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# Correlation between Valuation Results Obtained by Using Before-Tax and After-Tax Cash Flow Discounting Models

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## Abstract

This article discusses ways to obtain convergence of calculation results obtained using before-tax and after-tax cash flow discounting models. Various ways of achieving this goal are proposed. Recommendations are given on how to achieve a more correct discounting of cash flows at the rates of alternative income calculated via the CAPM and WACC models. Suggestions are provided on the analysis and adjustment of the observed risk premium in relation to after-tax and before-tax capital cash flows. The appendices consider the theoretical impact of inflation on the amount of the observed risk premium and provide examples explaining recommendations for the correct accounting of the tax factor.

## Keywords

Before-Tax Cash Flows, After-Tax Cash Flows, Before-Tax Discount Rates, After-Tax Discount Rates, Equivalence of After-Tax and Before-Tax Valuation Bases, Non-Equivalence of After-Tax and Before-Tax Valuation Bases

## 1. Introduction

Discounted cash flow models are widely used in the practice of valuation and the theory of corporate finance. These models are based on a number of principles. So, according to the first principle of corporate finance, money received or paid today is worth more than the same money received or paid tomorrow. According to the second principle of corporate finance, a reliable dollar (euro, yen or yuan) costs more than a risky dollar (euro, yen or yuan) (Brealey & Myers, 1997; Lee &

Finnerty, 1990). There is also something similar to the third principle of corporate finance—when calculating the total present value of the income flow for each type of cash flow, an appropriate rate of alternative return (discount rate) should be applied. At the same time, the type of cash flow should correspond to the estimated value (Damodaran, 2004; Kozyr, 2012, 2014). For example, if we estimate capital, we use capital cash flow, if we estimate invested capital, we use cash flow from all invested capital; and if we estimate capital cash flow, but use cash flow from all invested capital for this, then at the final stage of the calculations, we should make an additional adjustment (subtracting the amount of interest debt). This article examines the relationship between asset valuations based on before-tax and after-tax cash flows. To simplify the task, in the future, we will take into account only the impact of income tax on the aggregated values of before-tax ($CF_{BT}$) and after-tax ($CF_{AT}$) cash flows. The following dependencies exist between these flows:

$$CF_{BTi} - Tax_i = CF_{BTi} - t \cdot PBT_i = CF_{BTi} \cdot (1 - t_{ei}) = CF_{ATi} \quad (1)$$

where:

$CF_{BTi}$ = Before-tax cash flow[1] expected in the $i^{th}$ period;

$Tax_i$ = Income tax expected to be paid in the $i^{th}$ period;

$t$ = Income tax rate;

$PBT_i$ = Before-tax profit expected to be paid in the $i^{th}$ period;

$t_{ei}$ = The effective income tax rate expected in the first period, which, in the context of this article, is taken as the expected share of income tax in the before-tax cash flow;

$CF_{ATi}$ = After-tax cash flow expected to be received in the $i^{th}$ period.

The effective income tax rate shown in (1) can be calculated as follows:

$$t_{ei} = 1 - \frac{CF_{ATi}}{CF_{BTi}} = \frac{Tax_i}{CF_{ATi} + Tax_i} = \frac{Tax_i}{CF_{BTi}} \quad (2)$$

Anticipating the main content of this article, we will give a reasonable warning (Brealey & Myers, 1997): "Some firms do not deduct tax payments. They try to offset this mistake by discounting the cash flows before taxes at a rate higher than the opportunity cost of capital. Unfortunately, there is no reliable formula for making such adjustments to the discount rate". While generally agreeing with the difficulties of such an adjustment, nevertheless, we will try to find a number of possible recommendations on how to implement it and achieve convergence of the results of assessments carried out on both before-tax and after-tax valuation bases.

Further discussion of this topic will be divided into two conceptually different points of view. According to the first viewpoint, the value of an asset generating cash flow can be determined both on the basis of after-tax cash flows discounted at after-tax rates of alternative return and on the basis of before-tax cash flows

[1]At this stage of the review, we will not specify the type of cash flow (free cash flow in relation to shareholders or the company)—for us it is only important to distinguish them in terms of accounting or non-accounting of taxation.

discounted at before-tax rates of alternative return. Let's call this point of view *the concept of equivalence of after-tax and before-tax valuation bases*, or *the concept of equivalence* for short. This concept implies the theoretical equality of the calculation results of the value of an asset calculated on the basis of after-tax or before-tax cash flows. At the same time, this approach makes no clarification regarding the final valuation basis (after-tax or before-tax) for the estimated asset value.

According to the second viewpoint, the value of an asset that generates cash flows is determined from the required basis of the cost of the asset itself. For example, if it is necessary to determine the value of shares, it is advisable to apply the after-tax valuation basis, i.e. discount the expected after-tax cash flows at after-tax rates of alternative return, since companies issuing shares pay income tax before paying dividends. At the same time, if it is necessary to estimate the value of a production line or commercial building, it is advisable to apply the before-tax valuation basis—i.e. to discount before-tax cash flows at before-tax discount rates—since these assets themselves are not income tax payers. In the following, we will refer to this approach as *the concept of non-equivalence of after-tax and before-tax valuation bases*, or *the concept of non-equivalence* for short. This concept implies a distinction between asset estimates calculated on the basis of after-tax or before-tax cash flows. At the same time, within this approach, in relation to the estimated value of an asset, the emphasis is initially placed on its tax status, i.e. depending on how applicable income tax withholding is in relation to the estimated asset, the asset can be calculated on both before-tax and after-tax bases.

Let us now proceed directly with the consideration of these concepts.

## 2. The Concept of Equivalence of After-Tax and Before-Tax Valuation Bases

### 2.1. General Scheme

It is known that when using discount models, each type of cash flow should have its own type of discount rates. In relation to the subject under consideration, this means that after-tax discount rates ($r_{AT}$) should be used for after-tax income flows, and before-tax discount rates ($r_{BT}$) should be used for before-tax income flows. If we adhere to the concept of equivalence of after-tax and before-tax valuation bases, the following equality must be observed:

$$V = \sum_i \frac{CF_{ATi}}{\left(1 + r_{AT}\right)^i} = \sum_i \frac{CF_{BTi}}{\left(1 + r_{BTi}\right)^i} \tag{3}$$

where:

$r_{AT}$ = After-tax rate of alternative return (after-tax discount rate);

$r_{BT}$ = Before-tax rate of alternative return (before-tax discount rate).

In order for the present value of the income flows of the $i^{\text{th}}$ period calculated on both before-tax and after-tax bases to match, the following conditions must be met:

$$\frac{CF_{ATi}}{\left(1+r_{ATi}\right)^{i}}=\frac{CF_{BT_i}}{\left(1+r_{BTi}\right)^{i}} \tag{4}$$

From (4), a relationship can be obtained between before-tax and after-tax discount rates, the observance of which can lead to achieving a coincidence of the results of estimates obtained on both before-tax and after-tax bases:

$$r_{BTi}=\left(1+r_{ATi}\right)\cdot\left[\frac{CF_{BTi}}{CF_{ATi}}\right]^{\frac{1}{i}}-1=\frac{1+r_{ATi}}{\left(1-t_{ei}\right)^{\frac{1}{i}}}-1 \tag{5-1}$$

$$r_{ATi}=\left(1+r_{BTi}\right)\left(1-t_{ei}\right)^{\frac{1}{i}}-1 \tag{5-2}$$

Note that dependence (5) ensures the equivalence of the present value of the cash flows in the forecast period calculated on both before-tax and after-tax bases. We also note that in order to ensure non-negative after-tax discount rate in the $i^{th}$ period determined in accordance with (5), it is necessary to comply with the following condition:

$$r_{BTi}>\left(\frac{CF_{BTi}}{CF_{ATi}}\right)^{\frac{1}{i}}-1 \tag{6-1}$$

If the effective rate remains unchanged in all periods, in order to ensure non-negative after-tax discount rate in the $i^{th}$ period determined in accordance with (5), the following conditions must be met:

$$r_{BTi}>\frac{1}{\left(1-t_{e}\right)^{\frac{1}{i}}}-1 \tag{6-2}$$

In the special case, <u>if and only if $i=1$</u>, expression (5) can be reduced to the form:

$$r_{BT1}=\frac{1+r_{AT1}}{1-t_{e1}}-1=\frac{r_{AT1}+t_{e1}}{1-t_{e1}}=\frac{r_{AT1}}{1-t_{e1}}+\frac{Tax_{1}}{CF_{AT1}} \tag{7-1}$$

$$r_{AT1}=\left(1+r_{BT1}\right)\cdot\left(1-t_{e1}\right)-1=r_{BT1}\cdot\left(1-t_{e1}\right)-t_{e1}. \tag{7-2}$$

Note that in order to ensure the non-negativity of $r_{AT1}$ values in (7-2), the following conditions must be met:

$$r_{BT1}\geq\frac{t_{e1}}{1-t_{e1}} \tag{8}$$

*Note*. As follows from expressions (5) and (7), adhering to the concept of equivalence of the after-tax and before-tax valuation bases, the generally accepted expression $r_{AT}=r_{BT}\cdot\left(1-t\right)$ is generally incorrect to satisfy condition (4). Calculations show that the results of the assessment based on the after-tax rate defined according to expression (7-2) for multi-period models are almost always higher than the results of calculations based on the before-tax basis. This is reasoned by the non-linearity of the discounting procedure.

<u>Example 1</u>

Let the following data be known: After-tax cash flow of the 1st forecast period

($CF_{AT1}$) = 45 million Euros, before-tax flow of the 1st forecast period ($CF_{BT1}$) = 50 million Euros, $t_{e1}$ = 1 − 45/50 = 0.1, $r_{AT1}$ = 0.15.

It is necessary to determine a before-tax rate at which the before-tax income flows can be adjusted to provide a value equivalent to the present value of after-tax income flows.

The solution. Using (7-1), we obtain:

$$r_{BT1} = \frac{1+r_{AT1}}{1-t_{e1}} - 1 = \frac{1+0.15}{1-0.1} - 1 = 0.278$$

The checking:

$$PV\left(CF_{AT}\right) = \frac{45}{1.15} = 39.13 = \frac{50}{1.278} = PV\left(CF_{BT}\right)$$

In case there are grounds to believe that the effective tax rate will undergo changes in the future, thus circumstance must be taken into account when making estimation calculations. This can be done either by directly discounting the projected cash flows if such a change is directly indicated in the cash flows forecast or by taking an averaged value of the effective rate period.

Example 2

Let the following data be known (numerical indexes correspond to the numbers of the forecast period intervals):

$CF_{AT1}$ = 45 million Euro, $CF_{BT1}$ = 50 million Euro, $t_{e1}$ = 1 − 45/50 = 0.1, $r_{AT1}$ = 0.15.

$CF_{AT2}$ = 55 million Euro, $CF_{BT2}$ = 60 million Euro, $t_{e2}$ = 1 − 55/60 = 0.083, $r_{AT2}$ = 0.15.

It is necessary to determine such before-tax rates, which would allow obtaining the value equivalent of the present value of after-tax income flows, provided that before-tax income flows are discounted by these rates.

The solution. Using (5-1), we obtain:

$$r_{BT1} = \frac{1+r_{AT1}}{1-t_{e1}} - 1 = \frac{1+0.15}{1-0.1} - 1 = 0.278$$

$$r_{BT2} = \frac{1+r_{AT2}}{\left(1-t_{e2}\right)^{1/2}} - 1 = \frac{1+0.15}{\left(1-0.083\right)^{0.5}} - 1 = 0.201$$

The checking:

$$PV\left(CF_{AT}\right) = \frac{45}{1.15} + \frac{55}{1.15^2} = 80.72 = \frac{50}{1.278} + \frac{60}{1.201^2} = PV\left(CF_{BT}\right)$$

## 2.2. Using the Duration of Cash Flows to Approximate the Results of Calculations on Before-Tax and After-Tax Bases for a Finite Number of Periods

A better solution to the issue of convergence of the results of calculating the market cost of capital on both before-tax and after-tax bases may be the approach according to which the before-tax rate is assumed to be the same for all periods. In this case, it is calculated similarly to expression (5) with the only **difference**

**that the duration[2] of the forecast cash flows is used as parameter "*i*"**, i.e. the relationship between the after-tax ($r_{at}$) and before-tax discount rate ($r_{bt}$) may be as follows:

$$r_{at} = \left(1 + r_{bt}\right)\left(1 - t_e\right)^{\frac{1}{D}} - 1 \quad (9)$$

where:

$r_{bt}$ = Before-tax risk rate;

$t_e$ = Share of income tax in the before-tax flow;

$D$ = Duration of forecast cash flows.

This approach, firstly, practically ensures the convergence of the valuation results performed on both before-tax and after-tax bases (with a limited number of periods, i.e. no terminal cost), and secondly, it is possible to apply a single discount rate for all periods. At the same time, this approach has the following features and limitations: 1) in order to ensure the non-negativity of the after-tax discount rate, it is necessary to observe the inequality: $r_{BT} > 1/\left(1 - t_e\right)^{1/D} - 1$, which, in relation to current Russian conditions ($t$ = 25%), means $r_{BT}$ > 4 ÷ 6% (provided that the share of income tax in the before-tax cash flow is 25%, and the duration of cash flows is 5 - 7 years; for longer periods, the minimum value of $r_{BT\min}$ may be even lower); 2) the calculation is characterized by a cyclical pattern, because when calculating the duration, it is necessary to know the value of the discount rate, and it, in turn, depends [in this calculation—see above] depends on the duration value[3]. However, the problem is easily solved by parameter selection.

Example 3

Let the following data be known (numerical indexes correspond to the numbers of the forecast period intervals):

$CF_{AT1}$ = 75 million Euro, $CF_{BT1}$ = 100 million Euro, $t_{e1}$ = 1 − 75/100 = 0.25, $r_{BT1}$ = 0.35.

$CF_{AT2}$ = 225 million Euro, $CF_{BT2}$ = 300 million Euro, $t_{e2}$ = 1 − 225/300 = 0.25, $r_{BT2}$ = 0.35.

$CF_{AT3}$ = 75 million Euro, $CF_{BT3}$ = 100 million Euro, $t_{e3}$ = 1 − 75/100 = 0.25, $r_{BT3}$ = 0.35.

It is necessary to determine such after-tax rates, which would allow obtaining the value equivalent of the present value of before-tax income flows, provided that after-

[2]Duration definition and its use in fixed income investing, https://www.investopedia.com/terms/d/duration.asp.

[3]The author of this publication has developed an alternative method for estimating duration, which may be useful in some cases: $D \approx \frac{\ln\left(\left(CF_1 + CF_2 + \cdots + CF_n + TV\right)/MV\right)}{\ln\left(1 + R\right)}$, where $D$ = Duration, $CF_i$ = Forecast period cash flows, $TV$ = Terminal value, $MV$ = Total market value of the valued subject, $R$ = Applied discount rate. This formula is derived from another formula, which is an alternative way to calculate the present value of a series of upcoming payments (cash flows) and allows one to get a result close to the traditional calculation method. Here is the formula for this alternative method of calculating the present value: $PV = \frac{CF_1 + CF_2 + \cdots + CF_n + TV}{\left(1 + R\right)^D}$, where: $CF_i$ = Cash flow in the $i^{th}$ period, $R$ = Discount rate, $D$ = Payment duration. Remark: This formula should be used with caution (or not used at all) in the presence of negative cash flows during the forecast period.

tax income flows are discounted by these rates.

The solution. For the above conditions, the duration value is 1.94 years (the calculation is omitted). Using expression (9), we obtain:

$$R_{at} = (1+0.35)\times(1-0.25)^{1/1.94} - 1 = 0.16\,.$$

The checking:

$$PV(CF_{AT}) = \frac{75}{1.16} + \frac{225}{1.16^2} + \frac{75}{1.16^3} = 279 = \frac{100}{1.35} + \frac{300}{1.35^2} + \frac{100}{1.35^3} = PV(CF_{BT})$$

As we see, the problem is solved.

### 2.3. Ensuring the Equivalence of Cash Flows in the Post-Forecast Period Calculated on Before-Tax and After-Tax Valuation Bases

Ensuring the equivalence of cash flows in the post-forecast period calculated on various (before-tax and after-tax) valuation bases is a more difficult task. Its analytical solution can be obtained only if one of the two conditions listed below is met:

- In case before-tax and after-tax discount rates are equal in the last year of the forecast period:

$$r_{BTk} = r_{ATk} \tag{10}$$ [4]

where:

$k$ = The number of the last interval (year) of the forecast period.

The equivalence of cash flows in the post-forecast period calculated on various (before-tax and after-tax) valuation bases is achieved when equality is fulfilled:

$$r_{BTPF} = \frac{r_{ATPF} - g}{1 - t_{ePF}} + g \tag{11}$$

$$r_{ATPF} = (r_{BTPF} - g)(1 - t_{ePF}) + g \tag{12}$$

where:

$r_{BTPF}$ = The before-tax discount rate used in the post-forecast period;

$r_{ATPF}$ = The after-tax discount rate used in the post-forecast period;

$t_{Epf}$ = The effective tax rate expected in the post-forecast period;

$g$ = The expected growth rate of income flows in the post-forecast period.

- In case of non-compliance with condition (10), i.e. with $r_{BTk} > r_{ATk}$, ensuring the equivalence of the reduced cash flows in the forecast period calculated on various (before-tax and after-tax) valuation bases is achieved while observing the following equalities:

$$r_{BTPF} = r_{ATPF} \tag{13}$$

$$r_{BTk} = \frac{1 + r_{AT}}{(1 - t_{ek})^{1/k}} - 1 \tag{14}$$

[4]This condition means bringing the terminal values calculated on both before-tax and after-tax bases at a single discount rate.

where:

$r_{BTPF}$ = The before-tax discount rate used in the post-forecast period;

$r_{ATPF}$ = The after-tax discount rate used in the post-forecast period;

$k$ = The number of the last interval (year) of the forecast period.

Example 4

Let condition (10) be satisfied and the following data are known:

After-tax flow in the first post-forecast year ($CF_{ATk+1}$) = 45 million Euros, before-tax flow in the first post-forecast year ($CF_{BTk+1}$) = 50 million Euros, effective tax rate in the post-forecast period ($t_{ePF}$) = 1 − 45/50 = 0.1, after-tax discount rate applied in the post-forecast period ($r_{ATPF}$) = 0.15, expected growth rate of cash flows in the forecast period ($g$) = 0.05.

It is necessary to determine such a before-tax rate, the use of which, for before-tax income flows, would allow obtaining the same terminal value that is obtained by using after-tax cash flows and the discount rate.

The solution. Using expression (11), we obtain:

$$r_{BTPF} = \frac{0.15 - 0.05}{1 - 0.1} + 0.05 = 0.161$$

The checking:

$$TV_{AT} = \frac{45}{0.15 - 0.05} = 450 = \frac{50}{0.161 - 0.05} = TV_{BT}$$

Example 5

Let condition (10) not be fulfilled, but conditions (13)-(14) are fulfilled for the following data: After-tax flow in the first post-forecast year ($CF_{ATk+1}$) = 45 million Euros, before-tax flow in the first post-forecast year ($CF_{BTk+1}$) = 50 million Euros, effective tax rate in the post-forecast period ($t_{ePF}$) = 1 − 45/50 = 0.1, after-tax discount rate applied in the post-forecast period ($r_{ATPF}$) = 0.15, duration of the forecast period ($k$) = 3 years, expected growth rate of cash flows in the forecast period ($g$) = 0.05.

It is necessary to make sure that under the specified conditions, the values of the after-tax and before-tax terminal values are reduced by the valuation date, respectively, at the after-tax rate and at the before-tax rate calculated according to Formula (14), will be equal.

The solution. Let's first calculate the given value of the after-tax terminal value:

$$PV\left(TV_{AT}\right) = \frac{TV_{AT}}{\left(1 + r_{AT}\right)^k} = \frac{CF_{ATk+1}}{\left(r_{ATPF} - g\right)\cdot\left(1 + r_{AT}\right)^k}$$
$$= \frac{45}{\left(0.15 - 0.05\right)\cdot\left(1 + 0.15\right)^3} = 295.9 \text{ million Euros.}$$

Since, according to the condition of the example, equality (13) must be observed, the before-tax terminal value should be:

$$TV_{BT} = \frac{CF_{BTk+1}}{r_{BTPF} - g} = \frac{CF_{BTK+1}}{r_{ATPF} - g} = \frac{50}{0.15 - 0.05} = 500 \text{ million Euros.}$$

Using expression (14), we now calculate the value of the before-tax discount rate, at which the before-tax terminal value should be reduced by the valuation date:

$$r_{BT3} = \frac{1+r_{AT}}{\left(1-t_{e3}\right)^{1/3}} - 1 = \frac{1.15}{\left(1-0.1\right)^{1/3}} - 1 = 0.191$$

Now let's calculate the given before-tax terminal value:

$$PV\left(TV_{BT}\right) = \frac{TV_{BT}}{\left(1+r_{BT}\right)^3} = \frac{500}{\left(1+0.191\right)^3} = 295.9 \text{ million Euros.}$$

The checking:

$$PV\left(TV_{AT}\right) = 295.9 = PV\left(TV_{BT}\right)$$

## 2.4. The Convergence of the Results of Before-Tax and After-Tax Discount Models Containing Forecast and Post-Forecast Periods

Let's now try to apply the above methodology to obtain the same values (after-tax and before-tax), each of which covers the forecast and post-forecast periods.

Example 6

Let the following data be known (numerical indexes correspond to the numbers of the forecast period intervals):

$CF_{AT1}$ = 45 million Euros, $CF_{BT1}$ = 50 million Euros, $t_{e1}$ = 1 − 45/50 = 0.1, $r_{AT1}$ = 0.15.

$CF_{AT2}$ = 55 million Euros, $CF_{BT2}$ = 60 million Euros, $t_{e2}$ = 1 − 55/60 = 0.083, $r_{AT2}$ = 0.15.

Let us also know the following data regarding the post-forecast period:

After-tax flow in the first post-forecast year ($CF_{ATk+1}$) = 57.75 million Euros, before-tax flow in the first post-forecast year ($CF_{BTk+1}$) = 63 million Euros, effective tax rate in the post-forecast period ($t_{ePF}$) = 1 − 57.75/63 = 0.083, capitalization rate of after-tax flows in the forecast period ($r_{ATPF}$) = 0.15, expected annual growth rate of flows in the post-forecast period ($g$) = 0.05.

It is necessary to obtain the asset values calculated on both after-tax and before-tax valuation bases.

The solution. Let's first calculate the asset value on the after-tax basis:

$$\begin{aligned} V_{AT} &= PV\left(CF_{AT}\right) + PV\left(TV_{AT}\right) \\ &= \frac{45}{1.15} + \frac{55}{1.15^2} + \frac{57.75}{\left(0.15-0.05\right)\cdot\left(1+0.15\right)^2} \\ &= 517.39 \text{ million Euros.} \end{aligned}$$

Let's calculate the discount rate for before-tax flows in the post-forecast period:

$$r_{BTPF} = \frac{0.15-0.05}{1-0.083} + 0.05 = 0.159 \text{ million Euros.}$$

Now, we will calculate the value of the asset on the before-tax basis, while the discount rates of the before-tax flows in the forecast period will be calculated by using

Formula (5-1), and the terminal value will be reduced at the after-tax rate:

$$V_{BT} = PV\left(CF_{BT}\right) + PV\left(TV_{BT/AT}\right)$$
$$= \frac{50}{1.278} + \frac{60}{1.201^2} + \frac{63}{\left(0.159 - 0.05\right)\cdot\left(1 + 0.15\right)^2}$$
$$= 517.76 \text{ million Euros.}$$

This example shows that the convergence of calculations on both after-tax and before-tax bases was achieved only due to the unjustified reduction of the before-tax terminal value ($TV_{BT}$) at the after-tax discount rate.

Note also that in the above example, the task could be formulated differently: instead of obtaining two values calculated on both before-tax and after-tax bases, it might be necessary, for example, to determine the before-tax discount rate at which the valuation result calculated on the before-tax valuation basis would lead to the same valuation result that was obtained on the after-tax valuation basis. Such tasks can be solved by using the Excel option "Parameter selection". Applying this option to the condition of modified example 6 results in 0.159 (15.91%):

$$V_{BT} = PV\left(CF_{BT}\right) + PV\left(TV_{BT}\right)$$
$$= \frac{50}{1.159} + \frac{60}{1.159^2} + \frac{63}{\left(0.159 - 0.05\right)\cdot\left(1 + 0.159\right)^2}$$
$$= 517.39 \text{ million Euros.}$$

Let's consider one more example.

Example 7

Let the following data be known (numerical indexes correspond to the numbers of the forecast period intervals (years)):

$CF_{AT1}$ = 45 million Euros, $CF_{BT1}$ = 50 million Euros, $t_{e1} = 1 - 45/50 = 0.1$, $r_{AT1}$ = 0.15.

$CF_{AT2}$ = 55 million Euros, $CF_{BT2}$ = 60 million Euros, $t_{e2} = 1 - 55/60 = 0.083$, $r_{AT2}$ = 0.15.

$CF_{AT3}$ = 44 million Euros, $CF_{BT3}$ = 49 million Euros, $t_{e3} = 1 - 44/49 = 0.102$, $r_{AT3}$ = 0.15.

Let condition (10) not be fulfilled, but conditions (13)-(14) are fulfilled and the following data are known for the post-forecast period:

After-tax flow in the first post-forecast year ($CF_{ATk+1}$) = 46.2 million Euros, before-tax flow in the first post-forecast year ($CF_{BTk+1}$) = 51.45 million Euros, effective tax rate in the post-forecast period ($t_{ePF}$) = 1 − 46.2/51.45 = 0.102, capitalization rate of after-tax flows applied in the post-forecast period ($r_{ATPF}$) = 0.15, duration of the forecast period ($k$) = 3 years, expected annual growth rate of cash flows in the post-forecast period ($g$) = 0.05.

It is necessary to obtain the asset values calculated on both after-tax and before-tax valuation bases.

The solution. Let's first calculate the asset value on the after-tax basis:

$$V_{AT} = PV\left(CF_{AT}\right) + PV\left(TV_{AT}\right)$$

$$= \frac{45}{1.15} + \frac{55}{1.15^2} + \frac{44}{1.15^3} + \frac{46,2}{(0.15-0.05)\cdot(1+0.15)^3}$$
$$= 413.4 \text{ million Euros.}$$

Let's calculate the discount rate for before-tax flows of the third year in accordance with (14):

$$r_{BT3} = \frac{1+0.15}{(1-0.102)^{1/3}} - 1 = 0.192.$$

Let's now calculate the asset value on the before-tax basis. While calculating the before-tax terminal value, we use a discount rate numerically equal to the after-tax discount rate in the post-forecast period (due to the feasibility of condition (13) specified in the example), and we will reduce the terminal value at the after-tax rate used for the flows of the last year of the forecast period (calculated above and equal to 19.2%):

$$V_{BT} = PV(CF_{BT}) + PV(TV_{AT})$$
$$= \frac{50}{1.278} + \frac{60}{1.201^2} + \frac{49}{1.192^3} + \frac{51.45}{(0.15-0.05)\cdot(1+0.192)^3}$$
$$= 413.4 \text{ million Euros.}$$

As in the previous example, we note that using the Excel option "parameter selection" in relation to the conditions of this example allows you to select a single discount rate for before-tax income flows = 16.146%. At such a discount rate, the total value of the asset calculated on the before-tax basis will lead to the result obtained on the after-tax basis:

$$V_{BT} = PV(CF_{BT}) + PV(TV_{BT})$$
$$= \frac{50}{1.161} + \frac{60}{1.161^2} + \frac{49}{1.161^3} + \frac{51.45}{(0.161-0.05)\cdot(1+0.161)^3}$$
$$= 413.4 \text{ million Euros}$$

Since earlier, when deriving expression (11), the possibility of differences between the rates of $t_e$ (the share of income tax integral to the valued company as part of its before-tax cash flows) and $t_m$ (the average share of income tax as part of before-tax cash flows in the relevant sector) was not taken into account, in the case of differences between $t_e$ and $t_m$, with the possibility of using the constant growth model for the correct calculation with before-tax parameters, the following formula must be applied:

$$V_{BT} = \frac{FCFE_{BT1}}{\left[\frac{(r_{BT}-g)(1-t_m)}{1-t_e}\right]} \tag{15}$$

where the remaining parameters correspond to the previously accepted designations.

Example 8

Let's assume that the following parameters became known during the evaluation:

the expected after-tax rate of return is 14.375%, the expected before-tax rate is 16.5%, the average market "income tax rate" ($t_m$) is 25%, while the estimated company's share of income tax in the shareholders' before-tax cash flows ($t_e$) was 15%. Let's assume that, according to the forecast, a before-tax flow of 100 million Euros and an after-tax flow of 85 million Euros are expected next year. Let's also say that we have reason to believe that the company being valued will be able to maintain a growth rate of 8% per year indefinitely, and therefore we can use the perpetual constant growth model (the so-called Gordon model) to evaluate it.

Let's calculate the cost of a business generating such income flows.

The company's value on the after-tax basis is calculated as follows:

$$V_{AT} = \frac{FCFE_{BT1} \times (1 - t_e)}{r_{AT} - g} = \frac{FCFE_{AT1}}{r_{AT} - g} = \frac{100 \times (1 - 0.15)}{0.14375 - 0.08} = 1333 \text{ million Euros}$$

Let us now carry out the calculation on the before-tax basis in accordance with expression (15):

$$V_{BT} = \frac{FCFE_{BT1}}{\left[\frac{(r_{BT} - g)(1 - t_m)}{1 - t_e}\right]} = \frac{100}{\left[\frac{(0.165 - 0.08)(1 - 0.25)}{1 - 0.15}\right]} = 1333 \text{ million Euros}$$

As can be seen, the results of calculations carried out on both after-tax and before-tax bases coincide, which indicates their correctness.

It should be noted separately the situations when in several years of the forecast period (but not in its last year) negative cash flows are expected. In such years, companies do not pay income tax and, therefore, their after-tax cash flows become equal to before-tax cash flows. In these situations, the convergence of the results of valuations carried out on a before-tax and after-tax basis decreases. According to the valuations of the author of the article, this decrease is minimal when using the above-mentioned discounting technique, without making any special adjustments, i.e. when the same discount rate (before-tax or after-tax) is applied to all cash flows (both positive and negative) within a valuation basis (before-tax or after-tax).

Thus, the above arguments and calculation examples show that the application of the equivalence concept allows, under certain conditions, obtaining convergence of the calculation results on both before-tax and after-tax valuation bases. In general, complete convergence of the results may not be achieved, but nevertheless these estimates may be quite close. At the same time, the disadvantage of implementing this concept is the contrivance ("artificiality") of a number of propositions put forward, the main of which is the requirement that discount rates depend on the valuation period number (see expressions (5), (9), (14)) and the need to reduce the terminal value calculated on the before-tax basis at the after-tax discount rate (see Example 6), as well as the complexity of the corresponding calculations.

Let us now consider the concept of non-equivalence of the after-tax and before-tax calculation bases.

## 3. The Concept of Non-Equivalence of the After-Tax and Before-Tax Calculation Bases

As stated in the introduction, this concept implies a distinction between asset estimates calculated on the basis of after-tax or before-tax cash flows. At the same time, with respect to the estimated value of an asset, the concept initially focuses on its tax status—depending on how applicable income tax withholding is with respect to the asset being assessed, the asset can be calculated both on the before-tax and after-tax basis:

$$V_{AT} = \sum_i \frac{CF_{ATi}}{\left(1+r_{AT}\right)^i} \tag{16}$$

$$V_{BT} = \sum_i \frac{CF_{BT_i}}{\left(1+r_{BT}\right)^i} \tag{17}$$

where the designations correspond to the previously accepted ones.

What can be said about the economic essence of the before-tax and after-tax asset valuation? The market cost of assets and liabilities is most often estimated in relation to all the features of their functioning, including taxation. Therefore, most often the market cost is estimated on the after-tax basis. For example, the market cost of *capital* is estimated on the after-tax basis. Why is this so? Because it is impossible to get rid of paying income tax: by acquiring an asset, the owner also acquires obligations related to its ownership. From the point of view of the valuation methodology, this is especially clearly seen when the discounting factor tends to unity (e.g. for a business that will generate cash flow for one year, after which it will completely depreciate), when the market cost of the capital of an income-generating object will tend to the discounted after-tax cash flow of this business, and the difference between the after-tax and before tax results will tend to the discounted income tax (see further expression (22)). Since taxation is applicable to all periods and since the cash flow discounting model should work regardless of the number of valuation periods (intervals), the total cost of capital is the after-tax value. At the same time, when evaluating movable and immovable property, one can ignore the impact of income tax on their value and assume that their "true" market cost corresponds to the after-tax format, since these objects themselves are not subject to income tax, and therefore it is often difficult to determine the part of the total profit attributed to their share.

It should be noted that within the concept of non-equivalence between before-tax and after-tax discount rates, the traditional dependence is accepted[5] instead of dependence of (5).

Adhering to this concept, it can be argued that conditions (3)-(4), strictly speaking, do not seem to be quite correct, since, as is known, the type of cash flow must correspond to the estimated value. For example, if we estimate capital, we use capital cash flow, if we estimate invested capital, we use cash flow from all invested capital, and if we estimate capital cash flow, but use cash flow from all invested

[5] $r_{AT} = r_{BT}(1 - t_e)$.

capital for this, then in the final part of the calculations it is necessary to make an additional adjustment (subtracting the amount of interest debt). With this in mind, we can adjust conditions (3)-(4)—since the left side of this equality corresponds to calculations for obtaining the after-tax value and its right side corresponds to calculations for obtaining the before-tax value, we assume a priori that there may be some difference between the results of these estimates.

$$\Delta V = V_{AT} - V_{BT} \tag{18}$$

where:

$\Delta V$ = The difference between the results of asset valuations performed on the after-tax and before-tax bases.

Let's take a closer look at the difference between the results of estimates obtained on the before-tax and after-tax bases.

In general, this difference is equal to:

$$\Delta V = \sum_i \frac{CF_{ATi}}{(1+r_{AT})^i} - \sum_i \frac{CF_{BTi}}{(1+r_{BT})^i} \approx \frac{\sum_i CF_{ATi}}{(1+r_{AT})^{D1}} - \frac{\sum_i CF_{BTi}}{(1+r_{BT})^{D2}} \tag{19}$$

where:

$D1$ = The duration of after-tax cash flows;

$D2$ = The duration of before-tax cash flows.

The remaining designations correspond to the previously accepted ones.

In the limiting case ( $t \to 0$ ), this difference is:

$$\Delta V = \frac{CF_{AT0}}{(1+r_{AT})^0} - \frac{CF_{BT0}}{(1+r_{BT})^0} = CF_{AT0} - CF_{BT0} = -Tax \tag{20}$$

where:

$Tax$ = The amount of income tax payable as of the valuation date.

However, taking into account that the discounted cash flow method should take into account only future (relative to the valuation date) income flows, as well as the fact that cash flows and the amount of income tax are periodic quantities (i.e. quantities whose value is determined for one period, and not as of a certain date), we can conclude that as time tends to zero, the difference in the values of these quantities will also tend to zero:

$$\Delta V = \frac{CF_{AT0}}{(1+r_{AT})^{i\to 0}} - \frac{CF_{BT0}}{(1+r_{BT})^{t\to 0}} \to 0 \tag{21}$$

For one period and discounting at the end of the period (postnumerando), this difference will be:

$$\Delta V = \frac{CF_{AT1}}{1+r_{AT}} - \frac{CF_{BT1}}{1+r_{BT}} = \frac{CF_{BT1}(1-t_e)}{1+r_{AT}} - \frac{CF_{BT1}}{1+r_{BT}} = \frac{-Tax}{(1+r_{AT})(1+r_{BT})} \tag{22}$$

For two periods and discounting at the end of the period (postnumerando), this difference will be:

$$\Delta V = \frac{CF_{AT1}}{1+r_{AT}} + \frac{CF_{AT2}}{(1+r_{AT})^2} - \frac{CF_{BT1}}{1+r_{BT}} - \frac{CF_{BT2}}{(1+r_{BT})^2} \tag{23}$$

Example 9

Let's assume that the before-tax discount rate, determined by the market extraction method, is 30%, while the forecast of capital cash flows is as follows (Table 1).

Table 1. Forecast of the main components of cash flows.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax earnings (EBT), million Euros | 100 | 115 | 120 |
| Income tax at the rate $t = 0.2$ | 20 | 23 | 24 |
| After-tax earnings, million Euros | 80 | 92 | 96 |
| Depreciation, million Euros | 30 | 30.3 | 30.7 |
| Total investments, million Euros | 40 | 44 | 45 |
| Balance on newly attracted loans, million Euros | +5 | −10 | 0 |
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| After-tax capital cash flow ($FCFE$), million Euros | 75 | 68.3 | 81.7 |

In the future, it is assumed that cash flows will grow by 3% annually, and the income tax rate will remain unchanged.

We will determine the cost of capital using two approaches: on the before-tax and after-tax bases.

The cost of capital on the before-tax basis will be:

$$V_{BT} = \frac{FCFE_{BT1}}{1+r_{BT}} + \frac{FCFE_{BT2}}{\left(1+r_{BT}\right)^2} + \frac{FCFE_{BT3}}{\left(1+r_{BT}\right)^3} + \frac{FCFE_{BT3}\left(1+g\right)}{\left(r_{BT}-g\right)\left(1+r_{BT}\right)^3}$$
$$= \frac{95}{1.3} + \frac{91.3}{1.3^2} + \frac{105.7}{1.3^3} + \frac{105.7\times 1.03}{\left(0.3-0.03\right)\times 1.3^3} = 359 \text{ million Euros.}$$

To determine the value of the share capital on the after-tax basis, we will first determine the value of the discount rate on the after-tax basis, for which, first of all, we will calculate the values of effective tax rates in accordance with (2) (see Table 2).

Table 2. Calculation of effective tax rates.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| After-tax capital cash flows ($FCFE$), million Euros | 75 | 68.3 | 81.7 |
| Effective tax rate ($t_{efE}$) | 0.211 | 0.252 | 0.227 |

Let's calculate, in accordance with (7-2), the values of after-tax discount rates in each year of the forecast period:

$$r_1 = r_{BT}\times\left(1-t_{e1}\right) = 0.3\times\left(1-0.211\right) = 0.237$$

$$r_2 = r_{BT}\times\left(1-t_{e2}\right) = 0.3\times\left(1-0.252\right) = 0.224$$

$$r_3 = r_{BT} \times (1 - t_{e3}) = 0.3 \times (1 - 0.227) = 0.232$$

Let's now calculate the value of the after-tax discount rate for the forecast period in accordance with (12):

$$r_{ATPF} = (0.3 - 0.03)(1 - 0.227) + 0.03 = 0.239$$

All the necessary data is now available to calculate the cost of capital on the after-tax basis:

$$\begin{aligned} V &= \frac{FCFE_1}{1+r_1} + \frac{FCFE_2}{(1+r_1)(1+r_2)} + \frac{FCFE_3}{(1+r_1)(1+r_2)(1+r_3)} \\ &+ \frac{FCFE_3(1+g)}{(r_{PF}-g)(1+r_1)(1+r_2)(1+r_3)} \\ &= \frac{75}{1.237} + \frac{68.3}{1.237 \times 1.224} + \frac{81.7}{1.237 \times 1.224 \times 1.232} \\ &+ \frac{81.7 \times 1.03}{(0.239-0.03) \times 1.237 \times 1.224 \times 1.232} \\ &= 365 \text{ million Euros.} \end{aligned}$$

As can be seen from the data obtained, both valuation methods—on both before-tax and after-tax bases—lead to very similar results (359 and 365 million Euros). The observed difference in the results is understandable—the methods of linking before-tax and after-tax discount rates discussed in this section were originally derived on the basis of capitalization models. Therefore, in the considered example (where the discount model was used) there is no complete coincidence. The discrepancy in the results when using discount models will be all the more significant, the longer the forecasting period is and the more significant changes in cash flows are expected in this period.

In another limiting case ( $t \to \infty$ ), if the model of infinite cash flow generation with an annual growth rate ($g$) is applicable, the difference between the results on the before-tax and after-tax bases taking into account expression (12) will be:

$$\Delta V = \frac{CF_{AT1}}{r_{AT}-g} - \frac{CF_{BT1}}{r_{BT}-g} = \frac{CF_{BT1} \times (1-t_e)}{(r_{BT}-g)(1-t_e)} - \frac{CF_{BT1}}{r_{BT}-g} = 0. \qquad (24)$$

As follows from (24), with an infinite and constant growth rate of cash flows, difference between the two calculation bases becomes zero.

<u>Example 10</u>

Let there be every reason to believe that the asset will generate cash flows indefinitely, while they will be the same from year to year and equal to $CF_{AT}$ = 75 million Euros, $CF_{BT}$ = 100 million Euros, while the income tax rate = 25%, the before-tax discount rate = 20%, and the after-tax discount rate = 15%.

Let's estimate the value of the asset for these conditions on both before-tax and after-tax bases:

$$V_{AT} = \frac{75}{0.15 - 0} = 500 \text{ million Euros}$$

$$V_{BT} = \frac{100}{0.2 - 0} = 500 \text{ million Euros}$$

Thus, the arguments and calculation examples presented in this section show that the application of the concept of non-equivalence makes it possible to evaluate profitable assets without bothering to seek convergence of valuation results on both before-tax and after-tax valuation bases. At the same time, it is necessary to understand that the convergence of the results obtained on the before-tax and after-tax valuation basis is achieved only under the condition of applicability of the model of constant infinite growth. In general, the application of the concept of non-equivalence implies that there is some difference between the values obtained on both after-tax and before-tax bases and this difference cannot always be expressed analytically.

## 4. Making Tax Adjustments to the Risk Premium

Let's now consider another aspect, which is also related to discounting before-tax and after-tax income flows. It is known that the assessment of discount rates is often carried out by separately accounting (calculating) for the values of the risk-free rate and the risk premium (when discounting risky cash flows and in the absence of special "risk-correcting" adjustments):

$$r = r_f + pr \tag{25}$$

where:

$r$ = Discount rate applied to reduce (discount) risky income flows;

$r_f$ = Risk-free discount rate;

$pr$ = Risk premium.

Taking into account the consideration of tax aspects, expression (25) can be presented in the following specified forms:

$$r_{AT} = r_{fAT} + pr_{AT} \tag{26}$$

$$r_{BT} = r_{fBT} + pr_{BT} \tag{27}$$

where:

$AT$ = After-tax discount rates and their components;

$BT$ = Before-tax discount rates and their components.

It is believed that there is a simple relationship between before-tax and after-tax risk-free rates:

$$r_{fAT} = r_{fBT} \cdot (1 - t) \tag{28}$$

where:

$t$ = The income tax rate or, in general, the effective tax rate ($t_e$).

Despite the obviousness of expressions (26)-(27), there is usually a "mess" in practical calculations: when calculating the after-tax rate (to discount after-tax income flows), as a rule, a before-tax risk-free rate ($r_{fBT}$) is used, to which a risk premium is subsequently added. Of course, from a theoretical point of view, this in itself is incorrect. However, if we assume that the final value of the after-tax discount rate is determined correctly (e.g. due to the fact that the appraiser was guided by the values of other after-tax discount rates estimated by market extraction methods), it becomes obvious that in this case the value of the risk premium used in calculations is distorted—when using a higher value of the before-tax risk-free rate instead of the after-tax risk-free rate, a reduced risk premium should be applied:

$$r_{AT} = r_{fBT} + pr_{adj} \tag{29}$$

where:

$pr_{adj}$ = The observed and applied distorted value of the risk premium.

Taking into account the above expressions, the theoretical value of the premium applied to after-tax income flows can be calculated as follows:

$$pr_{AT} = pr_{adj} + t_e \cdot r_{fBT} \tag{30}$$

where:

$pr_{adj}$ = The value of the risk premium used in practical calculations (provided that it is subsequently added to the $r_{fAT}$).

<u>Example 11</u>

Let $r_{fBT} = 0.08$, $t_e = 0.25$, and the observed value of the risk premium added to the before-tax risk-free rate ($pr_{adj}$) = 0.05. It is required to determine the risk premium used to calculate the after-tax discount rate.

<u>The solution</u>. To get an answer, use expression (30):

$$pr_{AT} = 0.05 + 0.25 \times 0.08 = 0.07$$

It should be noted that when considering expressions (26)-(27), a reasonable question arises—should the risk premium depend on something else besides risk? After all, the amount of the risk premium itself should not seem to depend on the presence or absence of taxation. And if this is the case, then the values of the $pr_{AT}$ and $pr_{BT}$ risk premiums should not differ[6]. But if they do differ, what could be the reason for their difference? May the possibility of tax changes in the future be one of the uncertainties and, therefore, one of the types of risk? Without rejecting this answer, we note, nevertheless, that the main reason for the impact of taxation on the risk premium is that when the actual (realized) profitability of the calculated value, including the risk premium, is reached, a tax consequence arises in the form of an income tax liability. In addition, it can be shown (see **Appendix 1** at the end of the article) that the level of interest rates affects the observed (posteriori) risk premiums. Therefore, the answer to this question is positive—yes, the risk premium is also influenced by the income tax rate.

## 5. Errors Occurred When Using the CAPM Model Results

Let us now consider the impact of income tax when using the CAPM model to calculate the discount rate. First, let's give a complete list of assumptions on which this model is based (Sharpe et al., 1995):

1) Investors evaluate portfolios based on probable profit and dispersion (stand-

[6]Under this assumption, the relationship between before-tax and after-tax rates would be as follows: $r_{AT} = r_{fBT}(1-t) + Pr$, a $r_{BT} = r_{fAT}/(1-t) + Pr$, where: $Pr$ = Risk premium.

ard bending) during one holding period;

2) Investors who have a possibility of choosing two identical portfolios with the same profit (one and the same risk level), always choose the portfolio that supports its high profit;

3) Investors try to prevent from extra risk, i.e. from two identical portfolios with the same profit, they always choose the portfolio with lower level risk;

4) Each financial means can be divided into indefinite and unlimited parts and an investor can purchase any piece of his desirable shares or bonds;

5) There are some securities that have no risk. An investor may lend out any amount of his free money or borrow his desirable sum of money;

6) Taxes and commissions are not considered;

7) The duration of holding period is the same for all investors;

8) Risk-free percentage rate is the same for all investors;

9) All kinds of information are absolutely accessible for any investor;

10) Investors have homogeneous expectations. They evaluate the expected returns, standard deviations, and covariance of securities returns in the same way.

As follows from these assumptions, the CAPM considers the limiting case, i.e. potential obstacles such as limited divisibility, taxes, transaction costs, and the difference between risk-free borrowing and lending rates are considered absent.

Thus, the above statement made by one of the authors of the CAPM model clearly shows that this model does not take into account taxes (see assumption 6), considering them insignificant or absent. However, we know that taxes do exist in the real world, and they are quite significant, especially income taxes. Moreover, the first section of this article states that before-tax discount rates should be used to discount before-tax cash flows, and after-tax discount rates should be used to discount after-tax cash flows.

It would seem that, in the current order of things, the results of the CAPM calculation should be used to discount before-tax cash flows, since, as mentioned above, this model was developed under the assumption of no taxes. However, despite this, there is a practice in business valuation according to which the results of the yield calculation based on CAPM are used for discounting after-tax cash flows.

It is essential that in CAPM, risk is understood only as systematic risk, i.e. the correlation between fluctuations in the company's income and fluctuations in income from the "market portfolio of shares". Therefore, non-systematic risks are accounted for as additional "premiums"[7]. Perhaps, it would be appropriate to ask questions here: - Is it considered that the company's tax fluctuation is a systematic risk or not a systematic one? - Should we consider the possibility of additional taxes for years past as an unsystematic risk? The answer to the first of these questions seems to be positive, however, when studying the correlation between fluctuations in the

[7]In the later upgraded MCAPM model, two additional risk premiums were introduced: SCP = The premium for the small capitalization of the company (Small Cap Premium), SCRP = The premium for the specific risk of the company (Specific Company Risk Premium).

company's income and fluctuations in income from the "market portfolio of shares", the fluctuation of the company's taxes is taken into account a priori and therefore no additional risk premium component should be provided for it. As for the risk of additional taxes over the past years, it is obvious that this risk is not systematic and therefore, of course, its accounting is possible within the framework of additional accounting.

Let's note one more point regarding the application features of the CAPM model. As is known, the total return on investing in equities consists of the sum of exchange rates and dividend yields. However, the CAPM model deals with accounting only for exchange rate returns. Accordingly, when investing in equities that pay dividends, this model does not take into account the full return on investment by ignoring the dividend yield.

Accordingly, taking into account the above, the following consequences arise:

- When evaluating a company's capital, the use of the discount model for after-tax cash flows and the application of the discount rate calculated via the CAPM model leads to a distorted calculation result, because after-tax cash flows are discounted at a before-tax discount rate, since the CAPM model was created for a tax-free world and does not take into account dividend yields. However, this is what everyone who uses this calculation model does—they discount after-tax cash flows at a before-tax discount rate and do not take into account the dividend yield.
- When evaluating invested capital and/or capital, the use of the discount model for cash flows of invested capital (the so-called debt-free cash flows) and the application of the WACC discount rate to them leads to something unclear, since despite the fact that it is used to discount after-tax cash flows, one of the components (return on capital) in its calculation is taken on a before-tax basis and does not take into account the dividend yield.

The noted negative factors partially offset each other, since the use of a before-tax discount rate underestimates the present value of after-tax cash flows, while not accounting for dividend yields, on the contrary, overestimates their value.

Here are some recommendations that minimize these comments:

1) When using the CAPM model, it is proposed to discount after-tax capital flows at a rate calculated via the adjusted CAPM model, in which the risk-free return ($r_f$) and the average market return on investment in a market portfolio ($r_m$) are taken in the after-tax format (basis), i.e. multiplied by $(1 - t_e)$, and the dividend yield is added to the result. In other words, when using the CAPM model, it is proposed to discount after-tax capital flows at the rate calculated via the CAPM model multiplied by $(1 - t_e)$ and increased by the amount of the dividend yield. A demonstration of this proposal is presented in examples 12 and 14 (see **Appendix 2**).

2) It is proposed to discount after-tax flows on invested capital at the WACC rate, at which the return on equity is calculated as proposed above in paragraph 1.

3) It is proposed to discount before-tax capital flows (if applicable) at the CAPM rate, since, in essence, the rate of return calculated via the CAPM model is a before-

tax rate. At the same time, when the leveraged beta is calculated by using the Hamada formula, D/E shouldn't be multiplied by $(1 - t_e)$, and the dividend yield divided by $(1 - t_e)$ should be added to the calculation result. A demonstration of this proposal is presented in examples 13 and 15 (see **Appendix 2**).

4) It is proposed to discount before-tax flows on invested capital at the WACC rate, which is calculated in the usual way, except that it does not take into account the tax shield for borrowed capital (component "$(1 - t)$"), and the value of raising capital is calculated as indicated in paragraph 3.

## 6. Conclusion

1) When discounting, it should always be clearly understood which discount rate should be used: this rate should exactly match the type of discounted cash flow.

2) For long lasting (aiming to infinity) expected periods of cash flow generation, if the constant growth model (capitalization model) can be applied, there is an analytical relationship between the discount rates of before-tax and after-tax cash flows, which makes it possible to obtain convergence of the valuation results obtained on both before-tax and after-tax calculation bases.

3) For limited periods of cash flow generation, as well as when it is impossible to apply the cash flow capitalization model, the dependencies obtained in the first section of the article between discount rates calculated on both before-tax and after-tax bases allow us to obtain similar valuation results. However, in the general case (with chaotic dynamics of cash flows in the forecast period), it is possible to obtain convergence of these results only by using expressions (5) or (9) in the forecast period, and expressions (10)-(14) for the post-forecast period, which in itself is essentially a "fit", i.e. an artificial attempt to ensure the convergence of the valuation results obtained by using both before-tax and after-tax bases. Moreover, the convergence of valuation results carried out on a before-tax and after-tax basis decreases when negative cash flows are expected during the forecast period.

4) The results of the discrepancy between the valuation results obtained on both after-tax and before-tax bases are presented in the second section of this article as well as in **Appendix 2**. It is shown that the after-tax valuation results are higher than the before-tax ones.

5) The author has not specifically investigated the applicability of the convergence methods presented in this work across various fields, and therefore believes that such studies need to be conducted in the future.

6) The third section of the article shows that the observed (posteriori) risk premiums for capital investments in some cases may be distorted, which is why they should be analyzed in terms of their further use and adjusted if necessary.

7) Since in most cases (for private investors), after-tax cash flows are discounted, after-tax alternative income rates should be used as the discount rate[8], which means

[8] *Author's note*: Perhaps this is not entirely true for government corporations and organizations, which can be evaluated both on before-tax and after-tax bases.

that adjustments to alternative income rates calculated via the CAPM model (if used) are necessary, since one of the assumptions underlying this model is the assumption of the insignificance of taxes and because this model does not take into account the dividend yield. At the same time, in some cases (for example, when evaluating buildings or production lines), it may be advisable to use the before-tax valuation basis. Recommendations for appropriate adjustments are given at the end of the fourth section of this article, as well as in the demonstration examples provided in **Appendix 2**.

8) Currently, most calculations carried out by appraisers, stock and investment analysts contain methodological errors, which consist in the fact that they use before-tax discount rates for after-tax cash flows, since they take before-tax risk-free returns and before-tax premiums for the risk of investing in capital, and also often do not take into account the dividend yield. These errors lead to a systematic distortion of the evaluation results obtained within the framework of the income approach.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# Appendix 1

**Studies of the theoretical impact of inflation on the risk premium of investing in equities**

The process of determining discount rates[9] is largely based on two well-known dependencies:

- Equality reflecting the relationship of the nominal discount rate of risk flows ($r_n$) with the nominal risk-free rate ($r_{fn}$) and the risk premium ($pr$):

$$r_n = r_{fn} + pr \tag{31}$$

- The Irving-Fisher equation, reflecting the relationship of the nominal interest rate ($r_n$) with the real rate ($r_r$) and the inflation rate ($i$):

$$r_n = r_r + i + ir_r = r_r + i\left(1 + r_r\right) \tag{32}$$

When determining discount rates, a number of experts take into account inflation only in the risk-free component ($r_{fn}$) and do not take it into account in the risk premium ($pr$). Although this approach may be justified under certain assumptions, it cannot be considered entirely correct, since if we are talking about the observed risk premium, inflation also affects it. To substantiate the above statement, we introduce additional designation:

$r_{fr}$ = Real risk-free interest rate (i.e. the risk-free rate cleared of inflation);

$pr_t$ = "True" (implied) risk premium (i.e. a premium that should compensate *only* for risk costs).

Taking these designations into account, the following relations can be formulated:

$$\begin{aligned} r_n &= \left(1+i\right)\left(1+pr_t\right)\left(1+r_{fr}\right)-1 \\ &= \left(r_{fr}+i+ir_{fr}\right)+\left(pr_t+pr_t r_{fr}+pr_t i+ipr_t r_{fr}\right) \\ &= r_{fn}+pr_t\left(1+r_{fr}+i+ir_{fr}\right) \end{aligned} \tag{33}$$

The last equality in (33) is essentially equality (31), in which the risk premium ($pr$) is an observable (posteriori) premium:

$$pr = pr_t\left(1+r_{fr}+i+ir_{fr}\right) = pr_t\left(1+r_{fn}\right) \tag{34}$$

As can be seen from expression (34), the observed risk premium, in addition to the implied premium itself, also depends on the level of inflation and the real risk-free rate, as well as on their complex impact. At the same time, the above proportions are observed provided that inflation is fully accounted for in the nominal risk-free rate.

Note that expression (34) can be formulated in another way:

$$\begin{aligned} r_n &= \left(1+i\right)\left(1+pr_t\right)\left(1+r_{fr}\right)-1 \\ &= r_{fr}+pr_t\left(1+r_{fr}\right)+i\left(1+r_{fr}+pr_t\left(1+r_{fr}\right)\right) \\ &= r_r+i\left(1+r_r\right) \end{aligned} \tag{35}$$

[9]It is assumed that all of them, although being called percentages, are expressed in fractions of one and relate to the same period.

where:

$r_r$ = Real risk-adjusted interest rate:

$$r_r = r_{fr} + pr_t\left(1 + r_{fr}\right) = r_{fr}\left(1 + pr_t\right) + pr_t \quad (36)$$

Expressions (35)-(36) essentially reduce to the well-known expression (32)[10].

## Appendix 2

**Recommendations for adjusting the required return on capital investments based on the CAPM model**

This appendix provides demonstration examples with allowance for the recommendations 1 and 3 on considering the tax factor in the CAPM models, which were given in the fourth section of the article.

<u>Example 12</u>

<u>Recommendation 1</u>

*When using the CAPM model, it is proposed to discount after-tax capital flows at the rate calculated via the CAPM model multiplied by* $(1 - t_e)$ *and increased by the amount of the dividend yield.*

Let's assume that there is a cash flow forecast corresponding to the conditions of Example 9.

**Table A1.** Forecast of cash flows and effective tax rates 1.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| After-tax capital cash flow ($FCFE$), million Euros | 75 | 68.3 | 81.7 |
| Effective tax rate ($t_{efE}$) | 0.211 | 0.252 | 0.227 |

In the future, it is assumed that cash flows will grow by 3% annually, and the income tax rate will remain unchanged (Table A1).

Let's also say that the average market dividend yield in the considered market segment ($r_{\text{div}}$) is:

- Option 1: 4%;
- Option 2: 2%.

Let's now assume that we estimated the discount rate by using the CAPM model

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[10]If these equalities and (33) are true, the following proportions must be observed:

$$\frac{1 + r_{n1}\left(i_1\right)}{1 + r_{n2}\left(i_2\right)} = \frac{\left(1 + i_1\right)\left(1 + r_r\right)}{\left(1 + i_2\right)\left(1 + r_r\right)} = \frac{1 + i_1}{1 + i_2} \quad (*)$$

where:

$r_{n1}\left(i_1\right)$ = The nominal rate at the inflation rate $i_1$ and the real rate $r_r$;

$r_{n2}\left(i_2\right)$ = The nominal rate at the inflation rate $i_2$ and the real rate $r_r$.

Let's see if that's true. To do this, we substitute in (*) the expressions for $r_n$ from (35):

$$\frac{1 + r_{n1}\left(i_1\right)}{1 + r_{n2}\left(i_2\right)} = \frac{1 + r_r + i_1\left(1 + r_r\right)}{1 + r_r + i_2\left(1 + r_r\right)} = \frac{\left(1 + r_r\right)\left(1 + i_1\right)}{\left(1 + r_r\right)\left(1 + i_2\right)} = \frac{1 + i_1}{1 + i_2}$$

As can be seen, the conditions (*) for the obtained expressions (35) are met, which was required to be proved.

and got a value of 30%.

Let's determine the cost of capital based on after-tax cash flows for the two above-mentioned dividend yield options.

First, let's determine the cost of capital via the traditional approach by using after-tax cash flows discounted at the rate calculated via the CAPM model:

$$V = \frac{75}{1.3} + \frac{68.3}{1.3^2} + \frac{81.7}{1.3^3} + \frac{81.7 \times 1.03}{(0.3 - 0.03) \times 1.3^3} = 277 \text{ million Euros}$$

Let's now determine the cost of capital with allowance for Recommendation 1. As follows from Table A1 above, the average effective tax rate in the forecast period is 23% (=(0.211 + 0.252 + 0.227)/3). Let's take this value of the effective rate to adjust the before-tax discount rate to the after-tax format.

Option 1: The dividend yield is 4%.

Then, the value of the after-tax discount rate under the same conditions will be:

$$R_{AT} = 0.3 \times (1 - 0.23) + 0.04 = 0.271$$

Let's now calculate the cost of capital with allowance for Recommendation 1 using the after-tax valuation basis with the discount rate obtained above:

$$V = 1\frac{75}{1.271} + \frac{68.3}{1.271^2} + \frac{81.7}{1.271^3} + \frac{81.7 \times 1.03}{(0.271 - 0.03) \times 1.271^3} = 311 \text{ million Euros}$$

(+12.3% added to the result of the traditional approach based on the CAPM model).

Option 2: The dividend yield is 2%.

Then, the after-tax discount rate under the same conditions will be:

$$R_{AT} = 0.3 \times (1 - 0.23) + 0.02 = 0.251$$

Let's now calculate the cost of capital with allowance for Recommendation 1 using the after-tax valuation basis with the discount rate obtained above:

$$V = 1\frac{75}{1.251} + \frac{68.3}{1.251^2} + \frac{81.7}{1.251^3} + \frac{81.7 \times 1.03}{(0.251 - 0.03) \times 1.251^3} = 340 \text{ million Euros}$$

(+12.7% added to the result of the traditional approach based on the CAPM model).

As can be seen, due to the generally accepted logic of matching the discount rate to the type of cash flow, the calculation result turned out to be higher than the traditional valuation method (277 million Euros), which violates the above logic.

Example 13

Recommendation 3

*It is proposed to discount before-tax capital flows (if applicable) at the CAPM rate, since, in essence, the rate of return calculated via the CAPM model is a before-tax rate. At the same time, when the leveraged beta is calculated by using the Hamada formula, D/E shouldn't be multiplied by* (1 − $t_e$) *and the dividend yield divided by* (1 − $t_e$) *should be added to the calculation result.*

Again, let's assume that we have the initial data regarding the forecast of cash flows corresponding to the conditions of Example 9.

**Table A2.** Forecast of cash flows and effective tax rates 2.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| After-tax capital cash flow ($FCFE$), million Euros | 75 | 68.3 | 81.7 |
| Effective tax rate ($t_{efE}$) | 0.211 | 0.252 | 0.227 |

In the future, it is assumed that cash flows will grow by 3% annually, and the income tax rate will remain unchanged (Table A2).

Let's also say that the average market dividend yield in the considered market segment ($r_{div}$) is:

- Option 1: 4%;
- Option 2: 2%.

Let's now assume that we estimated the discount rate using the CAPM model and got a value of 30%.

Let's determine the cost of capital based on before-tax cash flows for the two above-mentioned dividend yield options.

First, let's determine the cost of capital via the traditional approach by using after-tax cash flows discounted at the rate calculated via the CAPM model:

$$V = \frac{75}{1.3} + \frac{68.3}{1.3^2} + \frac{81.7}{1.3^3} + \frac{81.7 \times 1.03}{(0.3 - 0.03) \times 1.3^3} = 277 \text{ million Euros}$$

Let's now determine the cost of capital with allowance for Recommendation 3. To do this, let's take a closer look at the structure of the discount rate obtained via the CAPM model. Suppose such a rate value was obtained based on the following data: risk-free rate of return ($r_f$) = 10%, unleverage $\beta_{ul}$ = 2.22, ratio of interest-bearing debt to capital ($D/E$) = 1, effective income tax rate = 0.23 (= average effective tax rates in Table A2 above), capital risk premium ($ERP$) = 0.05:

$$R_{CAPM} = 0.1 + 2.22 \times \left(1 + 1 \times (1 - 0.23)\right) \times 0.05 = 0.3$$

<u>Option 1</u>: The dividend yield is 4%.

Now let's apply Recommendation 3 to calculating the discount rate. Adjust the after-tax dividend yield to the before-tax format and get the value of the before-tax dividend yield: 0.04/(1 − 0.23) = 0.052. Then, taking into account these conditions and Recommendation 3, the value of the before-tax discount rate will be:

$$R_{BT} = 0.1 + 2.22 \times (1 + 1) \times 0.05 + 0.052 = 0.374$$

Let's now calculate the cost of capital with allowance for Recommendation 3 by using the before-tax valuation basis with the discount rate obtained above:

$$V = \frac{95}{1.374} + \frac{91.3}{1.374^2} + \frac{105.7}{1.374^3} + \frac{105.7 \times 1.03}{(0.374 - 0.03) \times 1.374^3} = 280 \text{ million Euros}$$

(+1.1% added to the result of the traditional approach based on the CAPM model).

Option 2: The dividend yield is 2%.

Now let's apply Recommendation 3 to calculating the discount rate. Adjust the after-tax dividend yield to the before-tax format and get the value of the before-tax dividend yield: 0.02/(1 − 0.23) = 0.026. Then, taking into account these conditions and Recommendation 3, the value of the before-tax discount rate will be:

$$R_{BT} = 0.1 + 2.22 \times (1+1) \times 0.05 + 0.026 = 0.348$$

Let's now calculate the cost of capital with allowance for Recommendation 3 by using the before-tax valuation basis with the discount rate obtained above:

$$V = \frac{95}{1.348} + \frac{91.3}{1.348^2} + \frac{105.7}{1.348^3} + \frac{105.7 \times 1.03}{(0.348 - 0.03) \times 1.348^3} = 304 \text{ million Euros}$$

(+9.7% added to the result of the traditional approach based on the CAPM model).

Let us now consider, within the framework of this example, a different structure of the alternative income rate, without changing its final value: risk-free rate of return ($r_f$) = 15%, unleverage $\beta_{ul}$ = 1.3, ratio of interest-bearing debt to capital market cost ($D/E$) = 1.2, effective income tax rate = 0.23, capital risk premium ($ERP$) = 0.06:

$$R_{CAPM} = 0.15 + 1.3 \times (1 + 1.2 \times (1 - 0.23)) \times 0.06 = 0.3$$

Let's now calculate the cost of capital for the two dividend yield options presented above.

Option 1: The dividend yield is 4%.

The value of the adjusted before-tax discount rate under the same conditions will be:

$$R_{BT} = 0.15 + 1.3 \times (1 + 1.2) \times 0.06 + \frac{0.04}{1 - 0.23} = 0.374$$

As can be seen from this example, when the income structure calculated via the CAPM model changes, the final value of before-tax income remains unchanged. It is not affected by the adjustments being made.

Option 2: The dividend yield is 2%.

The value of the adjusted before-tax discount rate under the same conditions will be:

$$R_{BT} = 0.15 + 1.3 \times (1 + 1.2) \times 0.06 + \frac{0.02}{1 - 0.23} = 0.348$$

Since the final values of the adjusted before-tax discount rates have not changed with the change in the rate structure under the CAPM model, the final cost of capital calculated in the before-tax format will not change (they will also be 280 and 304 million Euros). Thus, as can be seen from this calculation, a change in the structure of the rate of return does not change the total value of capital.

Example 14

Recommendation 1

*When using the CAPM model, it is proposed to discount after-tax capital flows at the rate calculated via the CAPM model multiplied by* $(1 - t_e)$ *and increased by the dividend yield.*

Let's have the following forecast of the company's capital cash flows.

**Table A3.** Forecast of cash flows and effective tax rates 3.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| After-tax capital cash flow ($FCFE$), million Euros | 57 | 52.9 | 65.5 |
| Effective tax rate ($t_{efE}$) | 0.4 | 0.42 | 0.38 |

In the future, it is assumed that cash flows will grow by 3% annually, and the income tax rate will remain unchanged (**Table A3**).

Let's also say that the average market dividend yield in the market segment under consideration ($r_{div}$) is:

- Option 1: 4%;
- Option 2: 2%.

Let's now assume that we estimated the discount rate by using the CAPM model and got a value of 15%.

Let's determine the cost of capital based on after-tax cash flows for the two above-mentioned dividend yield options.

First, we will determine the cost of capital via the traditional approach by using after-tax cash flows discounted at the rate calculated via the CAPM model:

$$V = \frac{57}{1.15} + \frac{52.9}{1.15^2} + \frac{65.5}{1.15^3} + \frac{65.5 \times 1.03}{(0.15 - 0.03) \times 1.15^3} = 502 \text{ million Euros}$$

Let's now determine the cost of capital with allowance for Recommendation 1. As follows from **Table A3** above, the average effective tax rate in the forecast period is 40% (=(0.4 + 0.42 + 0.38)/3). Let's take this effective rate to adjust the before-tax discount rate to the after-tax format.

Option 1: The dividend yield is 4%.

The value of the discount rate under the same conditions will be:

$$R_{AT} = 0.15 \times (1 - 0.4) + 0.04 = 0.13$$

Let's now calculate the cost of capital with allowance for Recommendation 1 on the after-tax basis, which uses the discount rate value obtained above:

$$V = \frac{57}{1.13} + \frac{52.9}{1.13^2} + \frac{65.5}{1.13^3} + \frac{65.5 \times 1.03}{(0.13 - 0.03) \times 1.13^3} = 605 \text{ million Euros}$$

(+20.5% added to the result of the traditional approach based on the CAPM model).

Option 2: The dividend yield is 2%.

The value of the discount rate under the same conditions will be:

$$R_{AT} = 0.5 \times (1 - 0.4) + 0.02 = 0.11$$

Let's now calculate the cost of capital with allowance for Recommendation 1 on the after-tax basis, which uses the discount rate value obtained above:

$$V = \frac{57}{1.11} + \frac{52.9}{1.11^2} + \frac{65.5}{1.11^3} + \frac{65.5 \times 1.03}{(0.11 - 0.03) \times 1.11^3} = 759 \text{ million Euros}$$

(+51.1% added to the result of the traditional approach based on the CAPM model).

Example 15

Recommendation 3

*It is proposed to discount before-tax capital flows (if applicable) at the CAPM rate, since, in essence, the rate of return calculated via the CAPM model is a before-tax rate. At the same time, when the leveraged beta is calculated by using the Hamada formula, D/E shouldn't be multiplied by* $(1 - t_e)$ *and the dividend yield divided by* $(1 - t_e)$ *should be added to the calculation result.*

Let's assume that we have the following forecast of the company's capital cash flows (Table A4).

Table A4. Forecast of cash flows and effective tax rates 4.

| Year/Parameter | 1 | 2 | 3 |
|---|---|---|---|
| Before-tax capital cash flow ($FCFE_{BT}$), million Euros | 95 | 91.3 | 105.7 |
| Before-tax capital cash flow ($FCFE$), million Euros | 57 | 52.9 | 65.5 |
| Effective tax rate ($t_{efE}$) | 0.4 | 0.42 | 0.38 |

In the future, it is assumed that cash flows will grow by 3% annually, and the income tax rate will remain unchanged (Table A4).

Let's also say that the average market dividend yield in the market segment under consideration ($r_{div}$) is:

- Option 1: The dividend yield is 4%;
- Option 2: The dividend yield is 2%.

Let's now assume that we estimated the discount rate by using the CAPM model and got a value of 15%.

Let's determine the cost of capital based on before-tax cash flows for the two above-mentioned dividend yield options.

First, determine the cost of capital via the traditional approach by using after-tax cash flows discounted at the rate calculated via the CAPM model:

$$V = \frac{57}{1.15} + \frac{52.9}{1.15^2} + \frac{65.5}{1.15^3} + \frac{65.5 \times 1.03}{(0.15 - 0.03) \times 1.15^3} = 502 \text{ million Euros}$$

Let's now determine the cost of capital with allowance for Recommendation 3. As follows from Table A4 above, the average effective tax rate in the forecast period is 40% (=(0.4 + 0.42 + 0.38)/3). Let's take this value of the effective rate to adjust the CAMP rate and the before-tax dividend yield in the before-tax format.

Let us now consider that this discount rate (15%) was obtained under the following conditions: risk-free rate of return ($r_f$) = 5%, unleverage $\beta_{ul}$ = 1.25, ratio of interest-bearing debt to capital market cost ($D/E$) = 1, effective income tax rate = 0.4, capital risk premium ($ERP$) = 0.05:

$$R_{CAPM} = 0.05 + 1.25 \times \left(1 + 1 \times \left(1 - 0.4\right)\right) \times 0.05 = 0.15$$

Option 1: The dividend yield is 4%.

The value of the discount rate under the same conditions will be:

$$R_{BT} = 0.05 + 1.25 \times \left(1 + 1\right) \times 0.05 + \frac{0.04}{1 - 0.4} = 0.242$$

Let's now calculate the cost of capital with allowance for Recommendation 3 by using the before-tax valuation basis with the discount rate obtained above:

$$V = \frac{95}{1.242} + \frac{91.3}{1.242^2} + \frac{105.7}{1.242^3} + \frac{105.7 \times 1.03}{\left(0.242 - 0.03\right) \times 1.242^3} = 459 \text{ million Euros}$$

(−8.6% compared to the result of the traditional approach based on the CAPM model).

Option 2: The dividend yield is 2%.

The value of the discount rate under the same conditions will be:

$$R_{BT} = 0.05 + 1.25 \times \left(1 + 1\right) \times 0.05 + \frac{0.02}{1 - 0.4} = 0.208.$$

Let's now calculate the cost of capital with allowance for Recommendation 3 by using the before-tax valuation basis with the discount rate obtained above:

$$V = \frac{95}{1.208} + \frac{91.3}{1.208^2} + \frac{105.7}{1.208^3} + \frac{105.7 \times 1.03}{\left(0.208 - 0.03\right) \times 1.208^3} = 548 \text{ million Euros}$$

(+9.2% added to the result of the traditional approach based on the CAPM model).

**Conclusions made from the considered examples:**

1) Valuation results obtained on the after-tax basis are higher than the results obtained on the before-tax basis.

2) In the examples considered, the results obtained with allowance for the recommendations in Section 4 on adjusting the rates of return obtained via the CAPM model were most often higher than the results obtained by using the CAPM model in a traditional way, but the result of one such estimate, on the contrary, was lower (the result of Example 16 when calculated on the before-tax basis and at the 4% dividend yield rate).

3) Changing the structure of the rate of return does not change the total value of the capital.

4) As is expected, the results of the adjusted estimates depend on the values of the tax rate and the dividend yield.