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On the Properties of Elemental and High-T_c Superconductors in a Unified Framework I

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Abstract

For both elemental and high-T_c superconductors (SCs), we show that the empirical values of their T_c, gaps (Δs) and coherence lengths (ξs)—which are among more than a dozen parameters that characterize an SC of each category—are explicable in a unified framework of chemical potential (μ)-incorporated generalized BCS equations which provide an alternative to their explanation via the muti-band approach. Notable features of this study are: i) It sheds new light on the deviations of the gap-to-T_c ratios of the elemental SCs from the universal value. ii) Among the SCs it deals with is Bi-2223 which has the remarkable property of being characterized by three gaps as reported by Ponomarev, *et al.*, (Pis'ma v ZhETF (JETP Lett.) 100,126 (2014)). iii) It employs a novel approach of simultaneously solving three or four equations which yield the value of the Fermi velocity and the values of μ and the interaction parameters in the pairing equation, both at $T = T_c$ and 0 K, corresponding to the T_cs, Δs and ξs of these SCs, The other 12 SCs dealt with here are Al, Sn, Nb, Cd, Pb, MgB_2, YBCO, Bi-2212, Tl-2212, Tl-2223 and compressed H_3S and LaH_{10}.

Keywords

Generalized BCS Equations, Chemical Potential, Characteristic Parameters of a Superconductor and Their Dependence on Chemical Potential, Multiple Debye Temperatures

1. Introduction

In general, a superconductor (SC) is characterized by the values of its following parameters.

T_c: Critical temperature

Δ: Gap of an elemental SC

$\Delta_1 < \Delta_2 < \Delta_3$: Gaps of a composite SC with three gaps

θ: Debye temperature

E_F, v_F: Fermi energy, Fermi velocity

λ: Interaction parameter in the pairing equation due to the Coulomb repulsion between electrons and the attraction due to the ion-lattice

$m^* = \eta\, m_e$: Effective mass of an electron, m_e being the free electron mass

ξ: coherence length at $T = 0$

sf: Self-field, the field that exists in the absence of any applied field

H_c: Critical field of an elemental SC

H_{c1}, H_{c2}: Lower and upper critical fields of a type II SC

λ_m: Magnetic interaction parameter in the pairing equation for an SC in an applied field

N_L: Landau index, *i.e.*, the number of occupied levels when the (a, b) components of momentum are quantized when the SC is subject to an applied field in the c-direction

λ_L: London penetration depth at $T = 0$

n_s: Number density of charge-carriers

v_c: Critical velocity of Cooper pairs

j_c: Critical current density

To the above list may be added the chemical potential μ_1 at $t = T/T_c = 1$ (the chemical potential μ_0 at $t = 0$ is E_F and is therefore already included in the above list), because one of the objectives of this study is to find the values of μ_1 and μ_0 for each of the SCs while dealing with its: 1) T_c, Δ and ξ, 2) H_c or H_{c2} and λ_L, and 3) j_c. However, for reasons of length, we restrict ourselves in this paper to dealing with the properties noted in 1), deferring presenting the results of the remaining properties to a sequel. The SCs dealt with are Al, Sn, Nb, Cd, and Pb and the high-T_c SCs MgB_2, YBCO, Bi-2212, Bi-2223, Tl-2212, Tl-2223, and compressed H_3S and LaH_{10}.

A summary of the most widely followed theoretical approaches to address the T_c, Δ and ξ of SCs is given below (unless stated otherwise, the units employed in this paper are Gaussian).

For an elemental SC, the T_c and Δ are calculated via the BCS equations [1] which do not contain μ_0 because of the assumption that $\mu_0 \gg k\theta$ (k is the Boltzmann constant). For a high-T_c SC which invariably consists of two or more elements, T_c is calculated via the Migdal-Eliashberg-McMillan approach [2], which allows λ to be greater than unity because it is based on an integral equation the expansion parameter of which is not λ_{BCS} (which must be non-negative and less than 0.5 in order to satisfy the Bogoliubov constraint for stability of the system [3]), but (m_e/M), where m_e is the free electron mass and M the mass of an ion. For the multiple gaps of such an SC, one employs the multi-band approach (MBA)—or its variants—which was initiated by Suhl, *et al.* [4]. In this approach, the formation of Cooper pairs (CPs) can take place not only due to scattering in each of the bands individually, but also due to cross-band scattering. For the ξ of both type I and type II SCs is employed the following BCS relation

$$\xi = \hbar v_F / \pi\Delta_0 , \quad (1)$$

where

$$v_F = \sqrt{2\mu_0 / m^*}\, c \quad (\mu_0 \text{ and } m^* \text{ in units of electron-Volt}) \quad (2)$$

$$\mu_0 = \frac{\hbar^2}{2m^*}\left(3\pi^2 n_s\right)^{2/3} \quad (3)$$

and n_s is usually determined via the Hall effect. Note that (1) will lead to as many values of ξ for any SC as the number of gaps that characterize it. For an SC with two gaps, ξ_1 corresponding to $|\Delta_1|$ and ξ_2 corresponding to $|\Delta_2| > |\Delta_1|$ will in the following be identified with ξ_{ab} and ξ_c, respectively.

As an alternative to the above approach, we deal here with the T_c and Δ of SCs by employing the μ-incorporated generalized BCS equations (GBCSEs). GBCSEs are obtained via a Bethe-Salpeter equation (BSE) and provide a *unified* framework for dealing with both elemental and composite SCs. The features of the GBCSEs pertinent to the present study are:

1) The (T, W)-dependent GBCSE for an elemental SC is formulated in terms of the binding energy W of a CP rather than Δ. The equation for the T_c of the SC is obtained from it on putting $W = 0$ and is identical with the corresponding BCS equation when $\mu_0 \gg k\theta$. On the other hand, solving the equation at $T = 0$ shows that $\Delta \cong |W|$ (the equivalence being exact in the limit $\lambda \to 0$) [5]. In the language of field theory, the values of T_c and $|W|$ for an elemental SC are said to be obtained via the one phonon exchange mechanism (1PEM).

2) In a composite SC, the formation of CPs can be brought about not only via the exchanges of phonons due to one species of ions, but also due to more than one ion-species. The propagator of the BSE is then a 'superpropagator' which represents exchanges of multiple phonons between the electrons. Labelling the values of W as W_1 and W_2—the latter when pairing results from the 2PEM, it has been shown for a wide variety of SCs that $|W_1|$ can be identified with Δ_1 and $|W_2|$ with Δ_2 [5]. For this reason, $|W_1|$ and Δ_1 will henceforth be employed interchangeably, and likewise for $|W_2|$ and Δ_2. For an overview of how the MBA and the GBCSEs-based approach deal with the multiple gaps of an SC, we draw attention to [6].

This paper is organized as follows. Derived or recalled from earlier papers in Section 2 are the μ-incorporated GBCSEs in the form employed in this paper. The applications of these equations are taken up in Section 3. The final section sums up this study.

2. The Framework of the μ-Incorporated GBCSEs

The GBCSEs for the T_c and the $|W|$s

The parent BSE from which several equations are derived or recalled below is [5]

$$1=\frac{V}{(2\pi\hbar c)^3}\frac{1}{2}\int_{\mu-k\theta}^{\mu+k\theta}\mathrm{d}^3\boldsymbol{p}\frac{\tanh\left[\frac{1}{2kT}\left(p^2/2m_e-\mu-W/2\right)\right]}{p^2/2m_e-\mu-W/2}, \tag{4}$$

where V—which is non-zero only in the range of integration—is the same parameter as occurs in $[N(0)V]$ in the BCS theory and has the dimensions of electron-Volt-cm^3. However, $V/(2\pi\hbar c)^3$ now plays the role of a propagator.

In terms of $\xi = p^2/2m_e-\mu$, we can recast (4) as

$$1=\frac{\lambda}{2}\int_{-k\theta}^{k\theta}\mathrm{d}\xi\sqrt{1+\xi/\mu}\frac{\tanh\left[\frac{1}{2kT}(\xi-W/2)\right]}{\xi-W/2}, \tag{5}$$

where

$$\lambda=\frac{\left(2m_ec^2\right)^{3/2}\mu^{1/2}V}{4\pi^2(\hbar c)^3}. \tag{6}$$

When $T=T_c$ ($t=1$), and $W=0$, we label μ as μ_1 and λ as λ_1. We then have (5) as

$$1=\frac{\lambda_1}{2}\int_{-k\theta}^{k\theta}\mathrm{d}\xi\sqrt{1+\xi/\mu_1}\frac{\tanh(\xi/2kT_c)}{\xi}, \tag{7}$$

where

$$\lambda_1=\frac{\left(2m_ec^2\right)^{3/2}\mu_1^{1/2}V}{4\pi^2(\hbar c)^3}. \tag{8}$$

Equation (7) is identical with the BCS equation for the T_c of an elemental SC when $\mu_1 \gg k\theta$. Parametrizing μ_1 as

$$\mu_1=\rho k\theta \tag{9}$$

and employing $x=\xi/2kT_c$, we obtain (5) in the form employed in this paper in the 1PEM scenario as

$$1=\frac{\lambda_1}{2}\int_{-\theta/2T_c}^{\theta/2T_c}\mathrm{d}x\sqrt{1+\frac{2T_c}{\rho\theta}x}\frac{\tanh(x)}{x}. \tag{10}$$

When $T=0$, we label μ as μ_0 and λ as λ_0 in (4), whence

$$\lambda_0=\frac{\left(2m_ec^2\right)^{3/2}\mu_0^{1/2}V}{4\pi^2(\hbar c)^3}. \tag{11}$$

It follows from (8) and (11) that

$$\lambda_0=\lambda_1\sqrt{q}, \tag{12}$$

where μ_0 has been parametrized as

$$\mu_0=q\mu_1=q\rho k\theta. \tag{13}$$

In this case the equation for $|W|$ follows from (5) by choosing the signatures of W as follows [5]

$$W=+|W| \text{ when } \xi<0 \text{ and } W=-|W| \text{ when } \xi>0.$$

Therefore, we have (5) as

$$1=\frac{\lambda_1\sqrt{q}}{2}\left(I_1+I_2\right), \tag{14}$$

where

$$I_1=\int_{-k\theta}^{0}\mathrm{d}\xi\sqrt{1+\xi/q\rho k\theta}\,\frac{\tanh\left[\frac{1}{2kT}\left(\xi-|W|/2\right)\right]}{\xi-|W|/2}$$

$$I_2=\int_{0}^{k\theta}\mathrm{d}\xi\sqrt{1+\xi/q\rho k\theta}\,\frac{\tanh\left[\frac{1}{2kT}\left(\xi+|W|/2\right)\right]}{\xi+|W|/2}.$$

When T = 0, the tanh in I_1 = −1 and the tanh in I_2 = +1. In terms of $x=\xi/q\rho k\theta$, we can compactly combine these equations to obtain (14) as

$$1=\frac{\lambda_1\sqrt{q}}{2}\int_{-1/q\rho}^{1/q\rho}\mathrm{d}x\frac{\sqrt{1+x}}{|x|+|W|/2q\rho k\theta}, \tag{15}$$

which is the equation employed below for $|W|$ in the 1PEM scenario.

A composite SC characterized by two gaps has at least two ion-species that can potentially cause pairing, e.g., the Y and the Ba ions in YBCO (Bi-2212 has three such ion-species, viz., Bi, Ca and Sr). Operative in these SCs is not only the 1PEM, but also 2PEM, and in rare cases, even 3PEM. In the latter case, the equations for W_1, W_2 and W_3—which follow from (7) by replacing the propagator $V/(2\pi\hbar c)^3$ by $[V_A/(2\pi\hbar c)^3+V_B/(2\pi\hbar c)^3+V_C/(2\pi\hbar c)^3]$—are

$$\begin{aligned}&F1\left(\rho,T_c,\lambda_A,\lambda_B,\lambda_C\right)\\&\equiv 1-\frac{1}{2}\left(\lambda_A I\left(A,\rho,T_c\right)+\lambda_B I\left(B,\rho,T_c\right)+\lambda_C I\left(C,\rho,T_c\right)\right)=0\end{aligned} \tag{16}$$

$$\begin{aligned}&F2\left(\rho,W_3,q,\lambda_A,\lambda_B,\lambda_C\right)\\&\equiv 1-\frac{\sqrt{q}}{2}\left(\lambda_A J\left(A,\rho,W_3\right)+\lambda_B J\left(B,\rho,W_3\right)+\lambda_C J\left(C,\rho,W_3\right)\right)=0,\end{aligned} \tag{17}$$

where λ_A, λ_B and λ_C are the interaction parameters due to, respectively, the sub-lattices of the A, B and C species of ions at t = 1,

$$I\left(i,\rho,T_c\right)=\int_{-\theta_i/2T_c}^{\theta_i/2T_c}\mathrm{d}x\sqrt{1+\frac{2T_c}{\rho\theta_i}x}\,\frac{\tanh(x)}{x}$$

$$J\left(i,\rho,q,W_2\right)=\int_{-1/q\rho}^{1/q\rho}\mathrm{d}x\frac{\sqrt{1+x}}{|x|+|W_2|/2q\rho k\theta_i}$$

and i denotes the ion-species A, B, or C, with θ_A, θ_B and θ_C being their Debye temperatures. Note that when one of the λs is zero in (16) or (17), the equation pertains to the 2PEM, and when two of them are zero, it pertains to the 1PEM.

Equation (16) and Equation (17) are supplemented by (2) for v_F and (1) for ξ, written as

$$F3\left(v_F,\rho,q,\eta\right)\equiv 1-\frac{\sqrt{2\rho qk\theta_{SC}/\eta m_e}}{v_F}c=0\left(m_e \text{ in units of electron-Volt}\right) \quad (18)$$

with θ_{SC} being the Debye temperature of the SC, and

$$F4\left(\xi,W,\rho,q,\eta\right)\equiv 1-\frac{\hbar v_F\left(\rho,q,\eta\right)}{\pi W\xi}=0. \quad (19)$$

Composite SCs are usually characterized in the literature by a single value of the Debye temperature. We recall that Born and Karman [7] [8] had pointed out a long time ago in the context of the molar heats of salts that elastic waves in such anisotropic materials travel with different velocities in different directions and hence are characterized by more than one Debye frequency or temperature. Because composite SCs are invariably anisotropic materials, we have incorporated this feature in our work via the double pendulum model [9]. In this model the Debye temperature θ of a binary SC A_xB_{1-x} is resolved into θ_A and θ_B via the following equations

$$\theta\left(x\right)=x\theta_A+\left(1-x\right)\theta_B$$

$$\frac{\theta_A}{\theta_B}=\left[\frac{1+\sqrt{m_B/\left(m_A+m_B\right)}}{1-\sqrt{m_B/\left(m_A+m_B\right)}}\right]^{1/2}, \quad (20)$$

where A is the upper bob in the double pendulum and m_A (m_B) is the atomic mass of an A (B) ion. It is notable that the above equations are also applicable to an SC that has three ion-species that can potentially cause pairing. This is so because each of such species is found in a different sub-lattice and it is assumed that the Debye temperature of any sub-lattice equals the Debye temperature of the entire lattice. An example: θ(Bi-2212) = 237 K. The ions that can potentially cause pairing in this SC are: Ca, Bi and Sr, but Ca occurs in a sub-lattice that has no other occupant. Therefore θ_{Ca} = 237 K. Application of (20) to the BiO and SrO sub-lattices, respectively, yields, θ_{Bi} = 269 K (Bi as the upper bob; θ_{O} = 205 K) and θ_{Sr} = 286 K (Sr as the upper bob; θ_{O} = 188 K). The values of θ_{O} are not required in our work.

3. Addressing the T_c, W (or Ws) and ξ (or ξs) of SCs

We now proceed to show that if the values of the parameters in the heading of this section are employed as inputs into the equations given in the previous section, then the values of all the λs we obtain satisfy the Bogoliubov constraint. The significance of this result in the context of the high-T_c SCs is that it explicitly shows that the GBCSEs provide a viable description of their properties. Besides the values of the λs, this exercise yields the values of v_F and, notably, μ_1 and μ_0, which may prove to be valuable in fabricating SCs with bespoke properties.

Before undertaking the above task, we note that barring H_3S, the approach followed in this paper presents new results corresponding to the T_c and Δs of the remaining 12 SCs. In so far as the elemental SCs are concerned, the values of ρ, v_F, n_s, etc. obtained here differ radically from those that were reported in

[10]. The main reason for this is that the value of η employed in the present paper for any of these SCs is based on the empirical value of the Sommerfeld constant γ and its theoretical value obtained via the free-electron gas model, whereas in the earlier paper it was based on two alternative theoretical definitions of the density of states at the Fermi surface. An example: the value of η for Sn in the present paper is 1.26, whereas in the earlier paper it was 10. In order to choose between the two approaches, we must appeal to the empirical values of N_s of the SCs.

We now take up each category of SCs separately.

3.1. Elemental SCs

For these SCs, we solve (10), (15) and (19) simultaneously with the inputs of T_c, $|W|$ and ξ to obtain the values of ρ, q and λ_1 which then fix μ_1, μ_0, λ_0 and v_F via the equations noted in the caption of **Table 1**. Noted below are the conditions that the physically acceptable solutions must satisfy.

1) Both λ_1 and λ_0 must be non-negative and < 0.5 (Bogoliubov constraint).

2) λ_0 must be greater than λ_1.

3) μ_0 must be greater than μ_1, *i.e.*, q must be > 1. (21)

The results of the above exercise for all the elemental SCs being dealt with are given in **Table 1** where it can be seen that all the above conditions are met. Excepting η, the empirical values of the parameters in **Table 1** have been taken from [11]; those of η from [12] for Al, Sn, Cd and Pb and from [13] for Nb.

Table 1. The values of various parameters corresponding to ξ of the elemental SCs. Parameter-values in column 2 are employed as inputs into (18) and (19), which are solved simultaneously to obtain the results in column 3. The values of the first three parameters in column (4) then follow from the equations noted there; v_F follows from (18).

SC	$T_c(\mathrm{K}), \Delta(\mathrm{meV})$ $2\Delta/kT_c$ $\theta(\mathrm{K}), \eta$ $\xi(\mathrm{nm})$	ρ q λ_1	$\mu_1 = \rho k\theta(\mathrm{eV})$ $\mu_0 = q\mu_1(\mathrm{eV})$ $\lambda_0 = \lambda_1\sqrt{q}$ $v_F\left(10^5\ \mathrm{m/s}\right)$
1	2	3	4
Al	1.16, 0.179 **3.58** 428, 1.48 1550	199.2 **1.0042** 0.1657	7.347 7.378 0.1660 13.2
Sn	3.72, 0.593 **3.70** 195, 1.26 180	54.51 **1.0149** 0.2448	0.916 0.930 0.2466 5.09
Nb	9.25, 1.55 **3.89** 276, 12 39	115.0 **1.0382** 0.2480	2.735 2.840 0.2894 2.89

Continued

Cd	0.42, 0.072 **3.98** 210, 0.73 760	7.53 **1.0384** 0.1578	0.136 0.142 0.1607 2.61
Pb	7.2, 1.33 **4.29** 96, 1.97 96	230.1 **1.0926** 0.3682	1.904 2.080 0.3849 6.09

3.2. High-T_c SCs

Unlike the elemental SCs for which the values of the empirical parameters in the list given at the beginning of the paper vary negligibly from one source to another, the values of these parameters for high-T_c SCs vary widely. This is so predominantly because they depend on the form of the SC (bulk, thin film, etc.) and how it is doped. There would still be no problem in carrying out our study if all the properties in the list were available for the same sample, which is never the case. This circumstance necessitates a discussion of both the choice of inputs into the GBCSEs for any SC and the interpretation of the results it leads to. This task is taken up below for each of the high-T_c SCs being dealt with.

3.2.1. MgB_2

For the sake of concreteness, we employ the values of the T_c, $|W_1|$, $|W_2|$ and v_F for this SC given by Leggett [14] as

$$T_c \approx 40\text{ K}, |W_1| \approx 2.4\text{ meV}, |W_2| \approx 7\text{ meV}, v_F \approx 5\times 10^7\text{ cm/s} \tag{22}$$

For θ (MgB_2) which is not given in [14], we employ the value 815 K which is the mean of its values given in the review by Buzea and Yamashita [15], viz., 750 and 880 K. The values of θ_B and θ_{Mg} obtained by resolving θ (MgB_2) = 815 K via (20) are noted in column 1 of **Table 2**.

We now proceed to show how the GBCSEs not only provide an explanation of the properties noted in (22), but also shed light on several other related parameters. To this end, we undertake to solve simultaneously the four equations noted against MgB_2 in column 2 of **Table 2** and find that the values of the four parameters in (22) are insufficient to solve these equations—we also need the value of η ($\equiv m^*/m_e$) which is not given in [14] or [15]. On searching, we found that this parameter has been assigned widely different values by different authors, as is exemplified by Yelland, *et al.* [16] where its value is quoted as between 0.44 and 0.68, but with the remark that it is in the same ballpark as 1.25, 1.57, ≈ 1.1, 0.47, 0.50 and 0.33 quoted by several other authors. If we add to these values the range of η given by Mazin and Antropov [17] as (1.08 - 1.20), we have a rather bewildering situation. To cope with it, we ran our program for several of these values, beginning with $\eta = 1.57$, which led to $\mu_0 \approx 1116$ meV, an unacceptable value because it differs widely from its generally believed value of ≈ 100 meV as given in, e.g., Alexandrov [18]. Continuing the process with different values of η, we found that we needed not only to lower its value, but also that of v_F from the one noted in (22) to 2.7 × 10^7 cm/s given by Posazhennikova, *et al.*, [19].

In column 3 of **Table 2** are given the final values of the input parameters employed to simultaneously solve the four equations noted in column 2. In column 4 are given the values of the four parameters obtained by solving these equations and, in column 5, the values the of six parameters following from the solutions in column 4. It is thus seen that the λs and the μs satisfy the requisite conditions noted in (21) and that the value of μ_0 = 116 meV is very close to its value given as 122 meV in [18]. While our value of ξ_2 = 8.1 nm differs from its value given as 5 nm in [14], or (1.6 - 3.6) nm given in [15], as also is the case for ξ_1 for which our value of 22.6 nm differs considerably from its value given as (3.7 - 12) nm in [15] (only one value of ξ is quoted in [14]), we draw attention to values of ξ other than those given in [14] or [15] that can also be found in the literature. Notable among these are the values of ξ_2 given in several papers as 7 or 7.5 nm, e.g., [20] and that of (ξ_1/ξ_2) as 2.8 [21]. It is therefore seen that our value of ξ_2 is very close to the former values and that of (ξ_1/ξ_2) matches the latter exactly.

3.2.2. The Cuprate SCs

It has been shown that these SCs are characterized by a universal Fermi velocity in the range (2.7 ± 0.5) × 10^7 cm/s [22] and $\eta \simeq 3$ [23]. In the following we shall employ the values of these parameters in accord with these results.

We now deal with YBCO. Unlike MgB_2 for which the data are given as {T_c, Δ_1, Δ_2}, we found it hard to find a source where the two gaps of this SC are unequivocally given as corresponding to the same value of T_c. The data for YBCO are typically reported as {T_c (K), Δ (meV)} with the following values: {92, 20}, {60, 9} [11], {89, 29}, {79, 25} [24], and {68, 15.8} [25].

With θ(YBCO) = 410 K resolved via (20) into θ_Y and θ_{Ba} as noted in column 1 of **Table 2**, we dealt with this SC via the four equations noted against it in column 2 and adopting for it the values of the parameters given in column 3. Among these, the value of T_c = 90 K is chosen because four pairs of values of ξ are listed against it in [11]—vide **Table 3** below. Since only one value of the gap, viz., 20 meV, is given for this SC, the other value noted in column 3 is an assumed value. As concerns the value of v_F = 1 × 10^7 cm/s in column 3, we note that the range of v_F for cuprates has also been obtained as (2 − 8) × 10^6 cm/s [26] besides the one in [22] as noted above. The value employed by us is intermediate between the upper limit given in [26] and the lower limit given in [22]. Employed together with a value of η as specified above, it has the virtue of leading to μ_0 = 85.3 meV—as against its value obtained via an entirely different approach in [18] as 84 meV (for a sample of YBCO with T_c = 91.5 K)—and ξ_1 = 2.65 which matches exactly one of its listed values vide **Table 3**. In addition, it also leads to the values of the λs and the μs that satisfy the conditions specified in (21).

We now take up Bi-2212, Bi-2223, Tl-2212 and Tl-2223. For the values of the θs of these in **Table 2**, see [5]. The values of the T_c and $|W|$ of each of these are taken from [11], where only one value of $|W|$ is given. It is seen from column 2 of

Table 2 that while the rather large value of the gap of each of these SCs has been attributed to the 3PEM, the mechanism invoked for its T_c is the 2PEM. This is so because employment of the 3PEM for the $|W|$ of these SCs was found to be imperative in order that the values of the λs corresponding to them be in accord with the Bogoliubov constraint. However, we found that the T_cs of these SCs could still be accounted for by the 2PEM, *i.e.*, there was no compelling need to invoke the 3PEM for their explanation.

An important feature of the Bi- based SCs is that they are characterized by three values of λ, viz., λ_{Ca}, λ_{Bi} and λ_{Sr}, and likewise for the Tl-based SCs. This is unlike YBCO which is characterized by λ_Y and λ_{Ba} only. This implies that by employing the same set of equations as for YBCO, we can now find the values of the three λs and either ρ or q. We dealt with this situation by finding the values of the λs and q corresponding to an assumed value of ρ and then calculating the values of ξ via (19), and repeating the procedure by varying ρ until at least one of the values of ξ was obtained in close agreement with its listed value.

The {T_c (K), Δ (meV)} values for Bi-2212 and Bi-2223 given in [11] are {95, 38} and {105, 33}, respectively. It is seen here that the *increase* in the value of T_c in going from the former to the latter is accompanied by a *decrease* in the value of Δ, which is against the conventional wisdom about this feature. It therefore follows that if Δ = 38 meV for Bi-2212 is attributed to the 3PEM as we have done, then Δ = 33 meV for Bi-2223 cannot be attributed to the same mechanism. In other words, in the scenario of the 3PEM, Bi-2223 must have a gap-value greater than 38 meV which we found to be 51 meV.

For Tl-2212, (T_c = 114 K, $|W_2|$ = 30 meV) values in Table 2 are taken from [11], whereas $|W_1|$ = 4.9 meV is an assumed value. For Tl-2223, we have employed the empirical data of Ponomarev, *et al.* [27] who have reported three gap-values for it. Of these, we have employed the values of the largest and the smallest gaps in our study. The maximum value of the intermediate gap that we are then led to via $F2$ (ρ, W, q, λ_{Ca}, λ_{Tl}, 0) is 28.3 meV (which leads to ξ_2 = 1.78 nm) but does not match 45 meV quoted by the authors.

For SCs characterized by three values of λ, given in Table 2 are three values of ξ. For each of these SCs, the inputs employed to obtain the values of the λs and q and thence of the ξs, in fact, lead to four additional values of ξ which are not given in Table 2. This is so because for such SCs, in principle, seven different values of ξ can occur—three from the 1PEM, three from the 2PEM and one from the 3PEM. Among these, given in Table 2 are the values of ξ that are closest to the available empirical values as listed in [11] and/or [28].

3.2.3. The Compressed Super-Hydrides H_3S and LaH_{10}

The procedure followed for these SCs is similar to the one followed for MgB_2 or YBCO. For v_F of these SCs, we chose a value in the range derived by Talantsev [29] as (2.5 – 3.8) × 10^7cm/s, and for η the value 2.76 given by Durajski [30] which has been extensively employed in, e.g., [31] and [32]. The other inputs employed for these SCs and the results they lead to are given in Table 2. The values of the input parameters for H_3S are the same as were employed by us in [10]. For LaH_{10}, the value of $T_c = 246$ K is taken from Sun *et al.* [33]. For the larger gap of this SC, we have employed a value in the range given by Ruangrungrote, *et al.*, [34] as $4.32 \le 2\Delta/kT_c \le 5.25$ (*i.e.*, $45.8 \le \Delta_2 \le 55.6$ meV), whereas the value of Δ_1 is an assumed value which is justified by the overall results it leads to. Insofar as the role of pressure in these SCs is concerned, we should like to note that since the inputs employed for these SCs are pressure-dependent, so should be considered the results that they lead to.

Both our values of ξ for H_3S are within its range following from the work of different authors. It is notable that, generally, only one value of ξ for this SC is quoted in the literature which varies between 1.2 and 3.0 nm. For references to several papers where these values have been obtained, we refer the reader to [10] where nearly the same values of ξ were obtained as here. However, q was < 1 in [10], which led to complex-valued solutions of which the imaginary parts were neglected.

For compressed LaH_{10} too, our value of $\xi_1 = 1.56$ nm is in good agreement with its value given in [33] as 1.514 nm corresponding to the T_c under consideration.

Table 2. Results of solving simultaneous equations for high-T_c SCs noted in column 1, where are also given their Debye temperatures and those of their constituents that can potentially cause pairing. Specified in columns 2 and 3, respectively, are the equations invoked for each SC and the inputs employed to solve them. Column 4 gives the values of the parameters obtained by solving the equations. Employing the solutions in column 4, given in column 5 are the values of μ_1, μ_0, λ_{A0}, etc., obtained via (9), (12) and (16), respectively. The values of ξs are obtained via $\xi_i = \hbar c\sqrt{2\mu_0/\eta m_e}/\pi W_i$ ($i = 1,2,3$). The high-lighted values of ξ are those that are in good agreement with their values listed in sources noted in the text.

SC θ_{SC} θ_1 θ_2	The set of eqs. solved simultaneously	Inputs employed to solve the eqs. In column 2	Solutions of eqs. in column 2	μ_1, μ_0 (meV) λ_{A0}, λ_{B0}, λ_{C0} ξ_1, ξ_2, ξ_3 (nm)
1	2	3	4	5
MgB_2 $\theta_{MgB2} = 815$ $\theta_B = 1062$ $\theta_{Mg} = 322$	$F1$ (ρ, T_c, λ_B, λ_{Mg}, 0) $F2$ (ρ, W_2, q, λ_B, λ_{Mg}, 0) $F2$ (ρ, W_1, q, λ_B, 0, 0) $F3$ (v_F, ρ, q, η)	$T_c = 40$ K $\lvert W_2\rvert = 7.0$ meV $\lvert W_1\rvert = 2.5$ meV $v_F = 2.7 \times 10^7$ cm/s $\eta = 0.56$	$\rho = 1.5731$ $q = 1.0505$ $\lambda_{B1} = 0.2278$ $\lambda_{Mg1} = 0.1055$	110.5, 116.1 $\lambda_{B0} = 0.2335$ $\lambda_{Mg0} = 0.1081$ 22.6, **8.1**, -
YBCO/Y $\theta_{YBCO} = 410$ $\theta_Y = 410$ $\theta_{Ba} = 117$	$F1$ (ρ, T_c, λ_Y, λ_{Ba}, 0) $F2$ (ρ, W_2, q, λ_Y, λ_{Ba}, 0) $F2$ (ρ, W_1, q, λ_Y, 0, 0) $F3$ (v_F, ρ, q, η)	$T_{c2} = 90$ K $\lvert W_2\rvert = 20$ meV $\lvert W_1\rvert = 7.9$ meV $v_F = 1 \times 10^7$ cm/s $\eta = 3$	$\rho = 2.3193$ $q = 1.0408$ $\lambda_{Y1} = 0.4284$ $\lambda_{Ba1} = 0.4847$	81.9, 85.3 $\lambda_{Y0} = 0.4371$ $\lambda_{Ba0} = 0.4945$ **2.65**, 1.05, -
Bi-2212 $\theta_{Bi2221} = 237$ $\theta_{Ca} = 237$ $\theta_{Bi} = 269$ $\theta_{Sr} = 286$	$F1$ (ρ, T_c, 0, λ_{Bi}, λ_{Sr}) $F2$ (ρ, W_3, q, λ_{Ca}, λ_{Bi}, λ_{Sr}) $F2$ (ρ, W_2, q, λ_{Ca}, λ_{Bi}, 0) $F3$ (v_F, ρ, q, η)	$T_c = 95$ K $\lvert W_3\rvert = 38$ meV $\lvert W_2\rvert = 17.0$ meV $v_F = 2.2 \times 10^7$ cm/s $\eta = 3.0$	$\rho = 18.0$, $q = 1.1229$ $\lambda_{Ca1} = 0.3823$ $\lambda_{Bi1} = 0.0.3616$ $\lambda_{Sr1} = 0.4526$	367.6, 412.8 $\lambda_{Ca0} = 0.4051$ $\lambda_{Bi0} = 0.3831$ $\lambda_{Sr0} = 0.4796$ 6.59, **2.71**, 1.21

Continued

Bi-2223 $\theta_{Bi2223} = 275$ $\theta_{Ca} = 275$ $\theta_{Bi} = 312$ $\theta_{Sr} = 331$	$F1$ (ρ, T_c, λ_{Ca}, 0, λ_{Sr}) $F2$ (ρ, W_3, q, λ_{Ca}, λ_{Bi}, λ_{Sr}) $F2$ (ρ, W_2, q, 0, λ_{Bi}, λ_{Sr}) $F3$ (v_F, ρ, q, η)	$T_c = 105$ $\|W_3\| = 51.0$ meV $\|W_2\| = 33.0$ meV $v_F = 2.3 \times 10^7$ cm/s $\eta = 3.0$	$\rho = 16.0$, $q = 1.1899$ $\lambda_{Ca1} = 0.3539$ $\lambda_{Bi1} = 0.4668$ $\lambda_{Sr1} = 0.4637$	379.2, 451.2 $\lambda_{C0} = 0.3861$ $\lambda_{Bi0} = 0.5093$ $\lambda_{Sr0} = 0.5058$ 5.3, 1.46, **0.95**
Tl-2212 $\theta_{Tl2212} = 254$ $\theta_{Ca} = 254$ $\theta_{Tl} = 289$ $\theta_{Ba} = 296$	$F1$ (ρ, T_c, 0, λ_{Tl}, λ_{Ba}) $F2$ (ρ, W_3, q, λ_{Ca}, λ_{Tl}, λ_{Ba}) $F2$ (ρ, W_1, q, 0, 0, λ_{Ba}) $F3$ (v_F, ρ, q, η)	$T_c = 114$ $\|W_3\| = 30$ meV $\|W_1\| = 4.7$ meV $v_F = 2.3 \times 10^7$ cm/s $\eta = 3$	$\rho = 20.5$, $q = 1.0055$ $\lambda_{Ca1} = 0.1225$ $\lambda_{Tl1} = 0.4971$ $\lambda_{Ba1} = 0.4033$	448.7, 451.2 $\lambda_{Ca0} = 0.1229$ $\lambda_{Tl0} = 0.4985$ $\lambda_{Ba0} = 0.4044$ 6.23, **1.94**, 1.61
Tl-2223 $\theta_{Tl2223} = 290$ $\theta_{Ca} = 290$ $\theta_{Tl} = 330$ $\theta_{Ba} = 338$	$F1$ (ρ, T_c, λ_{Ca}, 0, λ_{Ba}) $F2$ (ρ, W_3, q, λ_{Ca}, λ_{Tl}, λ_{Ba}) $F2$ (ρ, W_1, q, 0, 0, λ_{Ba}) $F3$ (v_F, ρ, q, η)	$T_c = 118$ $\|W_3\| = 50.0$ meV $\|W_1\| = 5.5$ meV $v_F = 2.4\times10^7$ cm/s $\eta = 3$	$\rho = 18.3$, $q = 1.0742$ $\lambda_{Ca1} = 0.4877$ $\lambda_{Tl1} = 0.4249$ $\lambda_{Ba1} = 0.3938$	457.3, 491.3 $\lambda_{Ca0} = 0.5055$ $\lambda_{Tl0} = 0.4404$ $\lambda_{Ba0} = 4081$ 6.27, 1.78, **1.01**
H_3S* $\theta_{H3S} = 1531$ $\theta_H = 1983.2$ $\theta_S = 174.5$ *Under pressure	$F1$ (ρ, T_c, λ_H,, λ_S, 0) $F2$ (ρ, W_2, q, λ_H, λ_S, 0) $F2$ (ρ, W_1, q, λ_H, 0, 0) $F3$ (v_F, ρ, q, η)	$T_c = 203$ K $\|W_2\| = 39$ meV $\|W_1\| = 25$ meV $v_F = 2.5 \times 10^7$ cm/s $\eta = 2.76$	$\rho = 3.6668$ $q = 1.0137$ $\lambda_{H1} = 0.3703$ $\lambda_{S1} = 0.2644$	483.8, 490.4 $\lambda_{H0} = 0.3729$ $\lambda_{S0} = 0.2662$ **2.10**, **1.34**, -
LaH_{10}* $\theta_{LaH10} = 1156$ $\theta_H = 1248.6$ $\theta_{La} = 1146.7$ *Under pressure	$F1$ (ρ, T_c, λ_H,, λ_{La}, 0) $F2$ (ρ, W_2, q, λ_H, λ_{La}, 0) $F2$ (ρ, W_1, q, λ_H, 0, 0) $F3$ (v_F, ρ, q, η)	$T_c = 246$ K $\|W_2\| = 51.0$ meV $\|W_1\| = 33.6.$ meV $v_F = 2.5 \times 10^7$ cm/s $\eta = 2.76$	$\rho = 4.3932$ $q = 1.1206$ $\lambda_{H1} = 0.4723$ $\lambda_{La1} = 0.1043$	437.6, 490.4 $\lambda_{H0} = 0.5000$ $\lambda_{La0} = 0.1104$ **1.56**, 1/03, -

4. Discussion and Conclusion

In the BCS theory, no distinction is made between the values of the interaction parameter λ at $T = T_c$ and 0. Factually, however, these values are invariably different—as can be easily checked by calculating $\lambda(T_c)$ and $\lambda(\Delta_0)$ via, respectively, the equations for the T_c and Δ_0 of any elemental SC. The difference in the values of these λs is due to their dependence on μ. We now draw attention to Meservey and Schwartz [35] for an account of several diverse approaches devoted to explaining the deviations in the values of $R \equiv 2\Delta_0/kT_c$ of the elemental SCs from the universal value of 3.52. In this paper, we have shown it to be attributable to the difference between the values of μ at $T = T_c$ and 0, and that the greater the value of q ($\equiv \mu_0/\mu_1$) of an SC, the greater is the amount by which its value of R differs from the universal value. See, for example, the values of q for Al and Pb in **Table 1**.

A general feature of the values of μ_0 of the elemental and the high-T_c SCs is that while these are generally in the range of electron-Volt for the former, they are in the range of milli-electron Volt for the latter.

Table 3 pertains to important features about the reportage of the values of ξ in the literature.

Table 3. Attention is drawn herein to the properties of an SC that determine its ξ. While whether the SC is in the form of a powder, thin film, polycrystal or in bulk-form, etc., and how it is doped are not always specified, some of the other parameters that determine ξ are almost never mentioned. By taking the example of YBCO, we indicate below how the empirical data pertaining to its ξ are typically reported in the literature, as in, e.g., [21]. Δ below signifies that only one value of the gap was reported; NG means not given.

T_c (K)	E_F	η	Δ (meV)	Δ_1 (meV)	Δ_2 (meV)	ξ_{ab} (nm)	ξ_c (nm)
92	NG	NG	20	NG	NG	NG	NG
90	NG	NG	NG	NG	NG	2.5	0.8
89	NG	NG	NG	NG	NG	3.4	0.7
92.4	NG	NG	NG	NG	NG	4.3	0.7
92	NG	NG	NG	NG	NG	1.2	0.3
90	NG	NG	NG	NG	NG	1.3	0.2
90	NG	NG	NG	NG	NG	2.65	0.09
90	NG	NG	NG	NG	NG	12.9	4.0

5. Concluding Remarks

1) The elemental SCs studied in the present paper are the same as those dealt with in [10]. However, because of the difference in the values of v_F, the values of ξ in the two papers differ substantially. Since v_F depends on η, *i.e.*, the parameter which determines the effective mass of the charge carriers, it is imperative that its value be specified in order to make the definition of ξ unambiguous. Another difference between [10] and the present paper is that the incorporation of μ in the pairing equation in the latter leads to a dependence of the λs on T via the dependence of μ on T.

2) Despite the huge difference between the values of T_c of the elemental and the high-T_c SCs, surprisingly, the values of q—the ratio of the chemical potential at $T = 0$ and $T = T_c$—for most of the SCs of both types are generally very similar.

3) With **Table 3** in view, it will be seen that this paper is based on data taken from an assortment of sources. It is therefore remarkable that, for all the high-T_c SCs dealt with—despite the unusually large values of their Δ_2/Δ_3 and whether or not they are subject to ultra-high pressures, GBCSEs have led to values of all the λs, both at $T = T_c$ and 0, that essentially satisfy the Bogoliubov constraint.

4) The GBCSEs are based on the premise that an SC is characterized by as many interaction parameters (λs) as the number of ion-species in it that can potentially cause pairing. For an SC with three such species, we have pairing via the 3PEM when λ_1, λ_2 and λ_3 act in unison which lead to a single value of the gap Δ_3; when the λs act pairwise as (λ_1, λ_2), etc., they lead via the 2PEM to three values of the gap $\Delta_2 < \Delta_3$ and similarly, when the λs act individually, we have via the 1PEM three values of the gap $\Delta_1 < \Delta_2$. We believe that this picture is strongly supported by the observation of the three gaps of Tl-2223 reported in [26], and that Bi-2212, Bi-2223 and Tl-2212 should also exhibit this feature in appropriately controlled

experimental set-ups.

5) We also believe that if along with the values of the ξs of any composite SC are also reported the values of all the other properties as indicated in **Table 3**, then a study such as we have carried out here will shed greater light on the role that the chemical potential plays in determining the properties of the SC and that it will be a step towards fabricating SCs with bespoke properties.

6) Finally, we note that mechanisms other than the multiple phonon exchange mechanism have of course been proposed in the literature that also provide the glue for pairing in the cuprates. One of such proposals invokes spin fluctuations [36]. In this context, we should like to draw attention to Bohr's principle of complementarity, the essence of which is that exclusive or contradictory perspectives are not only compatible but essential for a deeper understanding of any physical phenomenon.

Conflicts of Interest

The authors declare no competing interests.

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