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A Mathematical Formulation of Residential Housing Prices in the United States

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Abstract

In the United States, house prices and corporate credit spreads are accurate predictors of GDP growth, outperforming both liquidity creation and residential investment. House prices also predict macroeconomic indicators such as inflation and unemployment. Therefore, this paper undertakes an examination of the determinants of house prices given that they indirectly affect GDP growth. Residential housing prices are modeled at the levels of affordable middle-class housing and luxury homes. Investor sentiments are modeled as risk-averse, moderate risk-taking, and risk-taking behavior of gradual increase in housing prices. Risk-aversion is presented as the Arrow-Pratt coefficient of risk-aversion. Moderating risk-taking is formulated as a Bessel function. Risk-taking behavior is indicated by an exponential distribution. These investor sentiment functions intersect with a rising Erlang pricing distribution to yield optimal home prices. The luxury home market consists of homes priced at >$1 million. For luxury homes, the same investor sentiments were employed, with housing prices being modeled by a downward-sloping gamma distribution. The mathematical formulations were empirically validated using federal housing data. Theoretical and practical implications are articulated.

Keywords

Residential Housing, Affordable Housing, Luxury Homes, US Home Prices, Housing Price Distributions, Investor Housing Price Sentiments

1. Introduction

Chatterjee (2023) demonstrated that housing prices and corporate credit spreads were stronger predictors of GDP growth than bank liquidity creation and residential investment. Moreover, shocks to housing prices and credit spreads had a stronger impact on GDP growth than other indicators. Miller et al. (2011) provided the conceptual argument that linked housing prices to economic growth. Home ownership results in wealth creation as homeowners have a large asset that can be used as collateral for future consumption, say through home equity loans. As consumption is the largest component of GDP, the increased consumption from home-equity-induced household spending translates into higher GDP growth. Further, homeowners frequently pay property taxes and pay for the upkeep and maintenance of their homes. This increased consumption fuels GDP growth, as property tax revenue is used to fund schools, and upkeep expenses increase the consumption of construction firms. Recent shocks to housing occurred in the form of the 2007-2008 subprime mortgage crisis, and the surge in mortgage rates from 2023-2024. The subprime mortgage crisis decreased housing prices, resulting in a sharp reduction in GDP growth. The surge in mortgage rates as the Federal Reserve has engaged in multiple increases in short-term interest rates, has limited trading up, or the purchase of new and more expensive homes by a family over time. Reduced new home purchases have a direct, adverse effect on economic growth.

Housing has a strong direct influence on consumption as indicated by multiple studies (see Lettau & Ludvigson, 2004, for a review). This finding results in housing influencing inflation and unemployment through consumption. As housing is a large component of household income (Saunders & Cornett, 2024), an increase in housing prices or an increase in mortgage rates increases consumption expenditure, fueling inflation. Yet, housing prices have a negative impact on unemployment (Chatterjee, 2023). Homeowners are more financially stable than renters, as they have the funds for down payment on the home and cash payments for upkeep and property taxes. Such financial stability may arise from stability of employment, suggesting that homeowners are in a better position to combat rising mortgage rates from higher housing prices. Thus, the links of housing prices with GDP growth, inflation and unemployment underscore its importance in an economy. Therefore, the purpose of this paper is to create mathematical models of housing prices based on investor sentiment in the face of economic shocks, interest rate changes, and zoning laws.

Two types of models are envisioned. The first model investigates the affordable housing market with houses priced at $500,000 or less. The second model pertains to housing that is priced above $1 million. As the former homeowner has significantly more financial constraints than the latter, the variables modeling each type of housing price will be different.

The paper provides a unique contribution to the housing economics literature in three ways. First, there is paucity in the literature about mathematical models of the variables that drive housing prices. Studies such as Aye et al. (2016) and Yang et al. (2020), which are among the few that address the predictor variables that influence housing prices, are empirical with no mathematical representation. This paper offers a more conceptual approach by first linking predictor variables to housing prices conceptually, then testing the proposed relationships empirically. Second, multiple cases of alternative investor sentiments have not been considered. This paper sets forth that the Bessel function of investor sentiment has both an optimistic and pessimistic position. The optimistic sentiment is the top of the circular Bessel function, while the pessimistic sentiment is the bottom of the circular Bessel function. Third, the combination of mathematical conceptualization followed by empirical validation completes the research process. Conceptualization alone is conjecture, as it is not validated with real data. Empirical validation by itself lacks a strong theoretical foundation. Therefore, this study combines both theoretical conceptualization and empirical testing.

2. Literature Review

2.1. The Demand for Mid-Priced Housing

Homeownership has historically been considered the American Dream, in the sense that individuals aspire to the ownership of homes for stability, security, and an asset whose price appreciation ensures wealth creation. Government policy has supported this demand for ownership through the mortgage interest deduction and subsidies to prevent foreclosure. The Tax Reform Act of 1986 codified the tax deductibility of mortgage interest, which effectively gave a tax credit to homebuyers to reduce the cost of financing their homes. Glaeser and Shapiro (2003) provided evidence that the mortgage interest deduction did not stimulate homeownership among renters who could not afford house purchases. Hilber and Turner (2014) proved empirically that the mortgage interest deduction had no effect on the homeownership rates of low-income households. Conversely, the mortgage interest deduction increased housing choice for high-income households. With a larger pool of homes to select from, high-income households increased their home purchases. Bourassa and Yin (2008) provided additional evidence of the ineffectiveness of the mortgage interest deduction in influencing home purchases among young, first-time homebuyers.

Demand shocks originate from sudden reductions in demand for housing on the part of homeowners. Zhang and Yang (2024) found that the demand for housing was disrupted during the COVID-19 lockdowns by unemployment, construction, spending, and the housing consumer price index. These demand shocks may also occur during recessions and other economic downturns. Intuitively, sudden unemployment strains household finances, particularly since mortgage payments constitute the largest component of monthly expenses. Irandoust's (2019) Granger Causality approach found that unemployment reduced housing values. Rapach and Strauss (2007) used an ARDL model to show that unemployment reduced housing price increases. Construction costs increase the prices of new homes, reducing demand due to reduced affordability. However, increased home values from higher construction costs increase the value of homes, thereby increasing the wealth of homeowners as houses sell at higher prices (Zahirovich-Herbert & Gibler, 2014). Zhang and Yang (2024) computed the housing consumer price index's contribution to home values, wherein the housing consumer price index measured the increase in housing prices. The sharp increase in the housing consumer price index, even during the COVID-19 lockdowns, acted as a demand shock in that homeowners were unable to purchase bigger homes at higher prices due to lack of affordability.

2.2. The Supply of Mid-Priced Housing

Lee (2024) set forth that house prices in the United States were driven by housing supply, credit availability, and speculation. The premium of housing prices over rents has risen steadily in the past two decades, suggesting that house prices are above rents, with this differential growing over time. Lee's (2024) vector autoregression compared these sources of house prices, using data from 1978-2021. He found that 52% of the variance in house prices could be explained by housing supply. One factor driving housing supply is zoning laws. Zoning laws established by municipalities determine the location of new construction. Restrictive zoning can significantly reduce the land area available for new home construction. Credit availability is driven by mortgage interest rates, with increases in mortgage rates decreasing consumer demand, which in turn, reduces the number of homes constructed as there are fewer buyers. The subsequent reduction in housing supply increases house prices. Glaeser et al. (2008) described the influence of speculators on house prices. When interest rates are low, corporate and individual speculators take advantage of favorable mortgage rates to purchase homes, which they subsequently sell at substantial profit. The demand for speculative purchases and sales rises as more investors enter the real estate market. A surfeit of speculation can cause a housing bubble, as occurred in 2007-2008, when inflated house prices secured by defaulting mortgages resulted in a market crash. About 39% of housing prices were driven by speculative demand (Kindelberger & Alber, 2000). Herd behavior leads speculators to band together behind a key investor, blindly following this individual's price projections, which are based on emotion and irrational assumptions that future prices will always be higher than current prices.

2.3. The Demand for Luxury Homes

Luxury residential homes are usually priced at >$1 million, and are often mansions on large lots in neighborhoods with similarly priced homes. Affluent individuals purchase these homes for the amenities provided, such as extra space for entertaining, tennis courts, and swimming pools. Sass (1988) modeled the demand for luxury homes, providing empirical validation of a theoretical model. The key variable in his model was the value of information on the part of prospective buyers. If buyers compare the price of the house with prior sales in the neighborhood, they become informed about the price of the homes for sale. The buyers will then bid at or above the price of recently sold homes, so that prices of homes for sale will remain high. For new homes, Noguchi's (2003) analysis of prefabricated luxury home sales in Japan showed that the addition of amenities, such as built-in furniture, fireplaces, and extra storage space, attracted and retained buyers willing to pay high prices. Sass's (1988) empirical results showed that price cuts were less steep for homes with prospective buyers who were informed about neighborhood sales prices, and buyers of new homes with unique features. The only homes in which price cuts increased significantly were mansions that had remained on the market for a long time, with buyers who were uninformed of existing sales prices.

2.4. The Supply of Luxury Homes

Luxury homes are built by teams composed of architectural firms, builders, designers, and interior decorators. Designers scour building plans for the most unique features, often traveling to other countries to seek out craftsmen and craftswomen who create the most original pieces of furniture, cabinets, flooring, and other design features. Handcrafted items are common, as buyers emphasize custom-made bespoke items capable of creating a unique living experience. For example, pink limestone from European quarries was used in the entryway of a home. The customized nature of these homes suggests that very few such homes are built, as the labor intensiveness of production increases costs, and consequently final prices, thereby restricting production to very few homes. In the Sass model (Sass, 1988), the limited number of homes available results in very few buyers. The buyers with high incomes have a variety of choices. They may purchase in any one of multiple locations, in small towns, large towns, cities, or in rural areas, on top of mountains, or beside beaches. They may use a variety of domestic and imported materials. They may add multiple amenities or a few amenities. In essence, demand is highly elastic due to buyer choice. As supply follows demand, builders are likely to build a few models to meet the varying tastes of buyers. It would not be feasible to mass-produce homes, given that each buyer wishes to own a home with different amenities from those provided by others. Limited supply may be a hallmark of bespoke homes.

The paucity of buyers may lead to significant price cuts. Sass (1988) cites the case of a total of 5 buyers from an overall population of 100 buyers. The 5 buyers may all reject the property as they may not be interested in the features of the house, which supported the lifestyle of the former owner. For example, a famous athlete's home has spent an excessive number of days on the market as it has courts, racket rooms, and customized memorabilia from the athlete's sporting career that are of no interest to future buyers. Price cuts will result from the long time that such properties remain unsold. Even with price cuts, the volume of sales will be low given the limited number of buyers.

3. Findings and Analysis

3.1. The Risk-Averse Investor in Mid-Priced Housing

The risk-averse investor wishes to avoid investment in increasingly unaffordable housing. The investor will increase housing investment as prices increase, suggesting that the Arrow-Pratt coefficient of relative risk aversion decreases as home buyers are willing to pay higher prices for homes with better amenities, such as proximity to schools and shopping centers. The objective function is

$$\text{Max}\quad u'(c)u''\frac{©-[u''(c)[u''(c)]}{u'(c)u'(c)}$$

St

The constraints are listed below.

$$W_t > 0 \tag{1}$$

The homebuyer's wealth, W, increases over time.

$$T_t > 0 \tag{2}$$

The homebuyer's tastes change in favor of more expensive houses with more amenities,

$$e^{\lambda}\alpha\sum t = 0\ to\frac{infinity(\lambda x)^i}{i!} > 0 \tag{3}$$

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)} > 0 \tag{4}$$

The skewness in Equation (4) characterizes an Erlang distribution.

$$2^{\lambda}(A + B\alpha > 0 \tag{5}$$

Equation (5) shows that there are tight bounds about the Erlang distribution (Lyon, 2021).

Taking Lagrangians,

$$\begin{aligned} &u'(c)u''\frac{©-[u''(c)[u''(c)]}{u'(c)u'(c)} - L_1W_t - L_2T_t - L_3[e^{\lambda}\alpha\sum t = 0\ to\frac{infinity(\lambda x)^i}{i!}] \\ &-L_{4\frac{2}{sqrt(\lambda)}} - 2^{\lambda}(A + B\alpha) \end{aligned} \tag{6}$$

Taking first derivatives of Equation (6),

$$\begin{aligned} &u''(c)u''\frac{©-[u''(c)[u''(c)]}{u''(c)u''(c)} - L_1W_t - L_2T_t - L_3[e^{\lambda}\alpha\sum t = 0\ to\frac{infinityi(\lambda x)}{i!}] \\ &-L_{4\frac{2}{\lambda}} - (\lambda - \lambda 2^{\lambda-1}(A + B\alpha) \end{aligned} \tag{7}$$

Taking second derivatives of Equation (6),

$$\begin{aligned} &u''(c)u''\frac{©-[u''(c)[u''(c)]}{u''(c)u''(c)} - L_1W_t' - L_2T_t' - L_3[e^{\lambda}\alpha\sum t = 0\ to\frac{infinityi(\lambda)}{i!}] \\ &-L_{4\frac{2}{\lambda}} - (\lambda - \lambda(\lambda-1)2^{\lambda-2}(A + B\alpha') \end{aligned} \tag{8}$$

Equation (8) represents the optimal price at which the risk-averse investor purchases the home. This price is point O in **Figure 1**, at the intersection of the Arrow-Pratt coefficient of relative risk-aversion, AB and the Erlang housing price distribution, *PQ*.

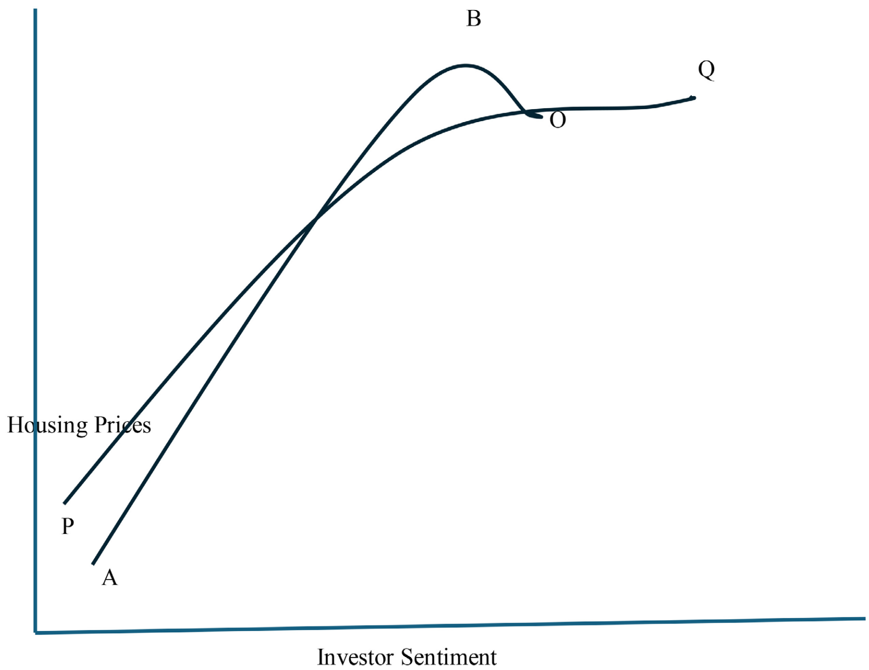

Figure 1. The optimal housing price for the risk-averse investor.

Point *O*, at the intersection of *AB*, investor sentiment, and *PQ*, housing price distribution, is the final price paid by the home buyer of residential real estate.

3.2. The Moderate Risk-Taker in Mid-Priced Housing

The moderate risk-taker is willing to pay higher prices than the risk-averse investor. At the same time, this type of investor exhibits greater flexibility in the sense that he or she may pay much higher prices or modestly higher prices, based on housing availability. If few houses are available, the investor will pay higher prices. Conversely, if a large inventory of houses is available for sale, the moderate risk-taker will become more price-conscious, bargaining prices down to a lower level. The aforementioned Lee's (2024) study listed credit availability and mortgage rates as determinants of housing demand. Both of these variables are important for the moderate risk taker, as greater credit availability means larger mortgages than this borrower can afford due to greater willingness to assume risk than the risk-averse investor.

The moderate risk-taker's sentiments are approximated by a Bessel function (Temme, 1996). In **Figure 2**, the point *O*, the top of the circular chain, is the point of acceptance of high housing prices, while the point *R*, the bottom of the circular chain, is the point of expectation of low housing prices. The pricing distribution is an Erlang distribution, of the family of gamma distributions, with positive shape parameter.

Figure 2 shows *S*, the point of intersection of the Bessel function of investor sentiment, *XY*, and the pricing Erlang distribution, *AB*, as the optimal housing price for the moderate risk-taker.

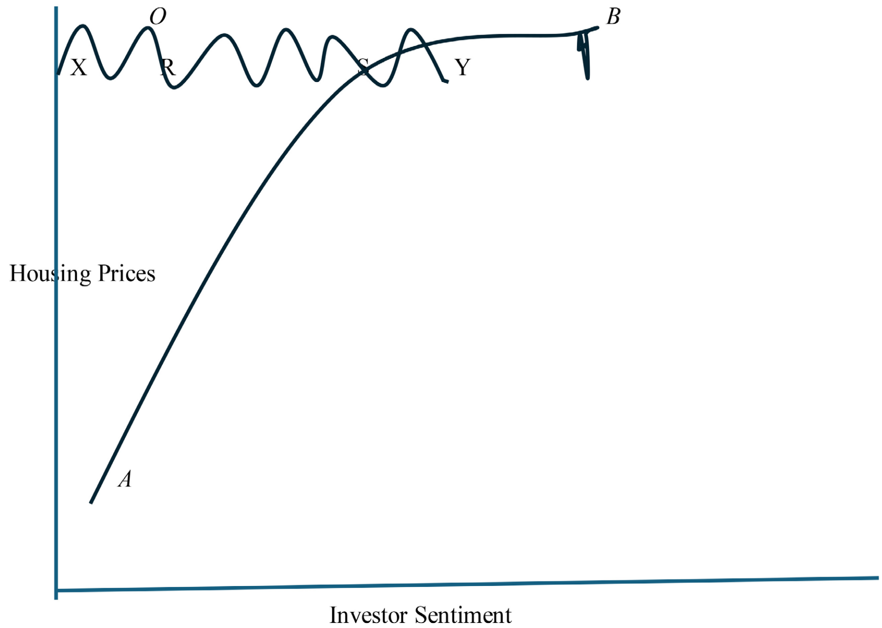

Figure 2. The optimal housing price of the moderate risk-taker.

The formulation is as follows,

<u>Case 1</u>:

$$\text{Max}\ \ [1/x[sine(x-\frac{n\pi}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}] \tag{9}$$

Equation (9) represents point *O*, the upper level of the Bessel function.

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)} > 0 \tag{10}$$

The skewness in Equation (10) characterizes an Erlang distribution.

$$2^{\lambda}(A + B\alpha > 0 \tag{11}$$

Equation (11) shows that there are tight bounds about the Erlang distribution (Lyon, 2021).

$$M_t < 0.36 Gross\ income \tag{12}$$

The monthly mortgage payment, M_t, must be less than 36% of gross income.

$$C_t > M_A + M_B + M_C + \cdots \tag{13}$$

Credit availability, C_t, must be sufficient to cover all mortgages.

Taking Lagrangians,

$$\begin{aligned}&[\frac{1}{x[sine(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}]}\\&-L_1[1e^{\lambda}\alpha\sum t=0\ to\frac{infinity(\lambda x)^i}{i!}]-L_2[\frac{2}{sqrt(\lambda)}-L_3[2^{\lambda}(A+B\alpha)\\&-L_4(M_t-0.36Goss\ income)-L_5(C_t-M_A-M_B-M_C)\end{aligned} \quad (14)$$

The first derivative of Equation (14) is as follows,

$$\begin{aligned}&[\frac{1}{x[cos(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(-r)(2r)!}{(r)!(n2r)!(2)}]}\\&-L_1'[1e^{\lambda}\alpha\sum t=0\ to\frac{infinity(\lambda x)^i}{i!}]-L_2'[\frac{2}{sqrt(\lambda)}-L_3'[2^{\lambda}(A+B)\\&-L_4'(1-0.36Goss\ income)-L_5'(C_t)\end{aligned} \quad (15)$$

The second derivative of Equation (14) is the optimal price for the moderate risk-taker at point *O*, the highest point of the spherical Bessel function.

$$\begin{aligned}&[\frac{1}{x[-sine(1-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(-r)(2)!}{(n2)!(2)}]}\\&-L_1''[1e^{\lambda}\alpha\sum t=0\ to\frac{infinity(\lambda)^i}{i!}]-L_2''[\frac{2}{sqrt(\lambda)}-L_3''[(2)]\\&L_3''[2^{\lambda}(A+B)-L_4''(1-0.36Goss\ income)-1\end{aligned} \quad (16)$$

<u>Case 2</u>:

$$\text{Max}\ \ [1/x[cos(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}] \quad (17)$$

Equation (17) represents point *R*, the lower level of the Bessel function.

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)}>0 \quad (18)$$

The skewness in Equation (18) characterizes an Erlang distribution.

$$2^{\lambda}(A+B\alpha>0 \quad (19)$$

Equation (19) shows that there are tight bounds about the Erlang distribution (Lyon, 2021).

Taking Lagrangians,

$$[\frac{1}{x[cos(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}]}$$

$$
\begin{aligned}
&- L_1[1e^{\lambda}\alpha \sum t = 0\ to \frac{infinity(\lambda x)^i}{i!}] - L_2[\frac{2}{sqrt(\lambda)} - L_3[2^{\lambda}(A + B\alpha) \\
&- L_4(M_t - 0.36Goss\ income) - L_5(C_t - M_A - M_B - M_C)
\end{aligned} \tag{20}
$$

The first derivative of Equation (20) is as follows,

$$
\begin{aligned}
&[\frac{1}{x[cos(x - \frac{n}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(-r)!}{(r)!!(2)}]} \\
&-L_1'[1e^{\lambda}\alpha \sum t = 0\ to \frac{infinity(\lambda x)^i}{i!}] - L_2'[\frac{2}{sqrt(\lambda)} - L_3'[2^{\lambda}(A + B) \\
&L_4'(M_t - 0.36Goss\ income) - L_5'(C_t - M_A - M_B - M_C)
\end{aligned} \tag{21}
$$

The second derivative of Equation (20) is the optimal price for the moderate risk-taker at point *O*, the highest point of the spherical Bessel function.

$$
\begin{aligned}
&[\frac{1}{x[-sine(x - \frac{n}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(-r)!}{(r)!!(2)}]} \\
&-L_1''[1e^{\lambda}\alpha \sum t = 0\ to \frac{infinity(\lambda)^i}{i!}] - L_2''[\frac{2}{sqrt(\lambda)} - L_3''[(2)] \\
&L_4''(M_t) - L_5''(-1)
\end{aligned} \tag{22}
$$

3.3. The Risk-Taker's Investment in Mid-Priced Housing

The risk-taker wishes to purchase housing with desired amenities, such as quality of neighborhood, proximity to work, and proximity to schools. This individual seeks to satisfy personal desires through purchases of housing, regardless of price. The only restriction on the risk-taker is affordability of housing, which after a certain point, may be excessive for the home buyer. Figure 3 models the sentiments of the risk-taker as an upward-sloping exponential distribution. The exponential

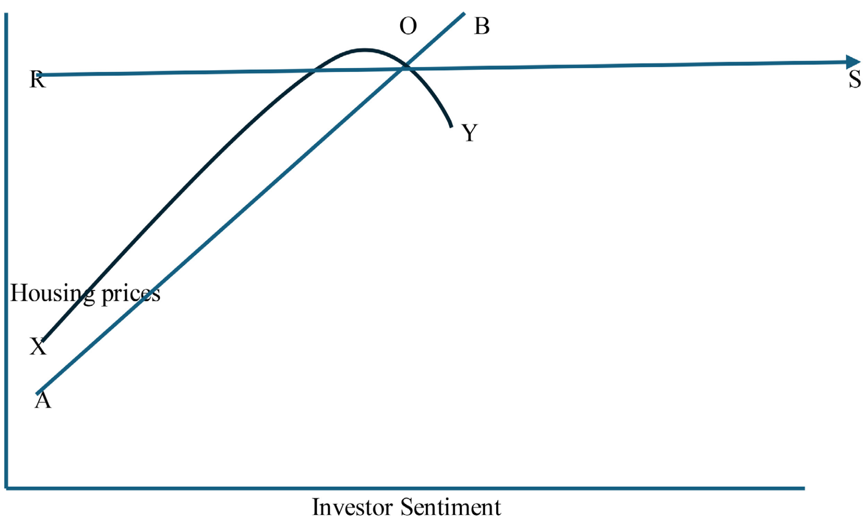

Figure 3. The optimal house price for the risk-taker.

distribution is a probability distribution of events in a Poisson process in which events occur continuously and independently (Weisstein, 2024). This paper views the events as news about housing prices, which social media brings continuously to the risk-taker. The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line *RS*, is the upper bound of housing prices based on affordability.

The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line *RS*, is the upper bound of housing prices based on affordability.

The formulation of the risk-taker's optimal housing price is given below,

$$\text{Max}\quad \begin{array}{l} \sum k = 0\ to\ j-1\left[\dfrac{1}{(n-k)\lambda}\right] + \sum k = 0\ to\ j-1\left[\dfrac{1}{(n-k)\lambda}\right] \\ + \sum k = 0\ to\ i-1\left[1/(n-k)(n-k)\lambda\lambda\right. \end{array} \tag{23}$$

where n is the number of random variables describing housing prices with rate of increase in prices λ. Equation (21) is the maximization of price expectations of housing prices by the risk-taker.

Subject to

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)} > 0 \tag{24}$$

The skewness in Equation (24) characterizes an Erlang distribution.

$$2\lambda(A + B\alpha > 0 \tag{25}$$

Equation (25) shows that there are tight bounds about the Erlang distribution (Lyon, 2021).

$$Y = a + Bx \tag{26}$$

A constraint is added to model line *RS* in **Figure 3**, the upper bound of housing affordability in Equation (24), where x = the highest price the risk-taker can pay for a house.

Taking Lagrangians,

$$\text{Max}\quad \begin{array}{l} \sum k = 0\ to\ j-1\left[\dfrac{1}{(n-k)\lambda}\right] + \sum k = 0\ to\ j-1\left[\dfrac{1}{(n-k)\lambda}\right] \\ + \sum k = 0\ to\ i-1\left[\dfrac{1}{(n-k)(n-k)\lambda\lambda} - L_1 \dfrac{2}{sqrt(\lambda)} - 0\right. \\ -L_2[2\lambda(A + B\alpha - 0] - L_3(Y - a - Bx \end{array} \tag{27}$$

Taking first derivatives of Equation (27),

$$\text{Max}\quad \begin{array}{l} \sum k = 0\ to\ j-1\left[\dfrac{(n-k)}{(n-k)\lambda}\right]_{-1} + \sum k = 0\ to\ j-1\left[\dfrac{(n-k)}{(n-k)\lambda}\right]_{-1} \\ + \sum k = 0\ to\ i-1\left[\dfrac{(n-k)}{(n-k)(n-k)\lambda\lambda} - L_1' 2\lambda - L_2'[2(A + B\alpha] - L_3'(Y - Bx)\right. \end{array} \tag{28}$$

Taking the second derivative of Equation (27) yields the optimal housing price for the risk-taker.

$$\text{Max}\quad \begin{gathered}\sum k = 0\ to\ j-1[\frac{n}{(n-k)\lambda}]_{-2} + \sum k = 0\ to\ j-1[\frac{n}{(n-k)\lambda}]_{-2} \\ +\sum k = 0\ to\ i-1[\frac{(n)}{(n-k)(n-k)\lambda\lambda} - L_1''2\lambda - L_2''[2(B\alpha] - L_3''(1-B)\end{gathered} \qquad (29)$$

3.4. The Risk-Averse Investor in Luxury Homes

The formulation of investor sentiment for the risk-averse investor is identical to Section 3.1. This is the rising Arrow-Pratt coefficient of risk-aversion as the home buyer increases expectations with rising prices for amenities, such as tennis courts, swimming pools, and customized furniture, or IJ in **Figure 4**. Yet, being risk-averse, the home buyer will cease spending on amenities after point *O* in **Figure 4**. Two price points are noteworthy. If there are few buyers, they will demand higher price cuts so that the final price is at point *P*. If there are many buyers, they will compete with each other to obtain the house's amenities, thereby driving the price upwards to point *O*. Given the decline in house prices due to illiquidity, i.e. few home sales, prices are modeled as a step-down declining gamma distribution, *AB*, where the shape parameter, $\lambda < 0$.

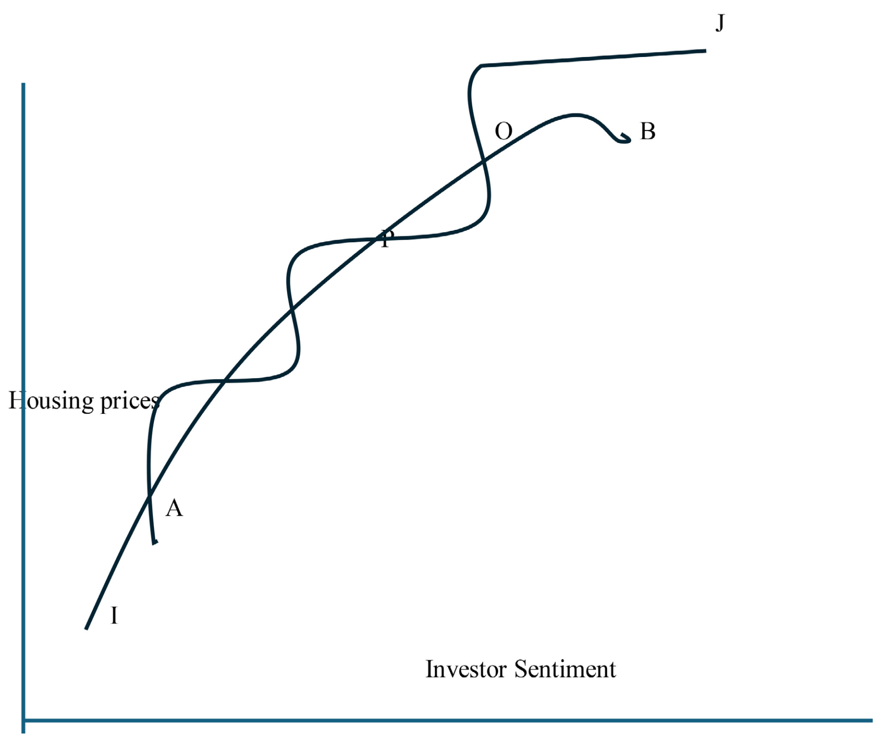

Figure 4. The optimal price for the risk-averse home buyer in luxury housing.

Optimal prices are at two points. The first one is at P with few buyers, while the second one is at O, when there are many buyers.

The formulation is listed below.

$$\text{Max} \quad u'(c)u'' \frac{© - [u''(c)[u''(c)]}{u'(c)u'(c)} \tag{30}$$

St

$$1/\alpha\gamma(\alpha, \frac{x}{\theta}) \tag{31}$$

Equation (31) shows the cumulative density function for a gamma distribution with $\lambda < 0$, where θ is the scale parameter, x is a random variable with mean α.

As price cuts determine the final price, prices with cuts diverge from normal prices. This divergence is expressed as a Kullback-Liebler divergence in Equation (32). The divergence is a gamma distribution with mean α_p and scale parameter, $\theta_{p.}$, while the standard gamma distribution has mean α and scale parameter, θ.

$$(\alpha_p - \alpha)\omega(\alpha_p) - \log(\alpha_p) + \log(\alpha) + \alpha(\log\theta - \log\theta_p) + \alpha_p(\theta_p - \theta)/\theta \tag{32}$$

Taking Lagrangians,

$$\begin{aligned} \text{Max} \quad & u'(c)u'' \frac{© - [u''(c)[u''(c)]}{u'(c)u'(c)} - L_1[\frac{1}{\alpha\gamma(\alpha, \frac{x}{\theta})}] - L_2[(\alpha_p - \alpha)\omega(\alpha_p) \\ & - \log(\alpha_p) + \log(\alpha) + \alpha(\log\theta - \log\theta_p) + \frac{\alpha_p(\theta_p - \theta)}{\theta}] \end{aligned} \tag{33}$$

Taking first derivatives of Equation (33),

$$\begin{aligned} & u''(c)u'' \frac{© - [u''(c)[u''(c)]}{u'(c)u''(c)} - L_1'[\frac{1}{\alpha\gamma(\alpha, \frac{x}{\theta})}] \\ & - L_2'[(\alpha_p - \alpha) - (1/\alpha_p) + 1/(\alpha) + \alpha(\log\theta - \log\theta_p) + \frac{\alpha_p(\theta_p - \theta)}{\theta}] \end{aligned} \tag{34}$$

Taking a second derivative of Equation (34) yields the optimal price,

$$\begin{aligned} & u''(c)u'' \frac{© - [u''(c)[u''(c)]}{u''(c)u''(c)} - L_1'[\frac{1}{\alpha\gamma(\alpha, \frac{x}{\theta})}] \\ & - L_2''[(\alpha_p - \alpha) - (1) + 1 + \alpha(\log\theta - \log\theta_p) + \frac{\alpha_p(\theta_p - \theta)}{\theta}] \end{aligned} \tag{35}$$

3.5. The Moderate Risk-Taker as Investor in Luxury Homes

As described in Section 3.2, the moderate risk-taker's sentiments may be modeled by a Bessel function. In Figure 5, this function, *AB*, intersects with a downward sloping gamma distribution, *SO*, at a high point *O* and a low point *P*. At *O*, home buyers are optimistic about finding a suitable home as there are many houses that suit their tastes. At *P*, the availability of houses for purchase is restricted, so that home buyers are less hopeful of finding a suitable house.

The intersection of the Bessel function, *AB*, and the gamma distribution *SO* yields the optimal luxury home prices at point *O* and point *P*.

<u>Case 1</u>:

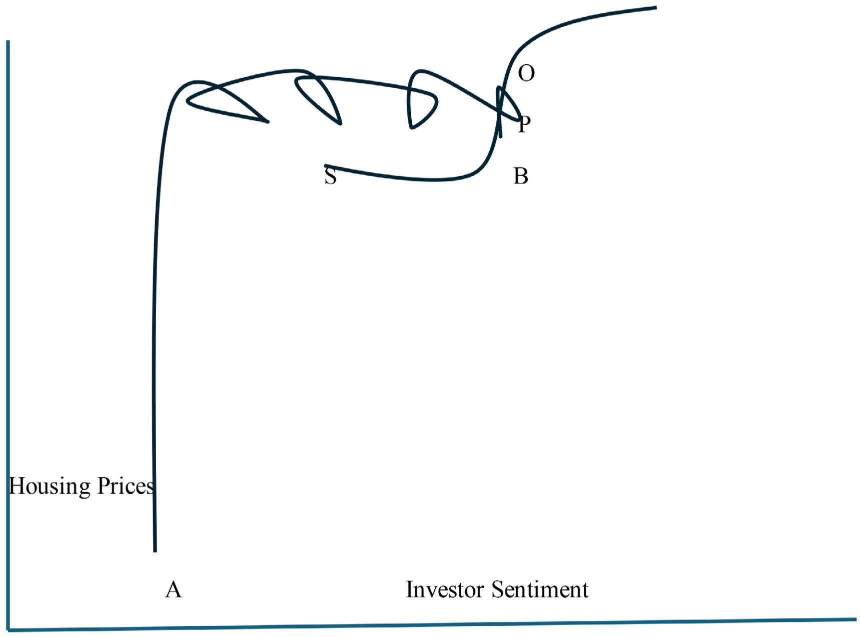

Figure 5. The optimal price for the moderate risk-taker as buyer of luxury housing.

$$\text{Max} \quad [1/x[sine(x-\frac{n\pi}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}] \tag{36}$$

Equation (36) represents point O, the upper level of the Bessel function,

$$1/\alpha\gamma(\alpha, \frac{x}{\theta}) \tag{37}$$

Equation (37) shows the cumulative density function for a gamma distribution with $\lambda < 0$, where θ is the scale parameter, x is a random variable with mean α.

$$M_t < 0.36 Gross\ income \tag{38}$$

The monthly mortgage payment, M_t, must be less than 36% of gross income.

$$C_t > M_A + M_B + M_C + \cdots \tag{39}$$

Credit availability, C_t, must be sufficient to cover all mortgages.

Taking Lagrangians,

$$\begin{aligned}&[\frac{1}{x[sine(x-\frac{n\pi}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}]} - L_1 1/\alpha\gamma(\alpha, \frac{x}{\theta}) \\ &- L_2[(M_t - 0.36 Goss\ income) - L_3(C_t - M_A - M_B - M_C)\end{aligned} \tag{40}$$

The first derivative of Equation (40) is as follows,

$$\begin{aligned}&[\frac{1}{x[cos(x-\frac{n\pi}{2})\sum r = o\ to \frac{(\frac{n}{2})(-1)(-r)(2r)!}{(r)!(n2r)!(2)}]} \\ &- L_1' 1/\alpha\gamma(\alpha, \frac{x}{\theta}) - L_2''[(1 - 0.36 Goss\ income) - L_3'(C_t)\end{aligned} \tag{41}$$

The second derivative of Equation (40) is the optimal price for the moderate risk-taker at point *O*, the highest point of the spherical Bessel function.

$$[\frac{1}{x[-sine(1-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(-r)(2)!}{(n2)!(2)}]} - L_1''1/\alpha(\alpha,\frac{x}{\theta})-L_2''[(1-0.36Goss\ income)-L_3''(C_t) \tag{42}$$

<u>Case 2</u>:

$$\text{Max}\ \ [1/x[sine(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}] \tag{43}$$

Equation (43) represents point *P*, the lower level of the Bessel function,

$$1/\alpha\gamma(\alpha,\frac{x}{\theta}) \tag{44}$$

Equation (44) shows the cumulative density function for a gamma distribution with $\lambda < 0$, where θ is the scale parameter, x is a random variable with mean α.

$$M_t < 0.36Gross\ income \tag{45}$$

The monthly mortgage payment, M_t, must be less than 36% of gross income.

$$C_t > M_A + M_B + M_C + \cdots \tag{46}$$

Credit availability, C_t, must be sufficient to cover all mortgages.

Taking Lagrangians,

$$[\frac{1}{x[sine(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(n+2r)!}{(r)!(n2r)!(2x)}]}+L_1 1/\alpha\gamma(\alpha,\frac{x}{\theta}) - L_2[(M_t-0.36Goss\ income)-L_3(C_t-M_A-M_B-M_C) \tag{47}$$

The positive sign for the first Lagrangian term indicates, point *O*, the bottom of the Bessel function.

The first derivative of Equation (47) is as follows,

$$[\frac{1}{x[cos(x-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(-r)(2r)!}{(r)!(n2r)!(2)}]} + L_1'1/\alpha\gamma(\alpha,\frac{x}{\theta})-L_2''[(1-0.36Goss\ income)-L_3'(C_t) \tag{48}$$

The second derivative of Equation (36) is the optimal price for the moderate risk-taker at point *P*, the lowest point of the spherical Bessel function.

$$[\frac{1}{x[-sine(1-\frac{n\pi}{2})\sum r=o\ to\frac{(\frac{n}{2})(-1)(-r)(2)!}{(n2)!(2)}]} + L_1''1/\alpha(\alpha,\frac{x}{\theta})-L_2''[(1-0.36Goss\ income)-L_3''(C_t) \tag{49}$$

3.6. The Risk-Taker's Investment in Luxury Housing

The risk-taker's desire for luxury housing is based on the increasing desire for amenities as approximated by an exponential distribution. The risk-taker will accept higher prices for the amenities. In Figure 6, the exponential distribution is *RS*. It intersects with the downward-sloping gamma distribution of luxury housing prices, *AB*, at point *O* and point *P*. The chief constraint is the number of buyers. With few buyers, sellers will reduce housing prices sharply so that the final price will be at point *O*. With many buyers, sellers can reject low bids, easily finding alternative buyers. The final price may be higher at point *P* due to the willingness of bidders to pay high prices.

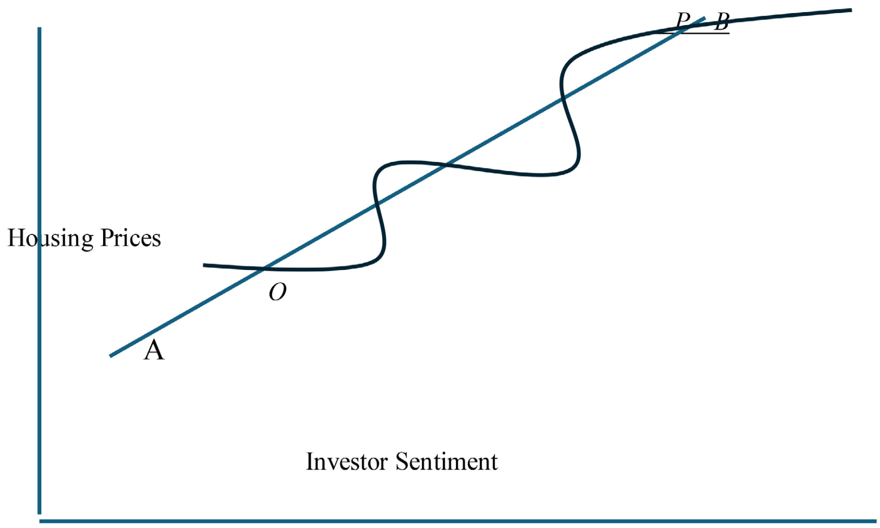

Figure 6. The optimal price for risk-taking buyer of luxury housing.

There are 2 optimal prices, one at *O* for markets with few buyers, and the other one at *P*, with many buyers. Final prices are at the intersection of an exponential distribution of investor sentiment and a gamma distribution of declining house prices.

The formulation is given below.

The formulation of the risk-taker's optimal housing price is given below,

$$\text{Max} \quad \begin{array}{l} \sum k = 0 \; to \; j-1 \left[\frac{1}{(n-k)\lambda}\right] + \sum k = 0 \; to \; j-1 \left[\frac{1}{(n-k)\lambda}\right] \\ + \sum k = 0 \; to \; i-1 \left[1/(n-k)(n-k)\lambda\lambda\right. \end{array} \tag{50}$$

where n is the number of random variables describing housing prices with rate of increase in prices λ. Equation (50) is the maximization of price expectations of housing prices by the risk-taker.

Subject to

$$1/\alpha\gamma\left(\alpha, \frac{x}{\theta}\right) \tag{51}$$

Equation (51) shows the cumulative density function for a gamma distribution with $\lambda < 0$, where θ is the scale parameter, x is a random variable with mean α.

A constraint is added to show the upper limit on the number of buyers,

$$Bx = H \tag{52}$$

where B is the number of buyers and H is the highest number of buyers, which will be a small number at point O, and a large number at point P in **Figure 6**.

Taking Lagrangians,

$$\text{Max} \begin{array}{l} \sum k = 0 \; to \; j-1 [\frac{1}{(n-k)\lambda}] + \sum k = 0 \; to \; j-1 [\frac{1}{(n-k)\lambda}] \\ + \sum k = 0 \; to \; i-1 [\frac{1}{(n-k)(n-k)\lambda\lambda} - L_1 [\frac{1}{\alpha\gamma(\alpha, \frac{x}{\theta})} - L_2 (Bx - H) \end{array} \tag{53}$$

Taking first derivatives of Equation (53),

$$\begin{array}{l} \sum k = 0 \; to \; j-1 [\frac{1}{(n)\lambda}] + \sum k = 0 \; to \; j-1 [\frac{1}{(n)\lambda}] \\ + \sum k = 0 \; to \; i-1 [\frac{1}{(n)(n-1)\lambda\lambda} - L_1' [\frac{1}{\alpha(\alpha, \frac{x}{\theta})} - L_2'(B) \end{array} \tag{54}$$

Taking second derivatives of Equation (53),

$$\begin{array}{l} \sum k = 0 \; to \; j-1 [\frac{1}{(n)}] + \sum k = 0 \; to \; j-1 [\frac{1}{(n)}] \\ + \sum k = 0 \; to \; i-1 [\frac{1}{(n)(n)\lambda\lambda} - L_1'' [\frac{1}{\alpha(\alpha, \frac{x}{\theta})} \end{array} \tag{55}$$

The coefficient of L_2, the second Lagrangian function, is not found at the optimal price.

Two additional models have been created to account for varying values of construction costs, zoning regulations, and interest rates. These conditions apply to mid-priced housing as it is more price-sensitive than luxury housing. The extreme situations of restricted housing supply and expanded housing supply are presented. Restricted housing supply occurs with high construction costs, strict zoning regulations, and high interest rates. When construction costs are high, builders are unable to finance the high cost of construction materials, so they reduce the construction of new homes. Zoning that prevents the building of homes in certain neighborhoods further reduces the supply of housing. High interest rates prevent developers from qualifying for mortgages to purchase land. Under these conditions, risk-takers may be the only individuals willing to pay the high prices for the few homes available. In contrast, low construction costs encourage new home construction, as do lenient zoning and low interest rates for developers to easily purchase property. Risk-averse homeowners will buy housing at reduced prices following the increase in housing inventory.

Case 1: Risk-Taking from High Construction Costs, Restrictive Zoning, and High Interest Rates

We reproduce the risk-taker's mid-priced housing formulation with the addition of constraints for construction costs, zoning, and interest rates.

Figure 7 models the sentiments of the risk-taker as an upward-sloping exponential distribution. The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line, *RS*, is the upper bound of housing prices based on affordability. This upper bound is higher than the same bound in Figure 3 to account for the reduced housing affordability. The new point *O* intersects with RS to reflect higher housing prices.

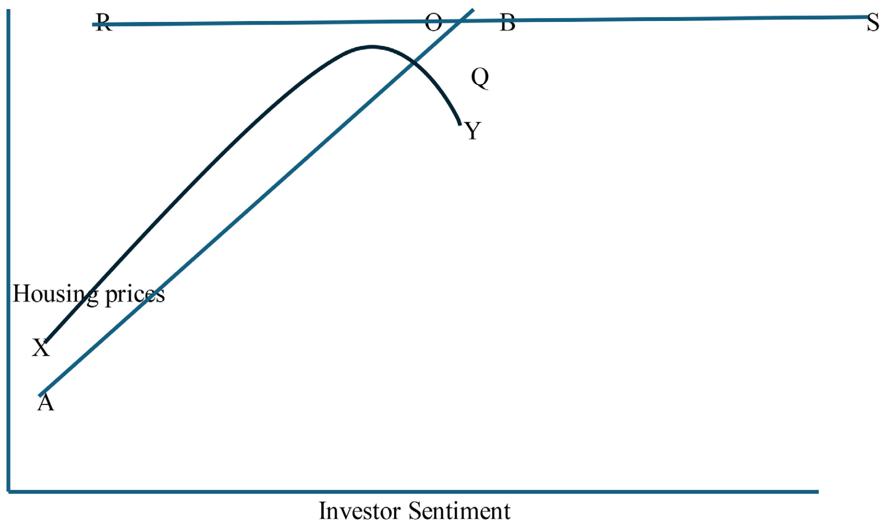

Figure 7. The optimal house price for the risk-taker with reduced housing supply.

The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line, *RS*, is the upper bound of housing prices based on affordability.

The formulation of the risk-taker's optimal housing price is given below,

$$\text{Max} \quad \begin{aligned} &\sum k = 0 \; to \; j-1\left[\frac{1}{(n-k)\lambda}\right] + \sum k = 0 \; to \; j-1\left[\frac{1}{(n-k)\lambda}\right] \\ &+ \sum k = 0 \; to \; i-1\left[1/(n-k)(n-k)\lambda\lambda\right.\end{aligned} \tag{56}$$

where n is the number of random variables describing housing prices with rate of increase in prices λ. Equation (56) is the maximization of price expectations of housing prices by the risk-taker.

Subject to

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)} > 0 \tag{57}$$

The skewness in Equation (58) characterizes an Erlang distribution.

$$2\lambda(A + B\alpha > 0 \tag{58}$$

Equation (59) shows that there are tight bounds about the Erlang distribution (Lyon, 2021).

$$Y = a + Bx \tag{59}$$

A constraint is added to model line *RS* in **Figure 7**, the upper bound of housing affordability in Equation (59), where *x* = the highest price the risk-taker can pay for a house.

Equation (60) is a constraint showing rising construction costs. *Cx* represents construction costs per housing unit on an upward trajectory,

$$Cx_1 > Cx_2 > Cx_3 > 1 \tag{60}$$

Equation (61) is a constraint showing zoning restrictions of *z* per unit on *X* total units, limiting housing supply to *H*.

$$zX = H \tag{61}$$

Equation (62) is a constraint showing rising interest rates, where interest rates are represented by *I*.

$$I_1 > I_2 > I_3 > 1 \tag{62}$$

Taking Lagrangians,

$$\text{Max} \begin{array}{l} \sum k = 0 \; to \; j-1\left[\frac{1}{(n-k)\lambda}\right] + \sum k = 0 \; to \; j-1\left[\frac{1}{(n-k)\lambda}\right] \\ +\sum k = 0 \; to \; i-1\left[\frac{1}{(n-k)(n-k)\lambda\lambda} - L_1 \frac{2}{sqrt(\lambda)} - 0\right. \\ -L_2[2\lambda(A+B\alpha-0] - L_3(Y-a-Bx-L_3[Cx_1 > Cx_2 > Cx_3 - 1 \\ -L_4(zX-H) - L_5(I_1 - I_2 - 1 \end{array} \tag{63}$$

Taking first derivatives of Equation (63),

$$\text{Max} \begin{array}{l} \sum k = 0 \; to \; j-1\left[\frac{(n-k)}{(n-k)\lambda}\right]_{-1} + \sum k = 0 \; to \; j-1\left[\frac{(n-k)}{(n-k)\lambda}\right]_{-1} \\ +\sum k = 0 \; to \; i-1\left[\frac{(n-k)}{(n-k)(n-k)\lambda\lambda} - L_1' 2\lambda - L_2'[2(A+B\alpha]\right. \\ -L_3'(Y-Bx) - L_3'(Cx_1 - Cx_2 - Cx_3) - L_4' zX - L_5'(I_1 - I_2) \end{array} \tag{64}$$

Taking the second derivative of Equation (63) yields the optimal housing price for the risk-taker.

$$\text{Max} \begin{array}{l} \sum k = 0 \; to \; j-1\left[\frac{(n)}{(n-k)\lambda}\right]_{-2} + \sum k = 0 \; to \; j-1\left[\frac{(n)}{(n-k)\lambda}\right]_{-2} \\ +\sum k = 0 \; to \; i-1\left[\frac{(n)}{(n-k)(n-k)\lambda\lambda} - L_1'' 2\lambda - L_2''[2(B\alpha]\right. \\ -L_3''(1-B) L_3''(Cx_1 - Cx_2 - Cx_3) - L''(zX) \end{array} \tag{65}$$

Case 2: Risk-Taking from Low Construction Costs, Lenient Zoning, and Low Interest Rates

We reproduce the risk-taker's mid-priced housing formulation with the addition of constraints for construction costs, zoning, and interest rates.

Figure 8 models the sentiments of the risk-taker as an upward-sloping exponential distribution. The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line, *RS*, is the lower bound of housing prices based on affordability. This lower bound is lower than the same bound in Figure 3 to account for the increased housing affordability. The new point *O* intersects with RS to reflect lower housing prices.

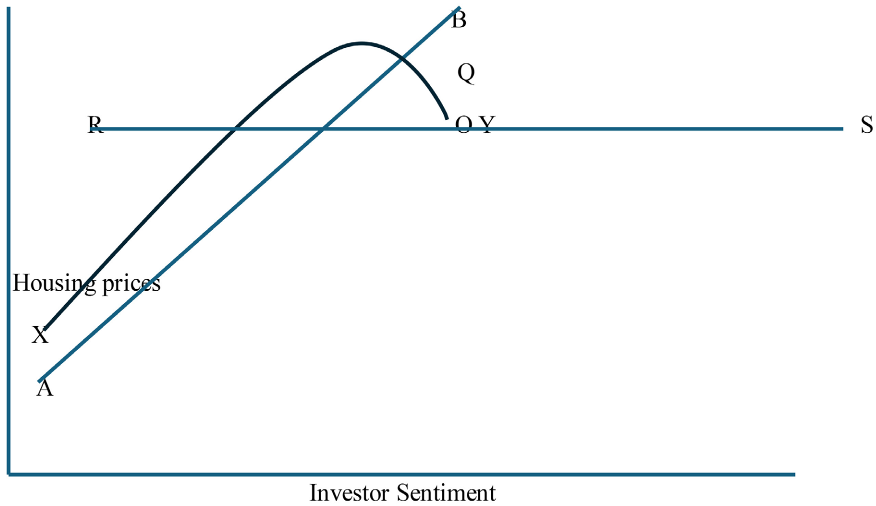

Figure 8. The optimal house price for the risk-averse homebuyer with reduced housing supply.

The exponential distribution, *AB*, intersects with the Erlang distribution of housing prices, *XY*, at point *O*, the optimal housing purchase price. The line, *RS*, is the lower bound of housing prices based on affordability.

The formulation of the risk-taker's optimal housing price is given below,

$$\text{Max} \quad \begin{array}{l} \sum k = 0 \; to \; j-1\left[\dfrac{1}{(n-k)\lambda}\right] + \sum k = 0 \; to \; j-1\left[\dfrac{1}{(n-k)\lambda}\right] \\ + \sum k = 0 \; to \; i-1\left[1/(n-k)(n-k)\lambda\lambda\right. \end{array} \tag{66}$$

where n is the number of random variables describing housing prices with rate of increase in prices λ. Equation (66) is the maximization of price expectations of housing prices by the risk-taker.

Subject to

An Erlang pricing distribution results in positive upward-sloping shape parameter α, and scale parameter, θ. Residential housing prices increase with positive shape, levelling off before prices become excessive.

$$\frac{2}{sqrt(\lambda)} > 0 \tag{67}$$

The skewness in Equation (68) characterizes an Erlang distribution.

$$2\lambda(A + B\alpha > 0 \tag{68}$$

Equation (69) shows that there are tight bounds about the Erlang distribution

(Lyon, 2021).

$$Y = a + Bx \tag{69}$$

A constraint is added to model line RS in **Figure 8**, the lower bound of housing affordability in Equation (69), where x = the lowest price the risk-averse homebuyer will pay for a house.

Equation (70) is a constraint showing decreasing construction costs. Cx represents construction costs per housing unit on an upward trajectory,

$$Cx_1 < Cx_2 < Cx_3 < 1 \tag{70}$$

Equation (71) is a constraint showing zoning restrictions of z per unit on X total units, increasing housing supply to H.

$$zX > H \tag{71}$$

Equation (72) is a constraint showing decreasing interest rates, where interest rates are represented by I.

$$I_1 < I_2 < I_3 < 1 \tag{72}$$

Taking Lagrangians,

$$\begin{aligned} & \sum k = 0\ to\ j-1\left[\frac{1}{(n-k)\lambda}\right] + \sum k = 0\ to\ j-1\left[\frac{1}{(n-k)\lambda}\right] \\ \text{Max} \quad & + \sum k = 0\ to\ i-1\left[\frac{1}{(n-k)(n-k)\lambda\lambda} - L_1\frac{2}{sqrt(\lambda)} - 0\right. \\ & -L_2[2\lambda(A + B\alpha - 0] - L_3(Y - a - Bx - L_3[Cx_1 - Cx_2 - Cx_3 - 1] \\ & -L_4(zX - H) - L_5(I_1 - I_2 - 1 \end{aligned} \tag{73}$$

Taking first derivatives of Equation (73),

$$\begin{aligned} & \sum k = 0\ to\ j-1\left[\frac{(n-k)}{(n-k)\lambda}\right]_{-1} + \sum k = 0\ to\ j-1\left[\frac{(n-k)}{(n-k)\lambda}\right]_{-1} \\ \text{Max} \quad & + \sum k = 0\ to\ i-1\left[\frac{(n-k)}{(n-k)(n-k)\lambda\lambda} - L_1'2\lambda - L_2'[2(A + B\alpha]\right. \\ & -L_3'(Y - Bx) - L_3'(Cx_1 - Cx_2 - Cx_3) - L_4'zX - L_5'(I_1 - I_2) \end{aligned} \tag{74}$$

Taking the second derivative of Equation (63) yields the optimal housing price for the risk-taker.

$$\begin{aligned} & \sum k = 0\ to\ j-1\left[\frac{(n)}{(n-k)\lambda}\right]_{-2} + \sum k = 0\ to\ j-1\left[\frac{(n)}{(n-k)\lambda}\right]_{-2} \\ \text{Max} \quad & + \sum k = 0\ to\ i-1\left[\frac{(n)}{(n-k)(n-k)\lambda\lambda} - L_1''2\lambda - L_2''[2(B\alpha]\right. \\ & -L_3''(1 - B)L_3''(Cx_1 - Cx_2 - Cx_3) - L''(zX) \end{aligned} \tag{75}$$

4. Empirical Validation

The theoretical formulations of Section 3 were validated using empirical data from the Federal Reserve Bank of St Louis. FRED data in three areas were the following Series IDs were obtained.

S&P Corel Logic Case-Schiller National Home Price Index	CSUSHP1NUSA
GDP	A19RP1A027NBEA
Business Expectations Sales Revenue Growth	ATLSBUSRGEP

Housing price data and GDP were collected monthly from 2010-2025 (April). Business survey data reflecting investor sentiment were collected over the same period. The dependent variable of housing prices was regressed on the independent variable of investor sentiment as a global measure, and as a specific measure of risk-aversion, moderate risk-taking, and risk-taking behavior. Risk-averse investors had scores of 1 or 2, moderate risk-takers had scores of 3 and 4, while risk takers had scores > 4.

The following regression equation was tested,

$$H_t = \alpha + \beta_1(IS) + \beta_2(GDP) + \varepsilon \tag{76}$$

where,

H_t = Housing prices,

IS = Investor sentiment (risk-averse, moderate risk-taking, or risk-taking),

GDP = Gross Domestic Product.

Table 1 provides the descriptive statistics for the input data. All skewness and kurtosis values are <20, indicating the distributions of housing prices and investor sentiment are normal.

Table 1. Descriptive statistics of variables.

Variable	Mean	Variance	Skewness	Kurtosis
Housing Prices (index)	1331.67	40.57	−1.75	11/41
Investor Sentiment (index)	4.41	1.46	−1.58	4.85
Risk-Aversion	1.32	0.21	0.78	−1.41
Moderate Risk-Taking	1.06	0.05	3.76	12.40
Risk-Taking	1.46	0.25	0.16	−2.01
Gross Domestic Product (in trillions of dollars)	21,964	0.740	1.34	1.23

Table 2. Results of regressions of housing prices on independent variables.

Variable	Coefficient	T value	Significance	Supports Theory?
Constant	489	1.51	0.13	
Panel A				
Investor Sentiment Global	13.85	2.08	0.03	Yes
Gross Domestic Product (GDP)	0.03	2.19	0.02	
N	100			
R^2	0.98			
Panel B				
Constant	1432	2.51	0.01	

Continued

Risk Aversion	111.13	−2.62	0.01	Yes
Gross Domestic Product (GDP)	−0.01	−0.51	0.60	
N	99			
R^2	0.98			
Panel C				
Constant	1.379	2.87	0.04	
Moderate Risk-Taking	55.20	0.99	0.03	Yes
Gross Domestic Product (GDP)	−0.00	−0.27	0.78	
N	99			
R^2	0.98			
Panel D				
Constant	150.1	3.01	0.003	
Risk-Taking	28.46	0.52	0.06	Yes
Gross Domestic Product (GDP)	−0.006	−0.32	0.74	
N	99			
R^2	0.98			

Table 2 shows the four regressions in Panel A, Panel B, Panel C, and Panel D, which were performed to validate the theoretical findings. Panel A supports the theory fully will housing prices rising with investor sentiment. Panel B supports the theory in that risk aversion significantly reduces housing prices, as risk-averse home buyers curb their purchases of high-priced real estate as they are averse to having large, risky mortgages. Panel C and Panel D support the theory that investor sentiment from buyers willing to accept higher mortgage payments, i.e. moderate to high-risk, will be sufficiently accepting of risk to drive housing prices to higher levels.

5. Conclusion

5.1. Discussion of Findings

The principal finding is that housing prices are driven by attitude to risk. Investor sentiment drives housing prices for both mid-priced housing and luxury housing. For mid-priced housing, risk-averse investors are the least likely to compromise on facing higher prices. Risk-aversion leads them to set fixed amounts for housing, with little desire for flexibility in determining final prices. In contrast, moderate risk-takers are more willing to accept higher purchase prices as they view additional amenities such as proximity to shopping and educational institutions. However, final prices will be contingent on the number of buyers. With mid-priced housing, there may be a large number of buyers, as prices are within the budgets of many prospective home buyers. Buyers may compare mortgage rates from different financial institutions to determine the rate that is affordable. They will then evaluate the benefits of amenities such as expensive landscaping and outdoor kitchens before they make the decision to purchase a property. However, the major distinguishing feature of moderate risk takers is that their positive sentiments toward higher-priced houses will result in increasing home prices over time.

The empirical validation data in Section 5 show that the slope of the equation relating moderate risk-taking to housing prices is steeper than the slope of the equation linking risk-taking to higher home prices. This finding suggests that housing prices rise faster with investor sentiment for moderate risk-takers than for risk-takers. This finding may be attributed to the difference between the mid-priced housing market and the luxury market. In the mid-priced housing market, there are many buyers who are moderate risk-takers. These buyers are able to afford larger mortgages that have payments < 36% of income. They are able to purchase housing that meets their tastes, as they have higher incomes. The large number of buyers who desire attractive properties with amenities drives housing prices higher. The market becomes a seller's market, in which sellers can charge higher prices by adding more features to their properties. Prices rise rapidly with investor sentiment.

In contrast, the luxury market consists of high-priced houses with prices > $1 million. The number of buyers declines at such high prices, as many prospective buyers will not qualify for such large mortgage payments. In the Sass model (Sass, 1988), the few buyers work diligently to determine prices of comparable properties. They will compare the prices of these recent home sales with the asking prices of homes in the area. If asking prices are > than that of recent sales, the buyers will refuse to pay higher prices. This action results in sellers giving price cuts to retain the buyers. The market becomes a buyer's market, in which buyers control prices. Investor sentiments drive housing prices lower. Buyer control yields buyer informativeness about housing prices as a variable that must be considered in future studies of the effects of investor sentiment on housing prices.

A puzzling finding is the non-significance of GDP as a determinant of housing prices, although Section 2 suggests the possibility of a relationship. This study explains this finding by the large size of the U.S. GDP at approximately $26 trillion. A 1% - 2% shift in this large base value is unlikely to have a significant impact on housing prices. The empirical study may be replicated for periods of considerable shifts in GDP, such as the immediate aftermath of recessions or economic booms, during which DP may significantly influence housing prices.

How must buyer informativeness be included in these models? Buyer informativeness may be considered as a predictor of price cuts. A few informed buyers will resist higher home prices, thereby reducing final house prices. In contrast, a large number of uninformed buyers will increase prices, as they will not be able to argue for price cuts with the seller. Since these findings were theorized by Sass (1988), over 35 years ago, future studies should update these findings for the modern era. Updating findings are necessary in a world of social media and instantaneous transmission of house prices over the Web, which did not exist 35 years ago.

5.2. Practical Implications

Real estate agents obtain earnings through the sale of properties. These agents must recognize that home buyer attitudes toward risk drive prices, and in turn, determine their income. Therefore, the real estate agents must employ different strategies to attract buyers with different attitudes to risk. In the mid-priced housing market, the following strategies for selling may be effective. For risk-averse home buyers, the real estate agent must emphasize utilitarian features of the property rather than aesthetic features, as these buyers will only pay additional amounts for useful features. They will not pay for beautification through manicured lawns and upscale furniture. Their aversion to risk is so acute that realtors must emphasize that buyers are paying for value, of true usefulness to themselves. With moderate risk takers, the real estate agent may include attractiveness of the property, history of the property, and community support. Buyers will be willing to pay for these features that make their lives more comfortable, while providing safety and security.

In the mid-priced housing market, risk takers are a unique quantity. They are less concerned about large mortgage payments if they can obtain the amenities of their choice. Realtors may act as a restraining influence on such buyers. As buyers continue to increase the size of mortgage payments by increasing the number of amenities, realtors must intervene. They must urge caution to the buyers, advising them of financial hardship due to job loss, health issues, or other personal emergencies. Borrowers must not assume excessively high mortgage payments, as there is real risk of foreclosure if payments are interrupted.

In the luxury real estate market, the number of buyers is a critical variable. When there are few informed buyers, buyers may obtain significant price discounts. Real estate agents must inform buyers of the number of buyers before the homeowner bids on a property. If there are few buyers, the homeowner can demand a large reduction in price. If there are a large number of buyers, the homeowner will need to bid near the asking price, or will lose the purchase to another buyer. The level of informativeness in the market is also an important indicator of housing prices. Informed buyers will be knowledgeable about home prices in the area, while uninformed buyers will not have this information. Being informed assists buyers in setting price cuts as they know the reasonable price for property in the area.

5.3. Research Limitations

A few research limitations exist. Regional differences in residential housing prices have not been considered. In the United States, such regional differences may be considerable. For example, median home prices in Northern California are in excess of $1 million for mid-priced housing. In contrast, in Southern locations, median home prices may be as low as $350,000. Future studies should consider such differentials in home prices.

Demographic influences on affordability were not considered, including age, income and gender. For example, young urban professionals may have high incomes, permitting them to afford more expensive homes than senior citizens living on lower, fixed pension income. Future studies should demarcate formulations based on age and income. Another variable of importance is family size. Small families of 1 - 2 persons have lower expenses, permitting them to afford larger home mortgage payments than large families with >5 members.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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