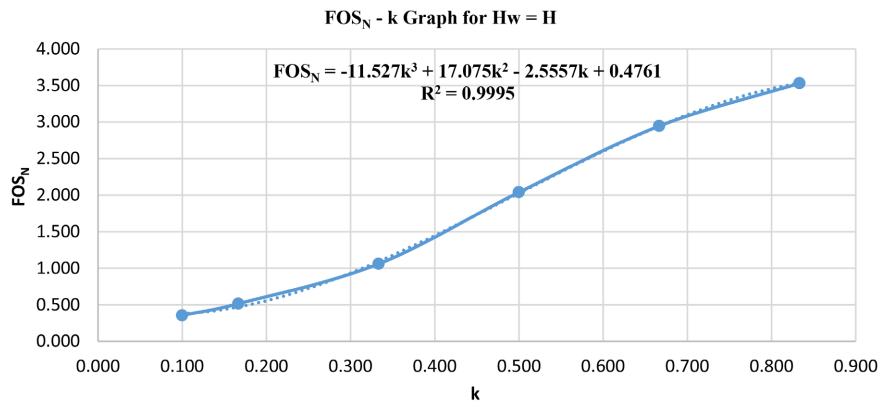


Figure 30. FOS_N - k graph for H_w = H.

From the equations obtained above, the relating factor of safety against overturning and k values for both Numerical and analytical solutions, where $Lc = kH$. A pre-dimensioning norm against overturning for upper and lower bound values of safety factors, which are between **2 and 3** (Table 7).

Table 7. Pre-dimensioning of heel length of Cantilever walls for different water levels in clayey sand soils.

Analytical solution	Numerical solution	Submerged depth
$[0.27H0.38H]$	$[0.29H0.43H]$	$0 \leq H_w \leq H/3$
$[0.33H0.45H]$	$[0.39H0.53H]$	$\frac{H}{3} < H_w \leq \frac{2H}{3}$
$[0.5H0.6H]$	$[0.50H0.67H]$	$\frac{2H}{3} < H_w \leq H$

- The table above shows us that, it's possible to pre-dimension the retaining wall against overturning while considering the possible levels of water table rise behind the wall. The higher the water table, the more the length of the heel must be added and thus increase the base of the retaining wall. Pre-dimensioning with the Analytical solution is more economical in terms of concrete volume than pre-dimensioning with Numerical solution. However, pre-dimensioning with the numerical method ensures more safety than the numerical method.
- This work is similar to the norm presented by Das Braja [9]. Comparing this numerical work for $H_w = 0$, the pre-dimensioning obtained is similar to that given. It seems the pre-dimensioning given by Das Braja is valid only for $0 \leq H_w \leq H/3$ according to this Numerical work.
- Pre-dimensioning of retaining wall base by Son and Adedokun [10] is not a function of the height of the wall but just the influence of surcharge load on the pre-dimensioning of retaining walls in hydro-mechanical conditions: **B [1.25 + 0.005q]** fully submerged, **B [0.881 + 0.00805q]** for 60% submerged soil and **B [0.7093 + 0.0091q]** for 20% submerged height. This work proposes pre-dimensioning as a function of wall height, and level of submerged soil.
- In order to solve overturning failure in hydro-mechanical conditions, the heel length is increased 10% and 17% more than Das Braja [9] states in the upper limit of pre-dimensioning, for fully submerged soil, for Analytical and Numerical methods, respectively.
- The numerical method shows that, for $2/3^{\text{rd}}$ submerged height, the heel increases 3% more than Das Braja [9] [11] [12] states in the upper limit of pre-dimensioning.

4. Conclusion

This work has also proposed new contributions to the study of SSI of Cantilever Retaining walls in Hydro-mechanical conditions. While analyzing the effect of water level on Factor of safety against overturning for different heel lengths, water level had very little or no effect for submerged depths: $0 \leq H_w \leq H/3$. However, for submerged depths: $H/3 \leq H_w \leq H$, a significant drop of factor of safety against overturning, implying that the retaining, was observed. Wall is safe against overturning for the aforementioned situation and unsafe for the lastly mentioned

situation, this result was coherent for both the analytical and numerical methods. Generally, it was observed that, even though both methods gave similar results, the analytical method gave lower safety factors compared to the analytical method for varying heel lengths. This is evident due to the fact that the numerical considers more soil and wall properties than the analytical method: the numerical method takes into consideration that the material is deformable before calculating the factor of safety while the analytical method does not. A proposition of pre-dimensioning criteria for heel length for different water level conditions for both the analytical and numerical was observed. For analytical and numerical methods respectively, the heel can be pre-dimensioned against overturning as: L_c : $[0.27H$ $0.38H]$, $[0.29H$ $0.43H]$ for $0 \leq H_w \leq H/3$; $[0.33H$ $0.45H]$, $[0.39H$ $0.53H]$ for $H/3 < H_w \leq 2H/3$; $[0.5H$ $0.6H]$, $[0.50H$ $0.67H]$ for $2H/3 < H_w \leq H$. The numerical method guaranteeing more safety than the analytical method, Cantilever retaining walls can thus be predimensioned considering Clayey-Sand soil in hydro-mechanical conditions. This work shows that the heel length is increased by 10% and 17% compared to previous works in the upper limit of pre-dimensioning, for a fully submerged soil, for analytical and numerical methods, respectively. The numerical method shows that, for 2/3 of the submerged height considered, the heel increases by 3% compared to previous work in the upper pre-dimensioning limit.

Acknowledgements

The authors gratefully acknowledge the support of the mechanical laboratory of HTTTC (high technical teacher training college) of University of Bamenda, the laboratory of National Higher Polytechnic Institute (NAHPI) of University of Bamenda and the mechanics laboratory of ENSET (Ecole Normale Supérieur D'Enseignement Technique) of Douala.

Author's Contributions

Kuma Moses Mbuu initiated the project and project administration, investigation and writing-original draft; Leonard Nsahlai and Bertrand Jules Penka: Supervision, formal analysis, validation, writing review and editing; Arnaud Nguessi Kouamou and Gilbert Tcherno: Methodology, Data curation, Visualization, writing review and editing; Elvis Agandeh and Abong Claret Phonchu: read and approved the final manuscript.

Funding

No funds, grants or other support was received.

Declarations Conflicts of Interest

There is no conflict of interest about this research.

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