Scientific Research Publishing

Theoretical Economics Letters, 2025, 15(6), 1456-1469
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

A Mathematical Formulation of the Valuation of Gold as an Inflation Hedge in the United States

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How to cite this paper: Abraham, R. (2025). A Mathematical Formulation of the Valuation of Gold as an Inflation Hedge in the United States. *Theoretical Economics Letters, 15,* 1456-1469.
https://doi.org/10.4236/tel.2025.156081

Received: July 25, 2025
Accepted: December 8, 2025
Published: December 11, 2025

Abstract

The literature has long viewed gold as an inflation hedge. The reasoning is that when inflation rises due to macroeconomic fluctuations, volatility in the financial markets fails to satisfy investor fears of instability in asset prices. Investors are attracted to gold due to its ability to hold its value during periods of market turbulence. The purpose of this paper is to value gold mathematically in terms of its ability to serve as an inflation hedge, and its value from an investor's risk-taking propensity. The paper assesses the perceptions of three types of investors about gold prices. Risk-averse investors are modeled as adopting the Arrow-Pratt coefficient of risk-aversion. Moderate risk-takers' expectations of gold prices are modeled as using Laplace transform to modify gold price expectations as additional information is received. Risk-takers' gold price expectations are modeled as an exponential distribution. As gold prices spike during periods of uncertainty, such as the financial crisis of 2007-2008 and the ten-month high interest rate-period of 2023-2024, this paper models gold prices as a Levy jump process, with jumps representing periods of surges in gold prices. Empirical validation of gold price changes over time are included to support the theoretical formulations. The paper makes the contributions of 1) providing a mathematical basis for the movement of gold prices over time, 2) including investor risk-taking as a determinant of gold prices, and 3) empirical validation of the theoretical contribution to provide validation of theoretical propositions.

Keywords

Gold Prices, Arrow-Pratt Coefficient of Risk-Aversion, Laplace Transforms, Levy Jump Process, Inflation Hedge, Investor Sentiments

1. Introduction

Gold is frequently viewed as a hedge against inflation. In essence, inflation is the rise in prices due to uncertainty about the future economic environment. Gold, as a commodity, holds its value during periods of macroeconomic uncertainty as the drivers of gold prices, i.e. the supply of gold bullion and the demand for gold are impervious to changes in commodity prices, oil prices, housing prices, and producer prices such as steel, which influence the prices of other goods. The supply of gold depends on the quantity of gold mined in a few lotions, such as South Africa. The demand for gold depends on the demand for gold by central banks to maintain reserves, and the use of gold in jewelry-making. Xu et al. (2021) used a stationary test with a flexible Fourier function to analyze the price of gold over time. Over a 20-year period, gold prices deviated from a long-run average rate over short 1 - 3 month time horizons. However, despite these deviations, gold reverted to the long-run hedge rate Their conclusion was that gold has maintained its purchasing power over 39 years, prior to 2021. It follows that gold is a reliable hedge against inflation, so that investors must hold some gold in their portfolios to diversify inflation risk. Other research has supported the ability of gold to serve as an effective inflation hedge. Using an ARDL framework, and LSTM deep learning, Binh (2024) found a strong positive relationship between gold and inflation during inflationary periods, indicating gold's ability to protect purchasing power during periods of high inflation. This long-term correlation of gold with inflation was observed by Chiang et al. (2023) and McCowan and Zimmermann (2010).

There is a paucity of research that values gold mathematically. The aforementioned literature examines gold prices over time, establishing its predictors and criteria, empirically. Gold prices from U. S. sources are collected over time, followed by fitting to econometric models using cointegration techniques, such as ARDL. The purpose of this paper is to proceed beyond the fitting of gold prices to time, to create the underlying conceptualization of gold prices in mathematical models. This paper views gold prices as being driven partly by investor sentiment. We examine three types of investor sentiment, based upon risk-taking propensity. Risk-averse investors abhor risk. Their conservatism shows in investments in low-yielding default free U.S. Treasury securities, repurchase agreements, and municipal bonds. They may hold large amounts of gold of up to 30% of their portfolios, as they fear the adverse impact of inflation on their portfolio returns. Risk-aversion, or dislike of risk, may lead to the precautionary motive of holding excess gold to protect the purchasing power of their portfolios from inflation. Moderate risk-takers are willing to accept a modest amount of risk as long as it boosts their portfolio returns. They hold riskier investments than risk-averse investors, such as blue-chip stocks with increasing dividends, and investment-grade bonds rated as AAA, i.e. bonds that have never defaulted on interest payments to bondholders. They hold modest amount of gold, of about 20% to reduce the adverse impact of inflation on portfolio returns. Risk-takers form the third type of investor. These individuals are not concerned about risk. Their portfolios contain risky commercial paper which is unsecured debt offering high short-term returns. As long-term investments they may hold bonds that may have defaulted in the past, i.e. BBB or lower grade, with the expectation of earning high returns. They may purchase risky securities in small firms, which promise high returns. As they are less concerned about inflation, their portfolio allocation to gold may be as low as 10%.

To summarize, this paper makes three significant contributions. The first contribution is the creation of three mathematical models that measure the optimum value of gold as the maximum value of gold at the intersection of investor expectations and gold prices. The second contribution is the empirical validation of the valuation of gold using macroeconomic data to support the mathematical models. The third contribution is the recognition that investor sentiment can drive gold prices, i.e., that commodity prices can be based on investor expectations.

2. Literature Review

2.1. A Stable Storehouse of Value

The value of gold in the literature is at the intersection of the supply of gold and the demand for gold. The intersection of these two forces establishes the price of gold at a point in time. Baur et al. (2021) describe the price of gold at a point in time as a stable ratio. Since time immemorial, gold has been a store of value. In the ancient world, gold coins and gold bullion were frequently used as a medium of exchange. Gold coins were traded for property, clothes, utensils, and livestock, thus acting as a medium of exchange. Gold bullion was used in trade, as traders exchanged it for spices, silk and luxury items along ancient trade routes. Gold emerged as a safe haven which held its value during declines in the value of cash, securities, and bonds. Both the store of value and safe haven characteristics of gold must be considered in the context of the value of alternate assets such as securities and bonds. Gold prices decline during periods in which other assets lose value. However, gold prices decline by less than the prices of alternative assets. Thus, gold maintains its position as a store of value. Investors who observe the rapid decline in prices of alternate investment will be attracted to gold which loses less value than alternate investments.

In the modern era, macroeconomic turbulence has characterized most economies. In the United States, two world wars, the Vietnam war, alternate boom and bust periods, and the rise in the volatility of security prices have adversely influenced the value of capital market assets. The dividend discount model is used in the valuation of stocks as the present value of a stream of future dividends. Stocks are considered to be more valuable if they have a stream of rising dividends, which act as a proxy for future cash flows. Yet, dividends may be uncertain given that they emerge from future cash flows which may be subject to declining corporate sales and rising business expenses. Tariffs, materials shortages, labor shortages, and product obsolescence may reduce future corporate profits and dividends. Gold is not subject to these conditions. The stream of cash flows from gold depends on the supply of gold from mines, and the demand for gold as an inflation hedge. Campbell and Shiller (2001) have shown that over a 20-year period, gold prices could be accurately modeled as a stream of future cash flows. If gold prices fluctuated in the short-term, they reverted to the mean in the long-term. This mean reversion suggests that gold prices do not significantly move away from mean long-term values. This fining suggests long-term stability in gold prices.

2.2. Gold as a Hedge Against Inflation

Inflation is uncertainty about future consumer prices and producer prices. The sudden rise in prices due to inflation results in consumers and producers paying higher prices for goods. The literature supports gold's ability to protect against rising prices due to inflation, as gold prices rise with inflation. The rise in gold prices with inflation permits gold to retain its value with rising prices unlike a portfolio of goods and services, which lose value (Baur et al., 2021). Baur et al. (2021) undertook a detailed assessment of gold prices over time by relating gold prices to oil, agricultural commodities, and the CPI (Consumer Price Index, which measures inflation). They found that the gold/oil ratio was remarkably stable in comparison to gold ratios with other commodities. This suggests that gold prices movement synchronously with oil prices, or that prices of the two commodities are not only mutually related, but that the stability of this ratio over time suggests that they are not affected by fluctuations in the prices of other commodities or changes in the CPI, which measures inflation. Thus, gold acts as a hedge against inflation.

Investors who hold gold in their portfolios are less likely to be affected by changes in inflation or inflationary expectations. Jastram and Leyland (2009), Beckmann and Czudaj (2013), and Batten et al. (2014) found a robust link between gold prices and inflation, supporting the thesis that gold is an inflationary hedge. Jastram's (2009) historical assessment of gold prices over four centuries underscored its safe haven status during surges in inflation. Batten et al. (2014) extended the earlier finding of gold being correlated with inflation to gold price changes during multiple time periods. They found that gold's relationship with inflation varied during time periods, being predicted by interest rates. During the early 1980s, gold prices were strongly associated with inflation due to high interest rates. Beckmann and Czudaj (2013) compared gold's relationship with inflation using a Markov-switching error correction model, finding that gold was an effective hedge against future inflation in the long-run for four economies. However, gold was significantly more closely associated with inflation in the united States and the United Kingdom from 1970-2011 than the Euro Area and Japan.

2.3. Risk-Aversion: The Concept and Its Relationship to Gold Prices

Risk-Aversion is the propensity of individuals to only accept risk to the extent that there is a guarantee of positive return. The highly risk-averse individual will invest in securities with certain returns, such as Treasury bills. In the event that the investor is in an inflationary environment, he or she will acquire gold or other assets whose prices rise with inflation, thereby protecting the value of the portfolio. This is absolute risk-aversion, or the extensive reduction of risk through the holding of riskless investments. Yet, moderate risk-takers will accept a certain amount of risk to boost returns. In a classical work, Arrow (1965) developed models that relative risk-aversion by varying the amount that the investor places into a risky asset and a riskless asset in a two-asset portfolio. By increasing the amount of the risky asset, the investor earned higher returns, or returns increased with decreasing relative risk-aversion. Pratt (1964) independently arrived at results similar to Arrow with decreasing relative risk-aversion for increasing investment in the risky asset. Cohn et al. (1975) surveyed investors to support the Arrow (1965) and Pratt (1964) finding of decreasing relative risk-aversion. Such individuals have high inheritance expectations. Inheritance expectations are secure in that they offer a secure stream of annuity payments (Guillermeite & Liu, 2022).

It follows that gold is expected to be in the portfolios of risk-averse individuals, who value its ability to retain value in inflationary environments. To a lesser extent, moderate risk-takers will acquire gold for their portfolios to reduce the adverse impact of inflation, as such risk-takers want to limit portfolio risk due to inflation, though to a lesser extent than highly risk-averse investors. Risk-takers are not concerned about inflation, in the quest for high returns, even from risky investments. They will acquire minimal amounts of gold as they lack interest in its inflation-reducing propensities.

4. Findings and Analysis

4.1. The Risk-Averse Investor's Gold Price Function

The risk-averse investor experiences increasing relative risk-aversion in an inflationary environment, leading to increased demands for gold as seen in the *AB* curve. *CD* is the Levy jump process of gold prices that intersects *AB* at point *X*, the optimal price of gold for the risk-averse investor.

Figure 1 depicts the optimal gold price at point *X*, the minimum point of the risk-averse investor's price expectations for gold and the Levy jump process of gold prices. The objective function and constraints of the risk-averse investor's demand for gold are presented below.

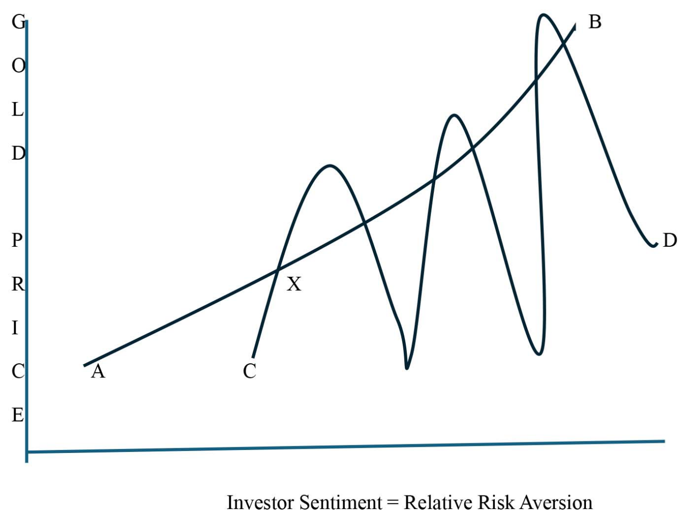

Figure 1. The risk-averse investor's gold price function.

Max

$$\frac{\partial R(c)}{\partial x} \tag{1}$$

Or

$$-u'(c)-[u(c)]u(c)]/u'(c)u'(c) \tag{2}$$

where $u'(c)$ = the change in utility of holding gold as an inflation hedge for $R(c)$, the Arrow-Pratt coefficient of risk-aversion. The Arrow-Pratt coefficient shows decreasing utility of returns due to increased risk. This describes the profile of the risk-averse investor who increases investment in gold as an inflation hedge due to increasing concern about risk.

Subject to,

$$Int\,x_1 + Int\,x_2 + Int\,x_3 > 0 \tag{3}$$

Interest rates are assumed to rise over time with increased inflation. Where *Int* = rising interest rates that influence the value of securities x_1, x_2, x_3 held in the portfolio. The securities may be commodities like silver, agricultural products such as soybeans, or oil. As all of these investments are commodities, the portfolios are commodity portfolios.

$$Inf\,x_1 + Inf\,x_2 + Inf\,x_3 > 0 \tag{4}$$

where *Inf* = rising inflation rates that influence the value of securities held in the portfolio.

$$\forall > 0 \tag{5}$$

Equation (5) is a gradient vector that accounts for short-term fluctuations in gold prices. As gold is a commodity, its price fluctuates in the short-term, with long-term mean reversion. A gradient vector shows the direction of gold price changes, positive and negative, per quarter, so serves as a measure of gold price fluctuations.

$$Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1)\prod dx > 0 \tag{6}$$

Equation (6) is a Levy-Khintchine formula that describes gold prices as a series of discontinuous jumps. Levy jump processes are used, as commodities like gold sharply increase and decrease in price over short period of time. Levy jumps, and by extension, Levy-Khintchine movements are diagrammed as discontinuous jumps, so effectively model gold price changes.

Taking Lagrangians,

$$\begin{aligned} &-u'(c) - \frac{[u(c)]u(c)]}{u'(c)u'(c)} - L_1(Int\,x_1 + Int\,x_2 + Int\,x_3 - 0) \\ &- L_2(Inf\,x_1 + Inf\,x_2 + Inf\,x_3 - 0) - L_3(\forall - 0) \\ &- L_4(Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1)\prod dx - 0) \end{aligned} \tag{7}$$

Taking the first derivative of the Lagrangian function in Equation (7),

$$\begin{aligned}&-u''(c)-\frac{[u'(c)]u'(c)]}{u'(c)u'(c)}-L_1'(Intx_1+Intx_2+Intx_3\\&-L_2'(Inf\,x_1+Inf\,x_2+Inf\,x_3)-L_3'(\forall)\\&-L_4'(Exp(t(ai\theta-0.5\sigma\theta+\int(e^{i\theta x}-i\theta x1_x)\prod dx)\end{aligned}\tag{8}$$

The optimal gold price is the minimum of the intersection of investor risk aversion and the lowest price of gold price jumps, as the risk-averse investor will only purchase gold at the lowest level of prices in order to reduce risk, This optimal point lies at the second derivative of Equation (7).

Taking the second derivative of the Lagrangian function in Equation (7),

$$\begin{aligned}&-u''(c)-\frac{[u''(c)]u''(c)]}{u'(c)u'(c)}-L_1''(Intx_1+Intx_2+Intx_3\\&-L_2''\left(Infx_1+Infx_2+Infx_3\right)-L_3''(\forall)\\&-L_4''(Exp(t(ai\theta-0.5\sigma\theta+(e^{i\theta x}-i\theta x1_x)\prod' dx)\end{aligned}\tag{8}$$

4.2. The Moderate Risk-Taker's Gold Price Function

Figure 2 diagrams the optimal price of gold for the moderate risk-taker at point C, the intersection of the moderate risk-taker's investor sentiment AB, with PQ, the price function. The objective function and constraints are listed in the following equations.

Max

$$\int f(t)e^{-st}dt \tag{9}$$

The objective function of investor sentiment is a Laplace transform that models revisions of expectations of gold prices. The moderate risk-taker gradually increases gold in his or her portfolio as inflation increases, revising expectations of gold prices as information about macroeconomic conditions is received. The revision of gold price expectations is modeled as s in Equation (9), where s is defined as follows,

$$s=\sigma+i\omega \tag{10}$$

The revisions of gold price expectations are modeled as Laplace transforms. The moderate risk-taker is open to revisions as he or she is not strongly in favor of risk-aversion or risk-taking. The revision is modeled by Laplace transforms which take the shape of a backward loop In which additional price information is shown to cause revisions of original portfolio positions. Where s = revision of gold price expectations due to higher inflation,

σ = spot price of gold,

ω = futures price of gold

As gold is a commodity, its price is a combination of the current price and the price over the next 3 months, where ω, the price over the next 3 months, captures fluctuations in prices in the immediate future.

Subject to,

$$Int\,x_1+Int\,x_2+Int\,x_3>0 \tag{11}$$

Where Int = rising interest rates that influence the value of securities x_1, x_2, x_3 held in the portfolio.

$$Inf\,x_1 + Inf\,x_2 + Inf\,x_3 > 0 \tag{12}$$

Where Inf = rising inflation rates that influence the value of securities held in the portfolio.

$$\forall > 0 \tag{13}$$

Equation (13) is a gradient vector that accounts for short-term fluctuations in gold prices

$$Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1) \prod dx > 0 \tag{14}$$

Equation (14) is a Levy-Khintchine formula that describes gold prices as a series of discontinuous jumps.

Taking Lagrangians,

Max

$$\begin{aligned} &\int f(t)e^{-st}dt - L_1(Int\,x_1 + Int\,x_2 + Int\,x_3 - 0) \\ &- L_2(Inf\,x_1 + Inf\,x_2 + Inf\,x_3 - 0) - L_3(\forall - 0) \\ &- L_4(Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1) \prod dx - 0) \end{aligned} \tag{15}$$

Taking the first derivative of Equation (15),

$$\begin{aligned} &f(t)e^{-st}dt - L_1'(Int\,x_1 + Int\,x_2 + Int\,x_3) - L_2'(Inf\,x_1 + Inf\,x_2 + Inf\,x_3) \\ &- L_3'(\forall) - L_4'(Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - i\theta x 1_x < 1) \prod dx) \end{aligned} \tag{16}$$

Taking the second derivative of Equation (15),

$$\begin{aligned} &e^{-st} - L_1''(Intx_1 + Intx_2 + Intx_3) - L_2''(Infx_1 + Infx_2 + Infx_3) - L_3''(\forall) \\ &- L_4''(Exp(t(ai\theta - 0.5\sigma\theta + (e^{i\theta x} - i\theta x 1_x) \prod' dx) \end{aligned} \tag{17}$$

Equation (17) is the optimal price function for the moderate risk-taker, or point C in **Figure 2**, at which the moderate risk-taker maximizes returns from gold price investments.

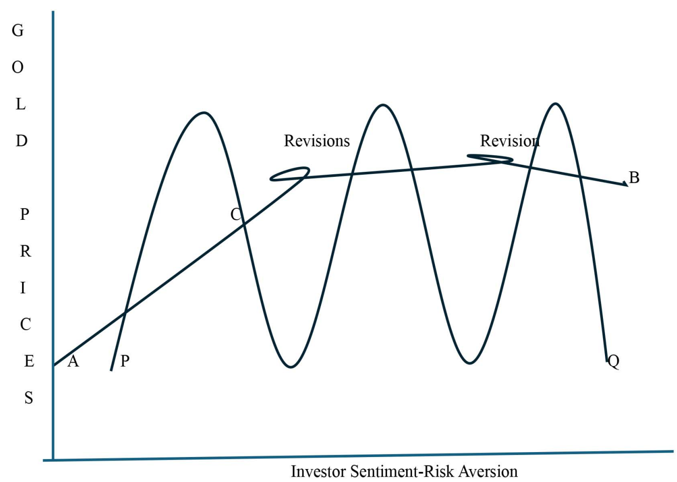

Figure 2. The moderate risk-taker's gold price function.

Figure 2 shows the optimal gold price for the moderate-risk taker at point C, the minimum point of the intersection of the Laplace transform of investor sentiment with the Levy jump process of gold prices.

4.3. The Risk-Taker's Price Function

The risk-taker is not concerned about risk, as he or she simply wishes to earn high returns. This paper models the risk-taker's sentiment as an exponential distribution, with increasing expectations of gold prices at a low level of risk-aversion. Figure 3 shows the optimal price *B* for the risk-taker at the intersection of the exponential distribution of investor sentiment and the gold price distribution of the Levy jump process.

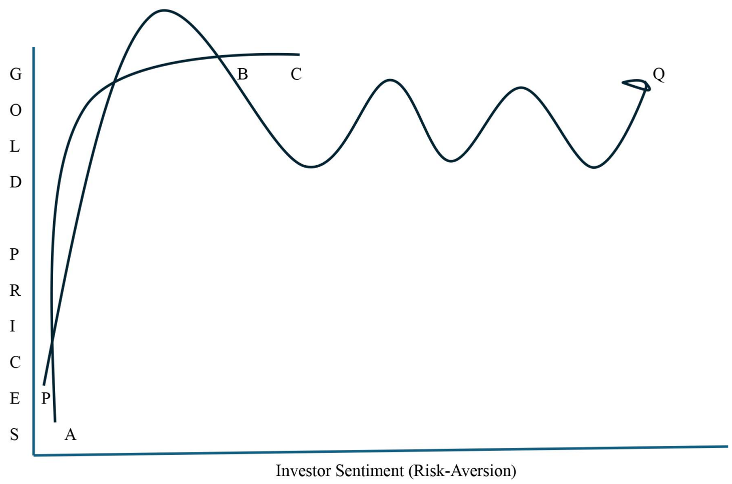

Figure 3. The risk-taker's optimal gold price function.

Figure 3 shows the optimal gold price for risk-taker at the intersection of investor price expectations *AB* and the Levy jump process *PQ*. The optimal price is represented by the point *B*.

The objective function and constraints of the risk-taker's formulation are given below,

Max

$$\lambda_k / (\lambda_1 \cdots \lambda_n) \tag{18}$$

Where $X_1 \ldots X_n$ are independently distributed random variables with parameters, $\lambda_1 \ldots \lambda_n$, that model the exponential distribution of risk-taker sentiments. The risk-taker is willing to accept more risk in the hope of earning higher returns. Figure 3 shows that the near-vertical positioning of increased return without consideration of risk is modeled by the exponential distribution, *AB*.

Subject to,

$$Int\,x_1 + Int\,x_2 + Int\,x_3 > 0 \tag{19}$$

Where *Int* = rising interest rates that influence the value of securities x_1, x_2, x_3 held in the portfolio.

$$Inf\,x_1 + Inf\,x_2 + Inf\,x_3 > 0 \tag{20}$$

Where *Inf* = rising inflation rates that influence the value of securities held in the portfolio.

$$\forall > 0 \tag{21}$$

Equation (21) is a gradient vector that accounts for short-term fluctuations in gold prices

$$Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1)\prod dx > 0 \tag{22}$$

Equation (22) is a Levy-Khintchine formula that describes gold prices as a series of discontinuous jumps.

Equation (23) is a Kullback-Leibler divergence to account for the short-term fluctuation in gold prices. This divergence is only modeled for the risk-taker as the risk-taker accepts the fluctuation without revision of investor sentiment. The risk-taker is unconcerned about risky fluctuations in gold prices, seeking only higher returns. Gold prices diverge from the mean in the short-term with the first two terms in Equation (23) showing the extent of divergence from the mean price.

$$\log(\lambda_0) - \log(\lambda) + \frac{\lambda}{\lambda_0} - 1 > 0 \tag{23}$$

$$-u'(c) - \frac{[u(c)]u(c)]}{u'(c)u'(c)} = 0 \tag{24}$$

As risk-aversion does not influence the risk-taker's expectations of gold prices, the utility of risk-aversion in Equation (24) = 0.

Taking Lagrangians,

$$\begin{aligned}&\frac{\lambda_k}{\lambda_1 \cdots \lambda_n} - L_1[Int\,x_1 + Int\,x_2 + Int\,x_3 - 0] - L_2[Inf\,x_1 + Inf\,x_2 + Inf\,x_3 - 0]\\ &- L_3[\forall - 0] - L_4[Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1)\prod dx > 0\\ &- L_5[\log(\lambda_0) - \log(\lambda) + \frac{\lambda}{\lambda_0} - 1 > 0] - L_6[-u'(c) - \frac{[u(c)]u(c)]}{u'(c)u'(c)} - 0]\end{aligned} \tag{25}$$

Taking the first derivative of Equation (25),

$$\begin{aligned}&\frac{\lambda_k}{\lambda_1 \cdots \lambda_n} - L_1'[Int\,x_1 + Int\,x_2 + Int\,x_3] - L_2'[Inf\,x_1 + Inf\,x_2 + Inf\,x_3]\\ &- L_3'[\forall] - L_4'[Exp(t(ai\theta - 0.5\sigma\theta + \int (e^{i\theta x} - 1 - i\theta x 1_x < 1)\prod dx > 0\\ &- L_5'[\frac{1}{-}(\lambda_0) - \frac{1}{\lambda} + \frac{\lambda}{\lambda_0} - 1 > 0] - L_6'[-u'(c) - \frac{[u(c)]u(c)]}{u'(c)u'(c)}]\end{aligned} \tag{26}$$

Taking the second derivative of Equation (25),

$$\begin{aligned}&\frac{\lambda_k}{\lambda_1 \cdots \lambda_n} - L_1''[Int\,x_1 + Int\,x_2 + Int\,x_3] - L_2''[Inf\,x_1 + Inf\,x_2 + Inf\,x_3]\\ &- L_3''[\forall] - L_4''[Exp(t(ai\theta - 0.5\sigma\theta + (e^{i\theta x} - i\theta x 1_x)\prod dx\\ &- L_5''[\frac{1}{-}(\lambda_0) - \frac{1}{\lambda} + \frac{\lambda}{\lambda_0}] - L_6''[-u''(c) - \frac{[u'(c)]u'(c)]}{u''(c)u''(c)}]\end{aligned} \tag{26}$$

Equation (26) is the optimal price function for the risk-taking investor in gold.

5. Empirical Validation

5.1. Data Collection and Analysis

World Gold Council data provided the prices and volumes of gold stocks traded in the United States and Canada. Thirty-five gold stocks were selected with prices and volumes measured. The Bureau of Economic Analysis provided US inflation, interest rates (the federal funds rate), and GDP for 2025. A single year's values were used in a cross-sectional analysis. The following regressions were tested,

The relationship tested was if an increase in risk aversion increased positively with inflation. Reverse causality exists here. Risk-averse investors anticipate inflation. They protect their portfolios by adding gold as an inflation hedge. Subsequently, when inflation raises prices, risk-averse investors add more gold to their portfolios to further immunize their portfolios from rising prices.

$$\text{Inflation} = a + b_1 \text{av} + b_2 \text{interest rate} + b_3 \text{gdp} \tag{27}$$

Where av = aversion, rated as 1 for risk-takers, 2 for moderate risk-takers, and 3 for risk-averse investors.

The second regression tested if risk aversion increased gold prices as investors who are more risk-averse will purchase a large volume of gold. The larger volume will increase gold prices which are a proxy for inflation.

$$\text{Price} = a + b_1 \text{volume} + b_2 \text{inflation} + b_3 \text{interest} + b_4 \text{gdp} \tag{28}$$

In essence, two proxies for risk aversion, av in Equation (1), and volume in Equation (2), are used to test if gold is an inflation hedge.

Table 1 shows descriptive statistics of key variables.

Table 1. Descriptive statistics of key variables.

Variable	Mean	Variance	Maximum	Minimum	Skewness	Kurtosis
Price	21.22	21.44	225.79	0.00	3.47	12.92
Volume (thousands)	1135	380.02	365,520	0.00	5.90	34.90
Risk-Aversion	2.11	0.39	3	1	−0.08	−0.35
Inflation	2.70	0.05	4.0	2.5	0.35	−1.81
Interest rate	4.34	0.46	4.87	2.3	−0.89	−0.70
Gross Domestic Product (in billions)	13,471	21,490	29,184	239	0.17	−2.09

Table 2 is a correlation matrix of key variables.

Table 2. Correlation matrix of key variables.

Variable	Price	Volume	Risk-Aversion	Inflation	Interest Rate	Gross Domestic Product
Price	1					
Volume	0.53	1				
Risk-Aversion	0.43	0.24	1			
Inflation	0.41	0.20	0.95	1		

Continued

Interest Rate	0.43	0.15	0.97	0.97	1	
Gross Domestic Product	0.22	0.73	0.73	0.73	0.57	1

5.2. Regression Results

Table 3 reports the results of two regressions. In the first regression the aversion measure did not significantly influence inflation (coefficient = 0.011, $p < 0.50$). In the second regression, volume of gold prices (proxy for risk-aversion) significantly influenced gold prices (proxy for inflation) with t value = 4.65, $p < 0.001$). Therefore, we can surmise that gold is a hedge against inflation.

Table 3. Results of regressions of inflation on investor sentiment, and regressions of gold prices on investor sentiment.

Variable	Coefficient	Significance
Regression 1 Dependent variable = inflation		
Constant	2.34	0.00
Risk-Aversion	0.01	0.50
Interest Rate	0.02	0.52
GDP	0.00001***	0.00
Regression 2 Dependent Variable = Price		
Constant	−68.26	0.29
Volume	4.65	0.00
Inflation	21.64	0.34
Interest Rate	8.08	0.09
GDP	−0.0006	0.25
N = 35, R^2 0.99 (Regression 1, 0.43 Regression 2)		

6. Conclusion

6.1. Discussion of Results

This paper has provided three mathematical formulations of gold price distributions as a function of investor risk-aversion. Three types of investors were identified. Risk-averse investors showed increasing risk aversion with inflation. They increased investment in gold as an inflation hedge as gold prices increased due to the strong correlation of gold prices with inflation (Binh, 2024). A caveat is that the categorical measure of risk aversion failed to find that gold was an inflation hedge, even though the volume measure of risk-aversion supported the notion of gold as an inflation hedge. Moderate risk-takers increased gold holdings with less frequency with inflation. Risk-takers were not concerned about gold's inflation protection properties, preferring to invest minimal amounts of gold for inflation protection. We may surmise that gold is an effective hedge against inflation in keeping with the findings of earlier studies (Binh, 2024; Chiang et al., 2023; McCowan & Zimmerman, 2010).

Several mathematical insights were obtained. The Levy jump process was used to reflect the short-term price volatility of gold as gold prices may fluctuate significantly in the short-term, while reverting to the mean in the long-term. Laplace transforms were used to model the moderate risk-taker's price expectations, as they permit revision of expectations as inflation information becomes available. Laplace transforms are a sophisticated tool that easily fit the formulation of the moderate risk-taker's price expectations.

Two measures of risk-aversion were employed. The more robust measure of risk-aversion, i.e. gold volume, significantly explained gold prices (proxy for inflation). The less robust measure of measuring risk-aversion as volume segments of 1 = risk-taking, 2 = moderate risk-taking, and 3 = risk-averse did not effectively explain inflation, so may not be used.

6.2. Practical Implications

Portfolio managers may add gold securities to portfolios to immunize portfolios from inflation. Many retirees are finding that their retirement portfolios did not grow significantly during their working years, so that their incomes during retirement are decreased. The addition of gold in 10% or 20% increments may protect these portfolios as returns from gold will rise from inflation. Currently, mutual funds do not hold significant levels of gold securities. Fund managers may increase holdings of U.S. and Canadian gold stocks to promote inflation protection.

Gold may form a part of long-term portfolios, due to its property of mean-reversion of prices. This suggests that gold investments promote long-term stability of a portfolio. Future research should replicate the current study for long-term gold prices to verify the mean0reversion of gold prices.

6.3. Research Limitations

Current market conditions in the United States and Canada were employed in the empirical validation of the mathematical formulations. Other countries may or may not have the same inflation protections of gold. In countries such as India, considerable amounts of gold are employed in jewelry-making for weddings and festivals. Demand and supply of gold in these countries may include additional dimensions that were not assessed in this study. Future research should replicate this study for other countries.

Central banks hold gold reserves in a variety of countries. Future research should formulate supply and demand functions for gold reserves owned by central banks.

Gold is but a single precious metal. Silver and platinum are also precious metals. Future research should replicate the study for these precious metals.

Gold usage has been viewed from the perspective of investors purchasing gold securities. Alternatively, gold is employed in industrial production. Supply and demand factors for gold usage in industrial production must be considered in future research.

The use of a cross-sectional design must be replaced by time series in future research. Levy jump processes, for example describe dynamic time series, which conform less to a cross-sectional design.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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