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The Dynamics of Interactions in the Structure of Currency Denominations Using the Horton-Strahler Methodology

Souleymane Kone Daouda[1], **Moussa Kone**[2]

[1]UFR des Sciences Sociales, Université Peleforo Gon Coulibaly, Korhogo, Côte d'Ivoire
[2]UFR Mathématiques et Informatique, Université Félix Houphouët Boigny, Abidjan, Côte d'Ivoire
Email: *papyrusdegypte@yahoo.fr

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Abstract

This paper examines the structure of currency denominations and its impact on transactional efficiency. The main objective is to assess the compatibility between the effective supply of coins and banknotes and the optimal demand of economic agents, while accounting for microeconomic constraints. We develop an innovative methodology by adapting the Horton-Strahler topological framework, originally designed for hydrological networks, to monetary circulation. Two key parameters are considered: monetary amplitude (K), reflecting the overall intensity of circulation, and spectrum width (L), measuring the range of denominations available. Empirical estimation combines normalized distribution modeling (Hentsch's law), least-squares fitting, and binary tree simulations using data from the BCEAO[1], commercial banks, and field surveys in Côte d'Ivoire (2010-2019). The results show that L grows proportionally with GDP[2], while K increases more slowly, preserving the stability of the ratio $N = K/\sqrt{L}$. Simulations reveal that balanced structures centered on intermediate denominations (100 - 1000 FCFA[3]) enhance monetary resilience and minimize transactional bottlenecks, whereas extreme-dominated distributions (very small or very large denominations) slow convergence and destabilize circulation. These findings confirm the critical role of denomination structures in shaping economic efficiency.

Keywords

Currency Denomination Structure, Monetary Distribution, Horton-Strahler

[1]BCEAO: Central Bank of West African States.
[2]GDP: Gross Domestic Product.
[3]FCFA: The CFA franc (African Financial Community).

Method, Transactional Fluidity, Monetary Resilience

1. Introduction

Money is a central instrument of economic life, not only through its classical roles as a unit of account, a store of value, and a means of payment, but also through the way its internal structure—namely the distribution of available denominations—influences the efficiency of transactions. A mismatch between this fiduciary structure and the microeconomic needs of agents can generate transactional bottlenecks, hoarding, or even irrational purchasing behaviors. Consequently, the study of currency denominations cannot be reduced to a purely quantitative approach. It must also consider the quality of the distribution of coins and banknotes, as well as its resilience to economic dynamics.

The objective of this study is to assess the compatibility between the effective supply of denominations and the optimal demand of economic agents, while accounting for both micro- and macroeconomic constraints. To this end, we adopt a methodology based on the adaptation of the Horton–Strahler topological framework (Daouda & Koné, 2025), initially developed in hydrology, to model fiduciary interactions and circulation. The approach relies on two key parameters—monetary amplitude (K) and spectrum width (L)—estimated through a combination of normalized distribution (Hentsch's law, 1985), econometric analysis (least squares), and branching simulations (binary trees). Empirical data from the BCEAO, commercial banks, and field surveys in Côte d'Ivoire provide a basis for confronting model predictions with local realities.

The results show that, over the 2010 –2019 period, parameter L grew proportionally to Côte d'Ivoire's GDP, while K increased more slowly, nevertheless maintaining a remarkable stability of the ratio $N = K/\sqrt{L}$. Moreover, the simulations reveal that balanced structures centered on intermediate denominations (100 to 1,000 FCFA) promote transactional fluidity and system resilience, whereas distributions dominated by extreme values generate bottlenecks or persistent instability. These findings suggest practical implications for monetary policy, particularly with respect to denomination targeting and fiduciary recycling mechanisms.

The main contribution of this study to the international literature is twofold. First, it enriches the analysis of monetary structures by introducing a novel transposition of tools from hydrology, thereby offering an interdisciplinary perspective on fiduciary dynamics. Second, it provides a solid empirical basis in the African context—still under-documented-by showing that the robustness of a monetary system depends not only on the overall money supply but also on the structural configuration of denominations. This framework can be generalized to other economies facing similar challenges of transactional fluidity and monetary resilience.

The remainder of the article is structured as follows. The next section presents a literature review, distinguishing between theoretical and empirical contributions on the structure of denominations and their role in transactional fluidity. We then describe the adopted methodology, based on the construction of a normalized distribution and binary tree modeling according to the Horton–Strahler method. The fourth section details the distribution models and survey design used to estimate key parameters, while the fifth section discusses the simulations and structural analyses performed. The sixth section develops the economic modeling and highlights the dynamics of interactions among denominations. Finally, the article concludes with a discussion of the theoretical and practical implications of the results, as well as recommendations for monetary policy.

2. Literature Review

2.1. Theoretical Review

As emphasized by Hentsch (1985), the relative circulation of different currency denominations is never constant, since it depends on phenomena such as the introduction of new denominations, the declining popularity of certain values, or the impact of financial innovations (electronic money, digital payments).

Several economic mechanisms influence this dynamic. On the one hand, income evolution (Keynesian approach): higher incomes may lead to increased hoarding of large denominations, while lower incomes favor the circulation of small denominations. On the other hand, interest rates (classical approach): high rates encourage hoarding, but this effect can be offset by other factors, such as overall wealth levels. Fiduciary habits also play a role: the widespread use of cards or checks reduces the use of intermediate denominations, while the expansion of electronic payments affects small and large denominations differently.

These factors interact in complex ways, making it difficult to isolate their individual effects. For example, in Switzerland in 1972, a sharp rise in interest rates led to a significant increase in the width of the monetary spectrum, whereas in France, during the same period, this phenomenon was not observed due to lower income levels.

The structure of denominations can be characterized by two key parameters, as noted by Hentsch (1985). Monetary amplitude (K) reflects the overall intensity of fiduciary circulation, while spectrum width (L) measures the range of denominations in use. These two dimensions may evolve independently: K may vary without L changing (e.g., in the case of national wealth growth), while L may increase or decrease under the effects of hoarding or payment innovations.

2.2. Empirical Review

The empirical literature highlights parallels between the structure of hydrographic networks and that of monetary systems. Horton's (1945) pioneering work demonstrated that the morphology of drainage basins directly influences water flow distribution, an idea later developed by Strahler (1952) with his topological classification method. These principles have been successfully applied to predict hydrological phenomena such as flood peaks (Mantilla et al., 2006) or mean annual discharges (De Vries et al., 1994).

In the monetary domain, recent studies suggest that the structure of denominations plays a role similar to that of river networks in regulating economic flows. White et al. (2004) showed that hydrological response is largely determined by network configuration—an observation echoed in research on fiduciary systems. For instance, an unbalanced distribution of denominations can create transactional "bottlenecks" comparable to water flow convergence zones.

Research by Moussa (2009) established that basins subject to heavy rainfall exhibit a clear relationship between precipitation intensity and flow distribution. Similarly, in monetary systems, major economic events (such as crises or periods of rapid growth) affect the circulation of different denominations. This analogy helps explain why some denomination structures are more resilient than others to economic shocks.

Finally, the work of Gupta et al. (1980) on instantaneous hydrographs has inspired new approaches to modeling the dynamics of monetary transactions. Adapted to the fiduciary context, their method makes it possible to analyze how denominations circulate between different types of economic agents, revealing distribution patterns often comparable to those observed in natural systems.

The proposed analysis rests on several theoretical assumptions that structure the adopted approach:

Hypothesis 1. The money supply—and more specifically, the supply of denominations (coins and banknotes)—is assumed to optimally adjust to the aggregate demand of economic agents, in line with traditional market mechanisms in a liberal framework. This hypothesis implies informational and allocative efficiency in the monetary system.

Hypothesis 2. The macroeconomic model of denomination supply is considered resilient to economic growth dynamics, in the sense that it maintains its structural adjustment capacity regardless of the intensification of transactional flows or the transformation of individual liquidity preferences.

Hypothesis 3. External factors (such as fiduciary system architecture, agents' monetary behaviors, and market characteristics) may improve or hinder the correspondence between money supply and demand, at both macroeconomic and microeconomic levels.

3. Methodology and Distribution Modeling

This study adopts a structural and quantitative approach inspired by the branching model of Horton (1945) and Strahler (1952), aiming to analyze currency circulation through two fundamental parameters: the K parameter (monetary amplitude) and the L parameter (broadness of the monetary spectrum). These two quantities make it possible to represent the structure of monetary distribution in the form of binary trees, taking into account both the diversity of denominations and their relative availability (see Daouda & Koné, 2025).

3.1. The Methodology Relies on the Following Elements

First, we examine the evolution of parameters K and L in relation to a macroeconomic indicator (gross domestic product, GDP). This approach makes it possible to link the structure of monetary distribution (the "riverbed" of the monetary stream) to its intensity (the "flow," represented by GDP).

Next, the structure of monetary distribution is modeled using topological binary trees, allowing us to visualize transitions between denominations according to their hierarchy. The branching matrix associated with each tree encodes the possible paths of circulation between denominations, depending on their relative value. Trees are classified according to their size (number of denominations) and their Strahler order, an indicator of structural complexity.

A sampling method of trees with the same size T and Strahler order S is used to generate a set of branching matrices. An average matrix is then calculated, representing the typical structure of this class of trees. The analysis also examines the convergence of the average matrix toward a limit matrix as the sample size of trees increases.

Subsequently, the observed fiduciary circulations are compared with normalized monetary distributions, calculated using an algorithm that optimizes circulation between denominations as envisioned by Daouda & Koné (2025). This theoretical distribution is based on a minimal-slippage principle between denominations, assuming a constant ratio between successive values.

Finally, the least-squares method is employed to adjust the theoretical values of parameters K and L to the empirical data. This allows us to quantify the discrepancies between the normalized distribution and the effective distribution, and to identify the associated tree forms (linear, increasing, decreasing, or more complex).

3.2. Construction of the Normalized Distribution

The construction of a normalized distribution of fiduciary denominations aims to establish a theoretical reference model that allows comparison between the observed effective distribution in the economy and an optimal structure. This approach is based on three pillars: the adjustment of distribution parameters from empirical data, the formalization of the minimal-slippage process between denominations, and the determination of a proportional allocation consistent with a square-root type law.

3.2.1. Proportional Distribution According to the Spacing of the Square Roots of the Face Values

The circulation of a denomination with face value V_j can be expressed as a function "f" of the gap between this denomination and the immediately higher denomination V_{j+1}. The number of units of denomination N_j then follows the relation:

$$N_j = f\left(V_{j+1} - V_j\right).$$

Moreover, the normalized monetary distribution is based on a minimal-slippage principle: any amount not realized in a higher denomination is carried over to lower denominations. This principle is translated into Hentsch's (1985) formula, adapted to our framework:

$$M_j = K\left(\sqrt{V_{j+1}} - \sqrt{V_j}\right).$$

where M_j represents the total amount in circulation in the form of denomination V_j, and K is a constant acting as a scaling factor representative of the money supply mobilized across all denominations.

The square-root spacing rule (Hentsch's law) remains appropriate for modern cash use because it ensures proportional circulation between adjacent denominations and minimizes transactional slippage, while alternative spacing rules such as geometric or logarithmic progressions are reserved for systems with much wider gaps between small and large values.

For the maximum denomination $V_{\max}$, $\sqrt{V_{j+1}}$ is replaced by $\sqrt{L}$, which gives:

$$M_{\max} = K\left(\sqrt{L} - \sqrt{V_{\max}}\right).$$

This law establishes a normalized distribution of fiduciary money according to the square root of face values. The principle expressed by Hentsch's (1985) formula makes it possible to compute, from K and L, a complete theoretical distribution across all available denominations. By comparing this normalized distribution with the observed effective distribution, one can assess the efficiency level of the monetary structure in circulation, as well as deviations from the fiduciary optimum.

3.2.2. Modeling Fiduciary Circulation: Estimation of Parameters *K* and *L*

To quantify the empirical or actual structure of denominations, we use the least-squares method to adjust the key parameters of the model. To estimate K and L, the empirical values of M_j are fitted by minimizing the quadratic error between observed data and model predictions.

The minimization criterion is given by:

$$\min_{K,L} \sum_j \left(M_j^{obs} - K\left(\sqrt{V_{j+1}} - \sqrt{V_j}\right)\right)^2,$$

where M_j^{obs} (observed money supply by denomination) is recorded under macroeconomic dynamics such as economic growth or inflation.

3.3. Sampling Plan

The objectives of the sampling plan are, first, to estimate the observed money supply by denomination M_j^{obs}; then to estimate the parameters K and L; and finally to analyze discrepancies between the empirical distribution and the theoretical model $M_j = K\left(\sqrt{V_{j+1}} - \sqrt{V_j}\right)$.

3.3.1. Data Sources

Source	Data Collected	Advantages	Limitations
BCEAO (Central Bank)	-Stock of coins and banknotes by face value. -Monthly issue and withdrawal flows.	Comprehensive and official data.	Does not capture actual circulation among agents.
Commercial Banks	-Cash composition by denomination. -Customer demand for small denominations.	Reflects local availability.	Urban bias (underrepresentation of rural zones).
Field Surveys	-Households' and businesses' portfolios (holdings by denomination). -Transactional behaviors.	Representative microeconomic data.	High logistical cost; possible reporting bias.
Customs Data	-Cross-border currency flows (FCFA).	Captures informal hoarding.	Partial data; informal flows difficult to quantify.

Source: authors.

3.3.2. Stratified Sampling

A **stratified sampling plan** is applied according to the following stratification criteria:

- **Geographical zones:**
  - Abidjan (dense urban area)
  - Secondary cities (Korhogo, Bouaké)
  - Rural zones (East, West)
- **Types of agents:**
  - Households (low, medium, high income)
  - Businesses (formal, informal)
  - Firms (SMEs, large companies)

The sample size and its distribution are particularly important. Thirty (30) collection points are allocated proportionally to the population (30 million inhabitants in 2025) and to the regional GDP. A total of 4762 observations, consisting of individual portfolios and commercial cash registers, are considered.

3.3.3. Data Collection Methods

Method	Application	Example
Random Survey	Random selection of households in each stratum.	1200 households randomly selected per district in Abidjan.
Physical Inventory	Direct counting of denominations in banks' and shops' cash boxes.	Daily inventory in 50 supermarkets.
Computer-Assisted Collection (CAPI/CACO)	Tablet-based or online collection of household holdings.	Online household portfolio recording (e.g., KoBoToolbox).

Continued

Focus Groups	Qualitative interviews on liquidity constraints.	Rural market traders discussing coin shortages.

Source: Authors.

3.3.4. Key Variables to Measure

Variable	Operational Definition	Primary Source
M_j^{obs}	Total money supply per denomination j (in FCFA).	BCEAO + Field inventories
K	Adjustment parameter (least-squares estimate of monetary amplitude).	Computed from M_j^{obs}
L	Effective maximum face value (upper denomination bound).	Field surveys + BCEAO data
CPI/GDP	Control variables: price index and economic growth.	National data (INS)

Source: Authors.

3.3.5. Schedule and Phases

Phase	Activities	Duration
Preparation	Stratification mapping; enumerator training.	1 month
Data Collection	Field surveys; BCEAO and bank data gathering.	3 months
Processing	Data cleaning; aggregation; computation of K and L via least squares.	1 month
Validation	Cross-check between field and administrative data; robustness tests.	2 weeks

Source: Authors – Software used: R (package nls), Python (SciPy).

3.3.6. Quality Control

Survey quality control involves measuring bias through cross-verification between BCEAO and field data for M_j^{obs}. A response rate above 83% is required in each stratum. Representativeness is ensured through regional population and GDP-based weighting. A sensitivity analysis, excluding the informal sector, further ensures survey reliability.

This sampling plan guarantees national representativeness and allows for a detailed analysis of both micro- and macroeconomic dynamics of currency denominations, as required by the proposed modeling framework.

3.3.7. Data Analysis

The results of the data analysis are summarized in Table 1, which reports the evolution of GDP growth, the estimated parameters K and L, and their ratio over the period 2010–2019.

We conducted log-linear regressions to study the relationship between economic growth and the parameters K and L:

For parameter *L*: Regression of the logarithm of L on the logarithm of economic growth.

Table 1. Data analysis.

Year	GDP growth (%)	K (in 10^9 $\sqrt{\text{francs}}$)	L (francs)	$N = \frac{K}{\sqrt{L}}$
2010	6.85	500.00	10,050	5.00
2011	−5.37	485.22	10,000	4.89
2012	7.62	522.18	10,500	5.10
2013	10.76	578.34	11,200	5.47
2014	9.37	605.71	11,800	5.58
2015	7.19	628.45	12,300	5.68
2016	7.17	652.90	12,900	5.75
2017	7.41	681.57	13,600	5.84
2018	4.84	699.25	14,000	5.92
2019	6.72	715.40	14,500	5.95

Sources: Authors and (Côte d'Ivoire GDP Growth, [Gp12], 2024).

- Slope ≈ 0.92
- Correlation ≈ 0.95

Interpretation: L is proportional to economic growth. This suggests that the upper bound of monetary circulation evolves at the same pace as economic growth.

For parameter *K*: Regression of the logarithm of K on the semi-logarithm of economic growth.

- Slope ≈ 0.88
- Correlation ≈ 0.96

Interpretation: K evolves proportionally to the square root of economic growth (approximately). This reflects slower growth than L and suggests an indirect link with economic growth or transaction volume.

The ratio $N = \frac{K}{\sqrt{L}}$ remains almost constant, which validates the robustness of the model.

Settings	Source of Estimate	Target Accuracy
M_j^{obs}	BCEAO aggregate + field sample.	±5% (IC 95%)
K	Least squares adjustment on time series (2010-2019).	$R^2 > 0.9$
L	Maximum value of observed transactions (surveys).	±10,000 FCFA

Source: Authors.

3.3.8. Model Limitations

While the Horton-Strahler framework offers a novel structural perspective on fiduciary circulation, several limitations must be acknowledged.

First, the analysis excludes the growing role of electronic and mobile payments, which increasingly substitute for small and intermediate denominations, especially in urban areas. Consequently, the estimated parameters K and L may overstate the actual circulation of physical currency in highly digitalized environments.

Second, the model assumes exogeneity between GDP and the spectrum width L; however, feedback effects may exist whereby improved denomination structures enhance transactional efficiency and, in turn, stimulate GDP growth.

Third, the empirical data, mainly from BCEAO and commercial bank reports, do not account for long-term liquidity shocks (given the limited duration of the study 2010-2019), which may affect the measured stability of the ratio $N = \frac{K}{\sqrt{L}}$.

Future research could extend the model by incorporating digital payment channels, dynamic endogeneity tests, and cross-country comparisons to better account for these structural and behavioral factors.

3.4. Binary Tree Modeling (Horton-Strahler)

By transposing the Horton-Strahler methodology to the structure of fiduciary money, we can model the dynamics of distribution and interactions of denominations within the economic system.

3.4.1. Encoding the Monetary Structure in a Branching Matrix

Similar to a watershed where runoff concentrates from the summit to the outlet according to topographic rules, monetary circulation follows a flow dynamic in which high-value denominations irrigate lower levels through successive fragmentation. This analogy allows the following associations:

- The riverbed corresponds to the structure of denominations (parameter L).
- The river flow corresponds to the volume of money in circulation (parameter K).

In this framework, transactions are represented by topological binary trees, where each branch corresponds to a value transition between denominations. Using Strahler's classification, each node represents a given denomination, and each branch encodes a possible transition between two denominations during a transaction (e.g., giving change, making exact payments, or combining units).

Example:

An illustrative example of a binary fragmentation tree starting from 10,000 FCFA is provided in Figure 1.

1) Strahler Classification

The Strahler classification assigns an order to each node of the tree:

- Leaves (terminal denominations, e.g., 5 FCFA) are rank 1.
- When a node has two children of the same rank r, its rank is $r + 1$.
- If the children have different ranks, the node takes the higher rank.

2) Economic interpretation

The higher the rank, the more central the denomination is in the circulation system. Small denominations (at the bottom of the tree) support transaction granularity. Large denominations (at the top of the tree) support high-value transactions but require downstream infrastructure (coins, small notes) to ensure efficiency.

```
                [10 000]
               /        \
        [5 000]        [5 000]
        /     \         /     \
  [2 000][2 000] [1 000][2 000]
     / \              ...      / \
[1 000][1 000]          [1 000][1 000]
   / \                            ...
[500][500]
```

Figure 1. Example of a binary fragmentation tree starting from 10,000 FCFA.

3.4.2. Classification by Size T and Strahler Order S

Trees are classified according to their size T (number of different denominations present) and their Strahler order S (maximum depth of nesting of exchanges between denominations).

$$C = \{1, 5, 10, 25, 50, 100, 200, 500, 1000, 2000, 5000, 10000\} \Rightarrow T_{\max} = 12$$

As shown in Table 2, the classification of trees according to their size (T) and Strahler order (S) highlights the increasing structural complexity of monetary denomination systems.

Table 2. Classification examples (T, S).

Size T	Example structure (including denomination)	Order of Strahler S	Remarks
1	[5]	1	Single denomination, sheet
2	[10] ← [5]	1	Linear tree, order unchanged
3	[25] ← [10, 15] ou [5 + 10 + 10]	2	Two leaves of rank 1 → parent node of rank 2
4	[50] ← [25, 25], [5 + 10 + 10 + 25]	2	Structured depth but not sufficient for order 3
5	[100] ← [50, 50] → descendants [25, 25, 25, 25]	3	Double duplication → increases symmetrical structure
6	[200] ← [100, 100]	3	Two subtrees of rank 3 → node of rank 4
7	[500] ← [200, 300], etc.	3 or 4	Depending on the balance of the subtrees
8	[1 000] ← [500, 500]	4	Balanced structure, with developed subtrees
9	[2 000] ← [1 000, 1 000] → [500 → … → 25]	4 or 5	Increasing complexity
10	[5000] ← [2000, 3000], rich descendants	5	Two subtrees of depth 4 or 5
11	[10000] ← [5000, 5000] with the entire available denomination base	5 or 6	Most complete tree on available denominations

Source: Authors.

3.5. Simulation and Analyses

Generation of typical matrices

We propose a generation and classification of typical branching matrices. Each branching matrix represents the tree of possible transitions between denominations during transactions. It encodes the frequency or probability that a denomination is used to obtain a smaller denomination in a fragmentation process (giving change, breaking down) or in a composition process (payment using several denominations).

The denominations used, in descending order, are:

$$C = \{10000, 5000, 2000, 1000, 500, 200, 100, 50, 25, 10, 5, 1\} \quad (n = 12)$$

Each row *i* corresponds to a denomination C_i (e.g., 10,000 FCFA). Each column *j* indicates a possible transition from C_i to C_j. The branching matrix is of size $n \times n$, denoted $R = [r_{ij}]$ with:

$r_{ij} > 0$ if denomination C_i can lead to C_j, $r_{ij} = 0$ otherwise.

When direct empirical data on denomination exchanges are unavailable, these weights are assumed proportionate to the face-value ratio $\frac{C_j}{C_i}$ and normalized so that $\sum_j r_{ij} = 1$.

Thus, higher probabilities are assigned to transitions between adjacent denominations (e.g., 1000 → 500 FCFA) and lower ones to distant pairs (e.g., 10,000 → 50 FCFA).

This rule produces realistic interaction patterns while keeping the total flow from each denomination constant across simulations.

1) Unitary descending fragmentation matrix

Assumption: each denomination breaks down into the next immediately smaller denomination.

This corresponds to a linear tree (minimal Strahler order: S = 1 or 2). An example of a unitary descending fragmentation matrix corresponding to a linear tree is provided in Table 3.

Table 3. Example of a linear tree.

Denomination\ Transition	10 k	5 k	2 k	1 k	500	200	100	50	25	10	5	1
10,000	0	1	0	0	0	0	0	0	0	0	0	0
5000	0	0	1	0	0	0	0	0	0	0	0	0
...												

Source: authors.

2) Balanced mixing matrix (symmetric binary tree)

Assumption: each denomination can break down into two or three immediately smaller denominations.

This corresponds to a balanced tree, ($T = 11$), Strahler order ($S = 4$) to 5.

As shown in Table 4, the balanced mixing matrix illustrates a symmetric binary tree structure in which each denomination can be decomposed into two or more adjacent lower denominations.

Table 4. Example of a balanced tree.

Denomination\ Transition	10 k	5 k	2 k	1 k	500	200	100	50	25	10	5	1
10,000	0	1/2	2/5	1/10	0	0	0	0	0	0	0	0
5000	0	0	2/5	2/5	1/5	0	0	0	0	0	0	0
...												

Source: Authors.

3) Complete interaction matrix (dense)

Assumption: each denomination can be converted into several smaller denominations.

In Table 5, the complete interaction matrix represents a dense tree structure in which each denomination can be converted into multiple lower denominations, reflecting complex transactional patterns.

Table 5. Example of a dense tree.

Denomination/ Transition	10 k	5 k	2 k	1 k	500	200	100	50	25	10	5	1
10 000	0	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{25}$	$\frac{1}{100}$	$\frac{1}{200}$	$\frac{1}{400}$	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{1}{2000}$
5 000	0	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{4}{25}$	$\frac{1}{50}$	$\frac{1}{100}$	$\frac{1}{200}$	$\frac{1}{500}$	$\frac{1}{500}$	$\frac{1}{1000}$
...												

Source: Authors.

This represents a highly branched tree (broad structure, S = 2 but (T ≫)), reflecting complex change-making or split-payment scenarios (e.g., in the informal sector).

4. Economic Modeling

4.1. Basic Hypothesis

The proposed modeling challenges the classical assumptions of homogeneous rationality and perfectly functioning markets. In reality, the diversity of socio-economic structures (sectors, income levels, governance) strongly influences the use of currency denominations.

The asymmetry between small and large denominations creates distortions: small denominations are indispensable for daily transactions, while large denominations cannot effectively substitute for them. This dynamic generates liquidity tensions depending on income cycles, which are further intensified by social preferences for certain means of payment (coins, banknotes).

The model thus highlights the need for differentiated allocation—coins oriented toward commercial circuits and banknotes toward households—in order to ensure fluid and resilient circulation.

4.2. Control Mechanisms for Denomination Structure

Control over denomination structure relies on the combined action of monetary authorities, banks, and businesses to ensure that coins and banknotes are available in forms suited to transactional needs. Households, depending on their income, favor different denominations (small, intermediate, or large), which creates tensions in availability. Businesses and retailers play a central role in regulation by recycling receipts and re-injecting denominations into the monetary circuit.

Three mechanisms are mobilized: conversion through the banking system, collection from customers, and pricing strategies designed to limit change-making problems. However, these strategies encounter constraints related to financial capacity, transaction diversity, and agent behavior. As a result, the effective distribution of denominations often remains unbalanced, generating temporary risks of shortage or surplus.

The success of the system therefore depends on coordination between monetary policies, the effective circulation of denominations, and the economic behaviors of agents.

4.3. The Dynamics of Interactions and the Structure of Denominations

We analyze how the distribution of currency denominations interacts with income structures and transaction dynamics, highlighting the consequences for exchange efficiency, monetary balance, and macroeconomic stability.

Perception shows that income has a homogeneous denomination structure: each income level draws primarily from a given set of denominations. The higher the income, the higher the value of the denomination used, for reasons of convenience. There are three income levels—high, medium, and low—thus three sets of homogeneity:

Large denominations (high income): with probability P_3, where K decreases.

Medium denominations (intermediate income): with probability P_2, where K is hyperbolic.

Small denominations (low income): with probability P_1, where K increases.

By principle, $P_1 + P_2 + P_3 = 1$. Theoretically, the money supply is equally distributed across all types of denominations, so:

$$P_1 = P_2 = P_3 = 1/3\,.$$

In practice, however, denominations of different values are unevenly distributed. As a result, small denominations are more likely (P_3') to be drawn than medium denominations (P_2'), which in turn are more likely than large denominations (P_3'). Thus:

$$P_1' > P_2' > P_3' .$$

This observed probability decreases exponentially:

$$P_3' = \left(\frac{1}{7}\right),\ P_2' = 2\left(\frac{1}{7}\right),\ P_1' = 2^2\left(\frac{1}{7}\right),$$

with 7 representing the sum of numerators, which reflects the number of possibilities.

$$P_3' = \frac{1}{7},\ P_2' = \frac{2}{7},\ P_1' = \frac{4}{7}.$$

Both income structures and transaction processes follow this logic: the higher the denomination, the greater the risk of needing to make change. In the case of a normal distribution of denominations, these probabilities translate into the proportional representation of each denomination in any cash register, allowing transactions of any price to be completed. Otherwise, this corresponds to the risk of encountering a given type of denomination shortage. Depending on the level of homogeneity, P' varies from 0 to 1.

During a transaction, there is an interaction between the income structure and the circulating denomination structure. Three situations emerge:

Low-income agents: They always find a denomination structure favorable to exchanges since they hold the currency.

$$P_1 / P' = 1$$

They are therefore insensitive to the degree of homogeneity of circulating denominations.

Middle-income agents: They face structures less favorable to exchanges, since they often lack the smallest coins.

P_2 / P' depends on the degree of homogeneity.

They are thus moderately sensitive to denomination homogeneity.

High-income agents: They face the least favorable structures, since they lack both small and intermediate denominations.

P_3 / P' depends strongly on the degree of homogeneity.

They are therefore highly sensitive to denomination structure.

4.4. Economic Development Generates Four Chronological Scenarios

1) A relatively homogeneous situation without a market.

2) A relatively heterogeneous situation without a market.

3) A relatively heterogeneous situation with a market.

4) A relatively homogeneous situation with a market.

Starting from a heterogeneous distribution of denominations, a return to homogeneous distribution is ensured by interactions among individuals with different denomination structures. Each structure tends to converge toward the other until homogeneity is achieved. The only denomination structure favorable to exchanges is one composed of small and complementary denominations (small and medium, small and large, or exclusively small).

The dominant ratio is therefore one coin of 4, for two coins of 2, for four coins of 1—that is, the set (1, 2, 4) with proportions (1/7, 2/7, 4/7). If denominations were equitably distributed, this result would emerge.

The speed of return to homogeneity depends on the number of interactions. The more possible structures exist, the longer convergence takes; isolated structures may even prevent interaction altogether. Simulations based on branching matrices are used to illustrate this.

In heterogeneous microeconomic structures, the number of possible denomination structures increases. The mean denomination structure no longer reflects the median, affecting the macroeconomic structure that must respond to new demands.

If the distribution is homogeneous, all income levels face no denomination shortages, and interactions maintain homogeneity so long as all income groups are geographically located in the same market. A homogeneous distribution among heterogeneous agents increases the likelihood of complementary matches, stabilizing circulation.

Markets may, however, be weakly interconnected. Without markets, agents exchange directly within local zones. Without financial markets, real markets remain fragmented, and homogeneous circulation is undermined. This creates sectoral imbalances:

- In high-income zones, P_3 increases at the expense of P_2' and P_1'.
- In middle-income zones, P_2 increases at the expense of P_3' and P_1'.
- In low-income zones, P_1 increases at the expense of P_2' and P_3'.

Each zone therefore gains what the others lose, creating sectoral prejudices and risks of exchange blockages. Consequently, heterogeneous macroeconomic structures may still be homogeneous at the sectoral level.

4.5. Development Has Two Opposing Effects

1) It creates inequality, which generates heterogeneity.

2) It improves market conditions (information, infrastructure, technology).

Which of these effects dominates? Both coexist: some agents operate under heterogeneous structures, others under homogeneous ones, with markets more or less connected and agents more or less rational.

We therefore perform additional simulations based on branching matrices to illustrate these dynamics.

4.6. Simulation

We simulated different circulation scenarios based on the initial distribution of denominations:

- **D1: Uniform distribution** – each denomination has the same initial frequency ($P_1 = P_2 = P_3$).
- **D2: Increasing distribution** – small denominations dominate (low incomes,

$P_1 > P_2 > P_3$).

- **D3: Decreasing distribution** – large denominations dominate (high incomes, $P_1 < P_2 < P_3$).
- **D4: Bimodal distribution** – extremes are favored (e.g., 10 FCFA and 10,000 FCFA).

4.7. Computation of Average Matrix and Convergence toward a Limit Matrix

We define an average matrix $\bar{R}$ as:

$$\bar{R} = \frac{1}{N} \sum_{k=1}^{N} R^{(k)}$$

where each $R^{(k)}$ is a matrix randomly generated according to a tree of size T_k and Strahler order S_k. This average matrix represents the typical structure of a given fiduciary system and makes it possible to study convergence, denomination centrality, and saturation zones.

4.8. Convergence Study of the Average Matrix $\bar{R}$

At each simulation (k), we compute the distance:

$$\left\| R^{(k)} - R^{(k+1)} \right\|, \text{ (Frobenius norm).}$$

where the norm used is the Frobenius norm. Convergence is considered satisfactory when this distance becomes negligible.

4.9. Results

The simulation results show that in the case of D1 (uniform distribution), convergence is rapid, with a stable structure reached once $N > 30$. For D2 (increasing distribution), convergence is slow, indicating the need for more transitions to achieve homogenization. In D3 (decreasing distribution), convergence is very slow, showing limited interaction with small denominations. Finally, in D4 (bimodal distribution with strong extremes), oscillations are observed, indicating divergence.

The more unbalanced the initial distribution, the slower—or more chaotic—the convergence of $\bar{R}$.

The analysis of the centrality of the denomination i (central role that a given cut plays in economic exchanges) gives (see Table 6):

Table 6. Centrality of the denomination.

Denomination (FCFA)	D1 (uniform)	D2 (increasing)	D3 (decreasing)	D4 (bimodal)
5	★★★★★	★★★★★★★	★★	★★★★★★★
50	★★★★	★★★★★★	★★★	★★★★

Continued

500	★★★★	★★★	★★★★	★★★
2000	★★★	★★	★★★★★★★	★★★★★★★
10,000	★★★	★	★★★★★★★★	★★★★★★★

Source: Authors.

5. Conclusion

The study reveals three key sets of results, derived from empirical analyses, structural modeling, and dynamic simulations.

First, the parameter L (monetary spectrum width) evolves proportionally to economic growth, as shown by statistical indicators (slope ≈ 0.92, $R^2 \approx 0.95$). More specifically, in Côte d'Ivoire between 2010 and 2019, L increased from 10,050 to 14,500 FCFA, reflecting a clear broadening of the range of denominations alongside GDP growth (+6.72% in 2019). In contrast, parameter K (monetary amplitude) grew at a significantly slower pace (slope ≈ 0.88, $R^2 \approx 0.96$), indicating that the overall fiduciary mass adapts less rapidly to transactional dynamics than the diversification of denominations does. Nevertheless, despite these differentiated evolutions, the ratio $N = \frac{K}{\sqrt{L}}$ remains remarkably stable, ranging only from 5.00 to 5.95 during the period. This confirms the robustness of the normalized distribution model proposed by Hentsch (1985), as it demonstrates that the structural relationship between these parameters persists despite economic fluctuations.

Second, binary tree modeling (Horton–Strahler) sheds light on the structural variety of fiduciary distribution. Branching matrix analysis identifies three distinct typologies:

- Linear trees ($S = 1$ - 2): characterized by sequential transitions, typical of less diversified systems (e.g., rural zones).
- Balanced trees ($S = 3$ - 5): with optimal symmetrical structures—such as [1000] → [500, 500]—ensuring maximum transactional fluidity ($T = 11$, $S = 4$ - 5).
- Dense trees ($S = 2$ - 3): marked by multiple transitions, sources of instability, observable in informal-sector fragmentation.

Moreover, the average matrix converges toward a limit structure once $N > 30$ samples, except in unbalanced distributions (D3 - D4), where convergence fails to occur.

Third, the dynamics of interactions and resilience reveal four scenarios:

- D1 (uniform distribution): rapid convergence toward homogeneity, with strong centrality of intermediate denominations (500 - 1000 FCFA).
- D2 (small-denomination dominance): slow convergence, with large denominations (10,000 FCFA) becoming saturated.
- D3 (large-denomination dominance): chaotic convergence, coupled with small-denomination shortages (50 - 100 FCFA) and transactional bottlenecks.

- D4 (bimodal distribution): divergence with oscillations, showing systemic bottlenecks (alternating saturation between 5/50 FCFA and 2000/5000 FCFA).

These results clearly demonstrate that denomination structure directly influences the stability and efficiency of the monetary system as a whole.

6. Theoretical and Empirical Interpretation

The findings confirm, refute, or nuance the initial hypotheses:

- Hypothesis 1 (optimal adjustment): Refuted in contexts of strong socio-economic heterogeneity. Discrepancies between theoretical and empirical distributions (M_j^{obs}) reach ±15% in rural areas, linked to both asymmetry in access to intermediate denominations (100 - 1000 FCFA) and differential hoarding (small denominations during crises, large denominations during growth phases).
- Hypothesis 2 (structural resilience): Partially confirmed. While the macroeconomic model resists growth (L & GDP), it is vulnerable to localized shocks (e.g., 2011 inflation in Côte d'Ivoire: K fell from 500 to $485.22 \times 10^9 \sqrt{\text{FCFA}}$). Resilience depends on the centrality of intermediate denominations (e.g., 500 FCFA), acting as essential relays between micro and macro levels, and on market connectivity (urban areas such as Abidjan converge twice as fast as rural areas like Korhogo).
- Hypothesis 3 (external factors): Confirmed. Fiduciary architecture (e.g., denomination ceilings) and monetary behaviors (greater hoarding at high interest rates) explain 96% of variations in K (multiple regression tests, $R^2 = 0.96$).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

(2024). *[Gp12] Côte d'Ivoire GDP Growth.* https://divercitytimes.com/economy/country/cote-dlvoire-gdp

Daouda, S. K., & Koné, M. (2025). *Analyse de la distribution de la monnaie fiduciaire entre les coupures et l'indice des prix, à partir de la méthodologie de Horton-Strahler.*

De Vries, H., Becker, T., & Eckhardt, B. (1994). Power Law Distribution of Discharge in Ideal Networks. *Water Resources Research, 30,* 3541-3543. https://doi.org/10.1029/94wr02178

Gupta, V. K., Waymire, E., & Wang, C. T. (1980). A Representation of an Instantaneous Unit Hydrograph from Geomorphology. *Water Resources Research, 16,* 855-862. https://doi.org/10.1029/wr016i005p00855

Hentsch, J. C. (1985). Distribution de la monnaie fiduciaire entre les coupures. Les paramètres et l'indice des prix. La distribution normalisée. *Journal de la société française de statistique, 126,* 139-144.

Horton, R. E. (1945). Erosional Development of Streams and Their Drainage Basins; Hydrophysical Approach to Quantitative Morphology. *Geological Society of America Bulletin, 56,* 275-370. https://doi.org/10.1130/0016-7606(1945)56[275:edosat]2.0.co;2

Mantilla, R., Gupta, V. K., & J. Mesa, O. (2006). Role of Coupled Flow Dynamics and Real

Network Structures on Hortonian Scaling of Peak Flows. *Journal of Hydrology, 322,* 155-167. https://doi.org/10.1016/j.jhydrol.2005.03.022

Moussa, R. (2009). Definition of New Equivalent Indices of Horton-Strahler Ratios for the Derivation of the Geomorphological Instantaneous Unit Hydrograph. *Water Resources Research, 45,* W09406. https://doi.org/10.1029/2008wr007330

Strahler, A. N. (1952). Hypsometric (Area-Altitude) Analysis of Erosional Topography. *Geological Society of America Bulletin, 63,* 1117-1142. https://doi.org/10.1130/0016-7606(1952)63[1117:haaoet]2.0.co;2

White, A. B., Kumar, P., Saco, P. M., Rhoads, B. L., & Yen, B. C. (2004). Hydrodynamic and Geomorphologic Dispersion: Scale Effects in the Illinois River Basin. *Journal of Hydrology, 288,* 237-257. https://doi.org/10.1016/j.jhydrol.2003.10.019