World Journal of Engineering and Technology, 2024, 12, 963-975
https://www.scirp.org/journal/wjet
ISSN Online: 2331-4249
ISSN Print: 2331-4222

Numerical Simulation of the Flow of a Molten Blast Furnace Slag in an Open Channel

Miguel A. Barron, Joan Reyes, Aristeo Garrido

Departamento de Materiales, Universidad Autonoma Metropolitana Azcapotzalco, Mexico City, Mexico
Email: bmma@correo.azc.uam.mx

How to cite this paper: Barron, M.A., Reyes, J. and Garrido, A. (2024) Numerical Simulation of the Flow of a Molten Blast Furnace Slag in an Open Channel. *World Journal of Engineering and Technology*, **12**, 963-975.
https://doi.org/10.4236/wjet.2024.124059

Received: September 30, 2024
Accepted: October 29, 2024
Published: November 1, 2024

Abstract

Despite its industrial importance, the flow of molten blast furnace slag in open channels has not been sufficiently studied. In this work, the unsteady non-uniform flow of a molten blast furnace slag in a rectangular open channel is numerically studied by solving the Saint-Venant equations by means of an explicit backwards finite difference scheme. An Arrhenius-type dependence of the viscosity of the slag on temperature is assumed. To calculate that viscosity, four temperatures are considered, namely 1450˚C, 1500˚C, 1550˚C and 1600˚C. To study the dynamic response of the system, a half-sinusoidal pulse with duration of 5 s is imposed at the channel entrance. According to the numerical simulations, for all the temperatures considered, the slag flow in the channel for an angle of 5 degrees is supercritical in nature. However, for an angle of 1 degree, the flow is transcritical, that is, it presents a transition from subcritical to supercritical.

Keywords

Blast Furnace Slag, Molten Slag, Open Channel Flow, Saint-Venant Equations, Supercritical Flow, Transcritical Flow

1. Introduction

The blast furnace is a countercurrent reactor which uses coke as the main fuel and reductant, sinter and pellets as the iron-bearing components, and slag-forming fluxes such as limestone and dolomite. Hot air is injected into the furnace through tuyeres located at the lower part of the furnace, coke is burned and the hot gases mainly containing N_2, CO and CO_2 rise in countercurrent with the solid charge to the top of the furnace. The solid charge is heated and the iron oxides are chemically reduced to metallic iron [1]. Molten slag is formed by physical and chemical interaction between added fluxes and iron ore gangue. Molten pig iron and molten slag are collected in the furnace crucible. Due to its lower density, molten slag floats on top of molten iron. In 2022, 1301 million tons of pig iron was produced in the world through the blast furnace, which accounts 91.2% of the primary iron obtained [2]. Approximately 300 kg of slag are formed for each ton of pig iron [3]. This means that approximately 390 million tons of blast furnace slag are produced worldwide per year. Due to the large quantities of blast furnace slag produced each year and its high environmental impact, from the point of view of sustainability and saving natural resources, in recent years, blast furnace slag has begun to be used more and more in pavement applications, concrete aggregate, construction fill, and so on [4].

Molten slag is tapped from the blast furnace and conveyed through an open channel to a pouring pot. From the pot, the molten slag is transported to a cooling station where it is solidified and subjected to a dry granulation process [5]. There are little published works on the transport of molten metals in open channels, however, most of them study the flow of molten metals subjected to magnetic fields [6]-[11]. Recently, two of the current authors have published some articles in which the flow of molten metals in open channels is studied from the point of view of the metallurgical industry [12]-[14]. Open channels for transporting molten metals and slag in the metallurgical industry differ from channels for transporting irrigation water and wastewater by their angle of inclination and length. Due to the tendency of molten materials to reoxidize and freeze with atmospheric air, their angle of inclination is greater and their length is shorter compared to water transport channels. In addition, the construction material of a metallurgical channel is made of refractory material that withstands high temperatures.

Despite the industrial importance of metallurgical slag, practically no studies have been reported on the flow of molten slag in open channels. Regarding the flow of a molten slag in open channels, two of the most important physical properties of a slag are viscosity and density. Among other things, molten slag differs from molten metal by the high viscosity and low density of the first one. The viscosity of a slag depends mainly on temperature and chemical composition [15]. It is expensive and difficult to measure the viscosity of molten slag in the laboratory. So, there are many empirical and mathematical models to predict the viscosity of molten blast furnace slag. For example, in [16], a model based on discrete data points is reported. The model prediction is compared with viscosity data of slag of compositions typical to blast furnace operations, and the results show that the viscosity can be predicted with average error of less than 10%. Other models are based on partial least-squares regression [17], on chemical composition [18], on the Vogel-Fulcher-Tammann equation [19], on experimental measurements by a rotating crucible viscometer [20], and on optical basicity [21].

In this work, the unsteady non-uniform flow of a molten blast furnace slag in a rectangular open channel is studied by solving the Saint-Venant equations. The transient Saint-Venant equations are numerically solved by means of an explicit backwards finite difference scheme in one-dimensional space [11] [22] [23]. As in

[14], the half-sinusoidal pulse is imposed on the channel inlet as a boundary condition in the volumetric flow rate in order to observe the dynamic and stable response of the system. An Arrhenius-type dependence of viscosity with temperature is used to take into account the effect of temperature on flow dynamics. The values of the activation energy and the pre-exponential factor are taken from the literature for a synthetic slag similar to that of a blast furnace [24]. The friction slope and the frictional resistance coefficient are calculated using the expressions reported in [25]. Froude numbers are monitored to determine the type of flow present, whether subcritical, critical, or supercritical, and the possible transition between such flows. Research on the type of flow in open channels is important from the point of view of the adequate management of molten slag, but also on the adequate maintenance of the channel given that depending on the type of flow, gravity waves emerge and can lead to pulsations and flow instability, and erosion issues may arise.

2. Mathematical Model

In an open channel, in which the depth of the transported liquid is small compared to the length, and the density of the liquid is constant, the height and the volumetric flow rate can be represented mathematically by the Saint-Venant equations. These equations are a one-dimensional form of the shallow water model [26], and consist of a continuity equation and a momentum equation [22]:

Continuity equation:

$$b\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial Q}{\partial t}+\frac{1}{b}\frac{\partial}{\partial x}\left(\frac{Q^2}{h}\right)+gbh\left(\frac{\partial h}{\partial x}+S_f-S_0\right)=0 \tag{2}$$

In Equations (1) and (2) $h(t, x)$ and $Q(t, x)$ are the height of the liquid and the volumetric flow rate, respectively. Besides, t and x are the coordinates in time and space, respectively. Also, b is the width of the channel, g is the gravitational acceleration, S_f is the friction slope, $S_0=\tan(\theta)$ is the channel slope, and θ is the channel inclination angle.

Algebraic manipulation of Equation (2) yields the following expression for the momentum equation [22]:

$$\frac{\partial Q}{\partial t}+\alpha\frac{\partial Q}{\partial x}+\beta=0 \tag{3}$$

where

$$\alpha=2\left(\frac{Q}{bh}\right)+\frac{gh-\left(\frac{Q}{bh}\right)^2}{\left(\frac{Q}{bh}\right)\left(\frac{5}{3}-\frac{4}{3}R_h\right)} \tag{4}$$

$$\beta = gbh\left(S_f - S_0\right) \tag{5}$$

$$R_h = \frac{bh}{b+2h} \tag{6}$$

The friction slope can be estimated from the Darcy-Weisbach formula [25]:

$$S_f = f_D \frac{u^2}{8gh} \tag{7}$$

where $f_D = 24/Re$ is the frictional resistance coefficient, u is the liquid velocity, and Re is the Reynolds number, which is determined is this way:

$$Re = \frac{D_h u \rho}{\mu} \tag{8}$$

where $D_h = 4R_h$ is the hydraulic diameter, and ρ and μ are the density and the viscosity, respectively.

Solving u from Equation (8) and substituting it into Equation (7) yields

$$S_f = \frac{f_D}{8gh}\left(\frac{Re\mu}{D_h\rho}\right)^2 \tag{9}$$

The viscosity of a slag depends strongly on temperature. Assuming Arrhenius-type dependence, the viscosity can be calculated in this way [24]:

$$\mu = \mu_0 \exp\left(\frac{Ea}{RT}\right) \tag{10}$$

where μ_0 is the pre-exponential factor (kg/m·s), Ea is the activation energy (J/mol), $R = 8.314$ (J/mol·K) is the gas constant, and T is the absolute temperature (˚K).

NUMERICAL SOLUTION

Saint-Venant equations were numerically solved by means of an explicit backwards finite difference scheme [13] [23]. The discretized continuity equation becomes:

$$h_i^{n+1} = h_i^n - \left(\frac{\Delta t}{b\Delta x}\right)\left(Q_i^{n+1} - Q_{i-1}^{n+1}\right) \tag{11}$$

And the discretized momentum equation is given by:

$$Q_i^{n+1} = Q_i^n - \left(\frac{\Delta t}{\Delta x}\right)\alpha_i^n\left(Q_i^n - Q_{i-1}^n\right) - \Delta t\beta_i^n \tag{12}$$

To solve the Saint-Venant equations, an initial condition and a boundary condition for h and Q are required.

Initial conditions for h and Q:

$$h(0,x) = h_0 \tag{13}$$

$$Q(0,x) = Q_0 \tag{14}$$

Boundary condition for h:

$$h(t,0) = h_0 \tag{15}$$

Boundary condition for Q:

A half-sinusoidal inflow pulse is imposed at the upstream of the channel on Q in order to analyze the dynamic response of slag in the channel [13]:

$$Q(t,0) = Q_0 + A\sin\left(\frac{2\pi t}{T}\right), t \le t_p \tag{16}$$

$$Q(t,0) = Q_0, t > t_p \tag{17}$$

where T = pulse period, t_p = pulse duration time, and A = 0.75Q_0 is the pulse amplitude.

The discretized Saint-Venant equations were programmed in Fortran 95 language and solved for a simulation time of 60 s using $Re = 4 \times 10^3$. Open channels used to transport metals and molten slag are shorter and steeper than channels used to transport irrigation and drainage water. A channel length of 10 m was assumed. In the numerical simulations, 500 nodes were employed, which produced a Δx value of 0.02 m. To have numerical stability, a time step Δt of 1×10^{-4} s was established [13].

3. Results and Comments

As was stated above, the viscosity of a slag depends fundamentally on temperature and chemical composition. The viscosity decreases as the temperature increases. The main components of a blast furnace slag are CaO, MgO, SiO_2 and Al_2O_3. The viscosity decreases with the increase of MgO and increases with the increase of Al_2O_3 [24]. The chemical composition of the considered blast furnace slag and its viscosity parameters, are shown in **Table 1** and **Table 2**, respectively. Viscosity parameters were calculated from the temperature-viscosity data reported in [24].

Table 1. Chemical composition of the blast furnace slag [24].

Component	CaO	SiO_2	Al_2O_3	MgO	MoO_3
Weight percent	41.6	38.0	17.2	3.0	0.2

Table 2. Viscosity parameters of the blast furnace slag, calculated from [24].

Parameter	Value
μ_0, kg/(m·s)	9.469×10^{-5}
Ea, J/mol	1.940×10^{5}

Four temperature values were considered, namely, 1450˚C, 1500˚C, 1550˚C, and 1600˚C, corresponding to 1723.15, 1773.15, 1823.15, and 1873.15˚K, respectively. The viscosity values as a function of temperature, calculated with Equation (10), are shown in **Table 3**.

Table 3. Calculated viscosities of the blast furnace slag.

Temperature, ˚K	Viscosity, kg/(m·s)
1723.15	0.7200

Continued

1773.15	0.4915
1823.15	0.3426
1873.15	0.2434

Values of the rectangular open channel parameters considered in the numerical simulation are shown in **Table 4** [13] [27].

Table 4. Open channel parameters [13] [27].

Parameter	Value
Initial height (h_0)	0.15 m
Volumetric flow rate (Q_0)	0.038 m^3/s
Channel width (b)	0.20 m
Initial velocity (u_0)	1.2667 m/s
Channel length	10.0 m
Inclination angle (θ)	5 degrees
Initial velocity (u_0)	1.27 m/s

The dimensionless Froude number Fr is essential for classifying flows in open channels. It is defined as follows [28]:

$$Fr = \frac{u}{\sqrt{gh}} \tag{18}$$

As will be seen later, numerical simulations show that the slag height and velocity of the slag evolve in time from a transient state until they reach a steady state at the channel discharge after a time of 60 s. These steady state values of height and velocity, together with the Froude number, are shown in **Table 5**. The Froude numbers in this table were calculated using the steady-state values of height and velocity through the expression:

Table 5. Values of height, slag velocity and Froude number in steady state as a function of temperature.

T (˚K)	h_{st} (m)	u_{st} (m/s)	Fr
1723.15	0.05246	3.62174	5.04854
1773.15	0.03800	5.00502	8.19795
1823.15	0.02854	6.66519	12.59620
1873.15	0.02196	8.66381	18.66719

$$Fr = \frac{u_{st}}{\sqrt{gh_{st}}} \tag{19}$$

If $Fr < 1$, $h_{st} > h_c$ and $u_{st} < u_c$, then the flow regime is called subcritical. In this case, the role of gravitational forces is pronounced and the flow has relatively large depths and small velocities. If $Fr > 1$, $h_{st} < h_c$ and $u_{st} > u_c$, then the flow regime is called supercritical, the inertial forces are dominant and the flow is characterized by relatively small depths and large velocities [29]. If $Fr = 1$, the flow regime is denoted as critical. In this case, gravity waves can lead to pulsations in channels, and stability and erosion issues may arise. To avoid the possibility of flow pulsations, it is important to avoid channel designs with $0.9 < Fr < 1.1$ [28].

The critical depth h_c is the flow depth at a section of the channel where the flow is critical. The expression for the critical depth is [30]:

$$h_c = \left(\frac{Q^2}{b^2 g} \right)^{1/3} \tag{20}$$

The velocity associated with the critical depth is called critical velocity, and is determined by making $Fr = 1$ in Equation (18):

$$u_c = \sqrt{g h_c} \tag{21}$$

Using the data in **Table 4**, the following values for critical height and critical velocity are obtained: $h_c = 0.1544$ m, $u_c = 1.23072$ m/s.

The results of the numerical simulations are described and commented below. **Figure 1** shows the evolution in time of the slag height at the channel discharge for different values of temperature. The height of the slag starts from a value $h_o = 0.15$ m, which is maintained for about 2 seconds. Then, it decreases until a time of approximately 3 seconds, time in which the height presents a peak due to the effect of the half-sinusoidal pulse of the volumetric flow. From the peak onwards, the height decreases apparently exponentially until reaching a steady-state value h_{st}. This stable value of the height decreases with increasing temperature, as can be seen in **Table 5** and **Figure 2**. In all cases, the stable value of the height is below the critical height, namely $h_{st} < h_c$. **Figure 3** shows that the Froude number Fr increases as the temperature of the slag increases or the viscosity of the slag decreases. For all temperatures considered, the value of Fr is greater than unity. This, together with the fact that $h_{st} < h_c$, means that the slag flow in the channel is a supercritical flow, regardless of the slag temperature.

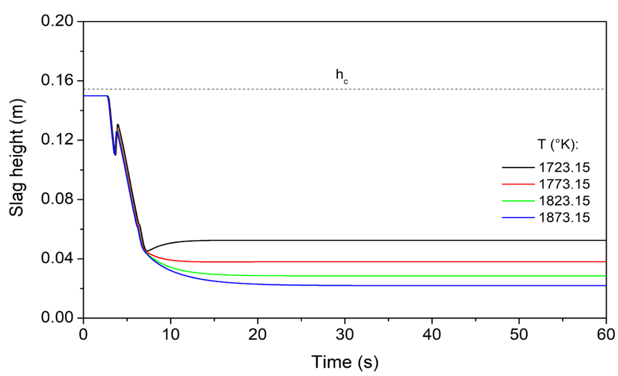

Figure 1. Evolution over time of the slag height at the channel discharge for different slag temperatures. The dashed line represents the critical height.

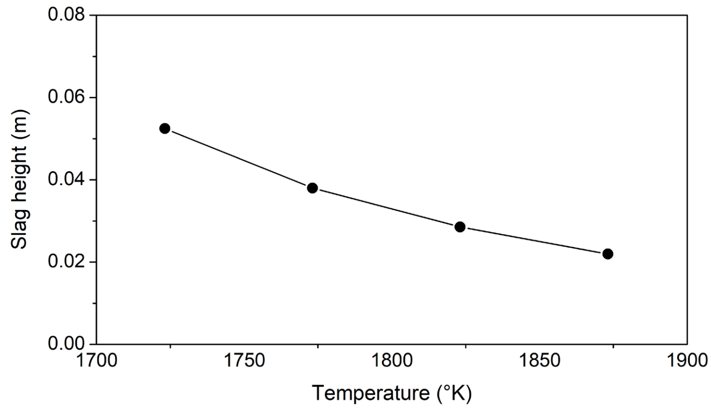

Figure 2. Steady-state slag height at channel discharge as a function of slag temperature.

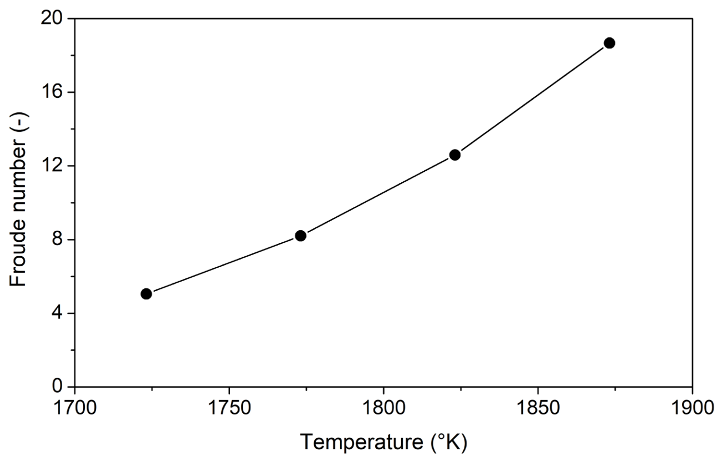

Figure 3. Froude number as a function of slag temperature.

The time changes of slag velocity in the open channel discharge for different slag temperatures are shown in Figure 4. The velocity of the slag increases progressively until it reaches a stable value that depends strongly on temperature. However, it is possible to observe two small peaks in the slag velocity, around a time of 3 and 7 s. Figure 5 explicitly shows the dependence of the steady state velocity on temperature. As can be seen in Table 5, Figure 4 and Figure 5, all steady-state velocity values increase with an increase in temperature, and all of them are greater than the critical velocity, *i.e.*, $u_{st} > u_c$. These results corroborate the supercritical nature of the slag flow in the open channel considered. Figure 6 shows the evolution in time of the volumetric flow rate. Two peaks are seen before 10 s, and from here, the values stabilize at Q_0. It means that for any temperature value, regardless of the different steady-state values of height and velocity, the steady-state volumetric flow remains constant. The presence of the two peaks can be explained, presumably, by the action of the gravitational forces that cause the first peak, and the inertial forces of the applied semi-sinusoidal pulse that cause the second peak. This is corroborated in Figure 7, which shows the behavior of the volumetric flow rate at 1723.15 K with and without pulse.

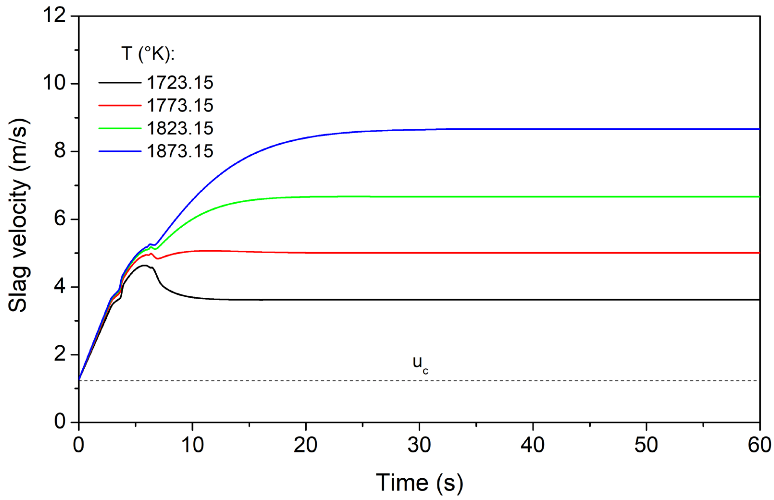

Figure 4. Evolution over time of the slag velocity at the channel discharge for different slag temperatures. The dashed line represents the critical velocity.

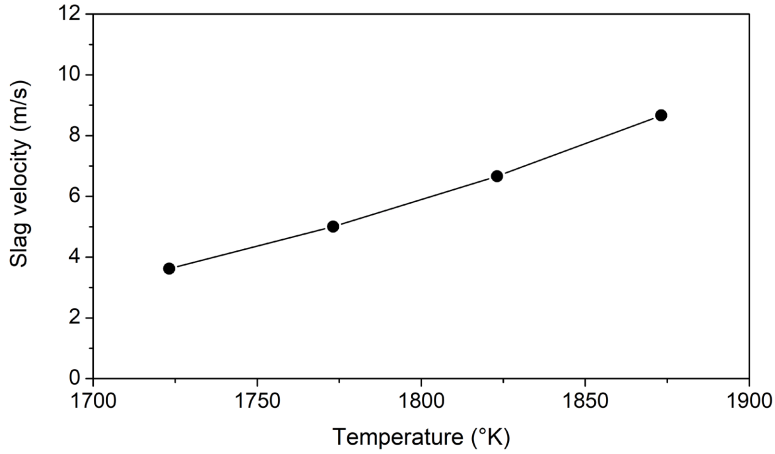

Figure 5. Steady-state slag velocity at channel discharge as a function of slag temperature.

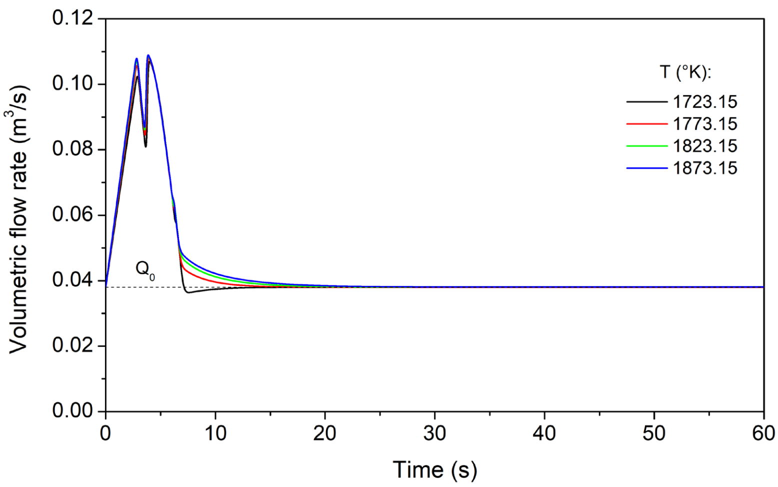

Figure 6. Evolution over time of the volumetric flow rate at the channel discharge as a function of the slag temperature.

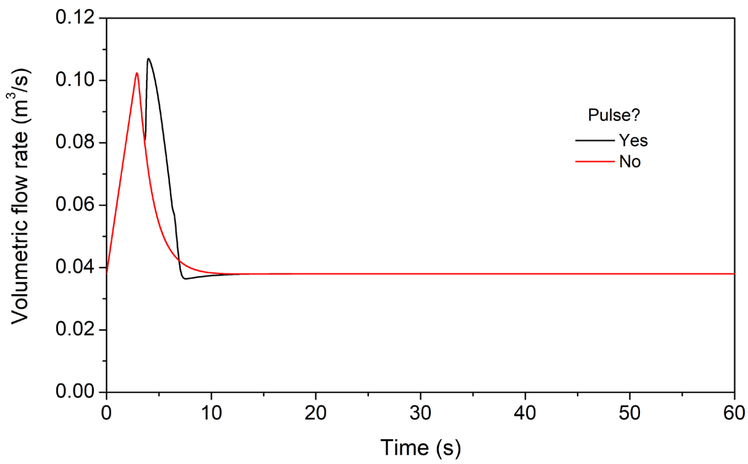

Figure 7. Volumetric flow rate in the channel discharge with and without the half-sinusoidal pulse.

In order to know the influence of the channel inclination angle θ on the type of flow, an additional numerical simulation was carried out using an angle of 1 degree and a temperature of 1723.15 K. The results for the slag height are shown in **Figure 8** for two angles of 1 and 5 degrees. As seen in this Figure, the height of the slag has an interesting behavior when $\theta = 1$. For this angle, when 5 s have elapsed, the slag height reaches a peak that exceeds the critical height, and thereafter decreases to a value below the critical height. This means that for a short time of about 3 s, the flow is subcritical, and then it becomes supercritical. In other words, the flow is temporary transcritical in nature [31], that is, the flow presents a temporary transition from subcritical to supercritical for an inclination angle of 1 degree.

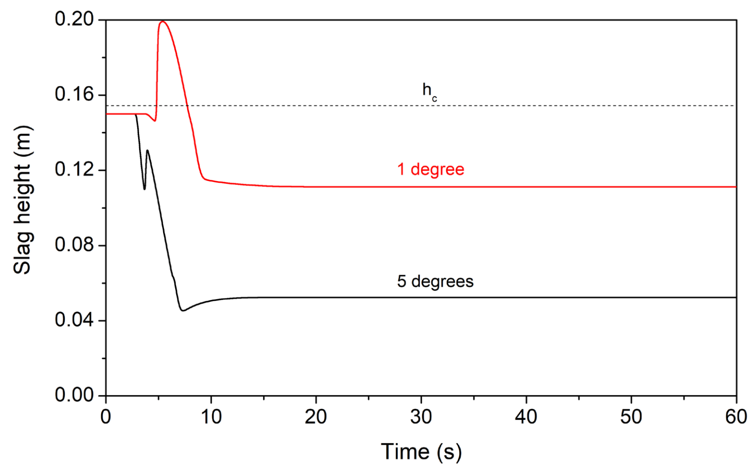

Figure 8. Evolution over time of the slag height at the channel discharge for two different channel inclination angles at $T = 1723.15$˚K.

4. Conclusions

The flow of molten blast furnace slag in an open channel was numerically analyzed.

For this, the transient Saint-Venant equations were solved using an explicit finite difference scheme. Four slag temperatures were considered. A length of ten meters, a width of 0.2 m, and an inclination angle of five degrees were also set. In order to know the influence of the channel inclination angle on the type of flow, an additional numerical simulation was carried out using an angle of 1 degree. A volumetric flow rate at the inlet was set at 0.038 m^3/s. A half-sinusoidal pulse in the volumetric flow of slag with a duration of five s and amplitude of 75% of the initial flow was established at the entrance of the channel in order to observe the response of the system. From the results of the numerical simulations, the following conclusions can be drawn:

1) For the channel considered, the slag flow is supercritical in nature, regardless of the value of the slag temperature.

2) The slag flow, in terms of height and velocity, reaches a steady state whose values are dependent on temperature.

3) Steady-state height decreases with temperature, while steady state velocity increases with temperature. In both cases, the volumetric flow remains constant.

4) The Froude number increases with temperature. This means that if the temperature of the slag increases, the supercritical nature of the flow becomes more intense.

5) An inclination angle of one degree of the channel produces a transient flow of transcritical nature, that is, it presents a temporary transition from subcritical flow to supercritical flow. An angle of five degrees does not present such a transition.

Of course, all numerical results and conclusions shown in this work should be verified with plant data or published works. However, to the authors' knowledge, that information is not available.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Geerdes, M., Chaigneau, R., Lingiardi, O., Molenaar, R., van Opbergen, R., Sha, Y. and Warren, P. (2020) Modern Blast Furnace Ironmaking. Fourth Edition. IOS Press.

[2] https://www.worldsteel.org

[3] Martins, A.C.P., Franco de Carvalho, J.M., Costa, L.C.B., Andrade, H.D., de Melo, T.V., Ribeiro, J.C.L., *et al.* (2021) Steel Slags in Cement-Based Composites: An Ultimate Review on Characterization, Applications and Performance. *Construction and Building Materials*, **291**, Article ID: 123265.
https://doi.org/10.1016/j.conbuildmat.2021.123265

[4] https://www.asa-inc.org.au

[5] Yu, P., Wang, S., Li, Y. and Xu, G. (2016) A Review of Granulation Process for Blast Furnace Slag. *MATEC Web of Conferences*, **68**, Article No. 06007.
https://doi.org/10.1051/matecconf/20166806007

[6] Morley, N.B. and Abdou, M.A. (1997) Study of Fully Developed, Liquid-Metal, Open-

Channel Flow in a Nearly Coplanar Magnetic Field. *Fusion Technology*, **31**, 135-153. https://doi.org/10.13182/fst97-a30816

[7] Morley, N.B. and Roberts, P.H. (1996) Solutions of Uniform, Open-Channel, Liquid Metal Flow in a Strong, Oblique Magnetic Field. *Physics of Fluids*, **8**, 923-935. https://doi.org/10.1063/1.868872

[8] Andreev, O., Kolesnikov, Y. and Thess, A. (2006) Experimental Study of Liquid Metal Channel Flow under the Influence of a Nonuniform Magnetic Field. *Physics of Fluids*, **18**, Article ID: 065108. https://doi.org/10.1063/1.2213639

[9] Wang, X.D., Klein, R., Kolesnikov, Y. and Thess, A. (2011) Application of Lorentz Force Velocimetry to Open Channel Flow. *Materials Science Forum*, **690**, 99-102. https://doi.org/10.4028/www.scientific.net/msf.690.99

[10] Fisher, A.E., Kolemen, E. and Hvasta, M.G. (2018) Experimental Demonstration of Hydraulic Jump Control in Liquid Metal Channel Flow Using Lorentz Force. *Physics of Fluids*, **30**, Article ID: 067104. https://doi.org/10.1063/1.5026993

[11] Stelian, C. (2014) Application of Manning's Formula for Estimation of Liquid Metal Levels in Electromagnetic Flow Measurements. *Metallurgical and Materials Transactions B*, **46**, 449-458. https://doi.org/10.1007/s11663-014-0206-9

[12] Barron, M.A., Reyes, J. and Medina, D.Y. (2022) Uniform Flow of Molten Metals in Rectangular Open Channels. *World Journal of Engineering and Technology*, **10**, 518-526. https://doi.org/10.4236/wjet.2022.103032

[13] Barron, M.A., Reyes, J. and Miranda, D.Y. (2022) Unsteady Non-Uniform Flow of Molten Metals in Rectangular Open Channels. *World Journal of Engineering and Technology*, **10**, 593-604. https://doi.org/10.4236/wjet.2022.103038

[14] Barron-Meza, M.A. and Reyes-Miranda, J. (2024) Temperature Drop of Molten Metals in Open Channels. *World Journal of Engineering and Technology*, **12**, 493-500. https://doi.org/10.4236/wjet.2024.123031

[15] Han, C., Chen, M., Zhang, W., Zhao, Z., Evans, T., Nguyen, A.V., *et al.* (2015) Viscosity Model for Iron Blast Furnace Slags in SiO_2-Al_2O_3-CaO-MgO System. *Steel Research International*, **86**, 678-685. https://doi.org/10.1002/srin.201400340

[16] Guo, H., Zhu, M., Li, X., Guo, J., Du, S. and Zhang, J. (2014) Improvement of the Blast Furnace Viscosity Prediction Model Based on Discrete Points Data. *Metallurgical and Materials Transactions B*, **46**, 378-387. https://doi.org/10.1007/s11663-014-0176-y

[17] Guo, H., Zhu, M., Yan, B., Deng, S., Li, X. and Liu, F. (2016) Dynamic Regional Viscosity Prediction Model of Blast Furnace Slag Based on the Partial Least-Squares Regression. *Journal of Metals*, **69**, 395-401. https://doi.org/10.1007/s11837-016-2195-4

[18] Iida, T., Sakai, H., Kita, Y. and Shigeno, K. (2000) An Equation for Accurate Furnace of Blast Furnace Type Slags from Prediction of the Viscosities Chemical Composition. *ISIJ International*, **40**, S10-S14.

[19] Gan, L. and Lai, C. (2013) A General Viscosity Model for Molten Blast Furnace Slag. *Metallurgical and Materials Transactions B*, **45**, 875-888. https://doi.org/10.1007/s11663-013-9983-9

[20] Saito, N., Hori, N., Nakashima, K. and Mori, K. (2003) Viscosity of Blast Furnace Type Slags. *Metallurgical and Materials Transactions B*, **34**, 509-516. https://doi.org/10.1007/s11663-003-0018-9

[21] Hu, X., Ren, Z., Zhang, G., Wang, L. and Chou, K. (2012) A Model for Estimating the Viscosity of Blast Furnace Slags with Optical Basicity. *International Journal of Minerals, Metallurgy, and Materials*, **19**, 1088-1092.

https://doi.org/10.1007/s12613-012-0675-2

[22] Chagas, P., Souza, R. (2005) Solution of Saint Venant's Equation to Study Flood in Rivers, through Numerical Methods. *Proceedings of the* 2005 *Annual AGU Hydrology Days*, Colorado, 7-9 March 2005, 205-210.

[23] Keskin, M.E. and Ağiralioğlu, N. (1997) A Simplified Dynamic Model for Flood Routing in Rectangular Channels. *Journal of Hydrology*, **202**, 302-314. https://doi.org/10.1016/s0022-1694(97)00072-3

[24] Chen, M., Zhang, D., Kou, M. and Zhao, B. (2014) Viscosities of Iron Blast Furnace Slags. *ISIJ International*, **54**, 2025-2030. https://doi.org/10.2355/isijinternational.54.2025

[25] Zhang, H., Zhang, F., Shen, K. and Yuan, M. (2015) A Surface and Subsurface Model for the Simulation of Rainfall Infiltration in Slopes. *IOP Conference Series*: *Earth and Environmental Science*, **26**, Article ID: 012025. https://doi.org/10.1088/1755-1315/26/1/012025

[26] Castro-Orgaz, O. and Hager, W.H. (2019) Shallow Water Hydraulics. Springer.

[27] Szekely, J. (1979) Fluid Flow Phenomena in Metals Processing. Academic Press.

[28] Tollner, E.W. (2021) Open Channel Design. Wiley. https://doi.org/10.1002/9781119664338

[29] Sturm, T.W. (2021) Open Channel Hydraulics. 3rd Edition, McGraw-Hill.

[30] Osman-Akan, A. (2006) Open Channel Hydraulics. Butterworth-Heinemann.

[31] Hager, W.H. and Castro-Orgaz, O. (2016) Transcritical Flow in Open Channel Hydraulics: From Böss to De Marchi. *Journal of Hydraulic Engineering*, **142**, Article ID: 02515003. https://doi.org/10.1061/(asce)hy.1943-7900.0001091