

Sovereign Default with Unobservable Physical Capital

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Abstract

We develop a model of sovereign default where borrowers' physical capital is not observable by the lenders and therefore the bond price schedule does not depend on capital accumulation. Borrowers take decisions on consumption, investment in physical capital, international assets, and whether to honor previous debt contracts (thus having an option to default). We calibrate the model on the Argentine economy and simulate the effects of productivity shocks. We compute the dynamics of equilibrium bond prices, physical capital, debt and consumption in addition to equilibrium default. We find that borrowing for consumption and investment is an optimal outcome, even if capital is unobservable, but differently from the literature with observable capital, countries do not over invest. Our results can inform international debt policies and conditionality clauses under imperfect information on borrowers' capital.

Keywords

Sovereign Default, Unobservable Physical Capital

JEL Classification

E21, E22, E32, F34, O16

1. Introduction

The canonical position of international lenders is that foreign assets should be used for fostering economic growth through investments and in turn for generating returns for debt repayment. Nevertheless, the experience of several countries, such as Argentina in 2001, Nigeria in 2004 and more recently Greece in 2015

whose borrowing was not followed by the agreed level of investment programs, raises the issue of whether international lending conditionality should stipulate that debtor countries use foreign assets for investment¹. Assessing whether it is optimal for a borrower to devote foreign assets to consumption and/or investment is therefore crucial for designing effective international debt policies.

The current literature on sovereign default mainly falls into two categories: models with no physical capital or models with observable physical capital. In models without physical capital, such as [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#), default probabilities are functions of total-factor productivity (or equivalently output). Here, countries borrow in order to smooth consumption (borrowing for consumption in bad states of nature) and may default if TFP reaches a relatively low level. The borrowing country is then excluded from the market and will be allowed back with a probability (probability of re-entry).

In models with physical capital, such as [Bai and Zhang \(2010, 2012\)](#), [Romero-Barrutieta et al. \(2015\)](#), [Park \(2017\)](#), and [Gordon and Guerron-Quintana \(2018\)](#), physical capital provides a buffer stock for consumption in bad states. Therefore, it lowers the probability of default. As capital is observable, the bond price becomes more favorable with higher levels of physical capital. Consequently, there is an incentive to invest more in physical capital to obtain more favorable loan terms (bond price). Therefore, there is an incentive to over invest, [Park \(2017\)](#). By over investment we mean choosing a capital stock at a level higher than would have been chosen if the bond price was constant.

An exception in the literature is [Marsiliani et al. \(2024\)](#), where physical capital is unobservable². There the focus is on optimal debt relief and optimal exclusion. It is shown that it is never optimal to try to avoid default, but setting the re-entry probability reasonably high, as well as allowing for partial default.

Our paper builds on the previous literature and, as is the case in [Marsiliani et al. \(2024\)](#), assumes that the borrower's physical capital is not observed by the lender and therefore the bond price schedule does not depend on capital. In this setting, borrowers can choose to borrow for consumption rather than for investment in physical capital, without altering the price of borrowing. Thus, an interesting question arises whether borrowers would use foreign assets for consumption only.

Consistently with most of the existing literature, see [Arellano \(2008\)](#), [Chatterjee and Eyigungor \(2012\)](#), [Mendoza and Yue \(2012\)](#) and [Lizarazo \(2013\)](#) among others, we calibrate our model to the Argentine economy, and provide numerical solutions (discrete state space). We compute the value function, policy functions, default probability and the bond-price schedule and present the impulse responses to positive and negative productivity shocks. We show results in terms of

¹On IMF and World Bank conditionality see [Sachs \(1989\)](#), [Kapur and Webb \(2000\)](#) and [Abbott et al. \(2010\)](#) among others. For the effects of IMF programs on economic growth, see [Easterly \(2005\)](#) and the meta-analysis of [Balima and Sokolova \(2021\)](#).

²Estimation of the capital stock of a country, especially for less developed economies, suffer from problems with data availability, recording and measurements, see [Blavy \(2006\)](#) and [Escribá-Pérez et al. \(2023\)](#). Therefore, unobservability of physical capital is a realistic assumption.

bond prices, borrower's capital accumulation, debt and consumption in addition to default options at the steady state and on the transition path. We finally validate the model through Monte Carlo simulations.

We find that, despite physical capital not being observable, it is still optimal for a country to borrow for both consumption and investment, not only consumption. However, differently from the previous literature with observable capital, in response to a shock, borrowers do not over invest. The reason is that the borrower has no incentive to over accumulate capital in order to influence the bond-price schedule.

Our results are important in shedding light on the borrowers' behavior and in informing international debt policies and conditionality clauses in the presence of imperfect information on the borrowers' capital.

The remaining of this paper is structured as follows: Section 2 describes our theoretical model of sovereign default with unobservable physical capital; Section 3 presents the data and calibration; the quantitative results and the Monte Carlo simulations are analyzed in Section 4; Section 5 concludes the paper.

2. The Model Economy

2.1. Consumption Preferences

The representative individual's utility from consumption c_t is as follows

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma} \quad (1)$$

with the parameters γ and β being relative risk aversion and the discount factor, respectively parameter and β the discount factor. Henceforth we will suppress time notation, indicating future quantities with ' (e.g. $c = c_t$ and $c' = c_{t+1}$).

2.2. Production Functions

Production y is assumed to be of the Cobb-Douglas form

$$y = ak^\alpha \quad (2)$$

with y , k , a denoting output, physical capital and total factor productivity (TFP), respectively.

TFP is stochastic (and the source of uncertainty in the model) and follows the AR (1) process:

$$a' = \mu + \rho a + \epsilon \quad (3)$$

where a' denotes with $\epsilon \sim N(0, \sigma_\epsilon^2)$.

We assume that a country defaulting on its debt is excluded from the international credit market for a period of time. A further assumption is that during this exclusion period the country's TFP is below the level it otherwise would have been. This assumption is common in the previous literature and captures the stylized facts that output during default tends to be lower³.

³See Borensztein and Panizza (2009), Reinhart and Rogoff (2011), Tomz and Wright (2007), Zymek (2012).

With TFP during default denoted by a^{def} , production during the exclusion period is

$$y^{def} = a^{def} k^\alpha \quad (4)$$

While the boundaries of TFP under non-exclusion are $a \in (\underline{a}, \bar{a})$, the boundaries of TFP under exclusion are $a^{def} \in (\underline{a}, \bar{a}^{def})$, where $\bar{a}^{def} \leq \bar{a}$.

2.3. The Resource Constraints

2.3.1. Repayment

If the country honors its debt, it has access to international borrowing and lending. We denote by b the amount lent. Consequently, if b is negative, the country borrows. Bonds have one-period maturity. If the country pays back $-b$, it can enter a new loan contract $-b'$. The funds raised from borrowing is $q(-b')$, with q being the bond price. This bond price in absence of default possibilities would just have been $1/(1+r)$, and given by the international interest rate r . In the case of default possibilities, risk neutral lenders seek to infer the next period's default probability. Since k is unobservable, q will depend on b' and a only.

The resource constraint is then

$$c = y + b - q(b', a)b' + (1 - \delta)k - k' - \Phi(k, k') \quad (5)$$

with δ being physical capital's depreciation rate, and $\Phi(k', k)$ the capital adjustment cost.

2.3.2. Default

Under the exclusion period, the economy only has access to one asset, physical capital, implying the resource constraint

$$c = y^{def} + (1 - \delta)k - k' - \Phi(k, k') \quad (6)$$

where k' denotes next period's physical capital, and δ the depreciation rate. We also assume that there is a capital adjustment cost $\Phi(k', k)$.

2.4. Value Functions

At any point in time, the borrowing country decides whether to honor the debt contract (repayment) or to default. Denote the value function of repaying by $v_g^r(b, k, a)$ and the value function of default by $v_g^d(0, k, a)$. Then, the value function for this decision is

$$v_g^0(b, k, a) = \max_{\{d, r\}} \{v_g^d(0, k, a), v_g^r(b, k, a)\} \quad (7)$$

2.4.1. Value Function of Repayment

The value function in case of repayment is

$$v_g^r(b, k, a) = \max_{\{b', k'\}} \{u(c) + \beta \mathbb{E} v_g^0(b', k', a')\} \quad (8)$$

subject to

$$c, k', q(b', a) \geq 0 \quad (9)$$

and Equation (5). Notice that $\mathbb{E}v_g^0(b', k', a')$ takes into account the option of default in the future (see Equation (7)).

2.4.2. Value Function of Default

Denoting by θ the (exogenous) probability of re-entry into the international financial market, the default value function is as follows

$$v_g^d(0, k, a) = \max_{\{k'\}} \left\{ u(c) + \beta \mathbb{E} \left[\theta v_g^0(0, k', a') + (1 - \theta) v_g^d(0, k', a') \right] \right\} \quad (10)$$

subject to

$$c, k' \geq 0 \quad (11)$$

and Equation (6).

2.5. Capital Adjustment Cost

For tractability, we assume that the capital adjustment cost $\Phi(k', k)$ is quadratic

$$\Phi(k', k) = \frac{\Phi}{2} \left(\frac{k' - k}{k} \right)^2 k \quad (12)$$

2.6. Bond Price Schedule

Denote by Ψ the perceived (by the lender) default probability. In equilibrium, since physical capital unobservable, this perceived default probability is only a function of b' and a . As is common in the literature, we assume that international lenders are risk neutral. This implies that they are indifferent between the gross return of $1 + r$ for certain and the expected return $(1 - \Psi)/q$ from lending to the risky country. Consequently, the bond price schedule is given by

$$q(b', a) = [1 - \Psi(b', a)] / (1 + r) \quad (13)$$

2.7. Perceived Equilibrium Default Probability

In a model of this kind, where the source of uncertainty is a TFP shock, physical capital tends to its steady state level relatively quickly. As we shall see later (Section 4.2), after a TFP shock capital approaches the steady state just after a few periods (between 3 and 6 quarters), see **Figure 5** and **Figure 6**. Also, in the simulations, see **Figure 7**, with shocks in virtually every period, capital is fluctuating around its steady state level (with no tendency to stay above or below). In absence of any other information, it is reasonable for the lender to act agnostically, as if capital was at its steady state level k^* ("best guess").

Consequently, we look at the equilibrium default probability of a borrower endowed with k^* . The borrower's choice of default or repayment is the given by

$$v_g^0(b, a; k^*) = \max_{\{d, r\}} \left\{ v_g^d(0, a; k^*), v_g^r(b, a; k^*) \right\} \quad (14)$$

The value function of repayment is

$$v_g^r(b, a; k^*) = \max_{\{b'\}} \left\{ u(c) + \beta \mathbb{E} v_g^0(b', a'; k^*) \right\} \quad (15)$$

subject to

$$c = ak^{*\alpha} + b - q(b', a)b' - \delta k^* \quad (16)$$

where $q(b', a)$ is given by Equation (13).

The value function of default is

$$v_g^d(0, k^*, a) = u(c) + \beta \mathbb{E} \left[\theta v_g^0(0, k^*, a') + (1 - \theta) v_g^d(0, k^*, a') \right] \quad (17)$$

subject to

$$c = a^{def} k^{*\alpha} - \delta k^* \quad (18)$$

Therefore, the sets of repayment and default decisions are given by

$$R(b) = \left\{ a \in A : v_g^r(b, k^*, a) \geq v_g^d(0, k^*, a) \right\} \quad (19)$$

$$D(b) = \left\{ a \in A : v_g^r(b, k^*, a) < v_g^d(0, k^*, a) \right\} \quad (20)$$

where $R(b)$ is the repayment set and $D(b)$ is the default set. The perceived default probability is

$$\Psi(b', a) = \int_{D(b')} f(a', a) da' \geq 0 \quad (21)$$

where $f(a', a)$ is the stochastic process (equation 3) and $0 \leq \Psi \leq 1$. When $D(b') = \emptyset$, the default probability is zero. Equation (21) is the perceived default probability from the viewpoint of the lender and gives the bond-price schedule (Equation (13)).

3. Data and Calibration

The model is calibrated to target the Argentine economy by using quarterly data from 1980 Q1 to 2017 Q4, with some parameters based on the recent RBC literature⁴.

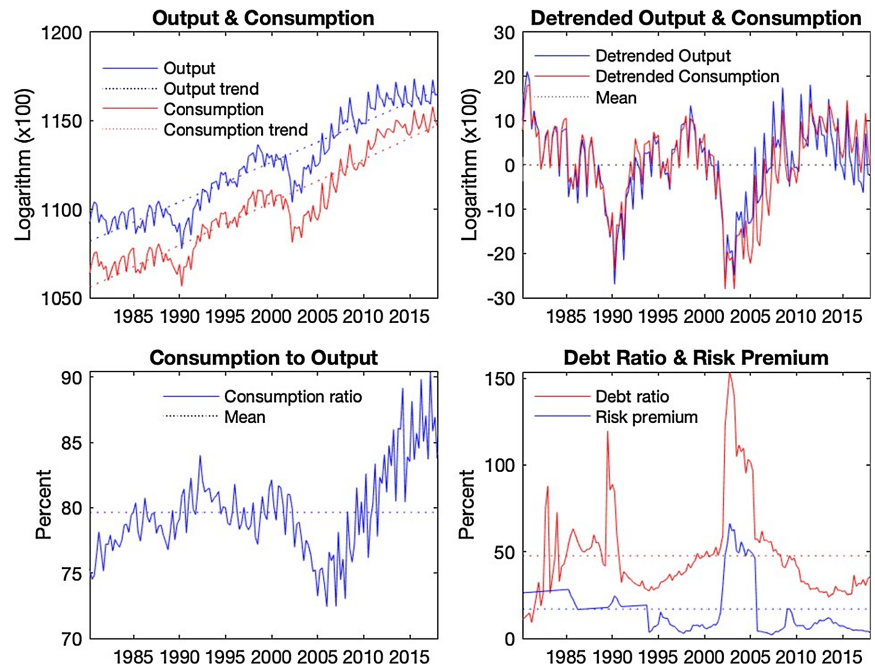
3.1. Descriptive Statistics

We present the properties of the data in **Figure 1** and **Table 1** below.

In **Figure 1**, the top left panel plots the log of GDP and consumption against time, together with their respective linear trends. The top right panel shows the de-trended series (with mean zero). The bottom two panels plot the consumption-output ratio (c/y), the debt-output ratio ($-b/y$), and the risk premium. The consumption to output ratio of Argentina was 75% in 1980 Q1, and peaked at 90% in 2017 Q4. Before the late 1980s, the debt to output ratio of Argentina had never exceeded 50%. It reached its lowest value at 9.07% in 1981 Q1 and increased to a peak of 153.63% in 2002 Q3. The spikes are clustered around the period of financial crisis and recession in 1990 and 2003.

Table 1 below presents business cycle statistics for the Argentine economy between 1980 Q1 and 2017 Q4. GDP and consumption are in log and linearly

⁴Data are from the Ministry of Economy of Argentina (MECON data). They are available quarterly at constant 2010 U.S. dollars, starting from the first quarter of 1980 to the last quarter of 2017 over 152 periods. We also use the overall risk premium on Argentine debt in USD from JP Morgan and compiled by Oxford Economics <https://www.oxfordeconomics.com/sovereign-risk-tool>.



Source: Authors' computations.

Figure 1. Output, consumption, trend, debt and risk premium.

Table 1. Business cycle statistics: Argentina (1980 Q1-2017 Q4).

Statistic	Value	Statistic	Value
$\mu_{\bar{y}}$	0	$\sigma_{\bar{y}}$	0.0917
$\mu_{\bar{c}}$	0	$\sigma_{\bar{c}}$	0.0969
$\mu_{\frac{c}{y}}$	0.7963	$\sigma_{\frac{c}{y}}$	0.0336
$\mu_{\left(\frac{-b}{y}\right)}$	0.4767	$\sigma_{\left(\frac{-b}{y}\right)}$	0.2719
μ_{Ψ}	0.1689	σ_{Ψ}	0.1420
$\frac{\sigma_{\bar{c}}}{\sigma_{\bar{y}}}$	1.0708	$\frac{\sigma_{c/y}}{\sigma_{\bar{y}}}$	0.3664
$\frac{\sigma_{(-b/y)}}{\sigma_{\bar{y}}}$	2.9651	$\frac{\sigma_{\Psi}}{\sigma_{\bar{y}}}$	1.5485
$\rho_{\bar{y}}$	0.6757	$\rho_{\bar{c}}$	0.7844
$\rho_{\left(\frac{-b}{y}\right)}$	0.8797	ρ_{Ψ}	0.9440
$\rho_{\bar{c}, \bar{y}}$	0.9223	$\rho_{\left(\frac{-b}{y}, \bar{y}\right)}$	-0.6552
$\rho_{\Psi, \bar{y}}$	-0.4713	$\rho_{\left(\frac{-b}{y}, \Psi\right)}$	0.6953
$\rho_{\left(\frac{-b}{y}, \bar{c}\right)}$	-0.7616	$\rho_{\Psi, \bar{c}}$	-0.5402

Source: Authors' computations.

de-trended with mean zero (denoted \tilde{y} and \tilde{c} respectively). The ratios debt/GDP ($-b/y$) and consumption/GDP (c/y) are also presented in **Table 1**.

The standard deviations for consumption and output are 0.0969 ($\sigma_{\tilde{c}}$) and 0.0917 ($\sigma_{\tilde{y}}$), respectively. Thus, consumption is 1.07 times more volatile than output ($\sigma_{\tilde{c}}/\sigma_{\tilde{y}}$). Moreover, we observe a positive correlation between output and consumption equal to 0.9223 ($\rho_{\tilde{c},\tilde{y}}$). Mean and standard deviation of the consumption/output ratio are 0.7963 ($\mu_{c/y}$) and 0.0336 ($\sigma_{c/y}$) respectively.

The auto-correlation of output is 0.6757 ($\rho_{\tilde{y}}$). The debt ratio ($-b/y$) has a mean of 47.67% ($\mu_{(-b/y)}$) and a standard deviation of 0.2719 ($\sigma_{(-b/y)}$). The correlation coefficient between output and debt ratio is negative at -0.6552 ($\rho_{(-b/y),\tilde{y}}$). This relationship has clearly been detected in 1990 and 2002, when the debt ratio increased to its peak at the same time as output experienced the largest drop for the Argentinian economy.

The debt ratio is 2.97 ($\sigma_{(-b/y)}/\sigma_{\tilde{y}}$) times more volatile than output and 2.81 ($\sigma_{(-b/y)}/\sigma_{\tilde{c}}$) times more than consumption. The mean and standard deviations of the risk premium are 0.1689 (μ_{Ψ}) and 0.1420 (σ_{Ψ}) respectively. The correlation coefficient between output and risk premium is negative at -0.4713 ($\rho_{\Psi,\tilde{y}}$) whilst debt ratio and risk premium have a positive correlation at 0.6953 ($\rho_{(-b/y),\Psi}$).

Overall, the signs of the correlation coefficients for Argentina are consistent with the RBC literature and stylised facts from developing countries (see [Aguiar and Gopinath, 2006](#); [Yue, 2010](#), among others).

3.2. Calibration

We take some of the parameters from the existing literature and calibrate others. First, we discretize the continuous stochastic process for TFP to obtain a 31-state Markov chain finite state Markov chain using the quadrature method (see [Tauchen, 1986](#); [Tauchen and Hussey, 1991](#))⁵.

The possible range of values of foreign assets (the negative of debt) and physical capital in the grids are set to $\{\underline{b}, \bar{b}\} = \{-4.5, 0.5\}$ and $\{\underline{k}, \bar{k}\} = \{2, 13\}$, respectively, with the number of grids being 51 and 71, respectively. Accordingly, the foreign assets ratio lies between $\frac{\underline{b}}{y}$ and $\frac{\bar{b}}{y}$, whilst the capital ratio lies within $\frac{\underline{k}}{y}$ and $\frac{\bar{k}}{y}$, to cover all recorded values of assets and capital for Argentina, as well as avoiding corners. The 51 grid points for foreign assets are equally distributed between the interval $\{\underline{b}, \bar{b}\}$, while the grid for physical capital is set to cluster around the expected steady state⁶. **Table 2** presents the parameters taken from the existing literature on Argentina.

⁵This method is commonly used in the literature. See e.g. [Aguiar and Gopinath \(2006\)](#), [Arellano \(2008\)](#), [Schaltegger and Weder \(2015\)](#), [Gordon and Guerron-Quintana \(2018\)](#).

⁶The total number of grids for capital is 71. There are 7 and 5 state spaces located with 0.5 distance for the value of capital at the intervals $\{2, 5\}$ and $\{11, 13\}$ respectively. The other 59 state spaces are evenly located within the interval $\{5, 11\}$. We thus make the grid finer around the steady state, where most of the dynamics of the model occur.

We calibrate the other parameters (shown in **Table 3**). The stochastic structure of TFP shock, which follows the AR (1) process, is specified by using the moment matching method to target the Argentine economy. We first target volatility of output ($\sigma_{\bar{y}}$) and consumption ($\sigma_{\bar{c}}$) and the correlation between them ($\rho_{\bar{c},\bar{y}}$). Secondary targets are the relative consumption and output volatility ($\frac{\sigma_{\bar{c}}}{\sigma_{\bar{y}}}$), the average consumption ratio ($\mu_{(c/y)}$) and the debt ratio ($\mu_{(-b/y)}$). Third target statistic is the frequency of default periods, 12.5%. The parameters in the TFP equation, ρ_a and σ_ε , are found to be 0.982 and 0.014 respectively. For θ (probability of re-entry after default) in the range 1% - 20%, the value 5% fits the targets statistics well; a higher (lower) value causes a too high (low) default frequency. For the capital adjustment cost parameter, Φ , after trying several values between 0 and 5 taken from the existing literature, we found that 2.4 fits well the actual changes in physical capital.

Table 2. Model specific parameter values from existing literature.

Parameters	Values	Source
Discount factor	β 0.95	Arellano (2008)
Risk aversion of borrower	γ 2	Aguiar and Gopinath (2006)
Risk-free rate	r 0.01	Aguiar and Gopinath (2006)
Capital share	α 0.35	Park (2017)
Capital depreciation	δ 0.05	Romero-Barrutieta et al. (2015)
Output default cost	χ 7%	Tomz and Wright (2007)

Source: Various sources as per table.

Table 3. Calibration parameters.

Parameters	Values
Stochastic structure (TFP)	$\rho_a, \sigma_\varepsilon$ 0.982, 0.014
Probability of re-entry	Θ 5%
Capital adjustment cost	Φ 2.4

Source: Authors' computations.

In Section 4.3 below, we run 100,000 Monte Carlo simulations to compare the model generated statistics to the target statistics (**Table 4**).

4. Numerical Results

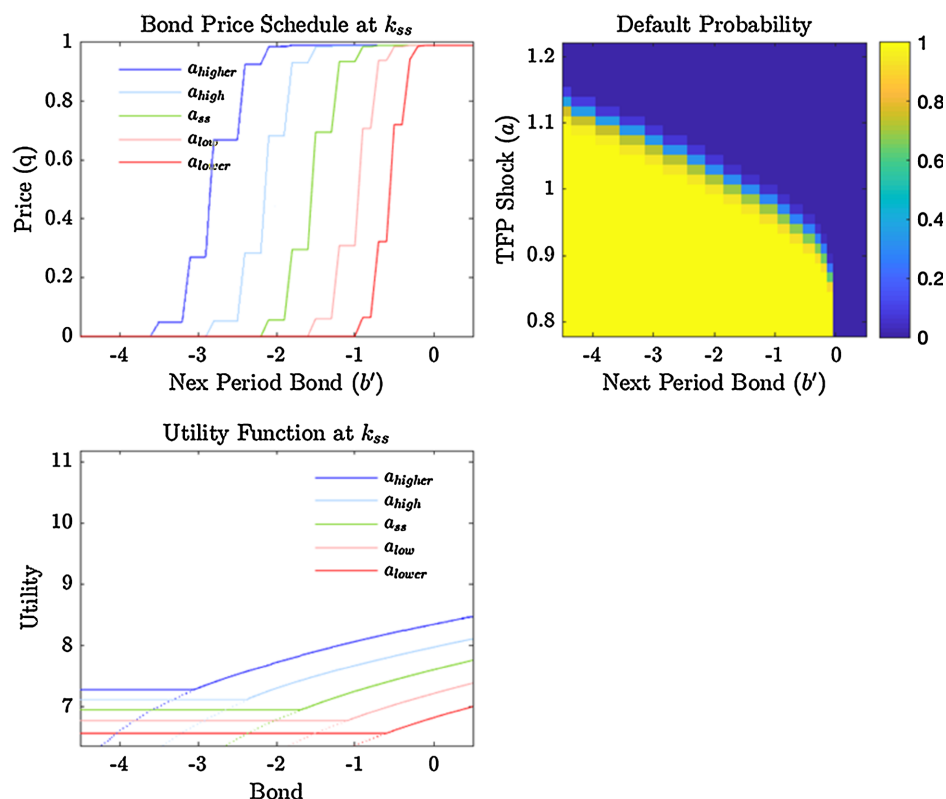
This section includes three main parts. In the first part we compute the equilibrium bond-price schedule, default probability, value functions and policy functions of a sovereign borrower. In the second part we analyze impulse response functions for positive and negative productivity shocks⁷. Finally, in the third part

⁷We follow the existing literature on foreign default (see Arellano, 2008; Bai and Zhang, 2012; Gordon and Guerron-Quintana, 2018) in modelling only productivity shocks.

we present the Monte Carlo simulations and provides a comparison between moments from the actual data for the Argentine economy from 1980 Q1 to 2017 Q4 and the moments from the simulations.

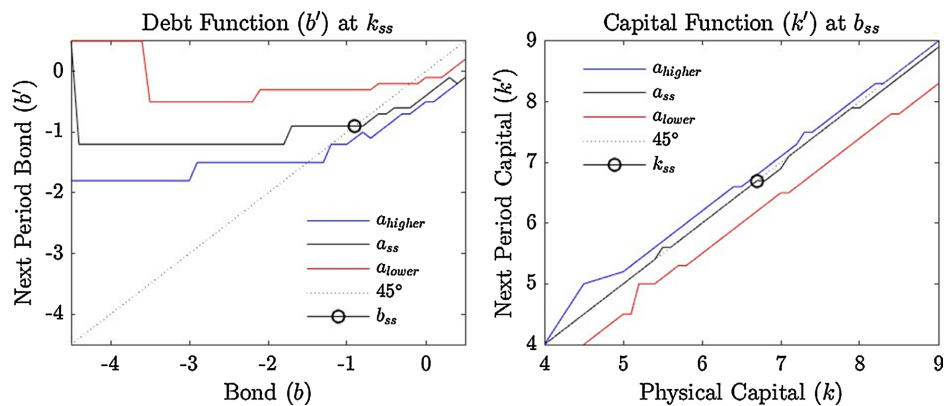
4.1. Equilibrium Functions

In this section we present the equilibrium bond price schedule, default probability, and value function (Figure 2), as well as the policy functions for debt and capital (Figure 3), and the default probability for different levels of capital (Figure 4).



Source: Authors' computations.

Figure 2. Bond price schedule, default probability and value functions.



Source: Authors' computations.

Figure 3. Policy functions for debt and capital.

The bond-price schedules, at the steady-state level of capital, are shown in the top left panel of **Figure 2**. Each schedule shows how the bond price (q) is a function of the amount being borrowed (as $b' < 0$) and corresponds to a different level of TFP. These plots indicate a negative relationship between next period level of debt and the cost of borrowing (recall that debt is the negative of b and a low q corresponds to higher cost of borrowing). If a country were to borrow more, the risk premium increases in the international market and the borrower will be charged a high price (low q). The bond price schedules are plotted for different levels of TFP, a_{higher} , a_{high} , a_{ss} , a_{low} and a_{lower} , representing the percentage difference from its steady state level, namely 6%, 3%, 0%, -3% and -6%, respectively. The sovereign country can borrow more at a lower risk premium when TFP is higher. Consequently, a negative shock to TFP can lead to an instant increase in bond risk premium for the same amount of debt. Thereby, a large negative shock can instantly induce a sovereign country with large debt to default because of an expensive borrowing rate (higher risk premium). The default probability is shown as a heat map chart in the top right quadrant of **Figure 2**.

The top right panel of **Figure 2** shows the default probability (Ψ), in color (zero indicated by dark blue and one indicated by light yellow) against bond selection (b') (with negative value being debt) and TFP (a). From this plot, we can notice that if a country intends to save with a bond (b') above or equal to zero, there will be no default in the following period. With $\Psi = 0$, the country can borrow or save money at the risk-free rate (without a risk premium; see the dark blue shaded area). However, if the country intends to borrow, there will be a positive default probability (light blue to yellow) in the next period with respect to the level of TFP. With a negative shock (lowering TFP), the borrower will be charged a higher risk premium in the following period because of a higher default probability. For a default probability of 100%, the borrower will be unable to borrow on the international market.

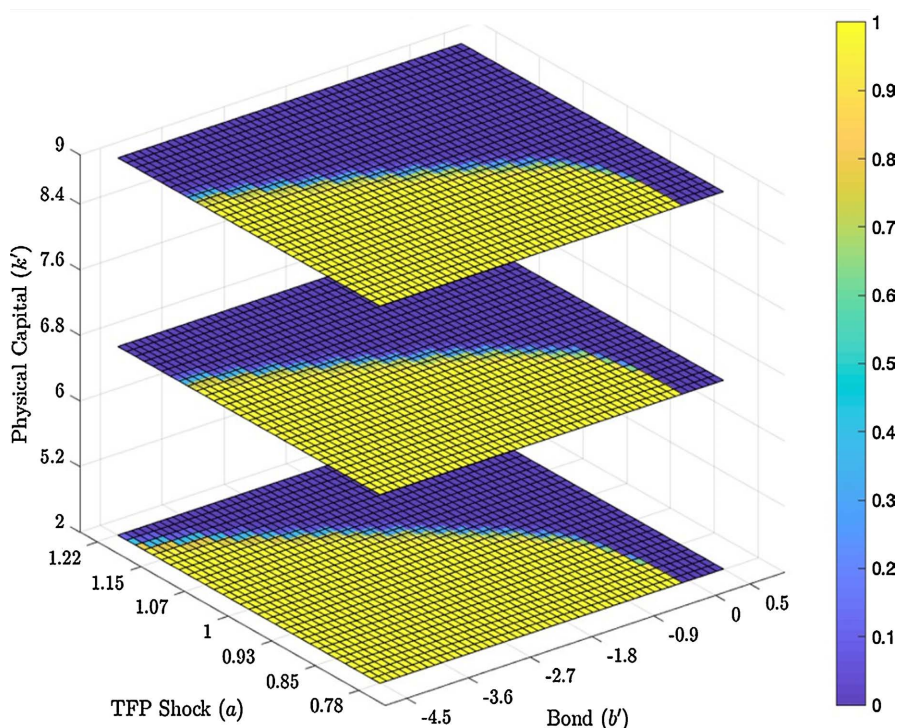
Finally, the bottom left panel of **Figure 2** shows the value functions (solid lines) with the horizontal segment left of the kink indicating default.

Figure 3 shows the policy (optimal choice) functions for debt and capital, for different levels of TFP.

The left and right-hand panels plot the optimal choice of b' and k' as a function of b and k , respectively. The steady state is where the policy function (at steady state TFP) crosses the 45° line. The policy functions also indicate convergence to the steady state.

Figure 4 shows the three-dimensional heat map charts of the default probability as a function of the borrower's selection of capital investment (k') and borrowing ($-b'$). Choices within the yellow area result in a positive probability of default in the next period. The top layer corresponds to high capital investment, whilst the middle and bottom layers correspond to the steady state and low capital investments, respectively. From the sliced layers in the chart, we notice that the *equilibrium* default probability in the following period depends on the amount of next

period's capital (k'). An increase in physical capital will increase future consumption possibilities in case of a future recession. Therefore, an increase in capital investment significantly reduces the default probability in the following period, as shown in **Figure 4** (the yellow area getting smaller). We should note that, since capital is unobservable, the bond price schedule is not affected.



Source: Authors' computations.

Figure 4. Default probability.

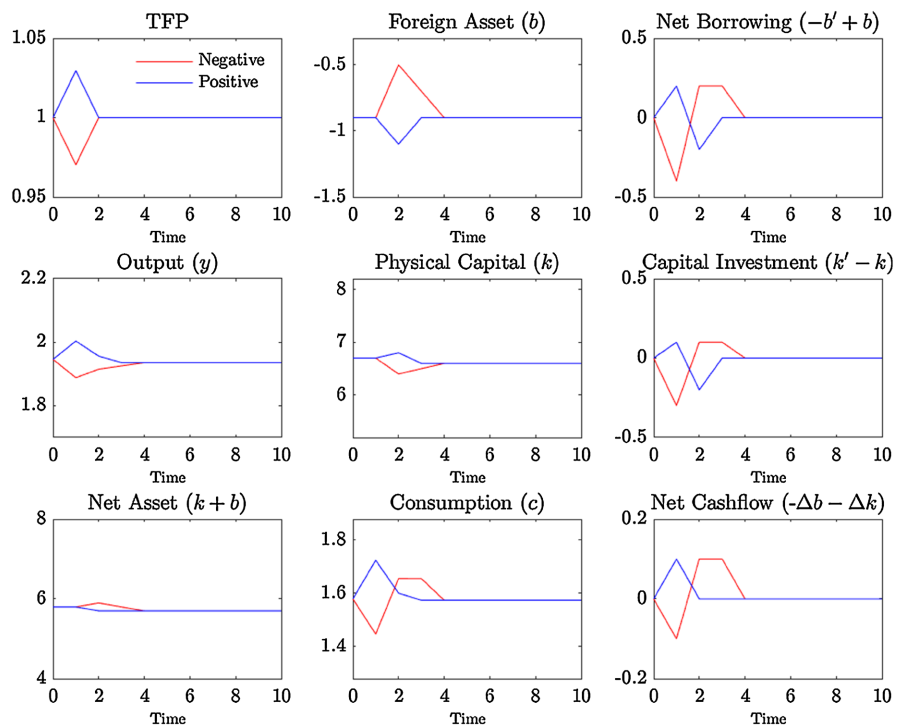
4.2. Impulse-Response Results

The second part of this section analyses the impulse response functions. **Figure 5** shows the response of foreign asset (b), net borrowing ($-b' + b$), output (y), physical capital (k), capital investment ($k' - k$), net asset ($k + b$), consumption (c) and net cash flow ($-\Delta b - \Delta k$), following a positive and a negative TFP shock of 3% (without persistence), when the economy is endowed with its steady state level of capital.

At the steady state, when a positive shock occurs in period 1, the country starts to borrow more for both consumption and investment; net borrowing becomes negative, while capital investment and consumption increase. After that, the country slightly decreases capital, repays its debt and reduces consumption until the TFP shock fades. Hence, we can deduce that a country tends to borrow for both investment and consumption during a period of positive TFP shock.

On the other hand, for a negative TFP shock in period 1, the country decides to repay with a sharp decrease in capital investment, while consumption is slightly reduced. Interestingly, **Figure 5** shows that it is optimal to sell physical capital in

response to a temporary negative technology shock, rather than to borrow. This result can be explained by the fact that borrowers will face a higher risk premium during a bad period (see **Figure 2**).



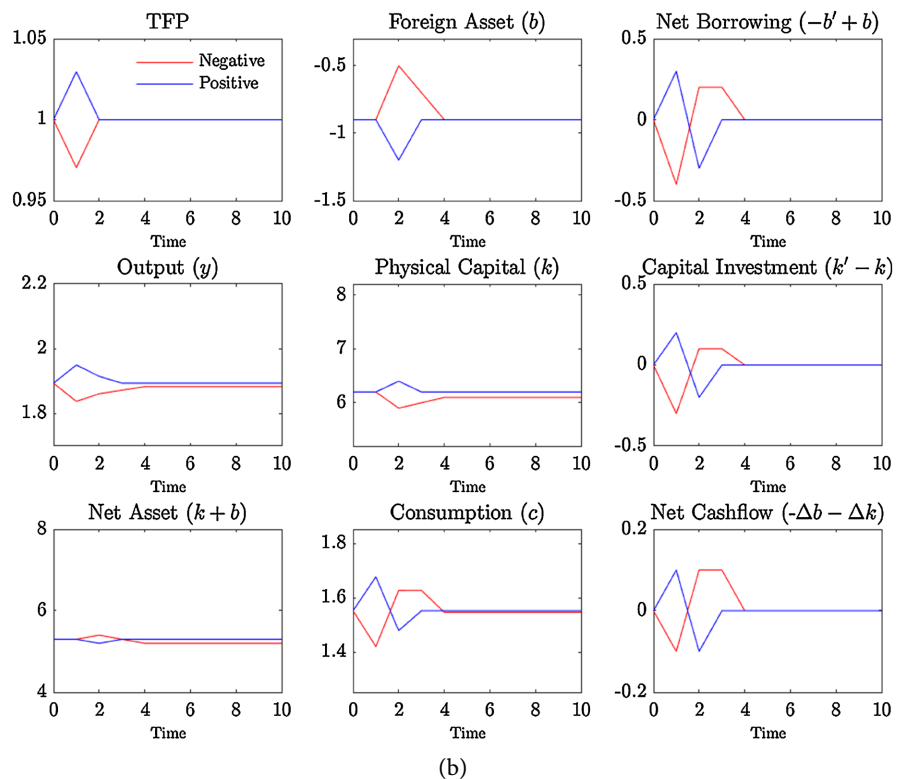
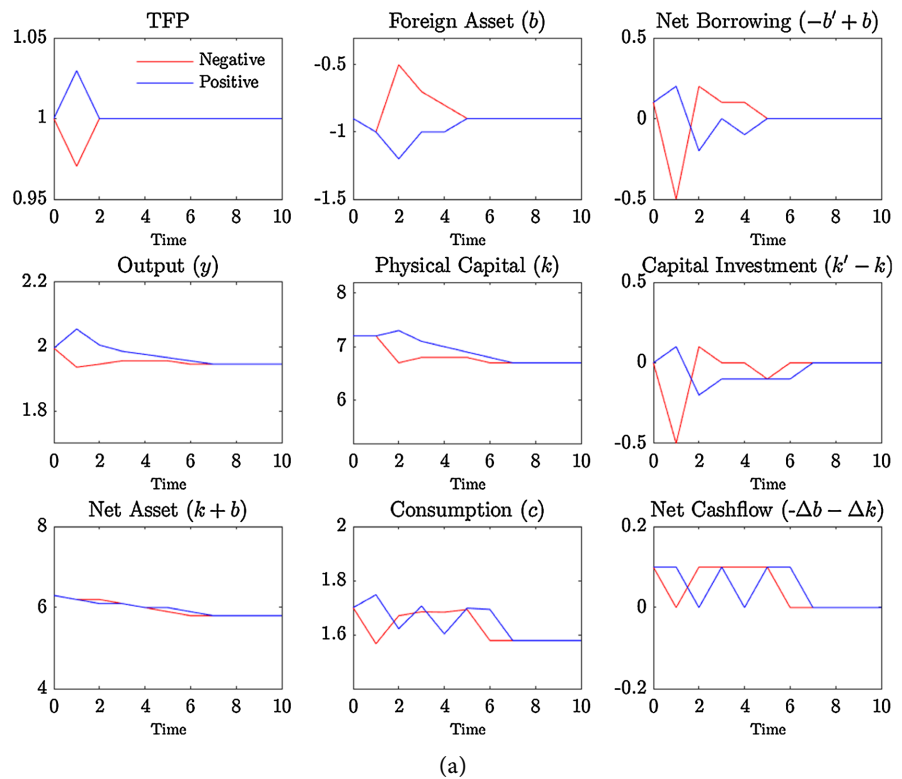
Source: Authors' computations.

Figure 5. Impulse response functions for initial capital at its steady state level.

We also provide the impulse response functions at above and below the steady state level of capital in **Figure 6**.

As shown in **Figure 6(a)**, the country begins with an initial level of capital above the steady state. When a positive shock occurs in period 1, the country will choose to borrow more for both investment and consumption. The country can maintain a higher level of debt while the physical capital is above its steady state. However, after period 1, the country starts to sell physical capital and repays its debt. As can be seen in **Figure 6(a)**, net assets continuously decline until they reach their steady-state level in period 7; the same happens to consumption. In comparison to the previous figure, consumption in **Figure 6(a)** can be maintained above the steady state for 6 periods. Thus, a country with capital above the steady state will sell assets for consumption.

Figure 6(b) shows opposite results for an initial capital below the steady state. With an increase in TFP in period 1, the country will start to borrow for both consumption and investment. However, as the initial capital is below the steady state, the borrower can borrow less than in the previous scenario. Hence, physical capital slightly increases after the positive TFP shock but still remains below its steady state level.

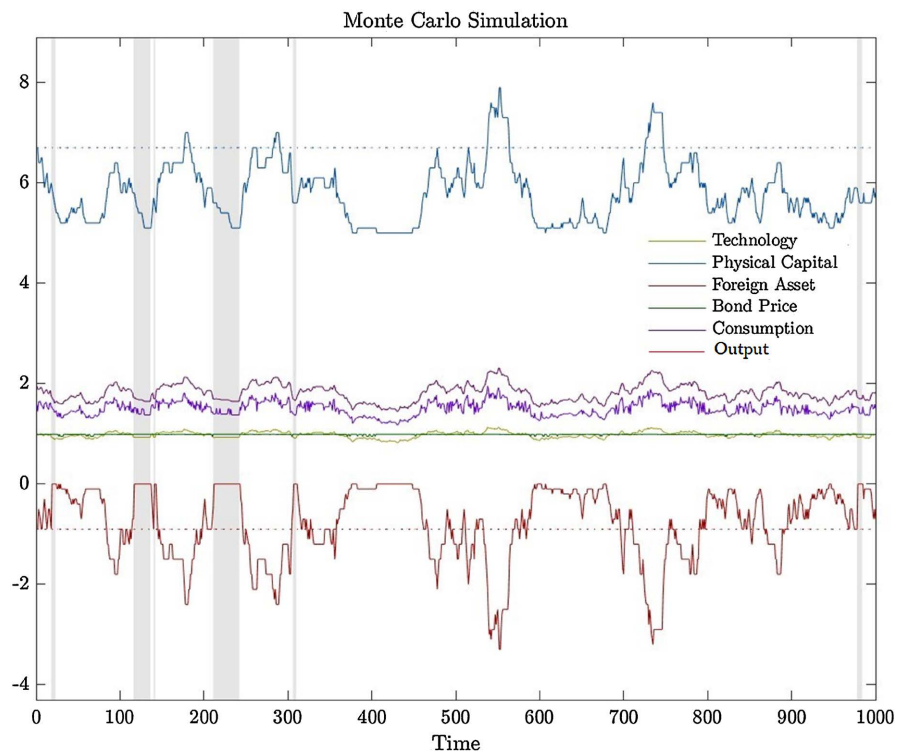


Source: Authors' computations.

Figure 6. Impulse response functions for initial capital above and below steady state level. (a) Impulse response functions for initial capital above its steady state level; (b) Impulse response functions for initial capital below its steady state level.

4.3. Monte Carlo Simulations

A Monte Carlo simulation over 1000 periods is shown in **Figure 7**. All variables are plotted in real terms.



Source: Authors' computations.

Figure 7. Monte Carlo simulation (1000 periods).

The simulation starts at the economy steady-state and targets the business cycle statistics of the Argentine economy, as shown in **Table 1**. The overlay default band (grey area) corresponds to the sovereign default decision in the model; the dotted lines represent the steady-state level of capital and debt (foreign assets).

From **Figure 7**, we can see that the government tends to default after facing a series of negative shocks or a large drop in output. After each default, the country will be excluded from the international bond market with a probability of re-entry (θ) equal to 5%. During the default periods output is lower. As can be seen in **Figure 7**, debt is always zero during defaults. Furthermore, after re-entering the bond market, the government will start borrowing and consuming more for a few periods, moving towards the steady state. In addition, the government will choose to stay in the bond market (honoring the debt) in absence of significant negative TFP shocks or if the physical capital stock is large enough. During a slight drop in output, the sovereign borrower tends to sell physical capital or reduce consumption in order to repay its debt. In the presence of high debt ratio, bond price fluctuates after a negative TFP shock. Interestingly, in some periods with a series of small negative shocks, the borrower sells physical capital to repay slowly its debt. With small debt and sufficient capital, the country can remain in the international fi-

nancial market during recession and wait for a positive shock.

We subsequently run 100,000 Monte Carlo simulations of 1000 periods each. We compare moments of the computed numerical results with the target moments from the actual data for the Argentine economy between 1980 Q1 and 2017 Q4 in **Table 4**.

Table 4. Model and target statistics.

Statistic		Model	Target
Output volatility	$\sigma_{\bar{y}}$	0.0940	0.0917
Consumption volatility	$\sigma_{\bar{c}}$	0.0924	0.0969
Consumption output volatility	$\frac{\sigma_{\bar{c}}}{\sigma_{\bar{y}}}$	0.9830	1.0563
Correlation coefficient (c, y)	$\rho_{\bar{c}, \bar{y}}$	0.9314	0.9223
Average consumption ratio	$\frac{\mu_c}{y}$	0.8273	0.7963
Average debt ratio	$\mu\left(\frac{-b}{y}\right)$	0.4625	0.4767
Fraction of default periods		0.0740	0.1250

Source: Authors' computations.

Output volatility ($\sigma_{\bar{y}}$), consumption volatility ($\sigma_{\bar{c}}$) and consumption output volatility ($\sigma_{\bar{c}}/\sigma_{\bar{y}}$) are 0.0940, 0.0924 and 0.9830 respectively. The correlation coefficient between consumption and output ($\rho_{\bar{c}, \bar{y}}$) is 0.9314. Most of the results are consistent with Argentine data. In particular, the model can target well the average consumption and debt ratio⁸ of Argentina. However, the fraction of default periods⁹ found by our model simulation is 7.40%, while the actual data shows a value of 12.50%.

Finally, we compare other moments of the simulated data from 100,000 Monte Carlo simulations with the corresponding moments from the actual data in **Table 5**.

The model generated volatilities of debt and risk premium are 0.3472 and 0.0083, respectively. The model autocorrelation of output and consumption are 0.9838 and 0.8569, respectively. Besides, the model shows a negative relationship between output and risk premium of -0.18806 , whilst the relationship between consumption and risk premium is -0.2138 . Model minimum and maximum values of the consumption and debt ratios are 71.28%, 95.21%, 0%, and 184.73%, respectively¹⁰. Overall, our model fits the actual data well.

⁸The model uses debt ratio at the steady state in order to target the average debt ratio of Argentina between 1980 Q1 and 2017 Q4.

⁹The fraction of default periods is defined as the number of periods in default over 100,000 periods of the simulation.

¹⁰We only model the choice between full repayment and full default ($b = 0$). In the case of Argentina, not all debt was defaulted on during our sample period. This is why $-b/y_{\min}$ shows up as 0.0907 in the data.

Table 5. Other statistics: Model and actual data.

Statistic		Model	Actual
Debt-ratio volatility	$\sigma\left(\frac{-b}{y}\right)$	0.3472	0.2719
Risk premium volatility	σ_{Ψ}	0.0083	0.1420
Output autocorrelation	$\rho_{\bar{y}}$	0.9838	0.6757
Consumption autocorrelation	$\rho_{\bar{c}}$	0.8569	0.7844
Correlation coefficient (Ψ, \bar{y})	$\rho_{\Psi, \bar{y}}$	-0.1881	-0.4713
Correlation coefficient (Ψ, \bar{c})	$\rho_{\Psi, \bar{c}}$	-0.2138	-0.5402
Minimum consumption ratio	$\frac{c}{y_{\min}}$	0.7128	0.7242
Maximum consumption ratio	$\frac{c}{y_{\max}}$	0.9521	0.9039
Minimum debt ratio	$-b/y_{\min}$	0	0.0907
Maximum debt ratio	$-b/y_{\max}$	1.8473	1.5363

Source: Authors' computations.

In order to highlight our contribution by modelling unobservable capital, we make a comparison to [Park \(2017\)](#), where physical capital is observable. In [Park \(2017\)](#), there is an extra incentive to capital in order to increase the bond price q , encouraging over accumulation of physical capital. If capital grows large, the probability of default increases, because with large k , the cost of exclusion (autarky) is less, relatively. Consequently, there is the possibility of default when k is high. In our model this does not happen. For large k the probability of default is lowered (see [Figure 4](#)). In our model, as lenders estimate k , they will offer less favourable loan conditions (lower q) when actual k is high, than they would have done if k was observable, and more favourable when actual k is low. Consequently, the country's incentive to borrow is higher (relative to a model with observable k) when actual k is low. Conversely, when actual k is high the country has an incentive of borrow less (relative to a model with observable k), meaning that debt would not be as high to move the country into the default region.

5. Conclusion

In this paper we provide a novel modelling framework where borrower's physical capital is not observable by the lender. As the equilibrium bond price does not depend on capital, the risk premium will be higher than otherwise. Borrowers have no incentive to accumulate capital in order to influence the bond price, which results in a lower steady-state level of physical capital than would be found in a model with observable capital.

We may ask ourselves if unobservable capital will cause countries to borrow

more for consumption rather than investment. Impulse response functions show that a positive technology shock will cause the borrower to borrow for both consumption and investment, regardless of initial capital being at, above or below its steady state level. This suggests that even though capital is not observable, countries have no incentive trying to borrow solely for consumption. It is enough that the bond price depends on the level of borrowing to discipline borrowing behavior. Furthermore, we find that borrowers choose to borrow more when it is cheaper (in good states of nature). In bad states of nature, we detect consumption smoothing behavior through sacrificing physical capital. When large negative productivity shocks occur, we find that borrowers are better off if they reduce consumption and choose to repay their debt in order to avoid default. Defaults, however, occur in the simulations if physical capital and consumption have reached relatively low levels. This happens around 7% of the time.

The simulation results show that our model fits well the default experience of Argentina. Therefore, we confirm the importance of modelling limited information about a borrower's physical capital for understanding the behavior of international borrowers and informing international debt policies.

There are limitations of our analysis. We have assumed the lenders act agnostically when not observing physical capital. Lenders may be able to do better, trying to learn the level of physical capital. For example, there is a correlation (though not perfect) between physical capital and TFP, so TFP could provide an estimate for capital. Still the bond price is a function of TFP, but the borrower cannot affect TFP through capital. Therefore, the incentive to accumulate capital in order to obtain favourable loan terms is still absent. However, the lender would be able to estimate the default probability more accurately. When TFP is high it is likely next period's capital will be high, and the default probability in the future is lower. Thus, the bond price schedule ought to respond more sharply to changes in TFP. Furthermore, given the production function, $y = ak^\alpha$, two variables need to be unobservable for the information problem to emerge. For example, if y was observable as well as TFP, the lender could perfectly determine k .

Our model can provide useful insights for sovereign debt decision making and policy. [Mitchener and Trebesch \(2023\)](#) identify several current trends in sovereign debt negotiations on which this paper can shed light, namely the increase in episodes of debt crisis without default and the surge in debt litigations with the latter in turn advocating for inclusion of investment protection and clear clauses for repayments.

Recent experiences with sovereign debt e.g. in Argentina, point to countries continuing to accumulate large debt instead of defaulting. This is confirmed by our simulation results, [Figure 7](#), where there are two episodes of large debt without default (total 4.76% of the time) and eight episodes of relatively large debt without default (total 12.70% of the time). [Mitchener and Trebesch \(2023\)](#) report the percentage of debt crises with and without default for advanced and emerging economies together. Their data show 27% debt crises with default, and 73% with-

out, for the period 1995-2006. Our model simulation is very close with 30% with default and 70% without (**Figure 7**).

According to the World Bank International Centre for Settlement of Investment Dispute (ICSID), in 2021 South America, Central Asia and Eastern Europe have seen an increase in debt restructuring litigations of 23%. Under current international legislation, the protection of sovereign bondholders is controversial (although arbitration is possible) and typically there are no binding rules dictating repayment priorities which instead is common practice in private finance law. **Gelpern et al. (2023)** report the use of novel contracts by Chinese state-owned banks that try to overcome this problem. These contracts include seniority clauses, lender-controlled cash accounts (acting as collateral), and possibility to ask for immediate repayment (cancellation clauses). One type of investment protection that has attracted attention in the economics literature on sovereign debt and restructuring is the seniority rule, giving repayment priority to creditors in the order in which they lent (see **Chatterjee and Eyigungor, 2015**). **Chatterjee and Eyigungor (2015)** show that when investment protection via seniority rules is applied to debt repayment an increase in welfare can be achieved for the case of Argentina. Thus, one of the implications of our paper for sovereign debt policymaking is that the welfare enhancing effect of inclusion of investment protection rules is reinforced in the case of unobservable capital (as the unobservability exacerbates inefficiency). This effect is particularly relevant for emerging economies where the problem of unobservable capital is more acute.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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