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Firms' Coalitions, Demand Systems and Asymmetric Prices

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Abstract

We extend Deneckere and Davidson's model on firms' incentives to form coalitions to predict which firms are more likely to enter a coalition. In Deneckere and Davidson, all firms have the same incentives to enter a coalition because they are all symmetric and therefore choose symmetric prices. We modify their demand specification to include different cross-price elasticities. With this, we can generate market equilibria where firms have different prices, and therefore different incentives to enter a coalition. Our modified model can inform the construction and estimation of empirical models of coalitions and price formation.

Keywords

Firms' Coalitions, Partial Cartels, Mergers and Acquisitions, Joint Ventures, Demand Systems, Asymmetric Prices

1. Introduction

In this paper, we extend Deneckere and Davidson's (1985) model on firms' incentives to form coalitions. We define coalitions in a broad sense, from cartels to mergers and acquisitions and joint ventures (see Jacquemin & Slade, 1989). To simplify the exposition, we refer to all these cases as coalitions. Our aim with this extension is to predict which firms are more likely to enter a coalition.

Deneckere and Davidson (1985) show that firms can have incentives to form partial coalitions (i.e., where only some firms in the market cooperate), instead of full coalitions (i.e., all firms in the market cooperate). In their model, when a coalition is formed, there are only two prices. All firms inside the coalition set the same price, and all firms outside the coalition set the same price. This is the case because all firms are symmetric. As a result, Deneckere and Davidson's (1985) model can determine whether or not a partial coalition is sustainable, but it is not possible to predict which firms enter the coalition because, by having symmetric prices, all firms have the same incentives to participate in a coalition.

To predict which firms have greater incentives to enter into a coalition, we modify the demand specification in Deneckere and Davidson (1985) to include different cross-price elasticities. We do this by using the demand specification outlined in Amir et al. (2017). We show that, as in Deneckere and Davidson (1985), firms can still behave as insiders and outsiders of the coalition. However, more interestingly, we can generate market equilibria where all firms have different prices, as we observe in the real world, and thus different incentives to enter a coalition.

The findings of this paper are particularly relevant for understanding real-world coalitions (such as cartels, mergers and acquisitions and joint ventures) and empirical investigations of Bertrand models with differentiated demand. They are also particularly relevant for markets where demand for products exhibits different price elasticities. Accordingly, products have different price elasticities if they are imperfect substitutes even when located next to each other along a horizontal attribute space. This is similar to the model of the car industry by Bresnahan (1987). One car type (or brand) is in direct competition only with cars of similar quality (and less in competition with cars of different quality), e.g., city cars versus luxury cars. Accordingly, two different city cars are closer substitutes than a city car and a luxury car. Another example is the newspaper media market, e.g., sports newspapers versus local newspapers. Accordingly, two different local newspapers are closer substitutes than a local newspaper and a sports newspaper.

Another example is a well-known cartel case, the interbank LIBOR lending market. In this market, banks lend money overnight at the interbank rate. The mechanism is as follows. If n banks submit bids, the LIBOR rate is set as the mean of the middle $n/2$ quotes. The submitted lending rates were clearly homogeneous because the chosen interbank rate was determined solely from the submitted bids. There was no evidence of cost differences between banks (see the discussion in Abrantes-Metz et al., 2011). Still, as observed in Abrantes-Metz et al. (2011) and Abrantes-Metz et al. (2012), bids differed between banks. This was true not only in periods of competitive market conditions but also during the period when the LIBOR market was cartelized.

The objectives of our paper are twofold. First, the theoretical predictions of existing cartels, mergers and acquisitions and joint venture models are inconsistent with real-world data regarding price patterns. Accordingly, as mentioned above, these models usually predict that firms (pre- and post-coalition) have symmetric prices. We aim to develop a theoretical model that can predict price patterns in better accordance with the data, i.e., asymmetric price patterns. Second, we intend to provide a novel set of hypotheses for essmpirical analyses regarding incentives to enter into coalitions, in particular, which firms are more likely to enter a coalition.

2. Model

In this section, we present the model, which as mentioned above, is a modification of Deneckere and Davidson (1985). In particular, whereas Deneckere and Davidson (1985) use the demand specification outlined in Shubik and Levitan (1980), we use the demand specification in Amir et al. (2017). As a result, whereas Deneckere and Davidson (1985) assume that the cross-price effects between product i and all other products, j, $j \neq i$, are identical, we assume that cross-price effects differ[1].

Demand and Supply

We illustrate the model with three firms. The model can be generalized to n firms, at the costs of exposition, clarity and complexity.

Assume then that there are three firms in the industry, $i = 1,2,3$, producing one good each. Without loss of generality, costs are normalized to zero. The products can differ along one (or more) characteristics. For instance, consumers may perceive products as differentiated according to quality, size, strength, geographical competition or another dimension. The utility function for the representative consumer is:

$$U(x) = \alpha\left(\sum_{i=1,2,3} q_i\right) - \beta\left(\sum_{i=1,2,3} q_i\right) - \delta_d \sum_{i \neq j} q_i q_j, \tag{1}$$

where q_i is the demand for product $i = 1,2,3$. The parameters α and β are the usual parameters from a linear demand function. The parameter δ_d represents the cross-price effect and the subscript d indicates how distant substitutes the products are, $\delta = |i - j|, i \neq j$.

From the above, we find the indirect demand functions for the products:

$$\begin{aligned} p_1 &= \alpha - \beta q_1 - \delta_1 q_2 - \delta_2 q_3 \\ p_2 &= \alpha - \delta_1 q_1 - \beta q_2 - \delta_1 q_3 \\ p_3 &= \alpha - \delta_2 q_1 - \delta_1 q_2 - \beta q_3. \end{aligned} \tag{2}$$

From here we can see that products 1 and 2 are close substitutes and the parameter takes the value, $\delta = |1 - 2| = 1$, whereas products 1 and 3 are more distant substitutes, $\delta = |1 - 3| = 2$. Assume for simplicity that $\beta, \delta_d > 0$, and $\beta > \delta_1 > \delta_2$. According to the definition of substitute (complementary) goods, two goods are substitutes (complements) in utility if:

$$\partial^2 U / \partial q_i \partial q_j \leq 0 \, (\geq 0). \tag{3}$$

Hence, in the above utility function—and in the indirect demand functions—all products are substitutes to one another, and the own-price effect is larger than the other-price effects. In the two-good case, substitute products are necessarily gross substitutes, and complement products are gross complements[2]. When there

[1] Boffa and Vannoni (2012) in turn look to incentives to forma cartel in a dynamic context.

[2] "Gross substitutes" describes goods where an increase in the price of one good cause a consumer to demand more of the other good, even if the consumer's overall real income decreases. "Gross complements" describes two goods where an increase in the price of one good leads to a decrease in the demand for the other good. This relationship is verified by observing a negative cross-price elasticity of demand.

are three or more goods, these conditions do not hold in general. When inverting the above indirect demand relations, we obtain the direct demand functions:

$$\begin{aligned} q_1 &= a_1 - b_1 p_1 + d_1 p_2 + d_2 p_3, \\ q_2 &= a_2 + d_1 p_1 - b_2 p_2 + d_1 p_3, \\ q_3 &= a_3 + d_2 p_1 + d_1 p_2 - b_3 p_3. \end{aligned} \tag{4}$$

Define $D = (\beta - \delta_2)(\beta^2 + \beta\delta_2 - 2\delta_1^2)$. The own-price effect for products 1 and 3 is $b_1 = b_3 = (-\beta^2 + \delta_1^2)/D$, whereas the own-price effect for product 2 is $b_2 = (-\beta - \delta_2)^2/D$. The cross-price effects between products 1 and 3 are $(\beta\delta_2 - \delta_1^2)/D$, whereas the cross-price effects between products 1 and 2 (and 2 and 3) are $(d_1 b - d_1 d_2)/D$. Two goods, i and j, are gross substitutes (complements) if:

$$\partial q_i / \partial p_j \geq 0 (\leq 0). \tag{5}$$

In the next section, we illustrate with numerical simulations the properties of our model.

3. Numerical Examples

We use two numerical examples to illustrate how substitute goods may also be gross complement goods, and how this may affect incentives to enter a coalition. Consider an industry with three firms with the following parameters:

Case 1: $\alpha = 10$, $\beta = 1$, $\delta_1 = 0.6$ and $\delta_2 = 0.5$

Case 2: $\alpha = 10$, $\beta = 1$, $\delta_1 = 0.6$ and $\delta_2 = 0.25$

In the first case (Case 1), all products are substitutes and also gross substitutes. In the second case (Case 2), all products are substitutes, but goods 1 and 3 are gross complements. Products 1 and 3 change from gross substitutes to gross complements when $\beta\delta_2 - \delta_1^2$ turns positive.

Consider now that entity k is a two-firm partial coalition consisting of firms 1 and 2; firm 3 is an outside firm. Entity k's profit maximization behavior is as follows:

$$\max_{p_1, p_2} p_1 \cdot q_1(p_1, p_{-1}) + p_2 \cdot q_2(p_2, p_{-2}).$$

The first-order conditions are:

$$\frac{\partial \pi_k}{\partial p_{k=1}} = q_1(p_1, p_{-1}) + \frac{\partial q_1}{\partial p_1} p_1 + \frac{\partial q_2}{\partial p_1} p_2 = 0, \quad \forall i, j$$

From the first-order conditions, we can solve for the equilibrium prices, and then solve for the equilibrium quantities and profits. Furthermore, using the first-order conditions, we can solve for the best response function, that is, the optimal price for coalition k given the prices of the outside firm, and the diversion ratio is $DR_{i,j} = \frac{\partial q_j}{\partial p_i} \Big/ \left| \frac{\partial q_i}{\partial p_i} \right|$, see e.g., Shapiro (1995).

In Case 1, in the three-firm industry outlined above, if all products are gross substitutes, all firms have incentives to enter into a coalition (see **Table 1**). The own-price elasticity for all products in both cases is −1. In this case, the diversion ratios[3] are all positive, $DR_{1,2} = DR_{3,2} = 0.47$, $DR_{1,3} = DR_{3,1} = 0.22$ and $DR_{2,1} = DR_{2,3} = 0.4$. In **Table 1**, we also document the aggregate diversion ratios, see Katz and Shapiro (2002) and Daljord et al. (2008).

Table 1. Diversion ratios, Case 1, all products are net and gross substitutes.

		To firm			Aggregate diversion
		1	2	3	
From firm	1	n.a.	0.469	0.219	0.688
	2	0.400	n.a.	0.400	0.800
	3	0.219	0.469	n.a.	0.688

Note: Diversion of sales from firm (in left column) to firm (in upper row). Aggregate diversion from firm (in left column) is provided in the rightmost column. n.a. stands for "not applicable".

Hence, a reduction in the sales of product 1 of one unit generates an increase in sales of about 0.47 units for product 2, and 0.22 units for product 3. A reduction in the sales of product 2 generates an increase in sales of 0.4 units for products 1 and 3. This is as expected because firms 1 and 2 (and firms 3 and 2) are close competitors, whereas competitors 1 and 3 are more distant competitors.

Assume now that firm 1 is deciding whether to form a coalition with either firm 2 or 3. Both cases are profitable, however, it is more profitable to form a coalition with firm 2 (see **Table 2**). This is seen from the diversion ratios, if firms 1 and 3 start a coalition, a high percentage of the sales will leave the coalition. If firms 1 and 2 form a coalition, a price increase will generate a smaller diversion percentage leaving the coalition.

Table 2. Profits, Case 1, all products are net and gross substitutes (prices in parentheses).

Firm	Competition	1 & 2	1 & 3	1, 2 & 3
1	8.344 (2.255)	9.486 (3.139)	8.758 (2.614)	12.821 (5.000)
2	6.957 (1.902)	8.037 (2.766)	8.046 (2.045)	9.615 (5.000)
3	8.434 (2.255)	10.705 (2.554)	8.758 (2.614)	12.821 (5.000)

However, the result is different in Case 2, where a mixed equilibrium exists (see **Table 3**). Whereas products 1 and 2 are gross substitutes $\partial q_1/\partial p_2 > 0$, products 1 and 3 are now gross complements, $\partial q_1/\partial p_3 < 0$. Compared with the competitive

[3]Diversion ratios measure the degree of substitutability between products, indicating how much sales would shift from one product to another if the price of the first product increased. Essentially, they quantify the fraction of lost sales from one product that would be captured by a competitor.

equilibrium, it is profitable for both firms 1 and 2 to enter into a partial coalition if firm 3 remains an outsider. The same is true for firms 2 and 3 if firm 1 stays outside the partial coalition. Firms 2 and 3 produce complementary goods, and do not have incentives to enter into a coalition. In contrast to the above case, the diversion ratios are now changed, and some diversion ratios become negative, $DR_{1,2} = DR_{3,2} = 0.7$, $DR_{1,3} = DR_{3,1} = -0.17$ and $DR_{2,1} = DR_{2,3} = 0.48$. This follows directly from the definition of the diversion ratios, and the fact that firms 1 and 3 produce complementary products.

Table 3. Diversion ratios, Case 2, all products net substitutes, products 1 and 3 are gross complements.

		To firm			Aggregate diversion
		1	2	3	
From firm	1	n.a.	0.703	−0.172	0.531
	2	0.480	n.a.	0.480	0.960
	3	−0.172	0.703	n.a.	0.531

Note: Diversion of sales from firm (in left column) to firm (in upper row). Aggregate diversion from firm (in left column) is provided in the rightmost column. n.a. stands for "not applicable".

Whereas the diversion ratios between firms 1 and 2, and 2 and 3 are positive, the diversion ratio between firms 1 and 3 is now negative. This reflects the fact that firms 1 and 3 produce complementary products. We also observe that the aggregate diversion from products 1 and 3 to other products falls to about 50%. In addition, the aggregate diversion from firm 2 increases to more than 95%.

As firms 1 and 2 (or 2 and 3) entered a partial coalition, it is profitable for firm 3 (or 1) to join the partial coalition, creating an industry-wide coalition (see Table 4). However, because firm 2's product is a gross substitute with both firm 1 and firm 3's products, it will be profitable for this firm to defect from the full coalition. Finally, because the two firms remaining in the partial coalition, firms 1 and 3, produce products that are gross complements, both have incentives to leave the partial coalition, and the industry then becomes characterized by Bertrand competition.

Table 4. Profits, Case 2, all products net substitutes, products 1 & 3 are gross complements (prices in parentheses).

Firm	Competition	1 and 2	1 and 3	1, 2 and 3
1	11.154 (2.632)	13.852 (4.126)	10.927 (2.407)	18.868 (5.000)
2	5.051 (1.463)	6.315 (2.903)	4.331 (1.355)	2.359 (5.000)
3	11.154 (2.632)	14.585 (3.010)	10.927 (2.407)	18.868 (5.000)

The incentive for firm 2 to join a partial coalition consisting of firms 1 and 3 is determined by the gross substitutability (complementarity) of the products, δ (see **Figure 1**). As long as δ is larger than 0.35, it is profitable for firm 2 to enter into a coalition consisting of firms 1 and 3. Furthermore, the full coalition is stable, no individual firm (or subset of firms) has incentives to leave the coalition. However, when δ is less than 0.35, the market configuration changes, as discussed above. Additional insights can be obtained from the diversion ratios, when firms 1 and 3 enter into an alliance: firm 2 has an incentive to enter into a full coalition because the cost of diversion for the other firms is very high. Hence, whereas firms 1 and 3 have strong incentives to enter into a coalition (negative interfirm diversion), firm 2 will lose from joining (or will gain from leaving) a full coalition.

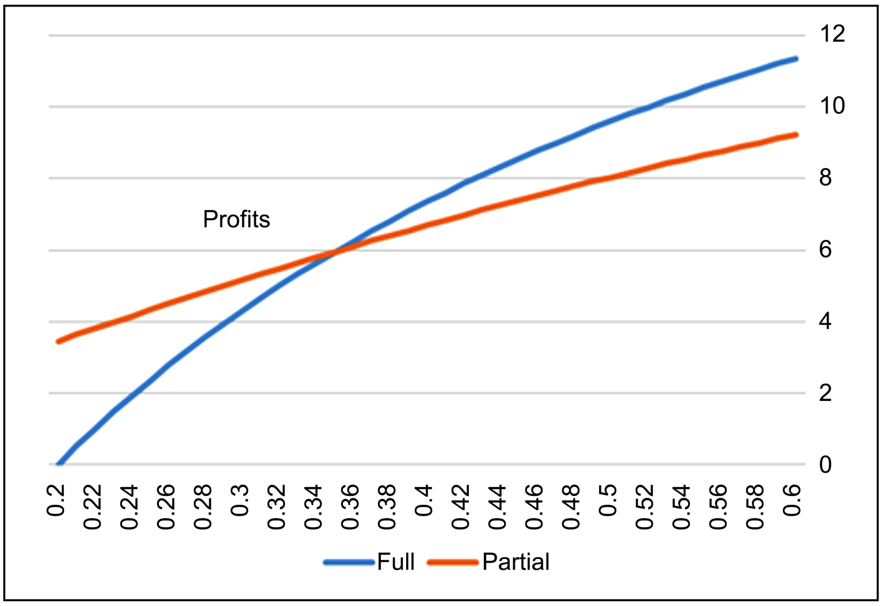

Figure 1. Profits for firm 2.

4. Conclusion

In this paper, we extended the model of Deneckere and Davidson (1985) to consider a richer demand structure that encompasses firms with different cross-price elasticities. By altering the demand specification in the model of Deneckere and Davidson (1985), we obtained a new set of predictions regarding firms' incentives to form coalitions, such as cartels, mergers and acquisitions and joint ventures.

First, we found that a subset of firms may find it profitable to enter into a coalition, whereas another subset maximizes profits on an individual basis. As in the cartel literature, it is more profitable to be outside than inside the cartel.

Second, although there are only two prices in the model of Deneckere and Davidson (1985), one price set by all insiders, and one price set by all outsiders, we found that firms inside the coalition may have incentives to set different prices. The same occurs for firms outside the coalition, i.e., they also have incentives to set different prices. In fact, in the model presented above, there may be as many different prices as there are firms, as we often observe in the real world.

Third, with the use of the demand specification of Amir et al. (2017), we also predicted which firms have the strongest incentives to enter into a coalition, and which firms have the strongest incentives to leave the coalition. This depends on the complementarity and/or substitutability between the goods produced by the firms.

Fourth, our model can help us understand why firms continue to set different prices after leaving a coalition. This is not possible in standard models of coalition formation, such as cartels, mergers and acquisitions and joint ventures, where all firms set the same price post-coalition. This can explain why in many cartels, firms inside the cartel can still accommodate different prices.

Fifth, our model can also be used to construct empirical models of coalition formation because it provides clear empirical predictions about the economic incentives to form alliances. In addition, the model can also be the basis for empirical models of asymmetric price patterns in markets.

In our view, future work should focus on two aspects that the literature on coalitions has not approached satisfactorily. First, most models of coalitions formation do not consider the n-firm case. Second, the literature in coalitions does not look deeply at other factors that may influence coalition formation, such as cost synergies, capacity constraints, and strategic considerations. Extending coalition models to n-firm case and the inclusion of a broader set of incentives on coalition formation can give new insights into the incentives and limits to form coalitions.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Abrantes-Metz, R. M., Kraten, M., Metz, A. D., & Seow, G. S. (2012). Libor Manipulation? *Journal of Banking & Finance, 36,* 136-150. https://doi.org/10.1016/j.jbankfin.2011.06.014

Abrantes-Metz, R. M., Villas-Boas, S. B., & Judge, G. (2011). Tracking the Libor Rate. *Applied Economics Letters, 18,* 893-899. https://doi.org/10.1080/13504851.2010.515197

Amir, R., Erickson, P., & Jin, J. (2017). On the Microeconomic Foundations of Linear Demand for Differentiated Products. *Journal of Economic Theory, 169,* 641-665. https://doi.org/10.1016/j.jet.2017.03.005

Boffa, F., & Vannoni, D. (2012). Entry and Collusion after Market Opening. *Theoretical Economics Letters, 2,* 316-322. https://doi.org/10.4236/tel.2012.23058

Bresnahan, T. F. (1987). Competition and Collusion in the American Automobile Industry: The 1955 Price War. *The Journal of Industrial Economics, 35,* 457-482. https://doi.org/10.2307/2098583

Daljord, O., Sorgard, L., & Thomassen, O. (2008). The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro. *Journal of Competition Law and Economics, 4,* 263-270. https://doi.org/10.1093/joclec/nhm032

Deneckere, R., & Davidson, C. (1985). Incentives to Form Coalitions with Bertrand Competition. *The RAND Journal of Economics, 16,* 473-486. https://doi.org/10.2307/2555507

Jacquemin, A., & Slade, M. E. (1989). Chapter 7. Cartels, Collusion, and Horizontal Merger. In *Handbook of Industrial Organization* (pp. 415-473). Elsevier. https://doi.org/10.1016/s1573-448x(89)01010-1

Katz, M. L., & Shapiro, C. (2002). Critical Loss: Let's Tell the Whole Story. *Antitrust, 17,* 49.

Shapiro, C. (1995). Mergers with Differentiated Products. *Antitrust, 10,* 23-30.

Shubik, M., & Levitan, R. (1980). *Market Structure and Behavior*. Harvard Univ. Press.