

# The Method of Thermoelectric Energy Generations Based on the Axial and Radial Flux Electromagnetic Inductions\*

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## Abstract

The traditional thermoelectric energy conversion techniques are explained in detail in terms of the axial flux electromagnetic (AFE) and the radial flux electromagnetic (RFE) inductions, and applications to heat engines for the energy-harvesting technologies are discussed. The idea is induced by the analysis of thermomechanical dynamics (TMD) for a nonequilibrium irreversible thermodynamic system of heat engines (a drinking bird, a low temperature Stirling engine), resulting in thermoelectric energy generation different from conventional heat engines. The mechanism of thermoelectric energy conversion can be categorized as the axial flux generator (AFG) and the radial flux generator (RFG). The axial flux generator is helpful for low mechanoelectric energy conversion and activations of waste heat from macroscopic energy generators, such as wind, geothermal, thermal, nuclear power plants and heat-dissipation lines, and the device contributes to solving environmental problems to maintain clean and sustainable energy as one of the energy harvesting technologies.

## Keywords

Axial Flux and Radial Flux Generators, Thermomechanical Dynamics (TMD), Thermoelectric Energy Conversions

## 1. Introduction

The prosperity of human society essentially requires advanced macroscopic energy generators (MEGs) such as turbines, motors and rotors for wind, hydroe-

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lectric, geothermal, thermal, nuclear power plants and so forth. However, the characteristic feature of MEGs is essentially directed to mass production and consumption of heat and energy, resulting in an enormous amount of abandoned waste heat and chemical substances, and so, our society needs to develop technologies for sustainable developmental goals (SDGs).

Based on very sensitive thermoelectric devices of low temperature heat engines (a drinking bird [1] and low temperature Stirling engines [2]), we proposed thermoelectric generators to activate discarded waste heat into usable electric power. The low temperature heat engines of a drinking bird and a Stirling engine can work at small temperature difference, producing usable electric energy [3]-[6]. It is possible to activate electric power from abandoned waste heat by the method of axial flux electromagnetic induction [3] [4].

## 2. The Traditional Thermomechanical Convertors, and the 3<sup>rd</sup> Kind New Convertor

### 1) Macroscopic high-power thermoelectric generators

The traditional mechanoelectric or thermoelectric convertors are categorized as the radial flux generator (RFG) by the classification of magnetic flux lines, which is suitable for huge energy productions and requires high speed rotations of turbines (Figure 1 and Figure 2).

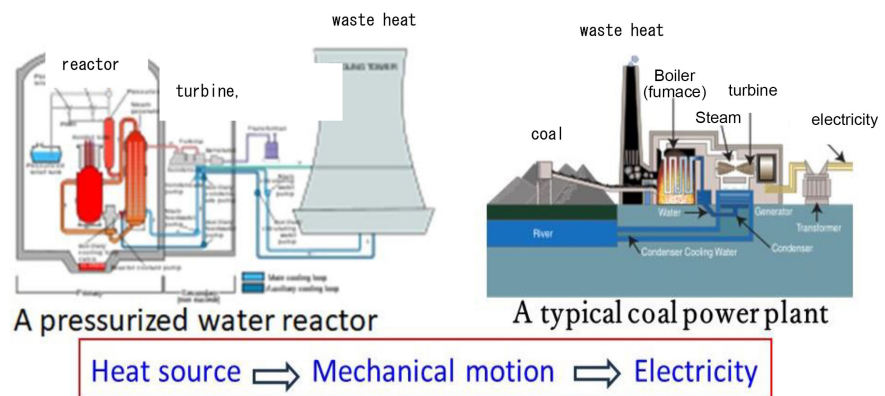


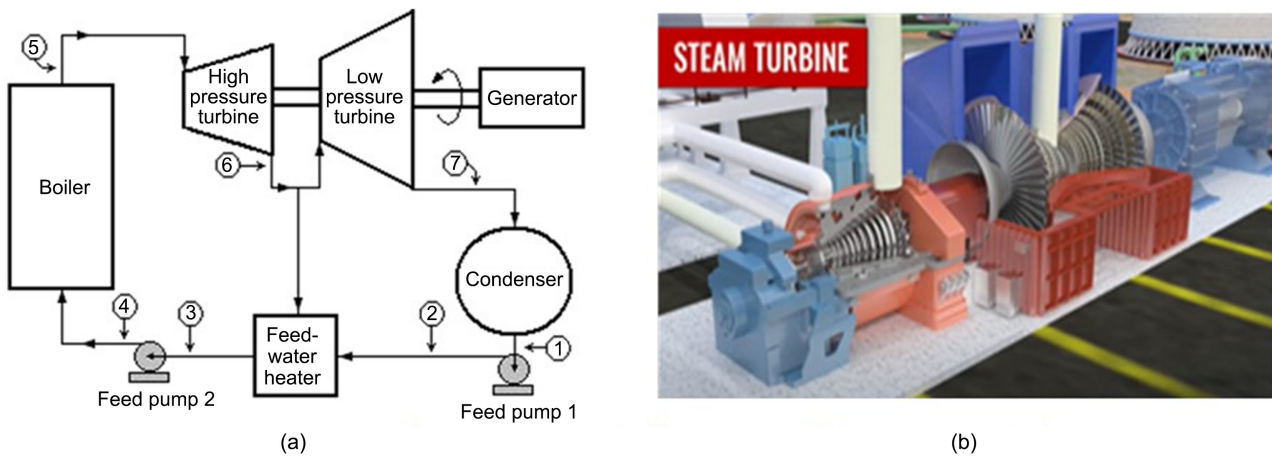
Figure 1. Macroscopic high-power thermoelectric energy generators.

Although the radial flux generators (RFG) are qualified for producing high electric power, it is not qualified for reactivating electric power from a low temperature heat flow, such as  $40^{\circ}\text{C} < T < 100^{\circ}\text{C}$  boiled water, waste heat from industries. In general, it is known that the system can only use 1/3 of produced total heat-energy, and 2/3 of heat-energy is not used or dissipated eventually in an external environment.

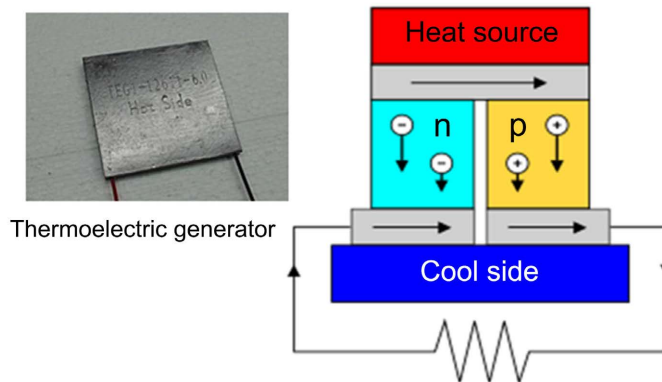
### 2) Microscopic, thermoelectric power generation.

The second traditional devices for thermoelectricity are microscopic power generators using crystal structure, transistors, a thermocouple (thermoelectrical thermometer), etc., known as Seebeck effect and Peltier effect (Figure 3). Mi-

microscopic and thermoelectric power generators are known as expensive and less efficient, and 2/3 of total heat-energy is dissipated as many kinds of waste.



**Figure 2.** (a) The system is large and heavyweight. It loses 2/3 of produced heat-energy to generate electric power. (b) The typical steam turbine. High speed and temperature are necessary.



**Figure 3.** This is a microscopic energy harvesting technology (EHT). The p-doped and n-doped semiconductors induce heat flows and electric current.

### 3) The 3<sup>rd</sup>-kind, new thermoelectric energy production

On the other hand, the axial flux generator (AFG) is most suitable for activating sensitive boiled water, waste heat from industries. This is an important consequence derived from the analysis of thermomechanical dynamics (TMD), which is proposed by the authors for nonequilibrium irreversible states (NISs) of heat engines [5] [6]. The disk-magnet electromagnetic induction (DM-EMI) technique with a low temperature Stirling engine revealed that electric power generation from low heat flows can be possible. This is assured by the theoretical analysis of TMD, proving that an optimal speed of mechanical rotation can exist in low rotational speed (about 30 - 60 rpm [3] [4]). Therefore, a low temperature, thermoelectric generation Stirling engine (TEG-Stirling engine) can be constructed. In this review, we explain and emphasize that the analysis of thermomechanical dynamics (TMD) applied to TEG-Stirling engine generates a

low-speed, low-weight, optimal TEG-Stirling engine.



**Figure 4.** A drinking bird (DB).

The idea of the 3<sup>rd</sup>-kind, new thermoelectric energy production is induced by the analysis of thermomechanical dynamics (TMD) for a nonequilibrium irreversible thermodynamic system of heat engines (a drinking bird, a low temperature Stirling engine), resulting in thermoelectric energy generation different from conventional heat engines, 1) and 2).

A drinking bird (DB) in **Figure 4** is also a simple and efficient heat engine. A cup of ordinary water produces mechanical motion, and the mechanical motion is changed to electric power (thermoelectric generation).

We proposed and solved thermomechanical equation of motion for the drinking bird [1].

A low temperature Stirling engine (LTSE) is shown in **Figure 5**. The upward heat-flow is changed into mechanical work (the flywheel rotations). A cup of hot water produces mechanical motion, and the nonequilibrium irreversible motion is solved by deriving a dissipative equation of motion [6].



**Figure 5.** A low temperature Stirling engine (LTSE).

The simple amusing toys are scientifically very fundamental. A drinking bird and a Stirling engine are heat engines, and the toys work in nonequilibrium irreversible states (NISs). The physical states of heat engines are not in thermodynamic nor mechanical equilibrium, and not simply explained by just thermodynamics + mechanics.

Therefore, one needs a method for NISs. It should be emphasized that electric power can be produced from low-speed revolutions, for example, about 30 rpm - 60 rpm. This is an important fact derived from TMD analyses.

### 3. The Method of TMD

The equation of motion and time-dependent physical quantities, such as internal energy  $\varepsilon(t)$ , work  $W(t)$ , entropy  $S(t)$  and temperature  $\tilde{T}(t)$  of heat engines are solved self-consistently by the method of thermomechanical dynamics (TMD). The method of TMD is a new classical approach proposed by the authors, along the work of Gibbs' thermodynamics which is based on fundamental thermodynamics and needs profound discussions on physical foundations. Hence, readers who are interested in theoretical discussions should be directed to references [5] [6].

The method of TMD requires three conditions.

#### 1) The dissipative equation of motion

In the case that mechanical and thermal states coexist, such as thermomechanical states of heat engines, the dissipative equation of motion for work must be constructed by considering phenomenological effects of frictional variations, time-dependent changes of physical quantities, thermal conductivity and efficiency.

Because time-symmetry is broken in the system of heat engines, there is no Euler-Lagrange type derivation of a correct dissipative equation of motion. It would be useful to make use of Hamiltonian or Lagrangian method at the beginning to find an approximate dissipative equation of motion and then, find an appropriate dissipative equation of motion.

#### 2) The total energy-flow conservation law

The thermodynamic work  $dW_{th}(t)$ , the internal energy  $d\varepsilon(t)$  and total entropy  $T(t)dS(t)$ , are related to one another by the energy conservation law:

$$d\varepsilon(t)/dt = T(t)dS(t)/dt + dW_{th}(t)/dt = dQ(t)/dt + dW_{th}(t)/dt \quad (1)$$

Thermodynamic equilibrium is defined by  $dW_{th}(t)/dt = 0$ : no thermodynamic power exists in thermodynamic equilibrium.

The expression of heat flow (entropy flow) is used

$$T(t)dS(t)/dt = dQ(t)/dt, \quad (2)$$

in the analysis of heat engines.

#### 3) Temperature, $\tilde{T}(t) = T\tau(t)$ , in a nonequilibrium irreversible state

The measure of a nonequilibrium irreversible state is defined by the ratio of entropy-flow against energy-flow:

$$\tau(t) = \frac{T(t) dS(t)/dt}{d\varepsilon(t)/dt} = \frac{dQ(t)/dt}{d\varepsilon(t)/dt}. \quad (3)$$

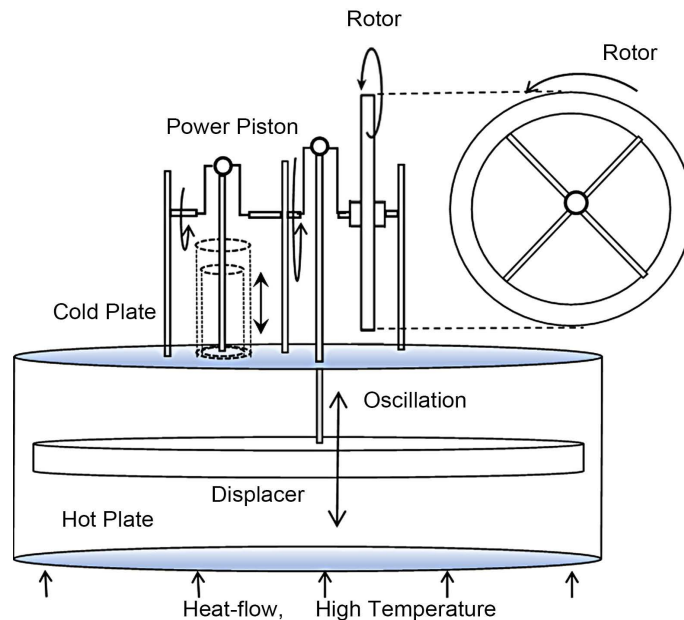
The value of  $\tau(t)$  is a dimensionless, positive-definite function,  $\tau(t) > 0$ . The temperature in nonequilibrium state (NISs) is defined by,

$$\tilde{T}(t) = T_0 \tau(t), \quad (4)$$

where  $T_0$  is the initial equilibrium temperature. When  $\tau(t) = 1$  holds identically with respect to time  $t$ , it defines thermodynamic equilibrium, which shows no work exists,  $dW_{th}(t)/dt = 0$ , at thermodynamic equilibrium. The conditions of near equilibrium states, local equilibrium, linearity of fluxes and forces of transport processes [7]-[9] are studied by the condition,  $\tau(t) = \frac{T(t) dS(t)/dt}{d\varepsilon(t)/dt} \sim 1$  in the TMD method.

#### 4. The Equation of Motion for Stirling Engine

A theoretical and schematic low temperature Stirling engine is shown in **Figure 6**, and the device consists of the following functions:



**Figure 6.** A theoretical and schematic structure of a low temperature Stirling engine [6].

**1) Heat source:** A homogeneous heat flow from boiled water ( $40^\circ\text{C} - 100^\circ\text{C}$ ) and geothermal heat, etc. The heat flow coming into the system is defined by  $dQ(t)/dt > 0$ .

**2) Heat exchangers:** The power piston is used to improve the heat flow and the flywheel rotation affected by friction losses.

**3) Regenerator:** The internal mechanism of heat exchangers between a hot plate and a cold plate. The thermomechanical conversion for work depends on

thermal efficiency, heat transfer, viscous pumping and friction losses.

**4) Heat sink:** The temperature difference between a hot plate and a cold plate is needed for internal heat flows.

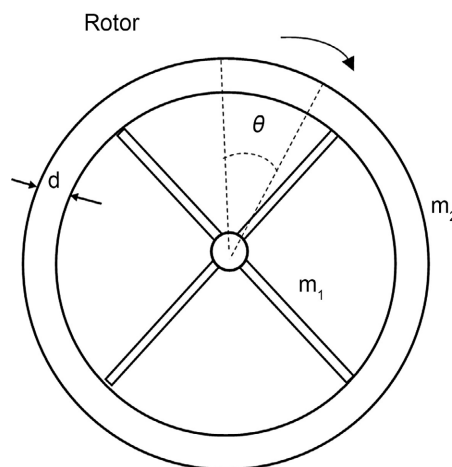
**5) Displacer:** The thermal heat flow from a hot plate to a cold plate exerts vertical oscillations of the displacer. The efficiency of displacer to maintain appropriate heat dissipations is essential for mechanical rotations of the flywheel.

It is essential to understand that heat engines in general are not in thermodynamic equilibrium, but in nonequilibrium irreversible states (NISs). Therefore, it is important to have a different theoretical approach for NISs, which is the reason why we proposed the method of TMD. The piecewise continuous driving forces produced by frictional and thermal fluctuations are assumed to couple to thermodynamic work,  $Q_w(t)$ , of the flywheel and power-piston with an associating dissipation of heat.

As the first requirement (1) of TMD, the *dissipative equation of motion* for a low temperature Stirling engine is proposed by:

$$I_0\theta''(t) + c\theta'(t) - \lambda_w Q_w(t) |\sin \theta(t)| = 0, \quad (5)$$

where  $\lambda_w$  is a dimensionless coupling constant for heat and mechanical work, and the angle,  $\theta(t)$ , is chosen as in **Figure 7**. The term  $|\sin \theta(t)|$  expresses piecewise continuous driving forces produced by rotations, frictional and nonequilibrium thermal fluctuations, and  $c$  is a friction constant.



**Figure 7.** The rotational angle,  $\theta(t)$ , starting from the vertical axis.

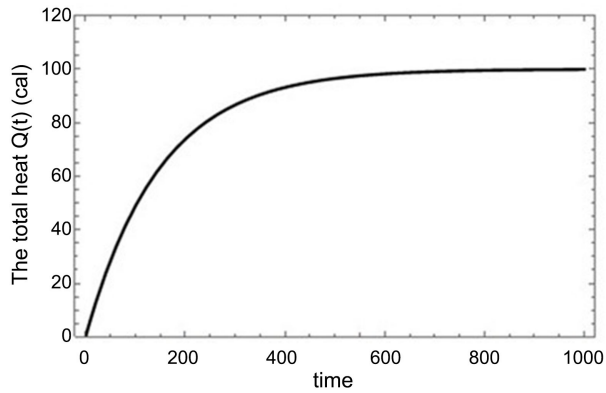
Although the fundamental equation of motion (5) seems simple, its mathematical and physical consequences are profound. The piecewise continuous driving force in (5) immediately indicates that the acceleration is not defined as differentiable and continuous quantity as supposed in Newtonian mechanics. The acceleration cannot be determined as the second-order derivative derived from the trajectory of motion, because the driving force contains jump discontinuities in the entire domains of motion.

### 5. The Solution to the Dissipative Equation of Motion of TEG-Stirling Engine

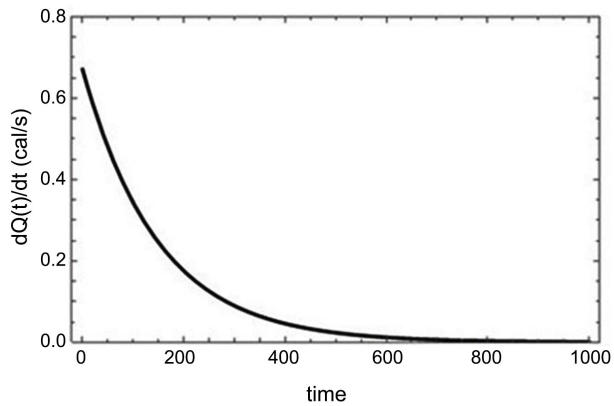
We show the computer simulations by employing the following incoming heat  $Q_{in}(t)$ ,

$$Q_{in}(t) = Q_H (1.0 - e^{-\xi t}) \tag{6}$$

and heat flow  $dQ_{in}(t)/dt$ , as shown in **Figure 8** and **Figure 9**, and  $Q_H$  and  $\xi$  are free parameters to adjust in the computer simulations, e.g.,  $Q_H \sim 100$  cal,  $\xi \sim 6.51 \times 10^{-3}$  (1/s) for the current simulations.



**Figure 8.** The total heat-in,  $Q_{in}(t)$ .

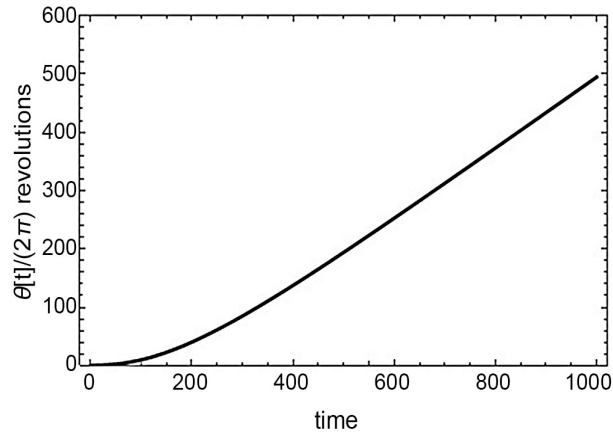


**Figure 9.** The heat flow,  $dQ_{in}(t)/dt$ .

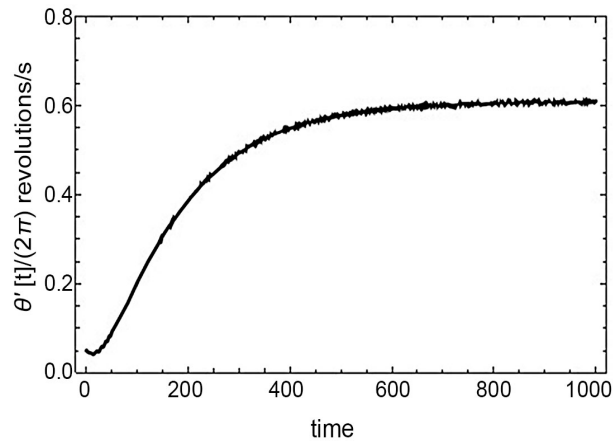
The dissipative equation of motion,  $Q_{in}(t)$  and  $dQ_{in}(t)/dt$  with  $Q_w(t) = \eta Q_{in}(t)$  ( $\xi$  and  $\eta$  are arbitrarily chosen small values) and Equation (5) are used to find the heat-energy solution for kinetic work,  $Q_{wk}(t)$ ,  $\theta(t)$  and  $\theta'(t)$  by maintaining the total energy-flow conservation law, (1) and (2). The computations should be repeated by taking different values of  $\xi$  and  $\eta$  until reasonable experimental values of angular velocity  $\theta(t)$  and  $\theta'(t)$  are obtained.

The number of rotations  $\theta(t)/2\pi$  (revolutions) and the angular velocity  $\theta'(t)/2\pi$  (revolutions/s) of the flywheel are respectively shown in **Figure 10**

and **Figure 11**. The maximum angular velocity seems stable and constant, but one can notice that the angular velocity in **Figure 11** has tiny fluctuations along the solution. The tiny fluctuations are caused by frictional variations and thermal fluctuations coming from the displacer and working fluid.



**Figure 10.** The number of revolutions,  $\theta(t)/2\pi$ , in the time range  $0 < t < 1000$ .

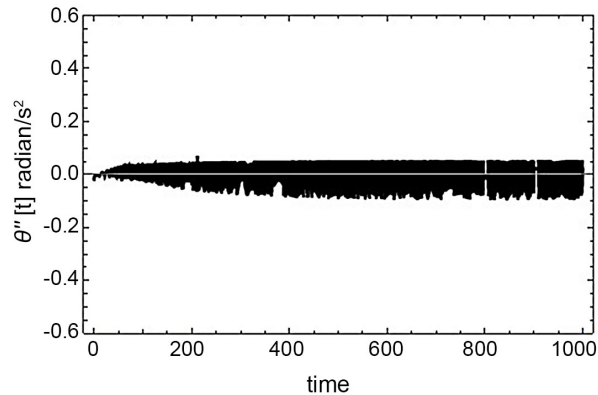


**Figure 11.** The angular velocity,  $\theta'(t)/2\pi$  (revolutions/s). Note the tiny fluctuations along the angular velocity.

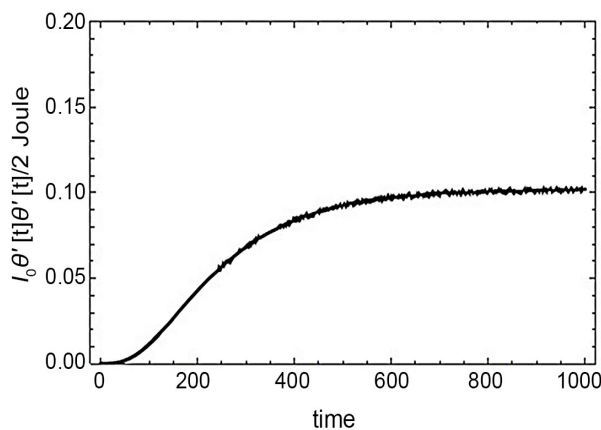
The trajectory  $\theta(t)$  and angular velocity  $\theta'(t)$  are continuous and differentiable, whereas the angular acceleration  $\theta''(t)$  is piecewise continuous and has finite numbers of jump discontinuities in a finite interval. The whole view of acceleration results in an assembly of hedgehog-like spiny lines as shown in **Figure 12**. The realistic flywheel thermal motion is produced reasonably well by the dissipative equation of motion (5). When heat exchangers and regenerators work properly, the flywheel rotation persists for a long period of time. Numerical calculations and self-consistency relations are discussed in detail in [5] [6]. Thermodynamic work,

$$Q_{wk}(t) = \frac{I_0}{2} \theta'(t)^2 \quad (\text{Joule}) \tag{7}$$

is shown in **Figure 13**. The rotational energy reaches a maximum stable value, which has a continuous, tiny-wiggly line because of  $\theta'(t)$ . The dissipative equation of motion is successful for producing thermomechanical flywheel rotations and applied to thermoelectric energy conversions [3] [4].



**Figure 12.** The piecewise continuous angular acceleration,  $\theta''(t)$  (rad/s<sup>2</sup>),  $0 < t < 1000$ .



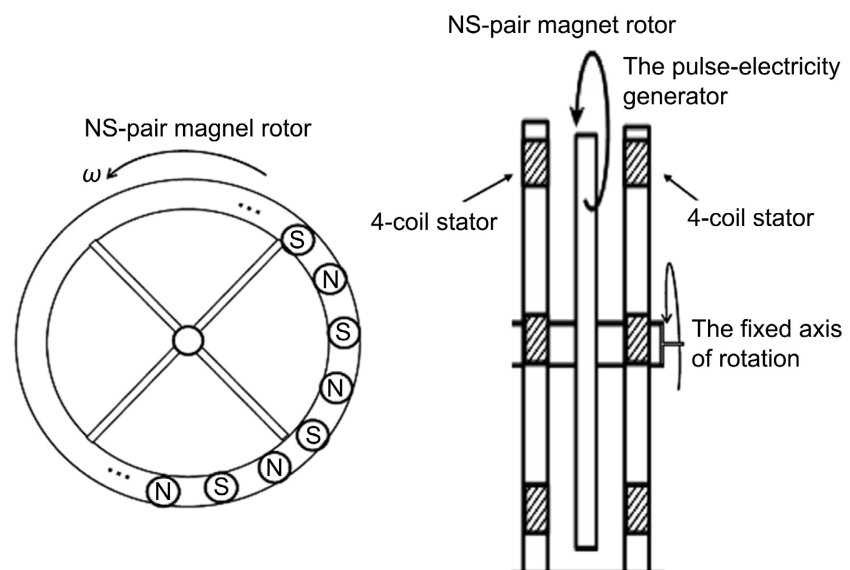
**Figure 13.** Thermodynamic work,  $Q_{wk}(t) = \frac{I_0}{2} \theta'(t)^2$ .

The thermomechanical states of the heat engine are in nonequilibrium irreversible states (NISs), and time-dependent thermodynamic work  $W_{th}(t)$ , internal energy  $\varepsilon(t)$ , energy dissipation or entropy  $T(t)dS(t)/dt$ , and temperature  $\tilde{T}(t)$ , are precisely obtained and computed in TMD, and physical quantities are numerically shown in [6]. We will focus on the DM-EMI applications to TEG-Stirling engine in the following section.

### 6. The DM-EMI Applied to TEG-Stirling Engine

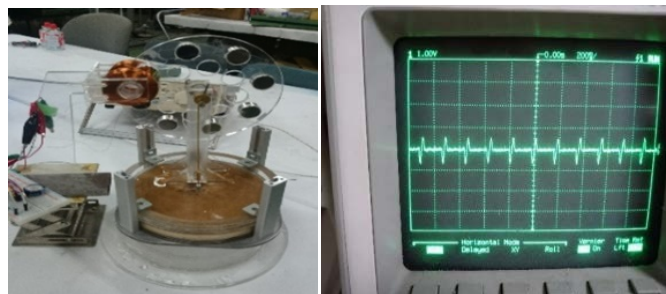
The computer simulations for the existence of optimal angular velocities (rpm) at low temperature and low heat flows are shown, and the fact is the proof of possibility for a low temperature TEG-Stirling engine as one of the sustainable environmental technologies (SETs). The large parts of technical as well as theo-

retical discussions are found in papers [3] [4]. The NS-pair disk magnet electromagnetic induction in a general schematic image is shown in **Figure 14**, and properties of electric current and power produced by the axial flux generator (AFG) are shown by changing angular velocity,  $\omega$  (rpm). The numerical simulations demonstrate the character of electric current and power. The numerical calculations with  $\omega = 120$  (rpm) and  $\omega = 30$  (rpm) are respectively compared. The axial magnetic flux of DM-EMI method produces pulse current (PC). The higher angular velocity driven by high temperature exhibits discrete properties of pulse electric current in very short ranges of time, whereas the lower angular velocities driven by a low temperature gradually demonstrate like a character of continuous electric currents. This also indicates one of the properties of AFG appropriate for a low temperature thermoelectric conversion.



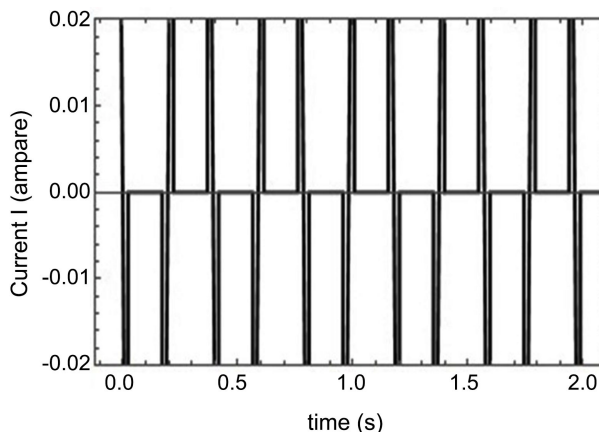
**Figure 14.** The image of NS-pair disk magnet electromagnetic induction.

A primitive experiment to show a pulse electric current is shown in **Figure 15**,  $\omega \sim 160$  (rpm), which is compatible with TMD theoretical calculations. Note that the pulse current direction in **Figure 15** is from down-to-up, which comes from choosing the direction of right- or left-rotations of the flywheel.

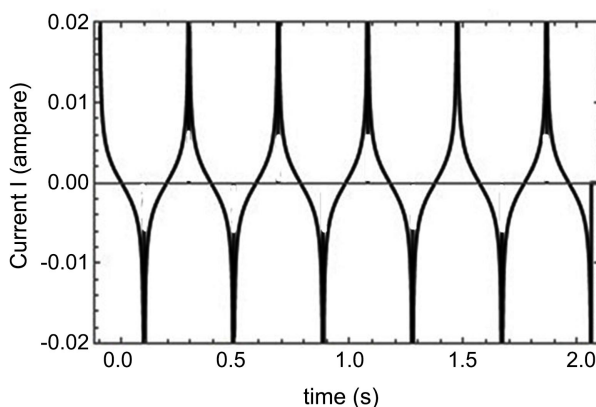


**Figure 15.** The primitive experiment (left) and pulse electric current (right).

The 4NS-pair disk-magnet rotor is composed of some pairs of N and S magnetic poles in a rotor, and numerical simulations produce an alternating pulse current as shown in **Figure 16** ( $\omega = 120$  rpm) and **Figure 17** ( $\omega = 30$  rpm).



**Figure 16.** The produced pulse electric current,  $\omega = 120$  (rpm).



**Figure 17.** The produced pulse electric current:  $\omega = 30$  (rpm).

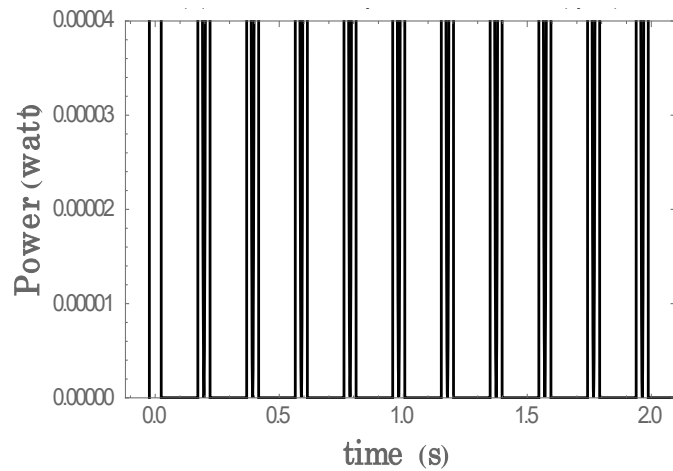
The N and S poles respectively induce a reversed pulse-current, examined by the theoretical analysis of electromagnetic induction. The direction of magnetic flux induced in the coils of the stators is completely opposite to the N and S poles, resulting in reversed pulse electric current. The current and voltage produced in a coil are inverse proportional against a produced electric energy in a time interval,  $(t_0, t_1)$ . It is understood from the relation:

$$E_e = \int_{t_0}^{t_1} V(t) I(t) dt, \tag{8}$$

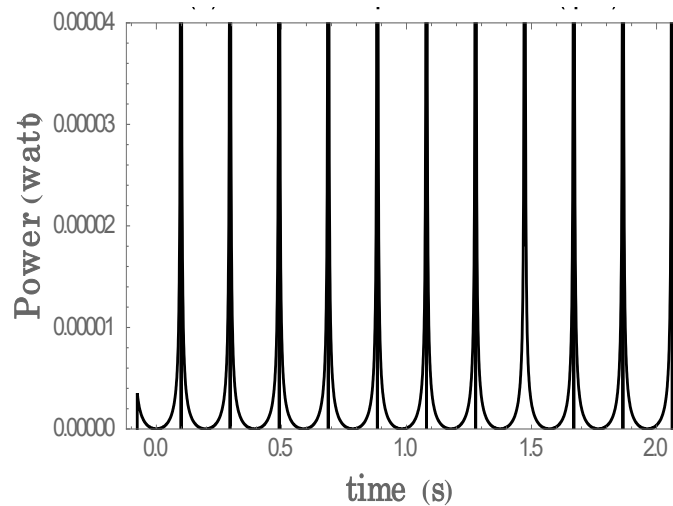
where  $E_e$  is a finite amount of electric energy produced by a magnet and a coil in the mechanism of axial flux generation. The electric energy  $E_e$  is finite and a constant average value of the time interval  $(t_0, t_1)$ , which is the property of axial flux DM-EMI. It immediately indicates that  $I(t)dt$  becomes small when  $V(t)$  is large, and vice versa.

The electric powers of  $\omega = 120$  (rpm) and  $\omega = 30$  (rpm) are specifically shown

in **Figure 18** and **Figure 19**. The electric energy is shown by the area which is visible in **Figure 19**, but the time interval for  $I(t)dt$  becomes small in **Figure 18** ( $\omega = 120$  rpm). The time interval of **Figure 19** becomes larger than that of **Figure 18**, indicating that electric power can be better extracted in a technical sense in case of  $\omega = 30$  (rpm). The result is essential for electric-power conversions, meaning that the electric power may be better extracted from low temperature heat flows ( $\omega \sim 30$  rpm) by employing the axial flux generator.



**Figure 18.** The produced pulse electric power:  $\omega = 120$  (rpm).



**Figure 19.** The produced pulse electric power:  $\omega = 30$  (rpm).

The important property of AFG concludes that an optimal angular velocity to produce electric power exists in a low angular velocity induced by a low temperature heat flow. This is one of the important results in the TMD analysis, which makes the extraction of electric power possible from  $50^{\circ}\text{C} - 100^{\circ}\text{C}$  boiled water. That is the reason why the heat-electric power conversion device is proposed by the authors as a thermoelectric generation Stirling engine [4]-[6]. It is remarkable that an optimal thermoelectric generation device of a drinking bird is specif-

ically constructed [10] only recently, as we discussed and expected theoretically [3] [4]. (Figure 20, Figure 21)



Figure 20. Vertical-axis wind turbines.

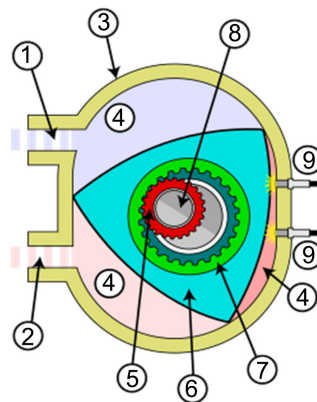


Figure 21. A rotary engine.

## 7. Conclusions

The huge power production and consumption of human societies and industries in the modern world have affected ecological systems on Earth, and it is imperative to develop clean energy and energy harvesting technologies. The DM-EMI technique proves that there exists an optimal speed of rotation (rpm) to extract electric power, even in a low temperature heat flow. The property of a low-(rpm) electric-power conversion and applications of the axial flux generator are one of the new findings and should be investigated further.

The new types of heat-electricity conversion devices are possible but have not been constructed nor applied sufficiently. The applications to compensate electricity productions for macroscopic energy generators (MEGs), vertical-axis wind turbines (VAWTs), internal combustion engines, a low-temperature TEG-rotary engines, TEG-diesel engines with hydrogen-fuel could be theoretically possible. We are planning to develop optimal devices for electric energy production and seeking collaborations and an experimental budget to test several types of TEG engines.

The method of TMD helped us integrate very sensitive physical problems of nonequilibrium irreversible thermodynamics with technologies for thermoelectric energy conversions and understand the time-progress of internal energy

$d\varepsilon(t)$  and work  $dW_{th}(t)$ , heat-flow or entropy-flow,  $T(t)dS(t)$  and nonequilibrium temperature  $\tilde{T}(t) = T\tau(t)$ , producing testable specific ideas for heat engines.

The TMD analysis of heat engines suggests that energy can be more efficiently produced and used so that waste of energy should be dramatically decreased. The very high-temperature pressurized steam required in a traditional RFG is not necessary for AFG for thermoelectric energy conversions. The electricity should be directly used for sustainable social infrastructure, such as electrolysis to produce basic chemicals, such as  $H_2$ ,  $O_2$ , C, COOH,  $CH_3COOH$ , etc., which supports biological stability, symbiosis and ecology in nature, and sustainable environmental goals (SEGs) [11].

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### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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