Scientific Research Publishing

Theoretical Economics Letters, 2026, 16(2), 496-504
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Dynamic Entry, Competition, and Welfare Loss under Uncertainty

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How to cite this paper: Fujita, Y. (2026). Dynamic Entry, Competition, and Welfare Loss under Uncertainty. *Theoretical Economics Letters, 16,* 496-504.
https://doi.org/10.4236/tel.2026.162029

Received: January 20, 2026
Accepted: April 21, 2026
Published: April 24, 2026

Abstract

This paper examines irreversible market entry under uncertainty in a Cournot oligopoly framework. While standard static Cournot models imply that increased competition monotonically reduces deadweight loss, this paper shows that intensified competition can amplify dynamic welfare losses when entry is irreversible and demand conditions are uncertain. Using a real-options approach, we derive closed-form expressions for privately and socially optimal entry thresholds and demonstrate that the wedge between them increases with the number of firms. Although greater competition improves static allocative efficiency by lowering prices, it reduces private entry incentives more rapidly than social benefits, leading firms to delay entry inefficiently. As a result, dynamic welfare losses arising from delayed market creation grow even as static inefficiency vanishes. These findings highlight a fundamental tension between static and intertemporal welfare effects of competition.

Keywords

Dynamic Entry, Cournot Competition, Real Options, Entry Timing, Welfare Loss

1. Introduction

In static Cournot models, an increase in number of firms reduces market power and monotonically decreases deadweight loss. In the limit, the Cournot equilibrium converges to the perfectly competitive outcome, a result known as the Cournot limit theorem. This classical insight, originating in Cournot (1838) and formalized in modern treatments such as Vives (1999) and Tirole (1988), firmly establishes competition as welfare improving in a static sense.

A large literature has extended this static framework to study market structure and welfare. Early contributions include Dixit (1979) and Salop (1979), who emphasized excess entry and product differentiation, while subsequent work such as Mankiw and Whinston (1986) analyzed entry decisions under perfect foresight and certainty. In these models, welfare losses arise from excessive duplication of fixed costs, but the timing of entry itself plays no role.

More recently, a growing literature has incorporated uncertainty and irreversibility into entry and investment decisions using real options theory. Seminal contributions by McDonald and Siegel (1986) and Dixit and Pindyck (1994) demonstrate that uncertainty creates an option value of waiting, leading firms to delay irreversible investment. These insights have been applied to market entry, trade liberalization, and industrial organization (e.g., Dixit, 1989; Leahy, 1993; Grenadier, 2002). In these studies, delayed entry is privately optimal but may be socially inefficient.

A related strand of research examines the interaction between competition and investment timing. Abel and Eberly (1996) and Caballero (1991) show that competitive pressures affect investment thresholds, while Fudenberg and Tirole (1985) and Grenadier (1996) emphasize strategic preemption and waiting in oligopolistic settings. However, much of this literature focuses on strategic interactions among a small number of firms or on preemption games, rather than on the welfare implications of entry timing under intensified competition.

This paper contributes to these literatures by highlighting a novel tension between static and dynamic welfare effects of competition. While increased competition reduces static deadweight loss in Cournot markets, we show that, under uncertainty and irreversible entry, intensified competition can amplify *dynamic welfare losses arising from delayed entry*. The key mechanism is a growing wedge between the privately and socially optimal entry thresholds, which expands monotonically with the number of firms. Importantly, this result does not contradict the Cournot limit theorem; rather, it complements it by showing that competition can simultaneously mitigate static inefficiency and exacerbate intertemporal inefficiency.

The remainder of the paper is organized as follows. Section 2 introduces the market environment. Section 3 derives the Cournot equilibrium and static welfare. Sections 4 and 5 characterize private and socially optimal entry timing, respectively. Section 6 analyzes the dynamic welfare loss from delayed entry and relates our findings to the Cournot limit theorem. In Section 7, we show that the excess entry theorem (e.g., Mankiw & Whinston, 1986) holds in the setting of this study as well. Section 8 concludes.

2. Market Environment

Consider a market where n symmetric firms exist. x_i and P denotes firm i's output and market price, respectively. We specify the inverse demand function as

$$P = a - b_t \sum_{i=1}^{n} x_i, \tag{1}$$

where a is a positive constant. We also specify each firm's marginal cost as $c < a$.

The demand slope parameter b_t is assumed to evolve stochastically according to a geometric Brownian motion

$$db_t = -\mu b_t dt + \sigma b_t dz_t, \tag{2}$$

where $\mu > 0$ captures the expected rate at which market conditions improve (i.e., demand becomes less price sensitive), $\sigma > 0$ measures uncertainty, and z_t is a standard Brownian motion. The discount rate is $\rho > 0$.

Entry into the market is irreversible. When firms enter, each incurs a sunk cost $I > 0$. After entry, no further fixed costs are incurred and Cournot competition prevails forever.

We assume that firms are identical and follow identical threshold strategies. Consequently, entry occurs simultaneously when the state variable reaches the common threshold. This justifies the aggregate sunk cost term nI that appears in section 5.

3. Cournot Equilibrium and Static Welfare

Given n active firms and a realization of b_t, the standard Cournot analysis yields firm i's output, total output and price in the Cournot equilibrium,

$$x_i^* = \frac{a-c}{b_t(n+1)}; \tag{3}$$

$$Q^* = \frac{n(a-c)}{b_t(n+1)}; \tag{4}$$

$$P^* = \frac{a+nc}{n+1}. \tag{5}$$

Thus, firm i's profit, $\pi_i^*(b_t,n)$, consumer surplus, $CS^*(b_t,n)$ and total surplus, $TS^*(b_t,n)$ are determined, respectively, as

$$\pi_i^*(b_t,n) = \frac{(a-c)^2}{b_t(n+1)^2}; \tag{6}$$

$$CS^*(b_t,n) = \frac{n^2(a-c)^2}{2b_t(n+1)^2}; \tag{7}$$

$$TS^*(b_t,n) = \frac{n(n+2)(a-c)^2}{2b_t(n+1)^2}. \tag{8}$$

From (8), we see that for a given b_t, total surplus increases with n, and static deadweight loss decreases monotonically, converging to zero as $n \to \infty$.

4. Entry under Uncertainty: Private Incentives

If a firm i enters when the state variable equals b_t, it obtains a perpetual profit flow $\pi_i^*(b_t,n)$. Present discounted value of entry for an individual firm evaluated at the entry time is therefore $V_P(b_t) = \frac{\pi_i^*(b_t,n)}{\rho+\mu} - I$, which is rewritten as

$$V_P(b_t) = \frac{K_P}{b_t} - I, \tag{9}$$

where

$$K_P = \frac{(a-c)^2}{(n+1)^2(\rho+\mu)}. \tag{10}$$

Prior to entry, on the other hand, firm i holds an option to wait. The characteristic equation is obtained from the homogeneous part of the Bellman equation:

$$\frac{1}{2}\sigma^2 b_t^2 V'' + \mu b_t V' - \rho V = 0, \tag{11}$$

which yields the standard quadratic equation for the exponent.

Standard real-options arguments imply that the optimal entry policy is characterized by a threshold b_P^*: the firm enters when $b_t \le b_P^*$.

Let $\underline{\beta}$ denote the negative root of

$$\frac{1}{2}\sigma^2 \beta(\beta-1) - \mu\beta - \rho = 0, \tag{12}$$

which is

$$\underline{\beta} = \frac{\frac{1}{2}\sigma^2 + \mu - \sqrt{\left(\frac{1}{2}\sigma^2 + \mu\right)^2 + 2\rho\sigma^2}}{\sigma^2}, \tag{13}$$

Then the privately optimal entry threshold is

$$b_P^* = \frac{K_P}{I}\left(1 + \frac{1}{\underline{\beta}}\right). \tag{14}$$

We assume that $b_P^* > 0$, which requires $\underline{\beta} < -1$; for this purpose, we impose $\rho > \mu + \sigma^2$.

From (14), we see that an increase in the number of firms reduces K_P and therefore lowers b_P^*, implying that firms optimally delay entry as competition intensifies.

5. Socially Optimal Entry Timing

From a social perspective, entry generates total surplus, $TS^*(b,n)$, each period but requires the payment of the total sunk cost nI. Present discounted value of social surplus evaluated at the entry time is $V_S(b_t) = \frac{TS^*(b_t,n)}{\rho+\mu} - nI$, which is rewritten as

$$V_S(b_t) = \frac{K_S}{b_t} - nI, \tag{15}$$

where

$$K_S = \frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu)}. \tag{16}$$

Applying the same optimal stopping logic, the socially optimal entry threshold b_S^* is determined as

$$b_S^* = \frac{K_S}{nI}\left(1+\frac{1}{\underline{\beta}}\right). \tag{17}$$

A direct comparison of (14) and (17) yields

$$b_S^* = \frac{n+2}{2} b_P^*. \tag{18}$$

From (18), we have the following lemma.

Lemma. $b_S^* > b_P^*$ *for any* $n \geq 1$.

This lemma implies that society prefers entry to occur earlier (at a higher value of b) than firms do privately.

6. Dynamic Welfare Loss from Delayed Entry

The welfare loss associated with delayed entry is defined as the difference between sum of discounted present value of social welfare after entry occurs at the socially optimal threshold and sum of discounted present value of social welfare after entry occurs at the privately optimal threshold.

When entry occurs at $b_t = b_S^*$, the present discounted value of total surplus thereafter, evaluated at the time $b_t = b_S^*$, is given by

$$W_S\left(b_S^*\right) = \frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu)b_S^*} - nI, \tag{19}$$

which is equivalent to (15) with (16) and $b_t = b_S^*$ substituted into it.

On the other hand, when entry occurs at $b_t = b_P^*$, the present value of total surplus thereafter, evaluated also at the time $b_t = b_S^*$, is given by

$$W_P\left(b_P^*, b_S^*\right) = \left(\frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu)b_P^*} - nI\right)\left(\frac{b_S^*}{b_P^*}\right)^{\underline{\beta}}, \tag{20}$$

where $\underline{\beta}$ is the parameter defined in (13).

Therefore, the difference between (19) and (20) is

$$\begin{aligned} W_S\left(b_S^*\right) - W_P\left(b_P^*, b_S^*\right) = {} & \frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu)b_S^*} - nI \\ & -\left(\frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu)b_P^*} - nI\right)\left(\frac{b_S^*}{b_P^*}\right)^{\underline{\beta}}, \end{aligned} \tag{21}$$

which is rewritten as

$$W_S\left(b_S^*\right) - W_P\left(b_P^*, b_S^*\right) = \frac{n(a-c)^2}{(n+1)^2\rho} b_P^* \left(1-\left(\frac{n+2}{2}\right)^{\underline{\beta}+1}\right) - nI\left(1-\left(\frac{n+2}{2}\right)^{\underline{\beta}}\right). \tag{22}$$

by using (18).

Substituting (10) and (14), and introducing the notation

$$\alpha = -\underline{\beta}, \tag{23}$$

we obtain

$$W_S\left(b_S^*\right)-W_P\left(b_P^*,b_S^*\right)=nI\frac{\alpha-1}{\alpha}\left[1-\alpha\left(\frac{2}{n+2}\right)^{\alpha-1}+\left(\alpha-1\right)\left(\frac{2}{n+2}\right)^{\alpha}\right]. \tag{24}$$

Here, let

$$g\left(n\right)=1-\alpha\left(\frac{2}{n+2}\right)^{\alpha-1}+\left(\alpha-1\right)\left(\frac{2}{n+2}\right)^{\alpha}, \tag{25}$$

and let the right-hand side of (24) be denoted as a function of n by $f\left(n\right)$. Then we have

$$f\left(n\right)=\frac{\left(\alpha-1\right)I}{\alpha}n\,g\left(n\right), \tag{26}$$

and therefore,

$$f\left(0\right)=0, \tag{27}$$

$$f'\left(n\right)=\frac{\left(\alpha-1\right)I}{\alpha}\left(g\left(n\right)+ng'\left(n\right)\right). \tag{28}$$

Here,

$$g'\left(n\right)=\frac{\alpha\left(\alpha-1\right)n}{\left(n+2\right)^2}\left(\frac{2}{n+2}\right)^{\alpha-1}, \tag{29}$$

and thus, for $n>0$ and $\alpha>1$,

$$g'\left(n\right)>0. \tag{30}$$

On the other hand, let $t=\frac{2}{n+2}$. Then for $n>0$, we have $0<t\le 1$, and $g\left(n\right)$ can be written as

$$g\left(t\right)=1-\alpha t^{\alpha-1}+\left(\alpha-1\right)t^{\alpha}. \tag{31}$$

Now,

$$g'\left(t\right)=\alpha\left(\alpha-1\right)t^{\alpha-2}\left(t-1\right)\le 0, \tag{32}$$

so $g\left(t\right)$ is monotonically decreasing on $0<t\le 1$. Moreover,

$$g\left(1\right)=1-\alpha+\left(\alpha-1\right)=0. \tag{33}$$

Therefore, for $0<t\le 1$, that is, for $n>0$,

$$g\left(n\right)\ge 0. \tag{34}$$

Hence,

$$f'\left(n\right)=\frac{\left(\alpha-1\right)I}{\alpha}\left(g\left(n\right)+ng'\left(n\right)\right)>0 \tag{35}$$

is obtained.

Thus, we have the following proposition.

Proposition. While static deadweight loss vanishes as n increases, the dynamic welfare loss arising from delayed entry becomes more severe. More pre-

cisely, when welfare loss is evaluated as the difference between social value under entry at the socially optimal threshold b_S^* and social value under entry at the privately optimal threshold b_P^*, its absolute magnitude is strictly increasing in n.

The Cournot limit theorem concerns static allocative efficiency: for a given market, increasing the number of firms reduces price distortions and deadweight loss. The welfare loss identified in this paper is fundamentally different. It is intertemporal and arises from delayed market creation under uncertainty and irreversibility.

Stronger competition reduces private entry incentives more rapidly than it increases social benefits, because firms do not internalize consumer surplus. Consequently, competition mitigates static inefficiency while amplifying dynamic inefficiency. The two effects coexist and operate in opposite directions.

7. Excess Entry in the Present Setting

In this section, we show that the excess entry theorem holds in the setting of this paper.

Let n denote the number of firms. Suppose that all firms enter when $b_t = b_P^*$. Let $V(n)$ be the present discounted value of total surplus evaluated at the initial state $b_t = b_0$. Then,

$$V(n) = \left(\frac{n(n+2)(a-c)^2}{2(n+1)^2(\rho+\mu) b_P^*} - nI \right) \left(\frac{b_0}{b_P^*} \right)^{\underline{\beta}}, \tag{36}$$

where $\underline{\beta}$ is the parameter defined in (13).

Substituting (10) and (14) into (36) yields

$$V(n) = \left(\frac{n(n+2)\rho I}{2(\rho+\mu)} \frac{\underline{\beta}}{\underline{\beta}+1} - nI \right) \left(b_0 \cdot \frac{(n-1)^2 \rho I}{(a-c)^2} \cdot \frac{\underline{\beta}}{\underline{\beta}+1} \right)^{\underline{\beta}}. \tag{37}$$

Define the positive constant (independent of n)

$$L = I \left(b_0 \cdot \frac{\rho I}{(a-c)^2} \cdot \frac{\underline{\beta}}{\underline{\beta}+1} \right)^{\underline{\beta}}. \tag{38}$$

Then

$$V(n) = L n \left(\frac{\rho}{2(\rho+\mu)} \cdot \frac{\underline{\beta}}{\underline{\beta}+1} (n+2) - I \right) (n+1)^{2\underline{\beta}}. \tag{39}$$

Let

$$f(n) = n \left(\frac{\rho}{2(\rho+\mu)} \cdot \frac{\underline{\beta}}{\underline{\beta}+1} (n+2) - I \right) (n+1)^{2\underline{\beta}}. \tag{40}$$

Maximizing $V(n)$ is equivalent to maximizing $f(n)$. Taking logs and differentiating yields

$$\frac{f'(n)}{f(n)} = \frac{1}{n} + \frac{A}{An + 2A - 1} + \frac{2\underline{\beta}}{n+1}, \tag{41}$$

where

$$A = \frac{\rho}{2(\rho + \mu)} \cdot \frac{\underline{\beta}}{\underline{\beta} + 1}. \tag{42}$$

The first-order condition $f'(n) = 0$ implies

$$\frac{1}{n} + \frac{A}{An + 2A - 1} + \frac{2\underline{\beta}}{n+1} = 0, \tag{43}$$

which can be rewritten as

$$\rho\underline{\beta}(\underline{\beta}+1)n^2 + (\underline{\beta}+1)((2\underline{\beta}+1)\mu - \rho)n + (\underline{\beta}+1)\mu - \rho = 0. \tag{44}$$

Hence, the socially optimal number of firms is determined as

$$n^* = \frac{-(\underline{\beta}+1)((2\underline{\beta}+1)\mu - \rho) + \sqrt{\Delta}}{2\rho\underline{\beta}(\underline{\beta}+1)}, \tag{45}$$

where

$$\Delta = (\underline{\beta}+1)^2((2\underline{\beta}+1)\mu - \rho)^2 - 4\rho\underline{\beta}(\underline{\beta}+1)((\underline{\beta}+1)\mu - \rho). \tag{46}$$

Therefore, free entry (i.e., $n \to \infty$) is not optimal. We conclude that the excess entry theorem holds in the present setting.

8. Conclusion

This paper has analyzed irreversible market entry under uncertainty in a Cournot environment. Using a linear inverse demand system and constant marginal costs, we derived closed-form expressions for privately and socially optimal entry thresholds. The analysis reveals a systematic wedge between private and social entry incentives, reflecting the fact that firms internalize only their own profits, whereas society also values consumer surplus generated by entry.

Unlike the preemption literature (e.g., Fudenberg & Tirole, 1985; Grenadier, 1996), we abstract from strategic asymmetries and focus on symmetric threshold behavior. Incorporating preemption would likely lead to earlier entry and reduce the delay identified here.

A key result is that intensified competition has opposing effects on static and dynamic efficiency. Consistent with the Cournot limit theorem, an increase in the number of firms reduces static deadweight loss and brings market outcomes closer to perfect competition. At the same time, stronger competition lowers private entry incentives more rapidly than social benefits, thereby widening the gap between socially and privately optimal entry times. This gap gives rise to a dynamic welfare loss associated with delayed entry, which is conceptually distinct from static allocative inefficiency.

The paper therefore highlights that competition can simultaneously improve static efficiency and worsen dynamic efficiency when entry is irreversible and subject to uncertainty. This insight suggests that welfare evaluations based solely on static considerations may be incomplete. Competition policy, deregulation, and market-opening reforms should take into account their impact on entry timing, as intensified competition may exacerbate welfare losses arising from socially inefficient delays. Future research could extend the analysis to settings with endogenous firm numbers, strategic preemption, or open-economy interactions.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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