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Endogenous Growth, Firm-Specific Capital, Investors' Beliefs, and Involuntary Unemployment

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Abstract

Magill and Quinzii's stock-market, overlapping generations (OLG) model with firm-specific capital exhibits perfectly flexible investment and full employment whereby GDP growth is exogenous. To model involuntary unemployment in a perfectly competitive stock-market economy aggregate investment needs to be inflexible. In this paper, inflexible aggregate investment refers to investors' beliefs about the expected marginal efficiency of investment à la Keynes. To endogenize growth, human capital accumulation is introduced into Magill and Quinzii's OLG economy. After deriving the intertemporal equilibrium dynamics, the existence and dynamic stability of steady states are investigated. It is shown that more investors' optimism regarding the expected return on investment raises GDP growth and decreases unemployment.

Keywords

Endogenous Growth, Firm-Specific Capital, Investors' Beliefs, Involuntary Unemployment

1. Introduction

Although fierce opponents on the causes of the great depression, Hayek (1931) and Keynes (1932) agreed that there is a close, positive relationship between investment and employment. However, the subsequent neo-classical synthesis between Keynesian macroeconomics and Walrasian microeconomics lost sight of this seminal relationship. With the new-classical revolution in macroeconomics (Lucas, 1972) and the subsequent real business-cycle literature (Kydland & Prescott, 1982), optimal capital accumulation became the focus of macroeconomists' interest, without the above-mentioned investment-employment relationship being rediscovered. Indeed, optimal investment was conceptualized as gradual adjustment towards the optimal capital stock due to strictly rising marginal adjustment costs. However, as the German economist Jörg Ebel (1978) convincingly argued convex adjustment costs are best motivated by full employment of labor force clearly at odds with economy-wide unemployment.

Morishima (1977) and more recently Magnani (2015) referred involuntary unemployment à la Keynes (1936) to inflexible aggregate investment. In contrast, perfectly flexible aggregate investment as in neo-classical growth models (Solow, 1956, Diamond, 1965) ensures that aggregate output creates its own demand, and thus aggregate demand cannot fall short of aggregate output. In other words: involuntary unemployment cannot occur.

To model involuntary unemployment in intertemporal, general equilibrium, the unemployment rate needs to become endogenous and aggregate investment inflexible. The big question, however, is how an endogenous unemployment rate and the inflexibility of aggregate investment can be featured in intertemporal general equilibrium. As mentioned above: while strictly rising marginal adjustment costs imply the inflexibility of investment, they are incompatible with economy-wide unemployment. On the other hand, zero or constant marginal adjustment costs of investment are compatible with economy-wide unemployment but imply the perfect flexibility of investment. Here we run into the following dilemma: either firm-level and aggregate investment are inflexible but involuntary unemployment is infeasible or firm-level and aggregate investment is perfectly flexible and economy-wide unemployment is feasible.

The question is now how to find a way out of this dilemma in the field of intertemporal equilibrium models.

A possible answer is provided by the notion of a "belief function" à la Roger Farmer (2013, 2020). A belief function depicts the "mood of the market" (G. Soros) respectively the "state of long-term expectations" (Smith & Zoega, 2009, p. 5) or more precisely the mood respectively the confidence of market participants, in our case the mood or the confidence of business investors with respect to investment-related market quantities in the future along the intertemporal equilibrium path. Methodologically speaking, a belief function represents the fourth neo-classical primitive or fundamental in addition to consumer preferences, production technologies and economies' resources.

Farmer (2024) introduced a production-based belief function of business investors into a stock-market OLG model with firm-specific capital and involuntary unemployment whereby gross domestic product (GDP) growth was exogenous. More optimism of business investors regarding production-output demand in the long run decreases unemployment in the short and long run but does not impact the GDP growth rate in the long run.

In view of the rather weak GDP growth, especially in advanced economies since the Great Recession 2008/2009, it is interesting to investigate whether more investors' optimism would not only reduce unemployment but also increase GDP growth in the long run.

To be able to answer this question, GDP growth is endogenized through human capital accumulation (Lucas, 1988; Glomm & Ravikumar, 1992) in this paper. As will be seen below, human capital accumulation makes the intertemporal equilibrium dynamics four-dimensional instead of three-dimensional with exogenous growth (see Farmer, 2024)[1]. This makes the analysis of the existence, the dynamic stability and the comparative dynamics of steady states analytically more demanding as will be seen after we have set up the model and derived the intertemporal equilibrium dynamics.

The main objective of the present paper is thus to modify Magill and Qunizii's (2003) stock market OLG model such that the GDP growth and unemployment rate become endogenous and the collective beliefs of firm managers regarding the expected rate of return on investment govern firm's optimally indeterminate investment quantities.

Our first contribution to the literature is thus to show how the structure of the intertemporal equilibrium dynamics derived from households' and firms' optimization conditions, from government's budget constraint and the intertemporal market clearing conditions change when firms' investment quantities are both optimally indeterminate and determined by firm managers' collective beliefs regarding the expected rate of return on investment.

Our second contribution to the literature consists of proving the existence and the dynamic stability of the steady states of the three-dimensional intertemporal equilibrium dynamics in our stock-market OLG model with involuntary unemployment and endogenous GDP growth.

Our third contribution to the literature is to derive the steady-state effects of main parameter changes on endogenous variables such as the GDP growth rate, the capital-output ratio, the discount on equity price, and the unemployment rate.

In contradistinction to Farmer (2024), there are two important novelties in this paper: This is, firstly, the belief function of firms' managers which depicts how the expected rate of return on investment impacts firms' investment amounts. Secondly, and novel, GDP growth is endogenous through human capital accumulation.

The structure of the paper is as follows. The next section presents the model set-up. This is followed by derivation of the intertemporal-equilibrium dynamics and demonstration of sufficient conditions for the existence and dynamic stability of steady states. We then investigate the comparative dynamics of the steady-state responses of the capital-output ratio, the GDP growth rate, the equity-price discount, and the unemployment rate to main parameter changes. A numerical specification of model parameters is then used to calculate numerically the intertemporal equilibrium responses to the same parameter changes as before.

[1]To avoid confusion throughout the paper it is apt to state that the minimal state vector is three-dimensional when we assume for the sake of analytical simplicity that the adjustment parameter. χ . in the profit expectation dynamics equals 1. Otherwise, the full dynamic system is four-dimensional.

2. The Stock Market OLG Model with Endogenous GDP Growth and Involuntary Unemployment

Before embarking on a description of the model economy in technical terms, we shortly brush over the assumptions regarding agents' behavior and the market mechanisms coordinating their actions.

2.1. Informal Model Description

The economy operates over an infinite number of periods, and each period is 25 - 30 calendar years long. Growth of the gross domestic product (GDP) is governed by endogenous growth of labor productivity enabled by the accumulation of human capital à la Lucas (1988).

The economy is composed of infinitely lived firms, an infinitely lived government and two-periods lived households. In each period, a young generation enters the economy which overlaps for one period with the generation that entered the economy one period before (=old generation). Each generation is represented by a continuity of identical households where each consists of one agent. Agents are either employed or unemployed. Each employed agent sells one unit of labor service inelastically to firms in exchange for a real wage rate which is determined by perfectly competitive firms demanding labor services. Unemployed households are supported by unemployment benefits from the government. To balance government's budget, the government collects flat wage taxes on employed households' wage incomes. The unemployed agents do not pay any taxes. Both the employed and the unemployed agent maximize an intertemporal utility function comprising active and retirement consumption subject to budget constraints for the active and retirement period. Due to the former constraint, the active period consumption expenditures plus the expenses for buying government and corporate bonds and firms' equity shares are to be covered by the net wage rate. Due to the latter constraint retirement consumption expenditures are financed by the revenues from asset sales and the returns on holding assets one period long.

All firms are endowed with identical Cobb-Douglas production functions whereby the services of employed labor and physical capital invested in the previous period generate the production output for households' consumption and firms' investment in physical capital. Physical capital is durable, depreciates at a finite rate each period, and is non-shiftable and firm specific. It has no value for sale. Firms are owned by the young equity holders and are managed such that the net present value of investment is maximized. Firm investment is financed by the sale of corporation bonds but not through equity sales.

There are five markets operating under perfect competition in each period: the market for labor services, the market for government and corporation bonds, the equity market, and the market for production output. Except for the labor market, all markets are cleared in each period. On the equity market, older agents sell their equity shares to the younger households who become firms' owners. On the government bond market, the government sells bonds that younger agents buy to invest their savings. On the corporate bond market, firms sell bonds that younger agents buy to invest their savings. On the market for production output, firms supply their output, households demand the output for consumption, and government for governmental services.

The assumption of perfect competition on all markets does clearly not accord well with nowadays empirical reality. It is made to examine a major claim in the history of economic thought brought forth by Keynes (1936): the existence of an "underemployment equilibrium" or involuntary unemployment in a perfectly competitive market economy.

2.2. Technical Model Description

As mentioned above, Farmer (2024) is now extended by introducing human capital accumulation. In order to point out the growth enhancing effects of human capital accumulation most clearly, it is assumed here that there is no population growth denoted as g_t^L ($g_t^L = 0 \Leftrightarrow G_t^L \equiv 1 + g_t^L = 1$), and no exogenous growth in labor efficiency denoted as g_t^a, *i.e.*, $g_t^a = 0$, $G_t^a \equiv 1 + g_t^a = 1$. As a result of the first assumption, the number of households remains constant over time: $L_t = L_{t-1} = L$.

Each household consists of one agent, and lives two periods long, namely youth (adult) and old age. In youth age, each household starts with human capital h_t, accumulated by the household in period $t-1$. Individual human capital is inelastically supplied to firms which remunerate w_t in exchange for the labor supply. The former denotes the units of the produced good per efficiency unit of labor.

In contradistinction to Magill and Quinzii's (2003) original OLG model, not the total labor supply is employed but only $(1-u_t)L_t$, where $0 \le u_t < 1$ denotes the unemployment rate. The number of unemployed people is thus $u_t L_t$. The government collects taxes on wages, quoted as a fixed proportion of wage income, $\tau_t w_t h_t$, $0 < \tau_t < 1$. The unemployed do not pay any taxes. Young, employed agents, denoted by superscript E, split the net wage income $(1-\tau_t) w_t h_t$ each period between current consumption $c_t^{1,E}$ and savings s_t^E. Savings of the employees are invested in the shares of firms, where a share $\theta_t^{j,E}$ of firm $j(=1,\cdots,J)$ in period t is bought in the stock market at price Q_t^j by the younger households from the older households. Moreover, the younger households also invest their savings in bonds emitted by firms $j(=1,\cdots,J)$, denoted by $b_{t+1}^{j,E}$, with a rate of return i_{t+1}^j and in government bonds, denoted by b_{t+1}^E, with a rate of return i_{t+1}. In old age, the employed respective unemployed household sells the shares at the price Q_{t+1}^j to the younger household in period $t+1$, and D_{t+1}^j denotes the dividend paid by firm j in period $t+1$. In old age, both employed and unemployed households consume their gross return on assets:

$(1+i_{t+1})b_{t+1}^E + \sum_{j=1}^{J}(1+i_{t+1}^j)b_{t+1}^{j,E} + \sum_{j=1}^{J}\theta_t^{j,E}(D_{t+1}^j + Q_{t+1}^j)$ respectively

$(1+i_{t+1})b_{t+1}^U + \sum_{j=1}^{J}(1+i_{t+1}^j)b_{t+1}^{j,U} + \sum_{j=1}^{J}\theta_t^{j,U}(D_{t+1}^j + Q_{t+1}^j)$. To remain as simple as possible, we assume that rental and interest income are not taxed.

The optimization problem of the two-period lived household, which enters the economy in period t, includes now human capital h_t, accumulated by the household in period $t-1$, and which enters household labor supply and wage income.

Thus, the typical younger household maximizes the following intertemporal utility function subject to the budget constraints for the active period (i) and the retirement period (ii):

$$\text{Max} \to \varepsilon \ln c_t^{1,E} + \beta \ln c_{t+1}^{2,E}$$

subject to:

(i) $c_t^{1,E} + b_{t+1}^E + \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} Q_t^j \theta_t^{j,E} = w_t h_t \left(1-\tau_t\right)$,

(ii) $c_{t+1}^{2,E} = \left(1+i_{t+1}\right) b_{t+1}^E + \sum_{j=1}^{J} \left(1+i_{t+1}^j\right) b_{t+1}^{j,E} + \sum_{j=1}^{J} \theta_t^{j,E} \left(D_{t+1}^j + Q_{t+1}^j\right)$.

Here, $0 < \varepsilon \le 1$ depicts the utility elasticity of employed household's consumption in youth and $0 < \beta < 1$ denotes the subjective future utility discount factor. The intertemporally additive utility function involves the natural logarithm of employed household's consumption in youth weighted by ε, and the natural logarithm of employed household's consumption in old age weighted by β.

Obviously, a strictly positive and finite solution to maximizing the intertemporal utility function subject to constraints (i) and (ii) requires that the following no-arbitrage conditions hold:

$$\frac{D_{t+1}^j + Q_{t+1}^j}{Q_t^j} = 1 + i_{t+1}^j = 1 + i_{t+1},\ j = 1, \cdots, J. \tag{1}$$

Thus, the utility maximizing consumption and savings functions read as follows:

$$c_t^{1,E} = \frac{\varepsilon}{\varepsilon+\beta}\left(1-\tau_t\right) h_t w_t, \tag{2}$$

$$c_{t+1}^{2,E} = \left[\beta/\left(\varepsilon+\beta\right)\right]\left(1+i_{t+1}\right)\left(1-\tau_t\right) w_t h_t, \tag{3}$$

$$s_t^E = \left[\beta/\left(\varepsilon+\beta\right)\right]\left(1-\tau_t\right) w_t h_t. \tag{4}$$

The typical younger, unemployed household maximizes the following intertemporal utility function subject to the budget constraints of the active period (i) and the retirement period (ii):

$$\text{Max} \to \varepsilon \ln c_t^{1,U} + \beta \ln c_{t+1}^{2,U}$$

subject to:

(i) $c_t^{1,U} + b_{t+1}^U + \sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J} Q_t^j \theta_t^{j,U} = \varsigma_t$,

(ii) $c_{t+1}^{2,U} = \left(1+i_{t+1}\right) b_{t+1}^U + \sum_{j=1}^{J} \left(1+i_{t+1}^j\right) b_{t+1}^{j,U} + \sum_{j=1}^{J} \theta_t^{j,U} \left(D_{t+1}^j + Q_{t+1}^j\right)$.

Again, $0 < \varepsilon \le 1$ denotes the utility elasticity of consumption in unemployed youth, while $0 < \beta < 1$ depicts the subjective future utility discount factor and ς_t denotes the unemployment benefit per capita unemployed.

As above, the no-arbitrage conditions (1) hold. Moreover, the utility maximizing consumption and savings functions for unemployed households read as follows:

$$c_t^{1,U} = \left[\varepsilon/(\varepsilon+\beta)\right]\varsigma_t, \tag{5}$$

$$c_{t+1}^{2,U} = \left[\beta/(\varepsilon+\beta)\right](1+i_{t+1})\varsigma_t, \tag{6}$$

$$s_t^U = \left[\beta/(\varepsilon+\beta)\right]\varsigma_t. \tag{7}$$

All firms are endowed with an identical (linear-homogeneous) Cobb-Douglas production function which now reads as follows: $Y_t^j = M\left(h_t N_t^j\right)^{1-\alpha}\left(K_t^j\right)^{\alpha}$, $M > 0$, $0 < \alpha < 1$. Here, Y_t^j denotes firm $j's$ output, $M > 0$ stands for total factor productivity, N_t^j represents the number of employed laborers, while K_t^j denotes the input of capital services, all in period *t*, and $1-\alpha$ (α) depicts the production elasticity (= production share) of labor (capital) services. In line with the seminal paper of Magill and Quinzii (2003), we assume that (physical) capital is durable, depreciates at the rate $0 < \delta < 1$, and needs to be installed one period before it is used. Thus capital K_t^j used by firm j is the capital stock that has been carried over from the period before, *i.e.* period $t-1$. Moreover, we assume "that capital once installed in a firm cannot be 'unbolted' and transformed back into the homogeneous current output or transferred to another firm, without incurring significant adjustment costs—which for simplicity we take to be infinite" (Magill & Quinzii, 2003: p. 242). Consequently, such firm-specific capital has limited value in a resale market. In the extreme, it is completely firm-specific, so that no part of it has a positive value in the second-hand market. "In such an economy capital accumulation will only take place if the market structure permits firms to be infinitely lived. Invested capital has value only if the firm retains its identity as income generating unit in the economy. The natural market structure which permits short-lived agents to transfer ownership of long-lived firms from one generation to the next is an equity market for ownership shares of firms" (Magill & Quinzii, 2003: p. 243).

Consistent with the firm specificity of capital is that each firm is a corporation with an infinite life where ownership shares are transmitted from one generation to the next through the stock market. As already introduced above, Q_t^j denotes the equity price of firm *j* at date *t*. Firms are owned by the equity holders and are managed to maximize the payoffs of their current owners. These are the younger households who buy the shares of firm *j* endowed with a capital of $(1-\delta)K_t^j$ from the older households for the price Q_t^j. Firm managers decide on the investment $I_t^j \geq 0$ to be made. Magill and Quinzii (2003: pp. 244-245) show that an investment quantity larger than zero is chosen such that the net present value of the investment is maximized:

$$\underbrace{\max}_{\left\{I_t^j, N_{t+1}^j\right\}} \left\{-I_t^j + \frac{1}{1+i_{t+1}}\left[M\left(K_{t+1}^j\right)^\alpha \left(h_{t+1}N_{t+1}^j\right)^{1-\alpha} - w_{t+1}h_{t+1}N_{t+1}^j + Q_{t+1}^j\left(\left(1-\delta\right)K_{t+1}^j\right)\right]\right\}$$

$$\Leftrightarrow \underbrace{\max}_{\left\{I_t^j, N_{t+1}^j\right\}} \left\{-I_t^j + \frac{1}{1+i_{t+1}}\left[M\left(\left(1-\delta\right)K_t^j + I_t^j\right)^\alpha \left(h_{t+1}N_{t+1}^j\right)^{1-\alpha} - w_{t+1}h_{t+1}N_{t+1}^j \right.\right. \quad (8)$$

$$\left.\left. + \left(1-\delta\right)^2 K_t^j + \left(1-\delta\right)I_t^j - V_{t+1}^j\right]\right\}.$$

Here, the equivalence between the first and the second line in (8) comes from Magill and Quinzii's (2003: p. 244) insight that in an intertemporal equilibrium shareholders expect an affine (linear) relationship between the expected equity price of non-depreciated capital in period $t + 1$, $Q_{t+1}^j\left(\left(1-\delta\right)K_{t+1}^j\right)$ and nondepreciated capital stock at that time, *i.e.*:

$$Q_{t+1}^j\left(\left(1-\delta\right)K_{t+1}^j\right) = \left(1-\delta\right)^2 K_t^j + \left(1-\delta\right)I_t^j - V_{t+1}^j,\ j = 1,\cdots,J,\ V_{t+1}^j \ge 0, \quad (9)$$

where $V_{t+1}^j, j = 1,\cdots,J$ denotes the discount on the equity price of firm *j* at time $t+1$ due to the non-shiftability of firm *j*'s capital stock.

Maximization of the second line in (8) implies the following first-order conditions:

$$\alpha M\left[\left(1-\delta\right)K_t^j + I_t^j\right]^{\alpha-1}\left(h_{t+1}N_{t+1}^j\right)^{1-\alpha} = \delta + i_{t+1}, \quad (10)$$

$$\left(1-\alpha\right)M\left[\left(1-\delta\right)K_t^j + I_t^j\right]^{\alpha}\left(h_{t+1}N_{t+1}^j\right)^{-\alpha} = w_{t+1}. \quad (11)$$

Since all firms have the same production function and the capital depreciation rate is same with all firms, the optimal capital labor ratio will be the same for all firms: $\frac{K_t^j}{h_t N_t^j} = \frac{K_t^{j'}}{h_t N_t^{j'}} = \frac{K_t}{h_t N_t}$, $j \ne j' = 1,\cdots,J$. Moreover, since the number of employed workers is $N_t \equiv \sum_{j=1}^{J} N_t^j = L\left(1-u_t\right)$, we can rewrite the profit maximization conditions (10) and (11) as follows:

$$\alpha M\left[\left(\left(1-\delta\right)K_t^j + I_t^j\right)/\left(h_{t+1}L\left(1-u_{t+1}\right)\right)\right]^{\alpha-1} = \delta + i_{t+1}, \quad (12)$$

$$\left(1-\alpha\right)M\left[\left(\left(1-\delta\right)K_t^j + I_t^j\right)/\left(h_{t+1}L\left(1-u_{t+1}\right)\right)\right]^{\alpha} = w_{t+1}. \quad (13)$$

Finally, the GDP ($= Y_t = \sum_1^J Y_t^j$) - function can be rewritten as follows:

$$Y_t = M\left(h_t L\left(1-u_t\right)\right)^{1-\alpha}\left(K_t\right)^\alpha. \quad (14)$$

The government does not optimize, but is subject to the following constraint period by period:

$$B_{t+1} = \left(1+i_t\right)B_t + \Delta_t + L_t u_t \varsigma_t + \Gamma_t - \tau_t\left(1-u_t\right)w_t h_t L, \quad (15)$$

whereby B_t denotes the aggregate stock of real public debt at the beginning of period t, Γ_t now denotes human capital investment (HCI) expenditures, and Δ_t denotes all non-HCI expenditures of the government exclusive of government's unemployment benefits per period $L_t u_t \varsigma_t$.

In line with Glomm and Ravikumar (1992) human capital in period t is determined by human capital of the generation entering the economy in period $t-1$, and by government's HCI spending in period $t-1$, Γ_{t-1}:

$$h_t = H_0\left(h_{t-1}\right)^{\mu}\left(\Gamma_{t-1}/L\right)^{1-\mu}, H_0 = \underline{H} > 0,\ 0 < \mu < 1, \tag{16}$$

where $\underline{H}$ represents a level parameter, μ denotes the production elasticity of human capital, and $1-\mu$ features the production elasticity of public HCI spending. Multiplying Equation (16) on both sides by L, we thus obtain the aggregate version of (16):

$$Lh_t \equiv H_t = H_0\left(Lh_{t-1}\right)^{1-\mu}\left(\Gamma_{t-1}\right)^{\mu} \equiv H_0\left(H_{t-1}\right)^{1-\mu}\left(\Gamma_{t-1}\right)^{\mu}. \tag{17}$$

The economy grows, even in the absence of population growth and exogenous progress in labor efficiency. Using the GDP growth factor $G_t^Y \equiv Y_{t+1}/Y_t$ as well as Equations (14) and (17), the former can be written as follows:

$$G_{t+1}^Y = \frac{H_{t+1}}{H_t}\frac{\left(1-u_{t+1}\right)^{1-\alpha}}{\left(1-u_t\right)^{1-\alpha}}\frac{\left(k_{t+1}\right)^{\alpha}}{\left(k_t\right)^{\alpha}},\ k_t \equiv \frac{K_t}{H_t}. \tag{18}$$

As Magnani (2015, pp. 13-14) rightly states, aggregate investment in Solow's (1956) neoclassical growth model is not micro-, but macro-founded since it is determined by aggregate savings. The same holds true in Diamond's (1965) OLG model of neoclassical growth where perfectly flexible aggregate investment is also determined by aggregate savings of households. Deviating from those neoclassical growth models, Morishima (1977) and more recently Magnani (2015, p. 14) claim that "investments are determined by an independent investment function." But where does this independent investment function come from?

As a first step to provide an answer to this question we recall the no-arbitrage conditions between shares and corporation bonds (1):

$\left(D_{t+1}^j + Q_{t+1}^j\right)/Q_t^j = 1 + i_{t+1},\quad j = 1,\cdots,J$, with

$D_{t+1}^j = M\left(K_{t+1}^j\right)^{\alpha}\left(h_{t+1}N_{t+1}^j\right)^{1-\alpha} - w_{t+1}h_{t+1}N_{t+1}^j - \left(1+i_{t+1}\right)I_t^j,\quad j = 1,\cdots,J$. Respecting the first-order conditions for net present value maximization (12) and (13), and assuming that affine equity price expectations are rational, *i.e.* Equation (9) holds, then we can show, following Magill and Quinzii (2003, p. 247), that

$$\frac{D_{t+1}^j + Q_{t+1}^j}{Q_t^j} = \frac{K_{t+1}^j\left(\delta + i_{t+1}\right) - \left(1+i_{t+1}\right)I_t^j + \left(1-\delta\right)K_{t+1}^j - V_{t+1}^j}{\left(1-\delta\right)K_t^j - V_t^j}, \text{ if and only if}$$

$$V_{t+1}^j = \left(1+i_{t+1}\right)V_t^j,\ j = 1,\cdots,J, \forall t \geq 0. \tag{19}$$

As a second step in providing an answer to the seminal question above we recall R. Farmer's (2013, 2020) notion of a "belief" function which he sees as synonymous with the neo-classical fundamentals like consumer preferences, corporation technologies and the resource endowment of an economy. R. Farmer and collaborators suggest different expected price or income variables about which investors or firm managers form beliefs (See for an overview Farmer, 2023).

Here, we assume that business investors form social beliefs about the expected marginal efficiency of aggregate investment (Keynes 1936, chap. 12) in line with Smith and Zoega (2009) and more recently Harvey (2022) and Harvey and Pham (2024)[2] Keynes defines the marginal efficiency of capital "as being equal to that rate of discount which would make the present value of the series of annuities given by the returns expected from the capital asset during its life just equal to the supply price." (Keynes 1936, p. 135) The returns expected from the capital asset in the quote correspond, in the present model context, to the aggregate profit Π_{t+1}^{ex} in $t+1$ which corporation managers as a whole expect from current aggregate investment.[3] The supply price mentioned in the quote correspond, in the present model context, to 1. Hence, in terms of present model setting, the expected marginal efficiency of aggregate investment per human capital reads as follows: $\vartheta\Pi_{t+1}^{ex}/(I_t/H_t)$, whereby $\vartheta>1$ is a parameter indicating "animal spirits" of business investors. Setting $\vartheta\Pi_{t+1}^{ex}/(I_t/H_t)$ equal to $1+i_t$ acknowledging $\vartheta>1$, and rearranging we obtain an equation determining aggregate investment per human capital as follows:

$$\frac{I_t}{H_t}=\frac{\vartheta\Pi_{t+1}^{ex}}{(1+i_t)}. \tag{20}$$

Equation (20) does not appear in Magill and Quinzii's (2003) stock market OLG model, since they assume full employment of the labor force, which is equivalent to $u_t=0$, $\forall t$ in our model. For $u_t>0$ and u_t being endogenous, Equation (20) features as the intertemporal equilibrium condition which makes the whole set of intertemporal equilibrium equations *determinate*. In line with Morishima (1977, pp. 117-119) and Magnani (2015, p.14), inflexible aggregate investment is assumed to be macro-founded but also turns out to be consistent with an indeterminate, market-value maximizing investment amount of firm j. In this restricted sense, we are entitled to claim that inflexible aggregate investment is micro- and macro-founded in our modified stock-market OLG model with endogenous growth and involuntary unemployment.

Like Fazzari, Peri and Variato (2020, p. 601) for the expected growth rate of aggregate demand and Plotnikov (2019, p. 120) for the expected real permanent income, we assume a linearized version of adaptive expectations with respect to aggregate profit in period $t+1$ towards its long-run expected level $\hat{\Pi}^{ex}$.[4]

$$\Pi_{t+1}^{ex}=(1-\varphi)\Pi_t^{ex}+\varphi\hat{\Pi}^{ex},\ 0<\varphi\le 1. \tag{21}$$

In addition to the restrictions imposed by household and firm optimizations and by the government budget constraint, markets for labor, government and firm

[2]Harvey (2022) and Harvey and Pham (2024) propose an interesting method how to measure investors' expectations and the difference between what was expected and what transpired. Because we use steady-state calibration of model parameters later in the text, we make no use of this method.

[3]Since the model period comprises 25-30 calendar years, Π_{t+1}^{ex} is equal to the add-up of 25-30 yearly aggregate profits which are the sum of profits expected by individual firm managers each calendar year. Π_{t+1}^{ex} represents an approximation to the annuities mentioned in the quote from Keynes (1936).

[4]Since the focus of the present paper is on steady state results, we use this simplified form of adaptive expectation formation which is common in the literature originating with Farmer (2013).

bonds, and the equity market ought to clear in all periods (the market for output of production is cleared by means of Walras' law[5]).

$$L\left(1-u_t\right)=\sum_{j=1}^{J} N_t^j = N_t, \forall t, \tag{22}$$

$$L\left(1-u_t\right)\sum_{j=1}^{J} b_{t+1}^{j,E} + Lu_t \sum_{j=1}^{J} b_{t+1}^{j,U} = \sum_{j=1}^{J} b_{t+1}^{j}. \tag{23}$$

$$L\left(1-u_t\right) b_{t+1}^{E} + Lu_t b_{t+1}^{U} = B_{t+1}, \ \forall t. \tag{24}$$

The demand of the younger employed and the unemployed households for firm bonds (left-hand side of Equation (23)) balances with their supply (right-hand side of Equation (23)). The demand of the younger employed and the unemployed households for government bonds (left-hand side of Equation (24)) equals the supply of government bonds (right-hand side of Equation (24)).

Firms finance their investments by the sales of bonds:

$$\sum_{j=1}^{J} I_t^j = \sum_{j=1}^{J} b_{t+1}^j, \ \forall t. \tag{25}$$

The shares of employed and unemployed younger households add up to unity:

$$L\left(1-u_t\right)\theta_t^{j,E} + Lu_t\theta_t^{j,U} = 1, \ j=1,\cdots,J, \ \forall t. \tag{26}$$

The sales of equity shares by employed and unemployed older households are equal to the share purchases of employed and unemployed younger households:

$$L\left(1-u_{t-1}\right)\theta_{t-1}^{j,E} = L\left(1-u_t\right)\theta_t^{j,E}, \ j=1,\cdots,J, \ \forall t, \tag{27}$$

$$Lu_{t-1}\theta_{t-1}^{j,U} = Lu_t\theta_t^{j,U}, \ j=1,\cdots,J, \ \forall t. \tag{28}$$

Using the definition of savings for younger employed households in (4) and younger unemployed households in (7), together with the bond market clearing conditions (23)-(24), the investment financing constraint (25) and conditions (26)-(28) lead us to the following aggregate savings/investment equality:

$$L\left(1-u_t\right)s_t^E + Lu_t s_t^U = \sum_{j=1}^{J} I_t^j + \sum_{j=1}^{J} Q_t^j. \tag{29}$$

Finally, we have firm specific accumulation equations

$$K_{t+1}^j = \left(1-\delta\right)K_t^j + I_t^j, \ j=1,\cdots,J. \tag{30}$$

3. Intertemporal Equilibrium

To start with, assume in line with Magill and Quinzii (2003, p. 249) a balanced-growth intertemporal equilibrium in which firms always exhibit the same relative sizes and stock market values. Then, consider initial conditions $\left(K_0^j, V_0^j\right) = \nu_j\left(K_0, V_0\right)$ with $\nu_j > 0$ and $\sum_{j=1}^{J}\nu_j = 1$. If, for the sequence of (real) wage and interest rates $\left(w_t, i_{t+1}\right)_{t\ge 0}$, aggregate equity-price discounts $\left(V_t\right)\ge 0$ and employment-invest-

[5]The proof of Walras Law is delegated to Appendix A.

ment decisions $(N_t, I_t)_{t\geq 0}$ satisfy the Equations (9)-(11), (19), (20) and (29), then $(V_t^j, N_t^j, I_t^j) = \nu_j (V_t, N_t, I_t)$ also satisfy Equations (9)-(11), (19), (20) and (29)), such that for each firm (N_t^j, I_t^j) is market-value maximizing, its market value is larger than zero, and the return on equity equals i_{t+1}. Hence, the optimal choices of individual firms can be depicted by the market-value maximizing choice of aggregate employment and capital.

Acknowledging the linear homogeneity of firm production functions and the underemployment equilibrium condition (22), we can switch to the ratio of aggregate real capital to aggregate human capital $k_t = K_t/H_t$ and rewrite the first-order conditions (12) and (13) as follows:

$$\alpha M (k_{t+1})^{\alpha-1} (1-u_{t+1})^{1-\alpha} = \delta + i_{t+1}, \tag{31}$$

$$(1-\alpha) M (k_{t+1})^{\alpha} (1-u_{t+1})^{-\alpha} = w_{t+1}. \tag{32}$$

To be able to calibrate later the present theoretical model to empirical data we transform main endogenous variables into per GDP ratios. To start with, the government budget constraint (15) is rewritten as follows:

$$\begin{aligned} \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} \equiv b_{t+1} G_{t+1}^Y &= (1+i_t) \frac{B_t}{Y_t} + \frac{\Delta_t}{Y_t} + \frac{L_t u_t \varsigma_t}{Y_t} + \frac{\Gamma_t}{Y_t} - \frac{\tau_t (1-u_t) w_t h_t L}{Y_t} \\ &\equiv (1+i_t) b_t + \gamma_t + \zeta_t + \delta_t - \frac{\tau_t (1-u_t) w_t h_t L}{Y_t}, \end{aligned} \tag{33}$$

with $b_t \equiv \dfrac{B_t}{Y_t}$, $\zeta_t \equiv \dfrac{L_t u_t \varsigma_t}{Y_t}$, $\theta_t \equiv \dfrac{\Delta_t}{Y_t}$ and $\gamma_t \equiv \dfrac{\Gamma_t}{Y_t}$.

Since the profit maximization condition (32) can be written for period t as

$$w_t h_t L (1-u_t) = (1-\alpha) M (K_t)^{\alpha} (h_t L (1-u_t))^{1-\alpha} = (1-\alpha) Y_t, \tag{34}$$

government's budget constraint (33) can be rewritten as follows:

$$b_{t+1} G_{t+1}^Y = (1+i_t) b_t + \theta_t + \varsigma_t + \gamma_t - \tau_t (1-\alpha). \tag{35}$$

In line with empirical reality before the corona crisis we assume time-stationary government debt to GDP ratios: $b_{t+1} = b_t = b$. In addition, wage tax rates $\tau_{t+1} = \tau_t = \tau$, non-HCI expenditure ratios $\theta_{t+1} = \theta_t = \theta$ and unemployment benefit ratios $\zeta_{t+1} = \zeta_t = \zeta$ are assumed to be time-stationary. Acknowledging these assumptions, the government budget constraint reads eventually as follows:

$$\gamma_t = b\left[G_{t+1}^Y - (1+i_t)\right] + \chi, \tag{36}$$

with $\chi \equiv \tau(1-\alpha) - \theta - \zeta$ denoting the primary surplus ratio (excluding government HCI-expenditures) of the government.

The next dynamic variable we want to transform into a per GDP ratio is the real capital stock K_t. The capital stock to GDP ratio is known as capital output ratio denoted as κ_t. The relationship to the real capital to human capital ratio k_t is as follows:

$$\kappa_t \equiv \frac{K_t}{Y_t} = \frac{K_t}{M\left(H_t\right)^{1-\alpha}\left(1-u_t\right)^{1-\alpha}\left(K_t\right)^{\alpha}} = \frac{\left(K_t\right)^{1-\alpha}}{M\left(H_t\right)^{1-\alpha}\left(1-u_t\right)^{1-\alpha}} = \frac{\left(k_t\right)^{1-\alpha}}{M\left(1-u_t\right)^{1-\alpha}}. \tag{37}$$

Using the relationship between κ_t and k_t in (37), in terms of the transformed variables, the first-order condition (31) can rewritten as:

$$\frac{\alpha}{\kappa_{t+1}} = \delta + i_{t+1}. \tag{38}$$

The growth factor of human capital in terms of the transformed reads as follows:

$$\begin{aligned}\frac{H_{t+1}}{H_t} &= H_0\left(H_t\right)^{\mu-1}\left(\gamma_t\right)^{1-\mu}\left(Y_t\right)^{1-\mu} \\ &= H_0\left(H_t\right)^{\mu-1}\left(\gamma_t\right)^{1-\mu} M^{1-\mu}\left(K_t\right)^{\alpha(1-\mu)}\left(H_t\right)^{(1-\alpha)(1-\mu)}\left(1-u_t\right)^{(1-\alpha)(1-\mu)} \\ &= H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\gamma_t\right)^{1-\mu}\left(\kappa_t\right)^{\frac{\alpha(1-\mu)}{1-\alpha}}\left(1-u_t\right)^{1-\mu}.\end{aligned} \tag{39}$$

The GDP growth factor in terms of the capital output ratio can be rewritten as follows:

$$\begin{aligned}G_{t+1}^{Y} &= \frac{H_{t+1}}{H_t}\left(\frac{\kappa_{t+1}}{\kappa_t}\right)^{\alpha/(1-\alpha)}\frac{1-u_{t+1}}{1-u_t} \\ &= H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\gamma_t\right)^{1-\mu}\left(\kappa_{t+1}\right)^{\frac{\alpha}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha\mu}{1-\alpha}}\left(1-u_{t+1}\right)\left(1-u_t\right)^{-\mu}.\end{aligned} \tag{40}$$

To derive the dynamics of the aggregate capital to output ratio, the aggregate version of the capital accumulation Equation (30) is divided on both sides by GDP:

$$\frac{K_{t+1}}{Y_{t+1}} G_{t+1}^{Y} \equiv \kappa_{t+1} G_{t+1}^{Y} = \left(1-\delta\right)\frac{K_t}{Y_t} + \frac{I_t}{Y_t} = \left(1-\delta\right)\kappa_t + \frac{I_t}{Y_t}. \tag{41}$$

Using Equations (20), (37) and (38), we obtain for the investment GDP ratio I_t/Y_t:

$$\frac{I_t}{Y_t} = \frac{H_t}{K_t}\frac{K_t}{Y_t}\frac{\vartheta\Pi_{t+1}^{ex}}{\left(1+i_t\right)} = \frac{1}{k_t}\kappa_t\frac{\vartheta\Pi_{t+1}^{ex}}{\left(1+i_t\right)} = \frac{M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}}{\left(1-u_t\right)\left(1-\delta+\alpha/\kappa_t\right)}. \tag{42}$$

Inserting (42) into (41) and rearranging gives:

$$\kappa_{t+1} G_{t+1}^{Y} = \left(1-\delta\right)\kappa_t + \frac{M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}}{\left(1-u_t\right)\left(1-\delta+\alpha/\kappa_t\right)}. \tag{43}$$

or – inserting (40) into (43) –:

$$\begin{aligned}&H_0 M^{(1-\mu)/(1-\alpha)}\left(\gamma_t\right)^{1-\mu}\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\left(\kappa_t\right)^{-\alpha\mu/(1-\alpha)}\omega_{t+1}\left(\omega_t\right)^{-\mu} \\ &= \left(1-\delta\right)\kappa_t + \frac{M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}}{\omega_t\left(1-\delta+\alpha/\kappa_t\right)},\ \omega_t \equiv 1-u_t.\end{aligned} \tag{44}$$

Next, we insert the GDP growth factor Equation (40) and Equation (38) into Equation (36). This procedure yields:

$$\gamma_t = b\left[H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\gamma_t\right)^{1-\mu}\left(\kappa_{t+1}\right)^{\frac{\alpha}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha\mu}{1-\alpha}}\omega_{t+1}\left(\omega_t\right)^{-\mu} - 1 + \delta - \frac{\alpha}{\kappa_t}\right] + \chi. \quad (45)$$

Solving Equation (44) for γ_t, we obtain:

$$\gamma_t = \left[\frac{(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(w_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}}{H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\kappa_{t+1}\right)^{\frac{1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha\mu}{1-\alpha}}\omega_{t+1}\left(\omega_t\right)^{-\mu}}\right]^{\frac{1}{1-\mu}}. \quad (46)$$

To arrive at the first equation of motion along the intertemporal equilibrium path we form from Equation (46) $\gamma_t^{1-\mu}$ and insert it together with γ_t from (46) into Equation (45) such that we obtain after rearranging:

$$\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\omega_{t+1}$$
$$= \frac{\left(H_0\right)^{-1} M^{\frac{-(1-\mu)}{1-\alpha}}\left[(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}\right]\left(\omega_t\right)^{\mu}\left(\kappa_t\right)^{\alpha\mu/(1-\alpha)}}{\left\{b\left\{\left(\kappa_{t+1}\right)^{-1}\left[(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}\right] - \left(1-\delta+\frac{\alpha}{\kappa_t}\right)\right\}+\chi\right\}^{1-\mu}}$$

(47)

Inserting γ_t from (46) into Equations (B.5) and (45) we obtain the following equations:

$$\begin{aligned}\omega_t = M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(1-\delta+\alpha/\kappa_t\right)^{-1}\Bigg\{&\sigma(1-\alpha) - \sigma\theta - (1-\delta)\kappa_t + v_t \\ &+ \sigma b\left[(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(1-\delta+\alpha/\kappa_t\right)^{-1} - \left(1-\delta+\alpha/\kappa_t\right)\right] \quad (48) \\ &- \sigma\left[\frac{(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}}{H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\kappa_{t+1}\right)^{\frac{1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha\mu}{1-\alpha}}\omega_{t+1}\left(\omega_t\right)^{-\mu}}\right]^{\frac{1}{1-\mu}}\Bigg\}^{-1},\end{aligned}$$

$$\begin{aligned}&\left[\frac{(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}}{H_0 M^{\frac{1-\mu}{1-\alpha}}\left(\kappa_{t+1}\right)^{\frac{1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha\mu}{1-\alpha}}\omega_{t+1}\left(\omega_t\right)^{-\mu}}\right]^{\frac{1}{1-\mu}} \quad (49) \\ &= b\left[(1-\delta)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1} - 1 + \delta - \frac{\alpha}{\kappa_t}\right] + \chi.\end{aligned}$$

Considering (49) in Equation (48), this equation reads for $t+1$ as follows:

$$\omega_{t+1} = \frac{M^{\frac{-1}{1-\alpha}}\left(\kappa_{t+1}\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+2}^{ex}}{\left(1-\delta+\alpha/\kappa_{t+1}\right)\left[\sigma(1-\alpha) - \sigma(\theta+\chi) - (1-\delta)\kappa_{t+1} + v_{t+1}\right]}. \quad (50)$$

Equation (50) represents the second equation of motion along the intertemporal equilibrium path.

The third equilibrium dynamic equation pops up when Equation (19) is divided on both sides by Y_t and the growth Equation (40) is used:

$$\frac{v_{t+1}}{\kappa_{t+1}}\left[\left(1-\delta\right)\kappa_t + M^{\frac{-1}{1-\alpha}}\left(\kappa_t\right)^{\frac{-\alpha}{1-\alpha}}\vartheta\Pi_{t+1}^{ex}\left(\omega_t\right)^{-1}\left(1-\delta+\alpha/\kappa_t\right)^{-1}\right]=\left(1-\delta+\frac{\alpha}{\kappa_{t+1}}\right)v_t. \quad (51)$$

The fourth dynamic equation results from Equation (21):

$$\Pi_{t+1}^{ex}=\left(1-\varphi\right)\Pi_t^{ex}+\varphi\hat{\Pi}^{ex}. \quad (52)$$

The four-dimensional difference equation system (47), (50)-(52) becomes determinate if we assume initial values for the capital output ratio and the equity price discount like Magill and Quinzii (2003): $\kappa_0=\underline{\kappa}>0$ and $v_0=\underline{v}>0$, whereby $\underline{\kappa}$ and $\underline{v}>0$ are historically given. Moreover, $\Pi_0^{ex}=\underline{\Pi}^{ex}>0$ with $\underline{\Pi}^{ex}$ is also historically given. ω_0 derives from Equation (50) for $t=-1$.

4. Existence and Dynamic Stability of Steady State

The steady states of the equilibrium dynamics depicted by the difference Equations (47), (50)-(52) are defined as $\lim_{t\to\infty}\kappa_t=\kappa$, $\lim_{t\to\infty}\omega_t=\omega$, $\lim_{t\to\infty}v_t=v$, $\lim_{t\to\infty}\Pi_t^{ex}=\hat{\Pi}^{ex}$. As in Magill and Quinzii (2003), there are two different steady-state solutions of the equilibrium dynamics (47), (50)-(52): (1) The zero-discount, or so-called Diamond-solution $\lim_{t\to\infty}\kappa_t=\kappa_D$, $\lim_{t\to\infty}v_t=v_D=0$, $\lim_{t\to\infty}\Pi_t^{ex}=\hat{\Pi}^{ex}$, and $\lim_{t\to\infty}\omega_t=\omega_D$, and (2), the positive-discount steady state $\lim_{t\to\infty}\kappa_t=\kappa$, $\lim_{t\to\infty}v_t=v>0$, $\lim_{t\to\infty}\Pi_t^{ex}=\hat{\Pi}^{ex}$, and $\lim_{t\to\infty}\omega_t=\omega$. Here we focus on solution (2). This leads us to the following proposition 1:

Proposition 1. Suppose that for given structural and policy parameters $\left(\hat{\Pi}^{ex}\right)_{\max}$ is such that it is the solution of

$$M^{\frac{-1}{1-\alpha}}\vartheta\left(\hat{\Pi}^{ex}\right)_{\max}\left[\frac{\alpha}{\delta-1+\alpha^{\frac{\mu-1}{2-\mu}}\chi^{\frac{1-\mu}{2-\mu}}\left(H_0\right)^{\frac{1}{2-\mu}}\left(\vartheta\left(\hat{\Pi}^{ex}\right)_{\max}\right)^{\frac{1-\mu}{2-\mu}}}\right]^{\frac{1-2\alpha}{1-\alpha}}$$

$$=\alpha\left[\alpha+\frac{\alpha\left(1-\delta\right)}{\delta-1+\alpha^{\frac{\mu-1}{2-\mu}}\chi^{\frac{1-\mu}{2-\mu}}\left(H_0\right)^{\frac{1}{2-\mu}}\left(\vartheta\left(\hat{\Pi}^{ex}\right)_{\max}\right)^{\frac{1-\mu}{2-\mu}}}\right].$$ Then, for all $\hat{\Pi}^{ex}<\left(\hat{\Pi}^{ex}\right)_{\max}$ the following steady-state solution for $\left(G^Y,\kappa,w,v\right)\gg 0$, $\Pi^{ex}=\hat{\Pi}^{ex}$, and $0<u<1$ exists:

$$\kappa=\frac{\alpha}{\chi^{\frac{1-\mu}{2-\mu}}\left(H_0\right)^{\frac{1}{2-\mu}}\left(\vartheta\hat{\Pi}^{ex}\right)^{\frac{1-\mu}{2-\mu}}\alpha^{\frac{\mu-1}{2-\mu}}+\delta-1}, \quad (53)$$

$$G^Y=1-\delta+\alpha/\kappa, \quad (54)$$

$$v=\alpha+\left(1-\delta\right)\kappa+\sigma\left(\theta+\chi\right)-\sigma\left(1-\alpha\right), \quad (55)$$

$$\omega = 1 - u = \frac{\vartheta \hat{\Pi}^{ex} \kappa^{\frac{1-2\alpha}{1-\alpha}}}{M^{\frac{1}{1-\alpha}} \alpha \left[\alpha + (1-\delta)\kappa \right]}, \tag{56}$$

$$\Pi^{ex} = \hat{\Pi}^{ex}. \tag{57}$$

Condition (54) shows the Golden rule, namely that the endogenous GDP growth factor (= 1+ GDP growth rate) equals the interest factor (=1 + interest rate).

The next step is to investigate the local dynamic stability of the unique steady-state solution. To this end, the intertemporal equilibrium Equations (40), (41), (50) and (51) are totally differentiated with respect to $\kappa_{t+1}, \omega_{t+1}, v_{t+1}, \kappa_t, \omega_t, v_t$ while assuming that $\varphi = 1$ and $b = 0$.[6] Then, the Jacobian matrix $J(\kappa, \omega, v)$ of all partial differentials with respect to κ_t, ω_t, v_t is formed as follows:

$$J(\kappa,\omega,v) \equiv \begin{bmatrix} \frac{\partial \kappa_{t+1}}{\partial \kappa_t}(\kappa,\omega,v) & \frac{\partial \kappa_{t+1}}{\partial \omega_t}(\kappa,\omega,v) & \frac{\partial \kappa_{t+1}}{\partial v_t}(\kappa,\omega,v) \\ \frac{\partial \omega_{t+1}}{\partial \kappa_t}(\kappa,\omega,v) & \frac{\partial \omega_{t+1}}{\partial \omega_t}(\kappa,\omega,v) & \frac{\partial \omega_{t+1}}{\partial v_t}(\kappa,\omega,v) \\ \frac{\partial v_{t+1}}{\partial \kappa_t}(\kappa,\omega,v) & \frac{\partial v_{t+1}}{\partial \omega_t}(\kappa,\omega,v) & \frac{\partial v_{t+1}}{\partial v_t}(\kappa,\omega,v) \end{bmatrix}. \tag{58}$$

In contrast to the intertemporal equilibrium model in Farmer (2024) we are unable to present algebraically the entries of the Jacobian (58). Using the numerical parameters of the baseline calibration below and $b = 0$ [7] one can show[8] that the determinant of the Jacobian is zero. This implies that one of the eigenvalues of the Jacobian, $\lambda_i, i = 1,2,3$ is zero, say: $\lambda_1 = 0$. Moreover, we find that eigenvalues λ_2 and λ_3 are real and are strictly larger than zero and smaller than one, *i.e.* $\lambda_2 = 0.760969$ and $\lambda_3 = 0.393099$.

Proposition 2. Suppose the assumption of Proposition 1, $b = 0$ and $\varphi = 1$ hold. Then the eigenvalues of the Jacobian (58) $\lambda_i, i = 1,2,3$ are real, $\lambda_1 = 0$, $0 < \lambda_2 < 1$, and $0 < \lambda_3 < 1$.

In other words: the equilibrium dynamics with initial values $\kappa_0 = \underline{\kappa} > 0$, $v_0 = \underline{v} > 0$ and ω_0 in line with Equation (50) for $t = -1$ in the neighborhood of the positive-discount, steady-state solution in our stock-market OLG model with involuntary unemployment is non-oscillating and converges asymptotically stable towards the steady state (53)-(57) as time approaches infinity.

As a second step, we investigate the case $0 < \varphi < 1$. Clearly, the same steady-

[6]Assuming $b = 0$ is clearly at odds with the present reality of the global economy. However, since the focus of this paper is on steady state results and the intertemporal equilibrium dynamics greatly simplify, this assumption seems to be warranted. The equations of total differentials are delegated to Appendix C.

[7]Notice that the steady state is independent of the magnitude of b. This is however not true for the stability of the steady state. As numerical experimentation shows asymptotic stability changes to saddle path stability as b becomes larger than 0.01 which corresponds to a yearly debt to GDP ratio for the global economy of 30%.

[8]Using MATHEMATICA 12.2.

state solution as in (53)-(57) applies in this case. By inserting Equation (52) into Equations (50) and (51), the equilibrium dynamics remain three-dimensional although the entries of the Jacobian (58) change. Without showing again the algebraic details of the entries, it turns out that under $0<\varphi<1$ and $b=0$ the determinant of $J(\kappa,\omega,v)$ is non-zero and all eigenvalues of are real, larger than zero and smaller than one.[9]

5. Comparative Dynamics of the Steady-State Solution

As a next step it is apt to investigate the comparative dynamics of the positive-discount steady state. The effects of infinitesimal, isolated parameter changes on the positive-discount steady state.

Proposition 3. Suppose that the assumption of Proposition 1 holds. Then, the effects of infinitesimal, isolated changes of main model parameters on the positive-discount steady-state solution (53)-(57) read as follows:

$$\frac{\partial\kappa}{\partial\hat{\Pi}^{ex}}=\frac{-\Gamma(1-\mu)\left(\hat{\Pi}^{ex}\right)^{\frac{2}{\mu-2}}\kappa}{(2-\mu)\left[\Gamma\left(\hat{\Pi}^{ex}\right)^{\frac{\mu}{\mu-2}}-(1-\delta)\left(\hat{\Pi}^{ex}\right)^{\frac{1}{\mu-2}}\right]}<0,\Gamma\equiv\chi^{\frac{1-\mu}{2-\mu}}\left(H_0\right)^{\frac{1}{2-\mu}}\vartheta^{\frac{1-\mu}{2-\mu}}\alpha^{\frac{\mu-1}{2-\mu}},\tag{59}$$

$$\frac{\partial G^Y}{\partial\hat{\Pi}^{ex}}=\frac{\alpha\Gamma(1-\mu)\left(\hat{\Pi}^{ex}\right)^{\frac{2}{\mu-2}}}{(2-\mu)\kappa\left[\Gamma\left(\hat{\Pi}^{ex}\right)^{\frac{\mu}{\mu-2}}-(1-\delta)\left(\hat{\Pi}^{ex}\right)^{\frac{1}{\mu-2}}\right]}>0,\tag{60}$$

$$\frac{\partial v}{\partial\hat{\Pi}^{ex}}=\frac{-\Gamma(1-\delta)(1-\mu)\left(\hat{\Pi}^{ex}\right)^{\frac{2}{\mu-2}}\kappa}{(2-\mu)\left[\Gamma\left(\hat{\Pi}^{ex}\right)^{\frac{\mu}{\mu-2}}-(1-\delta)\left(\hat{\Pi}^{ex}\right)^{\frac{1}{\mu-2}}\right]}<0,\tag{61}$$

$$\frac{\partial\omega}{\partial\hat{\Pi}^{ex}}=\frac{\kappa^{\frac{1-2\alpha}{1-\alpha}}M^{\frac{-1}{1-\alpha}}\left(\hat{\Pi}^{ex}\right)^{-1+\frac{1}{2-\mu}}\left[\Gamma(1-\alpha\mu)\vartheta\left(\hat{\Pi}^{ex}\right)+(1-\alpha)(1-\delta)(\mu-2)\vartheta\left(\hat{\Pi}^{ex}\right)^{\frac{1}{2-\mu}}\right.}{(1-\alpha)(2-\mu)\alpha\left[\alpha+(1-\delta)\kappa\right]\left[\Gamma\left(\hat{\Pi}^{ex}\right)^{\frac{1}{\mu-2}}-(1-\delta)\left(\hat{\Pi}^{ex}\right)^{\frac{\mu}{\mu-2}}\right]}$$
$$\frac{\left.-\alpha(1-\alpha)(1-\delta)\Gamma w\right]}{(1-\alpha)(2-\mu)\alpha\left[\alpha+(1-\delta)\kappa\right]\left[\Gamma\left(\hat{\Pi}^{ex}\right)^{\frac{1}{\mu-2}}-(1-\delta)\left(\hat{\Pi}^{ex}\right)^{\frac{\mu}{\mu-2}}\right]}>0,\tag{62}$$

$$\frac{\partial\kappa}{\partial\sigma}=0,\frac{\partial G^Y}{\partial\sigma}=0,\frac{\partial v}{\partial\sigma}=-(1-\alpha)(1-\tau)-\zeta<0,\frac{\partial\omega}{\partial\sigma}=0.\tag{63}$$

$$\frac{\partial\kappa}{\partial\zeta}=\frac{\Delta(1-\mu)\kappa\chi^{-1+\frac{1-\mu}{2-\mu}}}{(2-\mu)\left[\Delta\chi^{\frac{1-\mu}{2-\mu}}-(1-\delta)\right]}>0,\ \Delta\equiv\left(H_0\right)^{\frac{1}{2-\mu}}\vartheta^{\frac{1-\mu}{2-\mu}}\left(\hat{\Pi}^{ex}\right)^{\frac{1-\mu}{2-\mu}}\alpha^{\frac{\mu-1}{2-\mu}},\tag{64}$$

[9]Using once more MATHEMATICA 12.2.

$$\frac{\partial G^{Y}}{\partial \zeta}=-\frac{\alpha\Delta(1-\mu)\kappa\chi^{\frac{1}{2-\mu}}}{\chi(2-\mu)\kappa\left[\Delta\chi^{\frac{1}{2-\mu}}-\chi^{\frac{\mu}{2-\mu}}(1-\delta)\right]}<0, \tag{65}$$

$$\frac{\partial v}{\partial \zeta}=\frac{\chi^{\frac{2}{2-\mu}}\left\{(1-\delta)(2-\mu)\sigma+\Delta\chi^{\frac{-1}{2-\mu}}\left[(1-\delta)(1-\mu)\kappa-\chi(2-\mu)\sigma\right]\right\}}{\chi(2-\mu)\left[\Delta\chi^{\frac{1}{2-\mu}}-\chi^{\frac{\mu}{2-\mu}}(1-\delta)\right]}>0, \tag{66}$$

$$\frac{\partial \omega}{\partial \zeta}=\frac{-\Delta(1-\mu)\chi^{\frac{1}{2-\mu}}\kappa\left[(1-\alpha)(1-\delta)\alpha w-(1-2\alpha)\kappa^{\frac{-\alpha}{1-\alpha}}M^{\frac{-1}{1-\alpha}}\vartheta\hat{\Pi}^{ex}\right]}{\chi\left[\Delta\chi^{\frac{1}{2-\mu}}-\chi^{\frac{\mu}{2-\mu}}(1-\delta)\right](1-\alpha)(2-\mu)\alpha\left[\alpha+(1-\delta)\kappa\right]}>0, \tag{67}$$

since $(1-\alpha)(1-\delta)\alpha w-(1-2\alpha)\kappa^{\frac{-\alpha}{1-\alpha}}M^{\frac{-1}{1-\alpha}}\vartheta\hat{\Pi}^{ex}<0.$

Considering the results of the comparative-dynamics experiment in (59)-(67), one encounters findings broadly in line with Keynesian insights as in Farmer (2024). From (63), we can see that a marginal change in the savings rate has changed neither the steady-state capital output ratio nor the GDP growth factor nor the unemployment rate. However, the steady-state equity price discount decreases with a rising savings rate which can be seen from rewriting steady-state condition (55) as: $v=\alpha+(1-\delta)\kappa-\sigma\left[(1-\alpha)(1-\tau)+\zeta\right]$. Moreover, also in line with short-run Keynesian findings in our neo-classical growth model are the impacts of changes in the expected profits from investments in physical capital. A marginal rise in long-term profit expectations raises the endogenous GDP growth factor and one minus the unemployment rate implying a lower unemployment rate. Notice, however, these are steady-state impacts or long-run results in addition to the well-known Keynesian short-run effects. New are, however, the steady-state effects of more investor optimism on the steady-state capital output ratio and the equity price discount. Both the capital output ratio and the equity price discount decrease with higher expected profits. The capital output ratio declines since the share of capital in output (α) remains constant while output rises due to more investment according to the denominator in (59). The equity price discount goes down because the capital output ratio declines and with a fixed rate of capital depreciation the equity price discount decreases according to (61). Most important for the main topics of the present paper are the effects of more investor optimism regarding the expected profits on the steady-state unemployment rate u. Steady-state inequality (62) makes clear that ω rises and u declines with higher $\hat{\Pi}^{ex}$ since expected profits directly impact positively ω acknowledging Equation (56) and a lower capital-output ratio affects positively (from the nominator in (56)) and negatively (from the denominator of Equation (56)) one minus the unemployment rate. The positive effects overwhelm the negative one as inequality (62) shows.

Finally, we investigate the steady-state effects of a higher unemployment benefit ratio. It reduces the primary surplus ratio and on account of steady-state Equation (53) the capital-output ratio increases. From steady-state Equation (54), we see immediately that the steady-state growth factor declines. Rewritten steady-state Equation (55), that is $v = \alpha + (1-\delta)\kappa - \sigma\left[(1-\alpha)(1-\tau)+\zeta\right]$, shows counteracting effects of the unemployment benefit ratio on the equity price discount: while the unemployment benefit ratio directly affects the equity price discount negatively, the former indirectly exhibits a positive impact on the equity price discount via the rising capital-output ratio. Inequality (66) ensures that the positive impact is larger than the negative impact. The same applies to the relationship between the unemployment benefit ratio and one minus the unemployment rate. A higher unemployment benefit ratio affects via the capital output ratio positively and negatively one minus the unemployment rate as steady-state Equation (56) transpires. However, inequality (67) ensures that the former effect is larger than the latter such that a larger unemployment benefit ratio raises one minus the unemployment rate and hence reduces the unemployment rate.

6. The Intertemporal Equilibrium Dynamics in Response to Parameter Changes

The steady-state responses to structural and policy parameter changes are one thing, the other are the reactions of main dynamic variables along the intertemporal equilibrium path to isolated parameter changes from one steady state to another. In contrast to the comparative dynamics of steady-state solutions, a general analytical representation of responses of the equilibrium dynamic variables to parameter changes is practically impossible. Thus, we switch to a numerical specification of our stock market OLG model with endogenous growth and involuntary unemployment. The main model parameters are chosen such that the assumptions of Propositions 1 and 2 hold. Moreover, we choose the following parameter set which accords rather well with medium-term stylized facts regarding the growth rate of global real gross domestic product per capita averaged over the time period between 1964 and 2024 (see IMF 1980, IMF 1990, IMF 2008, IMF 2024, IMF 2025): $G^Y = 1.69$ (=annual growth rate of 1.77%[10]), the real interest rate averaged over the time period between 1964 and 2024 (see IMF 1980, IMF 1990, IMF 2008, IMF 2024, IMF 2025): $1+i = 1.69$, the savings ratio: 0.22, the investment ratio: 0.22 and the unemployment rate of the global economy averaged over the time period between 1991 and 2024: 6%. $\alpha = 0.22$, $b = 0$, $\beta = 0.3$, $\vartheta = 1.1$, $\delta = 0.8$, $\theta = 0.05$, $\varepsilon = 0.9$, $H_0 = 6$, $\mu = 0.8$, $M = 1.12$, $\hat{\Pi}^{ex} = 0.2143$, $\zeta = 0.15$, $\tau = 0.26$. Inserting these parameter values into the steady-state Equations (53)-(57), these equations generate the following steady-state solution: $\kappa = 0.1476$, $G^Y = 1.6904$, $v = 0.0677$, $\omega = 0.9403$ $(u \approx 0.06)$

[10] $(1+0.0177)^{30} \approx 1.69$.

The adjustment coefficient with respect to expected profits φ will be fixed at: $\varphi = 0.5$.

Consider now a small negative and unexpected shock on β from 0.30 to 0.31 implying a small increase in the savings rate. Then, the following **Table 1** exhibits the intertemporal equilibrium path of main endogenous variables towards the new steady state: $\kappa = 0.1476$, $G^Y = 1.6904$, $v = 0.0632$, $\omega = 0.9403$ $(u \approx 0.06)$.

Table 1. Intertemporal equilibrium path of $\left(\kappa_t, G_t^Y, v_t, \omega_t, \Pi_t^{ex}\right)_{t>1}$ after a small positive savings-rate shock.

t	0	1	2	3	4	…	13	…	40
κ_t	0.1476	0.1474	0.1465	0.1455	0.1454		0.1474		0.1476
G_t^Y	1.6904	1.6917	1.7102	1.7255	1.7316		1.6939		1.6904
v_t	0.0677	0.0677	0.0674	0.0668	0.0661		0.0634		0.0632
ω_t	0.9403	0.9569	0.9398	0.9262	0.9235		0.9388		0.9403
Π_t^{ex}	0.2143	0.2245	0.2194	0.2169	0.2156		0.2143		0.2143

Source: Author's own Calculation using GAMS Studio 51.

A glance at **Table 1** reveals that a small increase in the savings rate in the first period after the shock reduces the capital-output ratio, raises the GDP growth factor and the expected profit and reduces the unemployment rate, while the equity-price discount remains unchanged. From the second period onwards the capital output ratio further declines, and the GDP growth factor and the unemployment rise. Moreover, the expected profits also rise while the equity price discount steadily declines. Between the fourth and thirteenth periods the capital output rises again, while the GDP growth factor, the equity price discount, the unemployment rate and expected profits decline. After theoretically infinite periods (practically after 40 periods) the capital output ratio, the GDP growth factor, the unemployment rate and expected profits return towards the pre-shock values, while the equity-price discount becomes smaller than in the pre-shock steady state. That the unemployment rate after a one-period rises with a higher savings rate towards its pre-shock value sounds again "Keynesian" in our neo-classical stock-market OLG model with involuntary unemployment.

Starting again from the same steady state solution as before the savings-rate shock, we next assume that firm managers expect larger long-run profits from current investment: $\hat{\Pi}^{ex}$ rises from 0.2143 towards 0.215: all other parameters remain on their pre-saving-rate-shock values. The effects of this small, positive investment shock on the capital output ratio, the equity-price discount, the unemployment rate and on expected returns from investment along the intertemporal-equilibrium path are depicted in **Table 2**.

As **Table 2** reveals, in the first period the positive shock in expected profits reduces the capital output ratio, raises the GDP growth factor and expected profits, increases the unemployment rate, while the equity price discount remains unaltered.

Table 2. Intertemporal equilibrium path of $\left(\kappa_t, G_t^Y, v_t, \omega_t, \Pi_t^{ex}\right)_{t>1}$ after a small positive expected profit shock.

t	0	1	2	3	4	…	13	…	40
κ_t	0.1476	0.1473	0.1475	0.1475	0.1476		0.1475		0.1475
G_t^Y	1.6904	1.6944	1.6933	1.6913	1.6906		1.6912		1.6913
v_t	0.0677	0.0677	0.0676	0.0676	0.0676		0.0677		0.0677
ω_t	0.9403	0.9400	0.9421	0.9430	0.9433		0.9430		0.9430
Π_t^{ex}	0.2143	0.2146	0.2148	0.2149	0.2149		0.2150		0.2150

Source: Author's own Calculation using GAMS Studio 51.

Afterwards the capital output ratio and expected profits increase, while the GDP growth factor and the unemployment rate decrease. The equity price discount declines only slightly. Between the fourth and thirteenth periods the capital output ratio declines, the GDP growth factor, the expected profits, the unemployment rate and the equity discount rise. In comparison to the initial steady state, the capital output ratio is slightly lower, and the unemployment rate is noticeably lower in the new steady state. Correspondingly, the GDP growth factor is larger than in the steady state before. The equity-price discount in the new steady state equals the discount in the pre-shock steady state. Summing up: Except for the effects on the equity price discount, the effects of more investor optimism on the capital output ratio, the GDP growth factor and on the unemployment rate along the intertemporal equilibrium path correspond roughly to (post)-Keynesian insights in our neo-classical stock market OLG model with endogenous GDP growth.

Our last shock experiment concerns the unemployment benefit ratio. Starting once more from the steady state before any shocks, we increase slightly the unemployment benefit ratio from 0.15 to towards 0.1502. The impacts on the capital-output ratio, the equity-price discount, the unemployment rate and on the expected profits along the intertemporal equilibrium path are depicted in **Table 3**.

Table 3. Intertemporal equilibrium path of $\left(\kappa_t, G_t^Y, v_t, \omega_t, \Pi_t^{ex}\right)_{t>1}$ after a small positive unemployment benefit ratio shock.

t	0	1	2	3	4	…	13	…	40
κ_t	0.1476	0.1509	0.1505	0.1503	0.1501		0.1497		0.1497
G_t^Y	1.6904	1.6746	1.6535	1.6586	1.6620		1.6693		1.6696
v_t	0.0677	0.0670	0.0673	0.0676	0.0677		0.0681		0.0680
ω_t	0.9403	0.9584	0.9552	0.9530	0.9514		0.9484		0.9482
Π_t^{ex}	0.2143	0.2143	0.2143	0.2143	0.2143		0.2143		0.2143

Source: Author's own Calculation using GAMS Studio 51.

A marginally higher unemployment benefit ratio firstly increases the capital output ratio and decreases the GDP growth factor and the unemployment rate, while the equity-price discount decreases in the post-shock period and expected profits remain unchanged along the intertemporal equilibrium. Between the fourth and thirteenth periods the capital output ratio declines, whereby the GDP growth factor, the unemployment rate and the equity price discount rise. In comparison to the former steady state, the capital output ratio and equity price discount are larger while the GDP growth factor and the unemployment rate are lower in the new steady state.

7. Conclusion

This paper introduces endogenous growth, an endogenous unemployment rate and managers' beliefs about expected profits from capital investment into Magill and Quinzii's (2003) stock market OLG model with non-shiftable capital and affine equity-price expectations. Firm managers use collective beliefs about Keynes (1936) marginal efficiency of investment to determine their optimally indeterminate investment amount. The expected profits from current capital investment are adaptively adjusted towards their long-run expected level.

In contradistinction to Magill and Quinzii's (2003) full employment model, in our model the unemployment rate appears as additional dynamic variable with the consequence that the intertemporal-equilibrium dynamics is four- instead of two-dimensional as in Magill and Quinzii (2003). The step-by-step derivation of the intertemporal-equilibrium equations from the first-order conditions for intertemporal utility and market value maxima, the government budget constraint, the belief function of firm managers and the market-clearing conditions bring forth that the unemployment rate is a slowly moving dynamic variable in addition to the capital-output ratio and the equity-price discount.

While there are in principle two steady-state solutions we focus on the positive-discount steady state whereby the capital-output ratio accords with the Golden rule of intertemporal consumption allocation: the interest rate equals the endogenous GDP growth rate. We find in Proposition 1 that a positive-discount steady state exists if the long-run expected profits from firm investment are not too large, made precise in Proposition 1.

To be able to perform comparative dynamics of the effects of parameter shocks on main variables we then check the dynamic stability of the equilibrium dynamics in the neighborhood of the positive-discount steady state. We find that local asymptotic stability of the three-dimensional equilibrium dynamics is ensured when the existence condition holds and $b = 0$. All three eigenvalues are then equal or larger than zero and smaller than unity.

Knowing the existence and dynamic stability of the positive-discount steady state, we are entitled to perform local, comparative-dynamic experiments whereby investigating the impacts of infinitesimal changes of the main model parameters on the steady-state capital-output ratio, the GDP growth factor, the equity-price discount and on the unemployment rate. We find that the expectation of higher long-run profits from investment reduces the capital output ratio, the equity price discount and the unemployment rate, while it raises the GDP growth factor. In contrast, a marginally higher savings rate reduces only the equity price discount, while it does impact neither on the capital output ratio, nor the GDP growth factor nor the unemployment rate. Moreover, a higher unemployment benefit ratio raises the capital output ratio and the equity price discount, and it reduces the GDP growth factor and the unemployment rate.

Finally, we investigate the effects of marginal parameter changes on the intertemporal-equilibrium path of the capital output ratio, the GDP growth factor, the equity-price discount, and the unemployment rate. Due to the analytical complexity of the algebra of the partial derivatives of these dynamic variables with respect to marginal parameter variations, we resort to a numerical specification of main model parameters which are in line with the assumptions of Proposition 1 and are representative for numerical parameter values for the global economy averaged over the period 1964 and 2024. We find that a marginally smaller saving rate temporarily reduces the capital output ratio and the unemployment rate, and raises the GDP growth rate, while the equity-price discount remains constant After about 40 periods (theoretically after an infinite number of time periods) the capital output ratio and the unemployment rate return to their pre-shock values, while the equity-price discount permanently decreases.

Moreover, we investigate the intertemporal-equilibrium effects of more managers' optimism. We find that higher expected profits from current investment temporarily diminish the capital output ratio and slightly raise the unemployment rate, while the GDP growth factor rises and the equity-price discount remains roughly constant in the short and long run. However, the positive investment shock reduces markedly the unemployment rate and raises the GDP growth factor in the long run.

Finally, a marginally higher unemployment benefit ratio firstly raises the capital output ratio, and reduces the equity-price discount, the GDP growth factor and the unemployment rate, while in the new steady state the capital output ratio is higher and the GDP growth factor is lower, the equity price discount is higher and the unemployment rate is lower than in the old steady state.

Obviously, there is ample space for future research. Highest on the agenda in this respect is the search for the other steady state solution in our stock market OLG model with endogenous growth and unemployment, and how different public debt to GDP ratios affect the stability of the steady state solution.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendices

Appendix A: Proof of Walras' Law

Walras' Law claims that the aggregate output of production equals the aggregate demand for production output:

$Y_t = L(1-u_t)c_t^{1,E} + L(1-u_{t-1})c_t^{2,E} + L u_t c_t^{1,U} + L u_{t-1} c_t^{2,U} + I_t + \Delta_t + \Gamma_t$, if the markets for labor, government and firm bonds and equity are cleared.

To prove this claim, we first add up the budget constraints of younger and older households (employed and unemployed) for period t on the left-hand and right-hand side of the budget equalities.

$$\begin{aligned} & L(1-u_t)c_t^{1,E} + L(1-u_{t-1})c_t^{2,E} + Lu_t c_t^{1,U} + Lu_{t-1} c_t^{2,U} \\ & = L(1-u_t)h_t w_t(1-\tau_t) - L(1-u_t)b_{t+1}^{E} - L(1-u_t)\sum_{j=1}^{J} b_{t+1}^{j,E} - L(1-u_t)\sum_{j=1}^{J} Q_t^j \theta_t^{j,E} \\ & \quad + L(1-u_{t-1})(1+i_t)b_t^{E} + L(1-u_{t-1})\sum_{j=1}^{J}(1+i_t^j)b_t^{j,E} + L(1-u_{t-1})\sum_{j=1}^{J}\theta_t^{j,E}(D_t^j + Q_t^j) \\ & \quad + Lu_t\zeta_t - Lu_t b_{t+1}^{U} - Lu_t\sum_{j=1}^{J} b_{t+1}^{j,U} - Lu_t\sum_{j=1}^{J} Q_t^j\theta_t^{j,U} + Lu_{t-1}(1+i_t)b_t^{U} \\ & \quad + Lu_{t-1}\sum_{j=1}^{J}(1+i_t^j)b_t^{j,U} + Lu_{t-1}\sum_{j=1}^{J}\theta_t^{j,U}(D_t^j + Q_t^j). \end{aligned} \tag{A.1}$$

Acknowledging bond market clearing conditions (23)-(25) for $t+1$ and t in text, (A.1) can be rewritten as follows:

$$\begin{aligned} & L(1-u_t)c_t^{1,E} + L(1-u_{t-1})c_t^{2,E} + Lu_t c_t^{1,U} + Lu_{t-1} c_t^{2,U} \\ & = L(1-u_t)h_t w_t(1-\tau_t) - B_{t+1} - \sum_{j=1}^{J} b_{t+1}^{j} - L(1-u_t)\sum_{j=1}^{J} Q_t^j\theta_t^{j,E} + (1+i_t)B_t \\ & \quad + \sum_{j=1}^{J}(1+i_t^j)b_t^j + L(1-u_{t-1})\sum_{j=1}^{J}\theta_t^{j,E}(D_t^j + Q_t^j) + Lu_t\zeta_t - Lu_t\sum_{j=1}^{J} Q_t^j\theta_t^{j,U} \\ & \quad + Lu_{t-1}\sum_{j=1}^{J}\theta_t^{j,U}(D_t^j + Q_t^j). \end{aligned} \tag{A.2}$$

Considering the firm financing condition (25) and equity market clearing conditions (26)-(28) (A.2) reads as follows:

$$\begin{aligned} & L(1-u_t)c_t^{1,E} + L(1-u_{t-1})c_t^{2,E} + Lu_t c_t^{1,U} + Lu_{t-1} c_t^{2,U} \\ & = L(1-u_t)h_t w_t(1-\tau_t) - B_{t+1} - I_t - \sum_{j=1}^{J} Q_t^j + (1+i_t)B_t \\ & \quad + (1+i_t)I_{t-1} + \sum_{j=1}^{J}(D_t^j + Q_t^j) + Lu_t\zeta_t. \end{aligned} \tag{A.3}$$

Acknowledging government's budget constraint (15), labor market clearing condition (22), (A.3) reads as follows:

$$\begin{aligned} & L(1-u_t)c_t^{1,E} + L(1-u_{t-1})c_t^{2,E} + Lu_t c_t^{1,U} + Lu_{t-1} c_t^{2,U} \\ & = L(1-u_t)h_t w_t(1-\tau_t) - \left[(1+i_t)B_t + \Delta_t + \Gamma_t + Lu_t\zeta_t - \tau_t L(1-u_t)h_t w_t\right] \\ & \quad - I_t - \sum_{j=1}^{J} Q_t^j + (1+i_t)B_t + (1+i_t)I_{t-1} + \sum_{j=1}^{J}(D_t^j + Q_t^j) + Lu_t\zeta_t \end{aligned}$$

$$= L\left(1-u_t\right)h_t w_t - \Delta_t - \Gamma_t - I_t + \left(1+i_t\right)I_{t-1} + D_t - Q_t + Q_t$$
$$= \sum_{j=1}^{J} N_t^j h_t w_t - \Delta_t - \Gamma_t - I_t + \left(1+i_t\right)I_{t-1} + D_t. \tag{A.4}$$

The last step is to consider $D_t = \sum_{j=1}^{J} D_t^j = \sum_{j=1}^{J} Y_t^j - w_t h_t \sum_{j=1}^{J} N_t^j - \left(1+i_t\right)\sum_{j=1}^{J} I_{t-1}^j = Y_t - w_t h_t \sum_{j=1}^{J} N_t^j - \left(1+i_t\right)I_{t-1}$ and labor market clearing condition (22), to rewrite (A.4) as follows:

$$L\left(1-u_t\right)c_t^{1,E} + L\left(1-u_{t-1}\right)c_t^{2,E} + Lu_t c_t^{1,U} + Lu_{t-1} c_t^{2,U} + I_t + \Delta_t + \Gamma_t$$
$$= \sum_{j=1}^{J} N_t^j h_t w_t + \left(1+i_t\right)I_{t-1} + D_t = \sum_{j=1}^{J} N_t^j h_t w_t + \left(1+i_t\right)I_{t-1} + Y_t \tag{A.5}$$
$$- \sum_{j=1}^{J} N_t^j h_t w_t - \left(1+i_t\right)I_{t-1} = Y_t.$$

Appendix B

Intermediate steps to build the intertemporal equilibrium dynamics

Since affine price expectations (9) in text also prevailed in period t, the savings/investment equality (29) can be re-written as follows:

$$Lh_t\left(1-u_t\right)s_t^E + Lh_t u_t s_t^U = \frac{M^{-1/(1-\alpha)}\left(\kappa_t\right)^{-\alpha/(1-\alpha)} Y_t \vartheta \Pi_{t+1}^{ex}}{\left(1-\delta+\alpha/\kappa_t\right)\left(1-u_t\right)} + \left(1-\delta\right)K_t - V_t. \tag{B.1}$$

Inserting the optimal savings functions for employed (4) respective unemployed (7) households into (B.1) yields:

$$Lh_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + Lh_t u_t \sigma \varsigma$$
$$= \frac{M^{-1/(1-\alpha)}\left(\kappa_t\right)^{-\alpha/(1-\alpha)} Y_t \vartheta \Pi_{t+1}^{ex}}{\left(1-\delta+\alpha/\kappa_t\right)\left(1-u_t\right)} + \left(1-\delta\right)K_t - V_t, \sigma \equiv \frac{\beta}{\varepsilon+\beta}. \tag{B.2}$$

Using government's budget constraint (15) and Equation (34), the left-hand side of equation (B.2) can be written as follows:

$$Lh_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + Lh_t u_t \sigma \varsigma$$
$$= Lh_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + Lh_t\left(1-u_t\right)\sigma w_t \tau_t$$
$$+ \sigma\left[B_{t+1} - \left(1+i_t\right)B_t\right] - \sigma\Delta_t - \sigma\Gamma_t \tag{B.3}$$
$$= Lh_t\left(1-u_t\right)\sigma w_t + \sigma\left[B_{t+1} - \left(1+i_t\right)B_t\right] - \sigma\Delta_t - \sigma\Gamma_t$$
$$= \sigma\left(1-\alpha\right)Y_t + \sigma\left[B_{t+1} - \left(1+i_t\right)B_t\right] - \sigma\Delta_t - \sigma\Gamma_t.$$

On dividing equation (B.3) on both sides by Y_t, and after introducing the capital output ratio and the debt to GDP ratio, one obtains the following equation:

$$\sigma\left(1-\alpha\right) + \sigma\left[\frac{B_{t+1}}{Y_{t+1}}\frac{Y_{t+1}}{Y_t} - \left(1+i_t\right)\frac{B_t}{Y_t}\right] - \sigma\frac{\Delta_t}{Y_t} - \sigma\frac{\Gamma_t}{Y_t}$$
$$= \sigma\left(1-\alpha\right) + \sigma b\left[G_{t+1}^Y - \left(1+i_t\right)\right] - \sigma\theta - \sigma\gamma_t \tag{B.4}$$
$$= \frac{M^{-1/(1-\alpha)}\left(\kappa_t\right)^{-\alpha/(1-\alpha)} Y_t \vartheta \Pi_{t+1}^{ex}}{\left(1-\delta+\alpha/\kappa_t\right)w_t} + \left(1-\delta\right)\kappa_t - v_t, v_t \equiv \frac{V_t}{Y_t}.$$

Solving equation (B.4) for $1-u_t$ and using the shorthand $\omega_t \equiv 1-u_t$, yields:

$$\omega_t = \frac{M^{-1/(1-\alpha)}(\kappa_t)^{-\alpha/(1-\alpha)} Y_t \vartheta \Pi_{t+1}^{ex}}{(1-\delta+\alpha/\kappa_t)\left\{\sigma(1-\alpha)+\sigma b\left[G_{t+1}^Y-(1+i_t)\right]-\sigma\theta-\sigma\gamma_t-(1-\delta)\kappa_t+v_t\right\}}. \quad \text{(B.5)}$$

Appendix C

Total differentials of intertemporal equilibrium conditions to build the elements of Jacobian matrix (58)

Under $b=0$ government's budget Equation (36) implies: $\gamma_t = \chi, \forall t$. Acknowledging this equality in Equation (40) together with intertemporal equilibrium conditions (41), (50) and (51) we obtain the following total differentials of the dynamic system:

$$\begin{aligned} G^Y d\kappa_{t+1} + \kappa dG_{t+1}^Y = &\left\{(1-\delta)+\frac{\alpha\left[G^Y-(1-\delta)\right]}{\kappa}\left[\frac{1}{(1-\delta+\alpha/\kappa)}-\frac{1}{(1-\alpha)}\right]\right\}d\kappa_t \\ &-\frac{\left[G^Y-(1-\delta)\right]}{\omega}d\omega_t, \end{aligned} \quad \text{(C.1)}$$

$$dG_{t+1}^Y - \frac{\alpha G^Y}{(1-\alpha)\kappa}d\kappa_{t+1} - \frac{G^Y}{\omega}d\omega_{t+1} = -\left[\frac{\alpha G^Y}{(1-\alpha)\kappa}\mu d\kappa_t + \frac{G^Y}{\omega}\mu d\omega\right], \quad \text{(C.2)}$$

$$\begin{aligned} d\omega_{t+1} = &\frac{\omega\left[(1-\delta)d\kappa_{t+1}+dv_{t+1}\right]}{\left[(1-\alpha)\sigma-\sigma(\theta+\chi)-(1-\delta)\kappa+v\right]} \\ &+\frac{\alpha\omega}{\kappa}\left[\frac{1}{(1-\delta+\alpha/\kappa)\kappa}-\frac{1}{(1-\alpha)}\right]d\kappa_{t+1}, \end{aligned} \quad \text{(C.3)}$$

$$G^Y dv_{t+1} + v dG_{t+1}^Y + \frac{\alpha v}{\kappa^2}d\kappa_{t+1} = (1-\delta+\alpha/\kappa)dv. \quad \text{(C.4)}$$

Solving simultaneously (C.1), C.2), (C.3) and (C.4) for $d\kappa_{t+1}, dG_{t+1}^Y, d\omega_{t+1}, dv_{t+1}$ and forming the partial differentials with respect to $d\kappa_t, d\omega_t, dv_t$ povide us with the elements of the Jacobian (58).