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# Multidimensional Trade and Golden Rule in Africa West: Analogical Econometrics versus Numerical Econometrics

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## Abstract

**Objectives:** Our aim is to assess whether analogical econometrics or numerical econometrics yield the best results. **Methods:** Through analogical econometrics, we show that, at the equilibrium level, each productive factor and each good or service is far from having the same price in each generation and country. This cannot realize an efficient trade of externalities that decouples growth and volatility. Second, we use numerical econometrics tests. Stressing the theory of multidimensional trade is sufficiently similar to a biological mechanism, we could say of one or more things that also apply to the other. We would then have more powerful analytical tools than the very approximate analogical econometric methods, and economic policy would become normative. **Results:** Using numerical tests, through laboratory experiments, we find that the human organism's multidimensional exchange mechanism, consisting of more or less integral compensation processes for negative and positive externalities, leads to volatility of human growth(ecosystems), the main determinant of life expectancy, analogous to the relationship between the processes of economic growth volatility and sustainable growth. Because the analogy is clearly established, the frontier of production possibilities of neurotransmitter secretion centers appears to be the sole determinant of life expectancy, as is the global technology frontier for sustainable growth. Finally, the assumption of multidimensional exchange becomes the optimal policy for managing ecosystems and biodiversity and is the only one to guarantee general equilibrium in all markets. Any imbalance in social ecosystems is transmitted to natural ecosystems, affecting any efficient exchange system at all levels due to functional or conflicting interdependencies.

## Keywords

International Trade (Exchange of Neurotransmitters between Secretion Centers), Intergenerational Trade (Exchange between Generations of Secretion Centers), Steady State (Optimal Life Expectancy), Numerical Econometrics, Analogical Econometrics, Growth Volatility, Exchange of Externalities

## 1. Introduction

The econometrician's main job is always to study how the statistics of the observable sample of an unobservable parent population are sufficiently similar to those of the parent population, from a certain point of view, so that their analogy makes it possible to say of one or do of one what also applies to the other. Why is it that what is wisdom in the conduct of analogical econometrics would be folly for the conduct of numerical econometrics (two totally observable phenomena)? Aren't Newton's laws the same as those that explain the lift generated by the wings of a bird and those of an airplane? By applying Bernoulli's and Newton's natural laws, Solow-Swan shows how a pressure differential exerted by the difference between consumption and production creates the lift of a given economy toward its steady state. In the golden rule approach, markets are always in equilibrium, and problems of outlets and effective demand do not arise; fluctuations are the result of agents' reactions to exogenous shocks. For example, while traditionally a decrease in employment leads to a decrease in wages, in the theory of the equilibrium cycle, there is a reversal of causality: employment decreases because when wages decrease, agents choose to work less. Our analytical framework is therefore that of Ramsey's (1928) dynastic utility function model, renovated by Solow (1956) and revised by Cass (1965), Barro (1983), Black (1987), Blackburn (1999), Blackburn & Galindev (2003), and the New Classical Economics in the theory of equilibrium cycles; however, unlike the latter, we endogenize technical progress, abandoning Ramsey's altruism and its corollary, Ricardian equivalence, to more fully endogenize growth and real cycles through a model of exchange of externalities known as "multidimensional exchange". This framework is necessarily that of sustainability. However, the only basis for any sustainable system, i.e., normal, is a natural analogy. It is very rare to find phenomena in nature that are not analogous. Thus, everything that flies is the true precursor of the airplane. The same is true of the ladybug and the car, the train and the centiped. The term analogy indicates that one thing is sufficiently similar to another, from a certain point of view, that their analog makes it possible to say of one or do with one that also applies to the other. If we could show that the theory of multidimensional trade is sufficiently similar to a biological mechanism, we could say of one or more things that also apply to the other. We would then have more powerful analytical tools than the very approximate analogical econometric methods do, and economic policy would become normative. Economics could move more confidently toward the status of an exact science. The reliability of data and, above all, of analogical econometric methods is one of the greatest challenges facing economics. In the 1970s, leading economists used the same statistical data to obtain contradictory results, bringing our science into great disrepute. For centuries, many economists have preferred positive analysis to normative analysis, as if human behavior were error-free, virtually neglecting the numerical econometric method, which is the source of the glory of the great sciences. The theory of multidimensional trade (Edgeweblime, 2019) states that factors of production that exist in abundance in one generation and are not intensively used to produce goods and services in that generation are exported to other generations in exchange for scarce factors of production intensively used to produce goods and services that should be scarce in the generation under consideration. Low-consumption goods and services are indirectly exported from one generation to the next, whereas high-consumption goods and services are indirectly imported from other generations. In this way, positive externalities (nonnatural resources) are exchanged for negative externalities (overconsumption of natural resources).

Production and consumption externalities affect the well-being of certain organs through the actions of other organs without biological compensation. Production externalities are explained by the interdependence of organ functioning, whereas consumption externalities are linked to consumption choices; these externalities can be positive or negative. Clearly, this exchange of externalities is the fundamental mechanism that generates links between growth and volatility. The abbreviated multidimensional exchange model ($2 \times 2 \times 2 \times 4$), i.e., two countries, two goods, two factors of production, and four generations, resembles the biological mechanism of consumption. The various organs of the organism function in dazzling interdependence, exchanging almost finished products with one another. If we follow the circuits of consumption of goods and services in the human body, many constants attract our attention. This biological mechanism is performed with the help of several neurotransmitter secretion centers, two of which, in particular, spark my curiosity: the functions of the brain and the intestine during the consumption process. These two secretion centers, along with the goods whose consumption activates them, are considered two distinct countries. The neurotransmitters (serotonin and dopamine) synthesized by both centers play important roles in the organism, transmitting the information needed to produce two goods that we call carnal satisfaction on the one hand and cerebral or spiritual satisfaction on the other hand. Therefore, we have two goods (carnal satisfaction and spiritual satisfaction) and two production factors: serotonin and dopamine. Because the secretion centers themselves are made up of cells that renew themselves every month, we have generations of neurotransmitter secretion centers throughout the life of the organism. For simplicity, we consider two generations of secretion centers, although all generations of secretion centers are also considered. This gives us the multidimensional exchange model ($2 \times 2 \times 2 \times 4$).

In the neoclassical model, which considers two countries, two goods, and two factors of production, each country has a fixed quantity of labor and capital, which remains unchanged even after trade opens. Wheat and cloth, for example, are naturally endowed with the capacity to induce the production of fixed quantities of serotonin and dopamine (factors of production of satisfaction), i.e., once consumed by a given consumer, they are unchanging.

In this way, the individual's immune system controls the production of fixed quantities of serotonin and dopamine. Let us remember that we are dealing with a normal individual, i.e., one whose conduct is regulated, systematic, sober, balanced, irreproachable, addiction-free, with unvarying tastes, in good health and whose characteristic aggregates grow at a constant rate of almost zero. The first, known as the carnal, is predominantly serotonin, whereas the second, cerebral or spiritual, is predominantly dopamine. In fact, the consumption of a food item controls the production of 90% intestinal serotonin and 10% cerebral serotonin, indicating a partial specialization of secretion centers in the production of both carnal and spiritual satisfactions, exactly as in the factor proportion model.

In this paper, we want to assess whether analogical econometrics or new numerical econometrics yield the best results through the following research question: How can the potential impact of the optimal policies of externalities trade to uncouple growth and volatility to generate sustainable growth and clean production be explored by means of agent-based modeling (ABM) or through numerical econometric tests? To this end, we use ABM as an analogical econometric method to answer the following question: How can the potential impact of the optimal policies of externalities trade uncouple growth and volatility to generate sustainable growth and clean production? Numerical econometrics will be a tool for answering this same question to determine which of the two types of econometrics would yield the most robust results. To determine whether the theory of multidimensional trade is sufficiently similar to a biological mechanism, what can be said of biological metabolism or done with biological metabolism also applies to multidimensional trade. Hence, in this paper, the following research question is answered: does a multidimensional exchange mechanism exist in the human organism, involving more or less complete compensation between negative and positive externalities, to determine either the volatility of human growth that reduces life expectancy or an optimal life expectancy analogous to either the volatility of economic growth or the optimal or sustainable growth of an economy? More specifically, I investigate the impact of non-Pareto-optimal Walrasian equilibria in the exchange of externalities between neurotransmitter secretion centers and/or between generations of neurotransmitter secretion centers as a fundamental mechanism of human growth volatility leading to life expectancy perturbations. This would confirm the theory of multidimensional exchange, enabling policymakers to use the same remedies as in modern medicine.

To our knowledge, this quasi-experimental study brings the two sciences together is the first of its kind, both theoretically and empirically, on the economic (biological) links between growth and volatility, on the basis of the trade in externalities on the scale of overlapping generations of nations (generations of neurotransmitter secretion centers) and between countries (secretion centers).

The paper includes the introduction section (1), material and analog (2), the section on theory and analogous tests (3), the results and discussion of the ABM model specification (4), and the conclusions and recommendations (5).

## 2. Materials and Methodology

The multidimensional trade theory to be tested is described in (Edgeweblime, 2019).

This theory states that generations import productive factors from other generations intensively used in the production of goods and services highly consumed in the current generations and export productive factors intensively used in the production of goods and services weakly consumed in the same generations. Even though the neoclassical growth model goes back to Ramsey (1928) or von Neumann (1935), the recent versions are closely related to the analysis of optimal growth by Cass (1965) and Koopmans (1965). Ramsey (1928), assuming that the population is constant and considering that global output is a function of capital and labor, admits that consumer utility has a superior final limit. He established the well-known Keynes-Ramsey rule of optimal saving, which characterizes steady-state optimal consumption. In his study of optimal growth, Rawls (1974) assigns the same weight to each generation by fixing a fair savings rate. However, it is now generally accepted that the implementation of the Rawls criterion for successive generations constitutes a growth limitation. However, if you want to bring to evidence that generations of a nation separated by a very long period (100, 200, 1000, 5000… years) exchange goods for productive factors so that current growth is a byproduct, what should be the best formula? In the original evidence on multidimensional trade theory, Edgeweblime's hard question was as follows: How can the potential impact of the optimal policies of externalities trade to uncouple growth and volatility to generate sustainable growth and clean production be explored by means of agent-based modeling (ABM)? Because of the crucial importance of cleaner production, more evidence on the fundamental mechanisms of sustainable growth is needed. In this paper, I turn to the exact science of biology, borrowing from electronics and aerodynamics, the invaluable method of analog y. Both fields use the analogy method to model and understand the complex phenomena that occur in their respective domains. This simplifies calculations while providing valid results (https://fr.wikipedia.org/wiki/%C3%89lectronique_analogue_ique). I would like to know whether the theory of multidimensional exchange is sufficiently similar to that of food metabolism, from a certain point of view, that their analog y makes it possible to say of one or do with one that also applies to the other.

## 3. Theory and Numerical Evidence

On the one hand, in numerical econometrics, analogical estimators are converted into numerical estimators using analog-to-digital converters. Once converted, numerical estimators to be used by decision-makers are error-free i.e. without any loss of information with regard to the phenomena with which the analogy is established. As a result, the forecasts and decisions they enable to be made or taken have the precision of biological laboratory analyses, even if the digital estimator has to revert to its original analogical form. On the other hand, in analogical econometrics, the impossibility of completely observing the parent population means that analogical estimators cannot be converted into numerical estimators, which means that information is lost to a greater or lesser extent.

## 3.1. Analog Model Configuration

### 3.1.1. Specification of the Analog System

The approach to factor proportions is based on the following assumption:

Everything happens in a market of pure and perfect competition (a multitude of competing goods to produce either carnal satisfaction or cerebral satisfaction).

The individual has at his disposal two secretion centers (the brain and the intestine) producing either dopamine or serotonin with the help of two goods, wheat or cloth, which are candidates for the production of carnal satisfaction or cerebral satisfaction. Each produces two homogeneous categories of goods (carnal satisfaction and spiritual satisfaction). These goods are produced from two homogeneous factors (serotonin "S" and dopamine "D"):

Each good is produced with a distinct relative intensity in dopamine or serotonin: the production of cerebral satisfaction is intensive in dopamine and that of carnal satisfaction in serotonin.

The factors of production available in fixed quantities are used for their full potential in production and in an optimal manner. It is assumed that each secretion center of the organism (using factor goods) produces both goods (partial specialization).

The production (secretion) function is the same in both secretion centers for a good; the production (secretion) functions are as follows:

Homogeneous of degree 1, with constant returns to scale and decreasing marginal productivities.

Production or secretion factors (serotonin and dopamine) are immobile between secretion centers.

The marginal utility of each good (carnal satisfaction and cerebral satisfaction) always decreases.

From this, we can define the following expressions: CdC, units of D required for carnal satisfaction; CsC, units of S required for carnal satisfaction; CdS, units of D is required for spiritual satisfaction, CsS: units of S required for spiritual satisfaction D: total supply of dopamine to the brain (controlled production per unit of wheat), S: total supply of serotonin to the intestine (controlled production per unit of canvas).

The production of spiritual satisfaction is D intensive, $\Rightarrow \mathrm{CdS}/\mathrm{CsS} > \mathrm{CdC}/\mathrm{CsC}$ or $\mathrm{CdS}/\mathrm{CdC} > \mathrm{CsS}/\mathrm{CsC}$.

### 3.1.2. Strict Parallel between the Theory of International Exchange and Metabolic Processes

"A metabolic process is a set of chemical reactions that occur in living organisms. There are two opposing types of metabolism: anabolism, where smaller molecules are synthesized to make larger ones, and catabolism, where larger molecules are broken down into smaller molecules composed into smaller ones". These processes appear to be analogous to multidimensional exchange.

Let us start by denoting by $\partial$ the rate of positive or negative change in dopamine in the body and by $\partial^*$ that of serotonin. The final goods are virtually mobile from one secretion center to another but not from one secretion center generation to another, whereas production factors (serotonin, dopamine) are mobile from one secretion center generation to another but not between current secretion centers at time t1. The mobility of production factors (dopamine or serotonin) is achieved by exchanging positive externalities for negative externalities. Positive externalities are produced when the consumption of a good factor by one neurotransmitter secretion center enables the adequate functioning of the other, and vice versa. In the opposite case, negative externalities occur. Bajona and Kehoe's (2006) model is compatible with what is described here.

The consumption of one of the two primary products (canvas or wheat) induces a subsequent wave of dopamine and/or serotonin flow across neurotransmitter secretion centers (generations of neurotransmitter secretion centers).

Their production sites differ across different parts of the body and indirectly between secretion centers. The initial endowment ratio of product *i* or secretion center *i* (where $y_i$ = neurotransmitter secretion capacity) is equal to $y_i / Y = \acute{y}$, where $Y$ is the body's overall capacity to secrete neurotransmitters. The body uses its $y_i / Y$ capacity to secrete serotonin and dopamine from secretion center *i* to determine which levels and types of satisfaction the individual wants to have and which to export (store) in exchange for importing which levels and types of satisfaction (use). These exports and imports follow metabolic processes (convergent, divergent, complex, anabolic and catabolic) and affect consumer health and life expectancy. The body's capacity to secrete neurotransmitters changes from $Y$ to $Y'$. The capacity of the intestinal center becomes $y_i'$, and $y_i'/Y' = \acute{y}'$ becomes the new ratio of neurotransmitter secretion capacity.

The organism uses the new capacities of each secretion center to produce new waves of neurotransmitters destined either for its own carnal satisfaction or for export against an import of spiritual satisfaction. At the end of this first wave, the secretion centers in the organism will have coownership

$$\Delta Y - \Delta Y\left[\beta + \delta \vdash Y - \Delta Y - \Delta Y\left[\beta + \delta \vdash Y\left[\beta + \delta \vdash \beta + \delta \vdash (1 - \beta)\right]\dashv. \qquad (1)$$

$\beta$ is the internal absorption rate (absorption per unit of secretion capacity), whereas $\delta$ is the intensity of the relationship between secretion centers ($\beta = (C_i + I_i + G_i)/y_i$, $\delta = (x_i + m_i)/y_i$). $C_i$ represents wheat consumption, $I_i$ represents wheat's capacity to produce serotonin, and $G_i$ represents the proportion of wheat consumption destined for serotonin production in the brain.

At the start of the second wave, the stock of additional serotonin is

$$\Delta Y-\Delta Y\left[\beta+\delta\vdash Y\left[\beta+\delta\vdash(1-\beta)(1-\delta)\right]\right. . \tag{2}$$

The second wave of processes generates dopamine. Neurotransmitter production was calculated as

$$\begin{aligned}&\Delta Y-\Delta Y\left[\beta+\delta\vdash Y\left[\beta+\delta\vdash(1-\beta)(1-\delta)\right]\left[\beta+\delta\vdash(1-\beta)(1-\delta)\right]\right.\\&=\Delta Y-\Delta Y\left[\beta+\delta\vdash Y\left[\beta+\delta\vdash(1-\beta)(1-\delta)\right]^{2}\right. .\end{aligned} \tag{3}$$

At the end of the process waves, the impact on the overall stock of neurotransmitters in the body is equal to the sum of the geometric Progression, with a reason of less than one. This sum can be described as follows:

$$\begin{aligned}&\Sigma\Delta Y-\Delta Y\left[\beta+\delta\vdash y_{it}/Y\left[\beta+\delta\vdash(1-\beta)(1-\delta)\right]\right.\\&=\Sigma\Delta Y-\Delta Y\left[\beta+\delta\vdash y_{it}\middle/\left[\beta+\delta\vdash\beta+\delta(1-\beta)\right]\right.\\&=\Sigma\Delta Y-\Delta Y\left[\beta+\delta\vdash Y_{it}\right. .\end{aligned} \tag{4}$$

The multiplier for optimal neurotransmitter production is
$1/\left[\beta+\delta\vdash\beta+\delta(1-\beta)\right]$.

As a result, the ratio of dopamine to serotonin is constant. The savings curve of the Slow-Swan golden rule is a point that is in the stationary state. In other words, each generation (secretion centers) fully repays its overconsumption of serotonin via its overproduction of dopamine. Because of the interdependence of all human organs (markets), including the technology market, no generation (secretion) can overconsume serotonin without inventing the appropriate technologies (body adaptation). "As you make your bed, so you lie in it", or that "nothing is created, nothing is lost, everything is transformed". Therefore, while production or consumption levels may vary from one generation (secretion center) to the next, the level of satisfaction is always the same at all times and in all places. The latest American generation cannot be happier than Adam and Eve's generation, hence the futility of outrageous development. For more development of technology endogenization, see Edgeweblime (2019).

At each instant, consumers of product $i$ decide how much of each of the two goods to consume, how much dopamine to accumulate for the appropriate neurotransmitter secretion center and, consequently, how much serotonin to borrow from the appropriate neurotransmitter secretion centers. Each consumption wave generates neurotransmitter flows throughout the neurotransmitter secretion centers (generations of neurotransmitter secretion centers), which function in a sinusoidal manner, represented as follows:

$$\Sigma\Delta y_{it}=y_{i0}\cos\left(W_{ijt}-\varphi_{1}\right)+y_{i1}\cos\left(X_{it}-\varphi_{1}\right).\quad\Delta Y_{t}=\Delta Y_{it}=\Sigma\Sigma\Delta y_{it}\,. \tag{5}$$

The study of periodic functions indicates that each ₽ periodic motion is a sum of sinusoidal motions whose subperiods are ₽, ₽/2, ₽/3..., ₽/n. These represent the harmonics of the system.

Following the proposal of Grossman and Helpman (1991), $w_{ij}(t)$ is modeled as the ratio of the total exchange of neurotransmitter secretion control capacities of secretion center *i* with secretion center *j*. This ratio is calculated for secretion center *i*'s bilateral exports and imports divided by secretion center *I*'s aggregate production. This ratio is calculated for the bilateral exports and imports of secretion center *i* divided by the aggregate production of secretion center *i*. This ratio is represented by $w_{ij}(t)$. It is expressed as follows:

$$\left(P_j(t)\big/\left(P_i(t)\right)L_i(t)g_{ij}(t)+L_j(t)g_{ji}(t)\right)\big/\left(L_i(t)y_i(t)\right), i \neq j. \tag{6}$$

$G_i(t)$ represents the actual imports by secretion center *i* of neurotransmitters generated by secretion center *j*. $P_i(t)$ is the price of factor *i*, where $L_i(t)$ is the weight of product *i* at period *t*.

We now define $a_{ij}\left(0 \leq a_{ij} \leq 1\right)$ as a constant, representing the share of serotonin accessible to the secretion center *j* that can be imported by product into the dopamine secretion process is attributable to its own ability to control the secretion of this neurotransmitter (dopamine). Using Abramovitz's (1986) social capacity, $a_i s_j$ determines a product's potential to adopt existing technologies (in this case, its ability to control neurotransmitter secretion). Using these definitions, the accumulation of dopamine due to product *i* can be written as follows:

$$X * i(t)=\Phi\left[\beta+\delta \vdash \Sigma a_{ij} w_{ij}(t) X_j(t)\right]+(\Phi-\delta X) X_i(t), \tag{7}$$

where Φ represents the common neurotransmitter secretion parameter and $\delta X$ represents the rate of dopamine stock depreciation (aging, inhibition, or not). It is assumed that $\Phi \geq \delta X > 0$. The measure of the exchange of product *i*, $C_i$, with secretion center *j*, $C_j\left(w_{ij}\right)$, is given by

$$W_{ij}=a_{ij}+a_{ji} a_{ir} / i, i. \tag{8}$$

If, as we assume here, the food ratio is balanced and each secretion center maintains a multilateral exchange equilibrium at all times, we have
$L_i(t) \Sigma P_j(t) c_{ij}(t)=\Sigma P_i(t) L_j(t) c_{ji}(t)$, where $i_w$ and $\pi_i$ are functions of

$$\hat{a}_{ij}=a_{ij} Q_i \big/\left[\beta+\delta \vdash 1+t_{ij}\right], \tag{9}$$

where $t_{ip}$ is the transmission rate of neurotransmitters from secretion center *i* to secretion center *j*, and $Q_i$ is the output. Depending on the state of the secretion centers, this speed may be low, normal, or high. The case of normal speed is illustrated by points $P_0$ and $C_0$. The metabolism of the A representative agent is represented by points $P_0$ and $C_0$. The agent produces more dopamine and less serotonin at $P_0$ than it consumes at $C_0$. By consuming the quantities corresponding to point $C_0$, he achieves a higher level of utility $I_0$ than he would achieve by simply consuming what he produces at any point on the production curve. He achieves this level of utility by exporting dopamine in exchange for importing serotonin ($M_0$), according to the ratio of equilibrium food intake values.

If the rate of transmission of serotonin from secretion center *j* to secretion center *i* is uninhibited, secretion center *i* imposes a customs duty on the serotonin imported from center *j*; this increases the dopamine value of serotonin in the areas inhibited by center *i* compared with that of normal food intake. This difference in concentration generates.

Osmotic pressures are balanced by transferring resources from the dopamine secretion center to the serotonin secretion center until the marginal cost of producing each unit of serotonin is equal to the ratio of values resulting from osmotic pressure. This situation corresponds to point $P_1$.

The cells, for their part, make an adjustment and set their consumption at $C_1$, where the ratio of the marginal utilities of the two neurotransmitters is equal to the ratio of the values resulting from osmosis. During this process, the exchange between secretion centers collapsed, and serotonin imports by center i decreased from $M_0$ to $M_1$.

It is clear from the figure that this obstacle to the circulation of serotonin reduces well-being. At point $C_1$, the representative agent only obtains utility level $I_1$, which is lower than the $I_0$ achieved under equilibrium food intake.

The representative agent records an overall loss made up of the "effect on serotonin production" (the switch from $P_0$ to $P_1$ (production cost higher than the equilibrium serotonin value)) and the "effect on consumption" (the difference in osmotic pressure forces cells to bring their consumption to a point where the indifference curve is tangent to the ratio of values resulting from osmotic pressure to point $C_1$). This overall loss, which is made up of two effects (b and d), corresponds to the results obtained via the supply and demand curves. Indeed, the vertical and horizontal projections allow us to represent the production and consumption of dopamine (bottom or vertical projection) and serotonin (horizontal projection). slowing the transmission of serotonin from center *j* to center *i* is a tax on the export of dopamine from center *i* to secretion center *j* and therefore a deceleration in the transmission of the neurotransmitter dopamine, a kind of disruption of the organism's efficiency frontier affecting the individual's well-being and life expectancy. In this respect, following the work of Pataky et al. (2021), "The gradual and progressive age-related decline in hormone production and action has a detrimental impact on human health by increasing risk for chronic disease and reducing life span." These authors have also shown how hormonal changes expose people to various diseases, such as diabetes, frailty and cardiovascular disease. Other authors like wildly studied these aspects. Lamberts et al. (1997), Greendale et al. (1999), Tomczak & Stachowiak (2015).

Considering the dynamic behavior of the secretion center *i*, the specification of the transversality condition

$$\lim_{t\to\infty} k\exp\left(\int_0^t\left[\beta+\delta\vdash f'(k)-\delta-x-n\right]\mathrm{d}v\right) \quad \text{gives} \quad X^*(t)=\Phi\cdot X(t) \qquad (10)$$

where $X^*(t)=X_1(t),\cdots,X_j(t)$.

The transversality condition is deduced from Hopf's bifurcation theorem. According to this theorem, there is a local birth or death of a periodic solution from an equilibrium point if a parameter crosses a critical value. We use the transversality condition to ensure that the eigenvalues of the bifurcation theorem cross the imaginary axis at a nonzero speed. In biology, this condition is essential for understanding stable changes and the appearance of periodic solutions in dynamical systems. Herlenius and Lagercrantz (2011) reported that "In particular, at birth a cascade of neurotransmitters and transcription factors is activated. For example, the norepinephrine surge at birth may be important for initiating the bonding of the infant to the mother by increasing the ability to sense odors (Sullivan et al., 1994). Imprinting at birth and visual input to form the ocular dominance columns also occur during critical periods and are probably dependent on the switching on and off of neurotransmitters". **Figure 1** describes this condition through neurotransmitter transmission rates and production over different stages of life of a human individual.

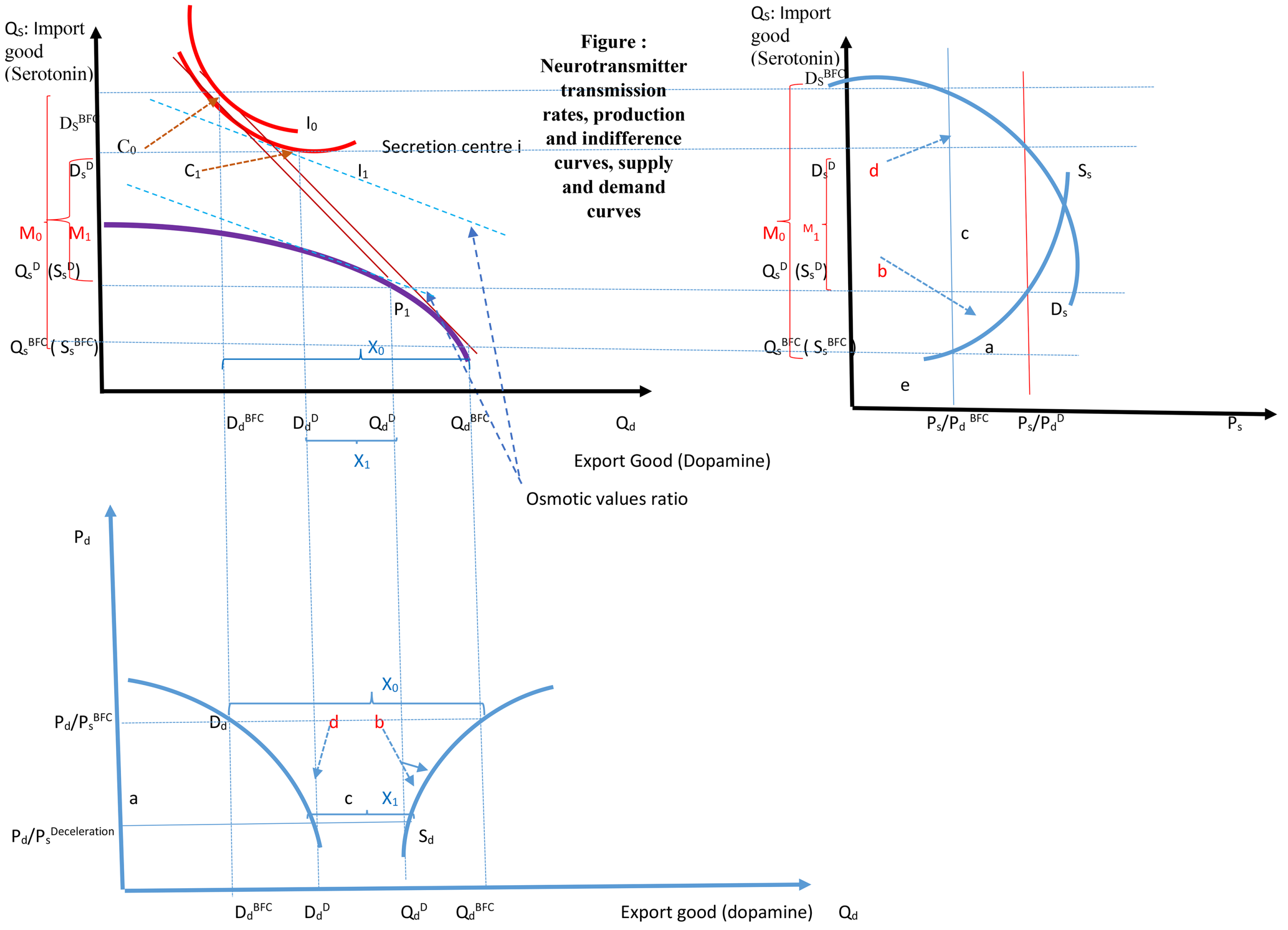


**Figure 1.** $D_d^{BFC}$: Balanced Dopamine or Balanced Food Consumption; $D_d^{D}$: Imbalanced Dopamine demand during transmission deceleration; $Q_d^{D}$: Imbalanced Dopamine production during transmission deceleration or imbalanced food consumption ratio; $Q_d^{RAE}$: Dopamine production in equilibrium food ratio.

Furthermore, by studying the equalization of the biological value of all exchanges, we can better understand the mechanisms of hormonal imbalance, the latter (biological value). Hormones are chemical messengers that regulate essential biological mechanisms so that a more or less permanent overall balance is maintained in a normal individual. However, hormonal imbalance can occur. For example, growth hormone regulates all metabolic and physiological processes so that a deficiency in the production of this hormone (due to acquired or inborn dysfunction of the pituitary gland) affects the physiological development of the individual, with numerous pathological consequences. For more details on these issues, see the work of Copeland et al. (2002), Heaney, Carroll, & Phillips (2013), Aldred, Rohalu et al. (2009), Sutton and Lazarus (1976).

Referring to the work of Viguié et al. (2012) on endocrine disruptors: consumer issues and scientific challenges, I begin with individuals consuming multiple products. Therefore, I can consider multiple interferences. In this case, if the R-rays are ( $R = R_0, R_1, R_2, \cdots, R_p$ ) with a neurotransmitter amplitude $\tau$ ( $\tau^2, \tau^2 p^2, \tau^2 p4, \cdots, \tau^2 p2p$ ) and the phases are $\left(0, \Phi + 2fr, 2\Phi + 4fr, \cdots, p\Phi + 2pfr\right)$, the induced amplitude is

$$\begin{aligned} A = {} & \tau^2 + \tau^2 p + \tau^2 p^2 e - j\left(\Phi + 2fr\right) + \tau^2 p4e - j2\left(\Phi + 2fr\right) + \cdots \\ & + \tau^2 p^2 e - jp^2\left(\Phi + 2fr\right), \end{aligned} \tag{11}$$

$$= \tau^2 \big/ \left(1 - p^2 e - j\Phi'\right), \tag{12}$$

and

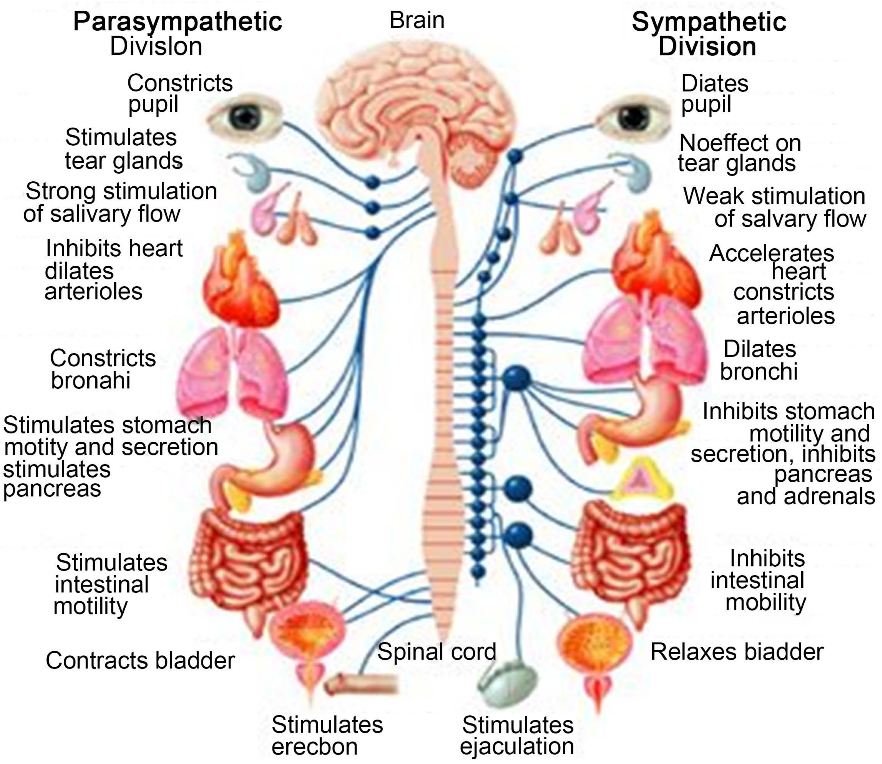
$$\Phi' = \Phi + 2fr. \tag{13}$$

### 3.2. The Strict Parallelism between the Theory of Exchange between Generations of Secretion Centers and Metabolic Processes

"The aging of an individual leads him to 3 states: vigorous, poly-pathological and dependent or frail". If every state involves cell renewal, we are indeed in the presence of a new generation of secretion centers in each state, with new neurotransmitter secretion capacities.

#### 3.2.1. Narrative Component

Thus far, I have analyzed the relationships among primary goods (clothing and wheat), secretion centers, and carnal and spiritual satisfaction. The consumption of these primary goods triggers metabolic processes in humans, the function of which is considered analogous to the factor proportion model. However, we have not considered the process of cell renewal, which affects the secretion capacities of the centers. Thus, there is a new type of exchange between these secretion centers throughout the life of the organism. The exchanges described were superficial since they only concerned the primary relationships between current secretion centers and elementary consumer goods, so the exchange of neurotransmitters between secretion centers was only virtual. An exchange of neurotransmitters between current and future secretion centers is inevitable because the latter have different comparative advantages due to the variation in their capacities to secrete dopamine and serotonin over time. This variation in secretion capacity is attributable either to endogenous internal metabolic processes or to the adoption of new behaviors related to increasing medical progress (exogenous processes). To fully understand this aspect, we assume the existence of debts and receivables between current and future secretion centers, as an individual’s consumption habits today can positively or negatively affect the neurotransmitter secretion capacities of future secretion centers. Consequently, each secretion center operated under intertemporal autarky conditions. Each secretion center has a different but partial specialization, insofar as what happens in the gut, for example, can influence cerebral serotonin activity, just as the substantia nigra reticulata (SNr) is composed mainly of GABAergic neurons but also of dendrites of dopaminergic neurons originating from the SNc. When the body lacks dopamine, the essential functions mentioned above are inhibited, whereas excess dopamine is often associated with psychiatric disorders. Likewise, because serotonin serves to inhibit many areas of the brain, the same areas are “uninhibited” when there is too little serotonin. In humans, impulsive, aggressive, or even violent behavior is generally associated with abnormally low serotonin levels. Serotonin could thus potentially be involved in the whole aging process via its links with various organs and the immune system in particular, as noted by Prof. Agid.



Source: https://i.pinimg.com/474x/00/5d/f7/005df7acf9c50c862e30177d6a7e633b.jpg.

**Figure 2.** Automatic nervous system in humans.

As Prof. Agid explained, inappropriate behavior in terms of the consumption of primary goods (canvas and wheat) generates negative Externalities that can affect our hormonal balance. As in the universe, global warming is perceived as an externality of production or consumption by certain economic agents.

The potential for inhibition or disinhibition of brain regions due to neurotransmitter shortages or excesses describes interactions between Current and future secretion centers. We therefore consider the following model: let us denote by $\partial$ the rate of positive or negative change in dopamine due to the evolution over time of the body's capacity to secrete this neurotransmitter and by $\partial^*$ under the same conditions, that of serotonin. Final goods (the level of satisfaction achieved by an individual in consuming a primary good) are directly mobile between current and future secretion centers in terms of approximately integral compensation between neurotransmitters over time. An individual's good consumption habits improve, or at least preserve, good secretion capacities for future generations of secretion centers. In the same way, poor consumption habits degrade the secretion capacities of future generations of secretion centers. Similarly, secretion centers that negatively affect an individual's life expectancy through difficult living conditions (negative externalities) can transmit a certain degree of immunity (positive externalities). The mobility of production factors (dopamine or serotonin) is achieved by this exchange of negative externalities for positive externalities.

### 3.2.2. Analog Specification

Intersecretary center exchanges are based exclusively on production factors and technology. Technology is therefore considered a productive factor, and its production depends only on the current capacity of the neurotransmitter secretion center to hoard serotonin. The technology production function

$$T(t)=\left(G(\rho),E(t),N(t)\right) \tag{14}$$

Is neoclassical, with the usual properties.

Let us consider two secretion centers in a human organism. The serotonin secretion center is represented by (Gs), and the dopamine secretion center is represented by (Gd). The two secretion centers are separate. According to Max Lugavere, the author of Brain Nutrition, Serotonin produced in the intestines does not cross the blood-brain barrier (international immobility here, "intersecretion centers" of the production of neurotransmitters, "here, serotonin"). However, what happens in the gut can influence brain serotonin activity through its ability to modulate inflammation. Therefore, there is direct exchange of brain serotonin for dopamine in the substantia nigra pars reticulata (SNr), which is composed mainly of GABAergic neurons, as well as dendrites of dopaminergic neurons from the SNc. Thus, each secretion center has different initial endowments, which are interdependent. We assume that all the secretion centers of an organism are co-owners of the neurotransmitters available at each instant, whose yield in terms of the organism's life expectancy is estimated at $y_i'$. If the compensation between brain serotonin and SNr dopamine is complete, i.e., just sufficient to ensure the vital functions of the organism so that it lives for $y_i'$ years, i.e., 100 years, we say that there is hormonal coherence or balance and that the prescribed life expectancy can be achieved. Otherwise, there is an exchange of externalities that are necessarily harmful to the organism, and life expectancy decreases. If the life expectancy of a representative agent was in the first period, then it would decrease, for example, according to a coefficient of noncompensation of neurotransmitter exchanges varying from $n = 0$ to 1.2, since, as some studies assert, "any death before the age of 120 is a premature death. Moreover, if the life expectancy at birth of each secretion center is from the first period, the life expectancy at birth of the organism is equal to 100n years; over the course of its life, each secretion center lends and borrows neurotransmitters from other secretion centers in more or less appropriate proportions (this lending and borrowing of neurotransmitters between secretion centers can be likened to exporting and importing). Consequently, the total amount of serotonin in a secretion center in the first wave is equal to

$$\frac{\Delta y_i'}{n} + \Sigma S_{j1}' \tag{15}$$

$\Sigma S_{ij}$ is the import of the first generation of secretion centers borrowed from subsequent generations (imported).

The second generation, at the beginning of the second period, is given by:

$$\frac{\Delta y_i'}{n} - S_{21} + k_{12} + \cdots + \Sigma S_{j2} \tag{16}$$

where $k_{12}$ represents the quantity of dopamine reimbursed from the first to the second generation. $k_{12}$ must be equal to $S_{21}$. $k_{12}$ represents exports from the first generation to the second generation, and $S_{21}$ imports from the second generation to the first generation. The total serotonin quantities for the last generation are equal to

$$\frac{\Delta y_i'}{n} - \Sigma S_{ni} + \Sigma k_{in} = \frac{S}{n} = S + K_n \tag{17}$$

The first generation uses all of its serotonin (dopamine) to compensate for the body's various deficiencies and to produce neurotransmitters for their own function. At the end of the first period, the second and subsequent generations have joint ownership.

$$\Delta y_i' - \Delta y' \left[ \beta + \delta (1 - \beta) \right]. \tag{18}$$

$\beta$ is the self-consumption ratio (consumption per neurotransmitter unit), whereas $\delta$ is the neurotransmitter ratio (the share of neurotransmitters to be returned to future generations).

At the start of the second period of an organism's life, the remaining quantities of serotonin are

$$\Delta Y - \Delta Y \left[ \beta + \delta \vdash y_i' \left[ \beta + \delta \vdash (1 - \beta)(1 - \delta) \right] \right. \tag{19}$$

The quantities of neurotransmitters in the second generation are given in equation (16). This generation proceeds as the first.

The remaining quantities of serotonin are given by

$$\Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'\left[\beta+\delta \vdash (1-\beta)(1-\delta)\right]\right.$$

$$- \Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'\left[\beta+\delta \vdash (1-\beta)(1-\delta)\right]\right]\left[\beta+\delta \vdash \beta+\delta(1-\beta)\right].$$ These are

$$=\Delta y \wedge y_i'\left[\beta+\delta \vdash (1-\beta)(1-\delta)\right]^2.$$

primitive neurotransmitters of the third generation. At the start of the body's third life period, the remaining quantities of serotonin are as follows:

$$\Delta y_i'\left[\beta+\delta \vdash (1-y\beta)(1-\delta)\right]^2. \tag{20}$$

We note that the new quantities of serotonin follow a geometric progression, with $(1-\beta)(1-\delta)$ as the reason.

The initial serotonin quantities of the *n*th generation are as follows:

$$\Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'\left[\beta+\delta \vdash (1-\beta)(1-\delta)\right]^{(n-1)}\right.. \tag{21}$$

The total amount of new serotonin is equal to the sum of the geometric progression for reasons of less than one. The limit of this sum is given by the following expression:

$$\begin{aligned}
&\Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'/\Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'\left[\beta+\delta \vdash (1-\beta)(1-\delta)\right]\right.\right. \\
&= \Delta Y - \Delta Y\left[\beta+\delta \vdash y_i'\middle/\left[\beta+\delta \vdash \beta+\delta(1-\beta)\right]\right. \\
&= \Delta Y - \Delta Y\left[\beta+\delta \vdash Y'.\right.
\end{aligned} \tag{22}$$

The optimal growth multiplier is

$$1\big/\left[\beta+\delta \vdash \beta+\delta(1-\beta)\right]. \tag{23}$$

In this way, each exchange wave generates neurotransmitter flows across generations, following sinusoidal functions such as

$$\Sigma\Delta y_{it} = y_{i0}' \cos\left(W_{ijt} - \varphi_2\right). \tag{24}$$

$$\Delta Y_t' = \Sigma\Delta y_{it}'. \tag{25}$$

The sinusoidal shape is due to the law of value added in each generation. When a good (in particular a productive factor) is imported into a secretion center, its value added increases in terms of life expectancy during the first period and decreases as cells age or as a result of the inhibition-disinhibition process. Starting at the age of 30, a decrease in organ function can be observed. Guilbaud A. et al. reported that "the aging of an individual leads him to 3 states: vigorous, polypathological and dependent or frail. The state of fragility is reversible. We have to be an actor in our aging and no longer suffer it. The centenarians of the blue zones have achieved, culturally, active aging, which has led them to successful aging." Although nerve cells can last a lifetime, the different phases of their aging are considered in this paper as successive generations of neurotransmitter secretion centers. Indeed, with age, there are changes or decreases in neurotransmitters due to behavioral, physiological or environmental factors.

The sinusoidal shape is also explained by the different comparative advantages of the secretion centers, leading to differences in the gains from the exchange (increases and decreases in neurotransmitter values in the different centers over time and in the eating and other habits of the representative agent).

Studies of periodic functions indicate that every periodic motion P is the sum of sinusoidal motions with p, p/2, p/3, ..., p/n as subperiods.

These represent the harmonics of the system...

$W'_{ij}(t)$ is the ratio of generation *i*'s total trade with generation *j* (i.e., generation *i*'s bilateral exports and imports divided by generation *i*'s aggregate production) is represented as follows:

$$W'_{ij} = \frac{\frac{P_j(t)}{P_i(t)} L'_i(t) g_i(t) + L'_j(t) g_i(t)}{L_i(t) y_i(t)}, i \neq j \tag{26}$$

$G_{ij}(t)$ represents the actual consumption per secretion center of generation *i* of neurotransmitters of generation *j*. $P_i(t)$ is the value of factor *i*, and $L'_i(t)$ is the quantity of neurotransmitters of generation *i* in each period *t*.

We now define $a_{ij}\left(0 \leq a_{ij} \leq 1\right)$ as a constant, representing the share of generation *j*'s accessible serotonin (dopamine) that can be consumed by Generation *i* as a part of its own serotonin (dopamine). According to Abramovitz (1986), social capacity air determines a generation's potential to adopt Existing technologies. Using these definitions, the quantities of dopamine accumulated by generation *i* can be written as follows:

$$X' * i(t) = \Phi\left[\beta + \delta \vdash \Sigma a_{ij} w_{ij}(t) X_j(t)\right] + \left(\Phi - \delta X\right) X_i(t), \tag{27}$$

where Φ represents the productivity parameter of common neurotransmitter secretion and $\delta X$ represents the aging rate of dopamine or serotonin.

Stock (obsolete or not), assuming $\Phi \geq \delta X > 0$. The measure of the exchange of generation $G_i$ with generation $G_j$, $W_{ij}$, is as follows:

$$W_{ij} = a_{ij} + a_{ji}\ i/j, i \neq j. \tag{28}$$

Assuming that each generation maintains a multilateral equilibrium at each instant, we have

$$L_i(t) \Sigma P_j(t) c_{ij}(t) = \Sigma P_i(t) L_j(t) c_{ji}(t), i \neq j, \tag{29}$$

where *i* is a function of $\hat{a}_{ij} = a_{ij} Q_i \Big/ \left[\beta + \delta \vdash 1 + t_{ij}\right]$, $t_{ij}$ is the rate of flow of imports from generation *i* to generation *j*, and $Q_i$ is the total secretion of neurotransmitters. Taking into account the dynamic behavior of generation *i*, the specification of Equation (33) gives $X^*(t) = \Phi \cdot X(t)$, where $X^*(t) = X_1(t), X_j(t)$ and, for example, subsequent generations use serotonin intensively in the production of carnal satisfaction or indirectly transmit carnal satisfaction to the current generation in exchange for intensive use of dopamine in spiritual satisfaction. This exchange occurs at the end of their lives or indirectly via spiritual satisfaction. Although the organism did not benefit from this degree of spiritual satisfaction during the previous generation, the latter indirectly ced this degree of satisfaction to the current generation (the organism's current state) by providing it with the neurotransmitters needed to produce this degree of spiritual satisfaction (positive externalities).

### 3.2.3. Strict Parallelism between the Theory of Multidimensional Exchange and Metabolic Processes

1) Assumptions common to exchange between consumed goods and secretion centers.

A1) The goods produced (degrees of satisfaction) available in fixed quantities in each generation are used for full consumption.

During each generation and in an optimal manner;

A2) At the opening of exchanges, quantities of neurotransmitters are immobile (mobile) between consumed goods (generations) but mobile (immobile) across generations (goods); produced goods are mobile (immobile) between consumed goods (generations) but immobile (mobile) across generations (goods).

A3) The exchange of neurotransmitters is characterized by perfect competition; quantities of serotonin can be used interchangeably in All productions; there is full employment in both goods and both generations.

A4) The satisfaction production function is the same for both goods and both generations; the production functions are homogeneous for degree 1, with constant returns to scale and decreasing marginal productivities; however, the satisfaction production technique is different.

A5) The marginal utility of each satisfaction is always decreasing.

A6) Neurotransmitter transport costs and other barriers to exchange are zero.

A7) The two goods consumed exchange only the satisfaction they produce; this satisfaction is perfectly mobile between the goods consumed (in the organism; in two generations (secretion centers), exchange only serotonin (dopamine) for dopamine (serotonin), these neurotransmitters are perfectly mobile between the secretion centers.

A8) Each satisfaction is produced with a different relative intensity of serotonin and dopamine. The production of spiritual satisfaction is Dopamine is intensive, and carnal satisfaction is serotonin intensive.

Our hypothesis contradicts the neoclassical model of international exchange. We propose that only the factors of production can be traded, However, the final goods (satisfactions) cannot be stored.

2) Steady state (or adult age)

If we consider that a generation of secretion centers that changes its serotonin management and, therefore, its ability to secrete, simultaneously acquires or loses the ability to secrete serotonin efficiently, it will pay back (transfer) to future generations of secretion centers a higher (less) level of technology (ability to secrete), we have

$$\delta = \left(\partial g_{ij}(t) + \partial' g_{ji}(t)\right) / \left(y_i(t)\right). \tag{30}$$

In this model, the net increase in the stock of dopamine at any given time is equal to the capacity to secrete raw dopamine minus the rate of cell aging plus miscellaneous effects (improvement in the general level of consciousness, advances in medicine and knowledge). If one generation grows at the rate of Ni, K increases in parallel, increasing the productivity of subsequent generations. The marginal secretion of K should be equal to the intergenerational transfer rate, Igc = Sgf. The rate of transfer is determined by the first generation, who determines how much serotonin future generations should use to produce satisfaction. This overconsumption of serotonin constitutes the consumption of the current generation and a debt to be paid to future generations in terms of dopamine. Insofar as serotonin serves to inhibit many areas of the brain, the same areas are "disinhibited" when there is too little serotonin. In fact, in these cases, dopamine-secreting capacities are spared or overused, depending on whether the current generation secretes more or too little serotonin. The more serotonin the current generation overconsumes, especially if it consumes a high level of carnal satisfaction, the more it should improve its secretory capacity and provide a higher level of secretory technology (ability to secrete) to future generations, thus ensuring that Igc = Sgf. It is not possible to have Igc < Sgf, or vice versa. Igc is the current generation's ability to secrete, whereas Sgf is a stock of future generations' ability to secrete. Since the production function of technology is neoclassical, technology clearly decreases over time.

i) Conditions and equilibrium

a) Conditions

The technical production function $T(t)=(G(\rho),E(t),N(t))$ is neoclassical and has the following properties: *g*(.) exhibits constant returns to scale, i.e., $G(\lambda E,E,\lambda E,N)=\lambda E,G(E,N)$, a property that is also known as degree-one homogeneity in $E$ and $N$.

When a secretion center chooses an initial secretion that is different from $W$, it must compensate for the overconsumption of serotonin with an equivalent supply of dopamine to establish or maintain a constructive multidimensional exchange. Otherwise, the secretion center and the organism may experience a hormonal imbalance. This hormonal imbalance varies according to the differential between effective secretion ($W_i$) and initial optimal secretion and the sensitivity of interdependencies between secretion centers. As a result, the FPP of the secretion center shifts around the organism's stechnological frontier. The derived growth is not Pareto optimal (see **Figure 1** and **Figure 2**).

b) Equilibrium

The function of the hormonal imbalance between secretion centers is described as follows:

$$(Xf-X)=f(Wf-W,\theta'). \tag{31}$$

$\theta'$ is the inter-center sensitivity factor. Hormonal imbalance becomes explosive (through other secretion centers) if the interdependencies are highly sensitive. In pure economics, Hsieh and Klenow (2009) and Klenow (2012) examine this point in detail. They used micro data from manufacturing establishments to quantify and compare potential resource misallocation between the USA and India. Research has indicated that resource misallocation can reduce total neurotransmitter productivity (TTP) and growth.

For the same reasons, when a generation initially chooses a secretion different from $W$, it must compensate for its overconsumption with an equivalent supply of dopamine. This will maintain or establish a constructive multidimensional exchange. If this compensation is not made, the generation and the organism suffer a potentially significant hormonal imbalance. This hormonal imbalance varies with the differential between effective secretion ($W_i$) and optimal initial secretion, as well as the sensitivity of intergenerational interdependencies. As a result, the generational PPF shifts around the organism's technological frontier. The derived growth is not Pareto optimal (**Figure 1** and **Figure 2**). The intergenerational hormone imbalance function can be described by the following relationship:

$$(Xf - X) = f(Wf - W, \theta'), \tag{32}$$

where $\theta$ is the intergenerational interdependence sensitivity factor. Hormonal imbalance becomes explosive (across other centers and generations) if the interdependencies are particularly sensitive. The factors of hormonal imbalance in the centers of secretion and generation (neurotransmitters and satisfactions) are the values, quality, and flexibility associated with them.

In general, the process of adjusting values, qualities, and quantities is widely described for inter-center and intergenerational exchange.

The values of goods stabilize, as does the quality of neurotransmitters in all secretion centers. We conclude that there is convergence toward a constant rate of equilibrium growth, where the serotonin and dopamine stocks are above their equilibrium levels. At general intergenerational equilibrium, all values will stabilize because their variations are symmetrically opposite in different periods. Productive factors in intergenerational exchange reduce the levels of scarce neurotransmitters in each period and enable the production of satisfaction consumed in a given period. The decline in the values and quality of satisfaction and neurotransmitters in a given period enables the consumers and producers of a given period to benefit from the gains of intergenerational exchange. As we can see, this general case is the rule, but many factors, such as hormonal imbalances in certain organisms (due to poor consumption habits), shift PPFs in such a way that the directions taken by these movements in each secretion center and/or generation interact with inter-center or intergenerational exchange to determine long-term growth per neurotransmitter. The direction of these movements depends on how consumption patterns and other shocks influence neurotransmitter allocation. Neurotransmitter levels may rise or fall, and secretion technologies or the marginal rate of neurotransmitter substitution between generations may change. Even if only the difference in neurotransmitter evolution of one secretion center/generation leads to a change in comparative advantages and the organic/intergenerational pattern of exchange, these differences in satisfaction and neurotransmitter values should disrupt the relationship between growth and hormonal imbalance. The relationship between growth and hormonal imbalance should then depend on these movements and their interaction within-central secretion and intergenerational exchange.

According to King et al. (1988), temporary disruption of PPFs can have permanent effects on the trajectory of secretion growth. The extent and nature of these effects depend on the type of disturbance.

c) Biological model calculation

At this stage, when the analogy is clearly established between multidimensional exchange and the biological metabolism of neurotransmitters secretion and transmission, the final step is to develop the statistics of the biological variables in the analogical model, in order to determine the estimators. Finally, the two types of analogy estimators (multidimensional exchange and biological model) will be converted into numerical estimators using appropriate converters. However, in this paper, we do not yet have the appropriate statistics for this calculation.

## 4. Results and Discussion of the ABM Specification

### 4.1. Agent-Based Modeling (ABM)

ABM is one of the few suitable tools to capture heterogeneity, relationships between individual actors, and nonrational preferences and behaviors in a single methodology. "This makes this methodology very suitable for the analysis of complex adaptive systems such as economies where local economic interactions influence regularities which in turn influence future interactions" (Tesfatsion, 2003) and for the analysis of public policy impacts on the behavior of social and economic actors" (Lempert, 2002; Chappin et al., 2007; Afman et al., 2010; Desmarchelier et al., 2013; Choi et al., 2012; Eppstein et al., 2013; Lengnick, 2012; van der Veen & van Oers, 2017).

In accordance with equations (8), (9), (24), and (25), each consumption wave generates neurotransmitter flows throughout the body, which sinusoidal functions follow, just as intergenerational exchange waves generate neurotransmitter flows across generations. The result is as follows:

For multidimensional horizontal exchange

$$\varphi_1 = \varphi_2 \quad \text{and} \quad \Delta Y_{o,t}^2 = y_{io}^{\,2} + y_{io}'^{2},$$

$$\begin{aligned} \ln\left(\frac{Y}{L}\right)_{i,t} &= \ln\left(A_i + A_i'\right) + \left(\alpha_E + \alpha_E'\right)\ln\left(\frac{E}{L} + \frac{E'}{L'}\right) + \left(\beta_N + \beta_N'\right)\ln\left(\frac{N}{L} + \frac{N'}{L'}\right) \\ &\quad + \left(a_{ij} + a_{ij}'\right)\left[\left(W_{ij}(t) + W_{ij}'(t)\right)\left(X_j(t) + X_j'(t)\right)\right] + \delta_X'' X_i'(t) \\ &\quad + \left(\alpha_E + \beta_N' + a_{ij} + \delta_X'\right)\left(W_{ij}\ln N\right) + \mu_{i,t}. \end{aligned} \tag{33}$$

For vertical multidimensional trade

$$\phi_1 = \phi_2 + \pi \tag{34}$$

We speak of indeterminate multidimensional exchanges when the exchanges are neither horizontal nor vertical, i.e. when:

$$\Delta Y_o^2 = y_{io}^{\,2} - y_{io}'^{2} + \tau + \frac{1}{\left[2\pi j\left(\partial\delta i(t)\right)\right]} \text{ or } \Delta Y_o^2 = y_{io}^{\,2} + y_{io}'^{2} + \tau + \frac{1}{\left[2\pi j\left(\partial\delta i(t)\right)\right]} \tag{35}$$

If we consider the latter form, we have

$$\begin{aligned}
\ln\left(\frac{Y}{L}\right)_{i,t} &= \ln\left(A_i + A_i'\right) + \left(\alpha_E + \alpha_E'\right)\ln\left(\frac{E}{L} + \frac{E'}{L'}\right) + \left(\beta_N + \beta_N'\right)\ln\left(\frac{N}{L} + \frac{N'}{L'}\right) \\
&\quad + \left(a_{ij} + a_{ij}'\right)\left[\left(W_{ij}\left(t\right) + W_{ij}'\left(t\right)X_j\left(t\right) + X_j'\left(t\right)\right)\right] + \delta_X'' X_i'\left(t\right) \\
&\quad + \left(\alpha_E + \beta_N' + a_{ij} + \delta_X'\right)\left(W_{ij}\ln N\right) \\
&\quad + \frac{1}{\frac{1}{\tau} - 2\pi j\delta\left(.\right)} + \mu_{i;t}, \quad \text{with } \left(.\right) = \ln\left(\frac{E}{L} + \frac{E'}{L'}\right) \times \ln\left(\frac{N}{L} + \frac{N'}{L'}\right).
\end{aligned} \tag{36}$$

which is neither vertical nor horizontal.

These three scenarios can be used to determine the relationship between growth and hormonal imbalance. In these cases, ( $\alpha_E + \alpha_E'$ ), ( $E + \alpha_E + \alpha_E'$ ), ( $'E$ ), ( $\beta_N\ \beta_N'$ ), ( $a_{ij} + a_{ij}'$ ), $\delta_X''$ , ( $\alpha_E + \alpha_E'$ ), $E + \beta_N' + a_{ij} + \delta'(X)$ ), ( $\alpha_E + \alpha_E'$ ), ( $E - \alpha_E + \alpha_E'$ ), ), ( $\beta_N - \beta_N'$ ), ( $a_{ij} - a_{ij}'$ ), $-\delta_X''$ , and ( $\alpha_E + \alpha_E'$ ), ( $E + \beta_N' - a_{ij} - \delta_X'$ ) are exogenous parameters whose sign and magnitude are crucial in determining the sign of the relationship between growth and hormonal imbalance in Equations (33), (35) and (36).

We know that $X_i\left(t\right)$ includes two scale effects. The first is the stock of serotonin and unnatural external effects, defined by the interaction between serotonin and dopamine, and the second is that dopamine and current secretion capacities determine the potential of a secretion center to adopt existing secretion capacities. The accumulation of dopamine (or secretion capacity) in secretion centers is related to these definitions.

Therefore, in the general case where the intersecretory center and intergenerational values balance, there is no hormonal imbalance in growth due to general equilibrium. This general balance means that dopamine secreted and "exported" to future generations will offset any imports (e.g., serotonin hoarding) used by the current generation to support growth. In other cases, the organism will experience a hormonal imbalance, and the choice of discount rate will ensure that exports and imports are compensated for.

The expression $u_{i,t}$ is very important in this analysis. It quantifies the uncertainty due to model specification errors or measurement problems. Its values verify the model.

### 4.2. Results and Discussion of the Golden Rule Model

Evidence of the effects of the golden rule model on the multidimensional exchange of human capital.

TEST: Step 1

We test this relationship on two samples. A sample of 25 OECD countries and a sample of 108 developing countries were used. The study period is 1980-2010.

These two groups of countries and the period are chosen because of the relative homogeneity of production technologies within the groups. All the statistics come from the World Development indicators.

Testing the spatial relationship between growth and volatility. In this first stage, it is important to examine the basic nature of the spatial relationship between growth and growth volatility. To this end, we calculate the average growth and standard deviation of growth rates by country and over a given period (1980-2011). The results of multidimensional trade and of average growth (*grgdp*) on growth volatility (vol) for the sample of 108 countries and over the period 1980-2010 are presented in **Tables 1-3** and **Tables 8-10**.

**Table 1.** Multidimensional trade and per capita GDP growth: Panel of three decades (1980-2010), *Dependent variable*: $\ln\left(\frac{Y}{L}\right)_i$.

| Independent Variables | Definition | Model1 | Model 2 | Model 3 | Specific effect on growth volatility |
|---|---|---|---|---|---|
| $A_i + A_i'$ | Time invariant factor −0.6581 | −0.6581<br>(−7.21) | 0.6567<br>(7.34) | 1.2584<br>(6.27) | |
| $\ln\left(\frac{E}{L}+\frac{E'}{L'}\right)$ | −Log of natural resources Authority per inhabitant (160.6) | 2.356037<br>(160.6) | 0.039851<br>(1.06) | 0.29278<br>(2.68) | + |
| $\ln\left(\frac{N}{L}+\frac{N'}{L'}\right)$ | Log of "unnatural resources" per inhabitant | 2.202217<br>(239.79) | 0.34942<br>(10.36) | 0.199865<br>(9.86) | + |
| $W_{ij}(t)+W_{ij}'(t)$ | Unnatural resources | 3.50e−23 | 3.57e−36<br>(−2.78) | 1.12e−25<br>(−3.59) | ± |
| $X_j(t)+X_j'(t)$ | Capital per worker | (2.31) | 1.70e−11<br>(4.23) | 1.03e−13<br>(5.97) | + |
| $X_j(t)+X_j'(t)$ (1) | Natural resources per worker | 8.76e−12<br>(1.99) | 4.20e−12<br>(0.91) | 2.81e−12<br>(−0.55) | ± |
| $X_j(t)+X_j'(t)$ (2) | Royalty and license fees payment | 8.55e−12<br>(1.93) | 0.7779<br>(11.0) | 0.6919<br>(8.40) | ± |
| $X_j(t)+X_j'(t)$ (3) | Royalty and license fees Receipt | 8.43e−12<br>(−4.26) | 2.80e−12<br>(−2.21) | 1.43e−12<br>(0.83) | ± |
| $X_j(t)+X_j'(t)$ (4) | Human Capital | 2.04e−12<br>(1.54) | 9.80e−12<br>(5.34) | 9.04e−12<br>(4.94) | ± |
| $X_j(t)+X_j'(t)$ (5) | Multidimensional trade scale effect | 0.3228<br>(−4.10) | 0.021633<br>(−3.66) | 0.00446<br>(−2.17) | − |
| $X_j(t)+X_j'(t)$ (6) | Multidimensional trade ratio | 0.3719<br>(−163.44) | 0.6581þ<br>(−7.21)þ | 0.34942<br>(239.79) | + |
| $W_{ij}\ln(N)$ | Interaction between | | 2.356037<br>(160.6) | | + |
| $W_{ij}$ | Natural and Unnatural resources' trade | | 2.202217 | | − |

Table 2. Relationship between mean growth and volatility, with Levin-Renelt control variables.

**The sample of 108 countries**

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | 0.6189 | −76.89 | 0.0080 | −0.6347 | −0.6032 |
| Gdppccp | Initial log GDP per capita | −0.003847 | −9.69 | 0.00039 | −0.0046 | −0.003068 |
| $\ln V$ | Average investment fraction of GDP | 0.0012 | 32.09 | 0.00038 | 0.001151 | 0.0013012 |
| aapgr | Average growth of the population | −0.002511 | −9.3 | 0.00027 | 0.00304 | −0.001982 |
| hc_residu | initial human capita | 0.003233 | 1.85 | 0.00017 | −0.00019 | 0.00666 |
| Intercept | Intercept | 0.0342 | 15.78 | 0.0021 | 0.029 | 0.03846 |

Log likelihood = 4624.73
Prob > chi$^2$(5) = 0.000
Prob > chi$^2$(5) = 0.000

Table 3. Relationship between mean growth and volatility, with Levin-Renelt control variables. The sample of 25 countries.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | −0.2956 | −23.97 | 0.01233 | −0.3197 | −0.2714 |
| Gdppccp | Initial log GDP per capita | −0.03305 | −4.77 | 0.006932 | −0.04664 | −0.01946 |
| $\ln V$ | Average investment fraction of GDP | 0.00027 | 0.64 | 0.00042 | 0.0005637 | 0.0011147 |
| aapgr | Average growth of the population | −0.03507 | −18.05 | 0.00194 | 0.03887 | −0.031264 |
| hc_residu | initial human capita | 0.04960 | 11.20 | 0.004429 | −0.04092 | 0.05828 |
| Intercept | Intercept | 0.1069 | 3.47 | 0.03077 | 0.04658 | 0.167234 |

Log likelihood = 677.85
Prob > chi$^2$(5) = 0.000
Prob > chi$^2$(5) = 0.000

### 2nd step

- introduction of Levin and Renelt control variables (Tables 4-6)

**Table 4.** Mean growth and growth volatility with a sample of France's 16 generations (1800-2000) with Levin-Renelt control variables.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | 0.02892 | 1.25 | 0.02312 | −0.0164 | 0.07424 |
| Gdppccp | Initial log GDP per capita | −0.0040 | 0.51 | 0.0078 | −0.0193 | 0.01134 |
| $\ln V$ | Average investment fraction of GDP | 0.00128 | 2.27 | 0.00056 | 0.00017 | 0.00238 |
| hc | initial human capita | −0.00652 | 1.63 | 0.00401 | −0.0143 | 0.00133 |
| aapgr | Average growth of the population | −0.0132 | 0.98 | −00679 | −0.0265 | 0.00008 |
| Intercept | Intercept | 0.0678 | 2.81 | 0.0433 | −0.0172 | 0.1528 |
| Log likelihood = 543.46 | | | | | | |

**Table 5.** Test of mean growth and growth volatility with a sample of 108 countries and its 16 generations (multidimensional trade).

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | 0.0125969 | 0.98 | 0.012909 | −0.012 | 0.0378 |
| Intercept | Intercept | 0.0159509 | 2.81 | 0.0056704 | 0.0048 | 0.02706 |
| F(1.127) = 0.01<br>R-squared: 0. 0001<br>Adj Rsquared = −0.0078 | | | | | | |

**Table 6.** Test of mean growth and growth volatility with a sample of 16 generations of France trading with all the generations of the 2 samples.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | −0.3836 | −186.22 | 0.00206 | −0.3876 | −0.379 |
| Gdppccp | Initial log GDP per capita | −0.0027 | −8.84 | 0.0003 | −0.0033 | −0.0021 |
| $\ln V$ | Average investment fraction of GDP | 0.0009 | 27.06 | 0.00003 | 0.00084 | 0.00097 |
| hc | initial human capita | −0.002121 | −15.92 | 0.00013 | −0.00238 | −0.0018 |
| aapgr | Average growth of the population | −0.0015 | −6.45 | 0.00023 | −0.00199 | −0.0010 |

**Continued**

| | | | | | | |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 0.03519 | −17.02 | 0.0020 | 0.0311 | 0.0392 |
| Log likelihood = 543.46 | | −0.3836 | −186.22 | 0.00206 | −0.3876 | −0.379 |

Step 2: introduction of Levin and Renelt. Control variables

The models to test are in the following form:

$$grgdp_{it} = \lambda vol_i + \theta X_{it} + \varepsilon_{it} \tag{1a}$$

$$\varepsilon_{it} \sim N\left(0, \sigma_i^2\right) \tag{1b}$$

$$i = 1, \cdots, I;\ t = 1, \cdots, T$$

$grgdp_{it}$ average annual growth in GDP/head for country $i$ and year $t$ (obtained in taking the differences of logarithm).

$\sigma_i$ : is the standard deviation of the residues, $\varepsilon_{it}$; $\varepsilon_{it}$ is the standard deviation of growth obtained from predicted values based on $X_{it}$ variables. $X_{it}$ variables differ from one country to another, From one year to another. $X_{it}$ : is the vector of the control variables Θ: is the vector of the coefficients common to the countries of the sample; $\lambda$ denotes the relationship between growth and volatility and is the most important parameter in this specification. The vector of control variables, X proposed by Levine and Renelt (1992) are the most important variables for the analysis of the growth of the countries. These variables are defined as follows: 1) "inv" Share of average investment in GDP; 2) (gdppccp): the logarithm of the GNP/initial head (at the beginning of the period); 3) hc or hc-residue when hc is purged of the difference between observed and predicted values obtained using a partial regression of hc on other control variables; aapgr: average growth rate of the population. In the sample of 108 countries, human capital is the average number of years of schooling of individuals in the population aged 25 and over. But in OECD countries, human capital is the secondary enrollment rate as a percentage of the relevant age group. For regressions, we will use the maximum likelihood method on panel data. The number of observations for the sample at 108 countries is 3240 and 630 for the sample of 25 developed countries.

Table 7: Relationship between the mean growth and the volatility with Levin-Renelt control variables.

NAKAMURA ENDOGENEITY TEST The test is done on two steps:

**Table 7.** Parameters sign and nature of multidimensional trade.

| | Model 1 | Model 2 | Model 3 | Nature of multidimensional trade |
|---|---|---|---|---|
| ($a_E$þ$a0_E$) or ( $a_E$ - $a'_E$ ) | 2.3560372 | 0.039851 | 0.29278 | Horizontal |
| ($b_N$ þ $b0_N$) or ð$b_N$-$b0_N$); | 0.202217 | 0.34942 | 0.03602 | Horizontal |
| ( $a_{ij}$ þ $a'_{ij}$ ) or ( $a_{ij}$ - $a'_{ij}$ ); | $3.50e^{23}$ | $3.57e^{1136}$ | 0.199865 | Horizontal and Vertical |
| (d$x_1$þd0$X_1$) or (d$x_1$-d0$x_1$) | $8.76e^{-12}$ | $1.70^{e}$ | $1.12e^{25}$ | Horizontal |

**Continued**

| | | | | |
|---|---|---|---|---|
| (d$x_2$þd0$x_2$) or (d$x_2$-d0X2) | $8.55e^{-12}$ | $4.20e^{-12}$ | $1.03e^{13}$ | Horizontal and vertical |
| (d$x_3$þd0$x_3$) or (d$x_3$-d0$x_3$) | $8.43e^{12}$ | 0.04297 | $2.81e^{12}$ | Vertical and Horizontal |
| (d$x_4$þd0$x_4$) or (d$x_4$-d0$x_4$) | $2.04e^{12}$ | 0.7779 | 0.6919 | Horizontal and vertical |
| (d$x_5$þd0$x_5$) or (d$x_5$-d0$x_5$) | 0.3719 | $2.80e^{12}$ | $1.43e^{12}$ | Vertical |
| ($a_E$þ $b$0$_N$þ $a_{ij}$ þ $d'_X$ ) or<br>($a_E$þ $b$0$_N$- $a_{ij}$ - $d'_X$ ) | 3.74 | 0.021633 | 0.00446 | Horizontal and vertical |
| | 0.04685 | $2.96e^{-23}$ | 0.00446 | Vertical |

Therefore, in the general case where the inter-secretory centre and inter-generational values balance, there is no hormonal imbalance in growth due to general equilibrium. This general balance means that dopamine secreted and "exported" to the future generation will offset any imports (e.g., serotonin hoarding) used by the current generation to support growth. In other cases, the organism will experience a hormonal imbalance, and the choice of discount rate will ensure that exports and imports are compensated for the expression (i,t)i,t) is very important in this analysis. It quantifies the uncertainty due to model specification errors or measurement problems. Its values verify the model.

In the present study, the relationship between growth and volatility is positive. This indicates that countries may have a choice between high-variance, high expected return technologies and low-variance, low expected return technologies. As stated above, the parameters ( $a_E$ þ b0$_N$þ $a'_{ij}$ þ d0$_X$) and ( $a_{ij}$ þ $a'_{ij}$ ) are essentially positive, but d is negative. This result means that the future generations should suffer more negative multidimensional trade spillover effects than positive external effects, which they could expect to receive from current generations. The negative relationship between human capital and growth is very instructive. It means human capital contributes more to intergenerational than international trade. Human capital tends to transform multidimensional trade vertically and destructively, indicating that an accumulation of resources is favorable for current generations (developed countries). Intergenerational knowledge and technology barriers (or knowledge and technology barriers between developed and developing countries) harm long-run growth. Although the accumulation of different resources (physical capital, human capital, natural resource endowments, institutional capital, and wealth distribution), generates comparative intergenerational or international trading advantages and gains, it harms global welfare in the long term. This conclusion is a high-level generalization of the Lerner symmetry theorem, which states that a country limiting imports through barriers tends to discourage exports (Table 1 and Table 7).

**Table 8.** Test of mean growth and growth volatility with a sample of 8 generations of France trading with all the generations of the 2 samples.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | −0.3836 | −186.22 | 0.00206 | −0.3876 | −0.379 |
| Gdppccp | Initial log GDP per capita | −0.0027 | −8.84 | 0.0003 | −0.0033 | −0.0021 |
| $\ln V$ | Average investment fraction of GDP | 0.0009 | 27.06 | 0.00003 | 0.00084 | 0.00097 |
| hc | initial human capita | −0.002121 | −15.92 | 0.00013 | −0.00238 | −0.0018 |
| aapgr | Average growth of the population | 0.03519 | −6.45 | 0.00023 | −0.00199 | −0.0010 |
| Intercept | Intercept | −0.0015 | 17.02 | 0.0020 | 0.0311 | 0.0392 |
| Log likelihood = 543.46 | | | | | | |

**Table 9.** Test of mean growth and growth volatility with a sample of 108 countries and its generations (multidimensional trade).

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | 0.0125969 | 0.98 | 0.012909 | −0.012 | 0.0378 |
| Intercept | Intercept | 0.0159509 | 2.81 | 0.0056704 | 0.0048 | 0.02706 |
| F(1.127) = 0.01<br>R-squared: 0. 0001<br>Adj Rsquared = −0.0078 | | | | | | |

**Table 10.** Test of mean growth and growth volatility with a sample of 8 generations of France trading with all the generations of the 2 samples.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Vol | Std Dev. Of growth (volatility) | −0.3836 | −186.22 | 0.00206 | −0.3876 | −0.379 |
| Gdppccp | Initial log GDP per capita | −0.0027 | −8.84 | 0.0003 | −0.0033 | −0.0021 |
| $\ln V$ | Average investment fraction of GDP | 0.0009 | 27.06 | 0.00003 | 0.00084 | 0.00097 |
| hc | initial human capita | −0.002121 | −15.92 | 0.00013 | −0.00238 | −0.0018 |

**Continued**

| | | | | | | |
|---|---|---|---|---|---|---|
| aapgr | Average growth of the population | −0.0015 | −6.45 | 0.00023 | −0.00199 | −0.0010 |
| Intercept | Intercept | −0.0015 | 17.02 | 0.0020 | 0.0311 | 0.0392 |
| Log likelihood = 543.46 | | | | | | |

Step 3: Test of the relationship between innovation variance and growth.

In order to examine the stochastic part of the relationship between growth and investment, we take the above model while changing the content of the control variables. Thus, we have two types of variables: the measure of variables at the beginning of the period and the predictors X. The variables to be taken into account in this new specification are:

Variables measured at the beginning of the period 1) Inv: the average share of investment in GDP at the beginning of the period; aapgr: the average annual growth rate of the population at the beginning of the period.

Predicted variables:

1) GDP per capita delayed by two periods;

2) The trend of time;

3) The trend of time squared;

4) Four dummy seasonal variables ( $Q_{1t}$ , $Q_{2t}$ , $Q_{3t}$ and DOt) whose role is to capture the specific effects. These variables are defined below.

**Table 11**: Sample of 21 OECD panel data countries;

**Table 12**: The 108 country sample and panel data regression (see Dynamics of trade and volatility.

$$\begin{aligned} grdp_{it} = {} & \beta_0 + \lambda\sigma_i + \beta_1 Gdpcc - re - 2 + \beta_2 Gdppcc + \beta_3 Inv + \beta_4 aapgr \\ & + \beta_5 Hc + re - 2 + \beta_6 Q1_t + \beta_7 Q2_t + \beta_8 Q3_t + \beta_9 T \\ & + \beta_{10} DT1_t + \beta_{11} DT2_t + \beta_{12} DT3_t + \beta_{13} DO_t + \varepsilon_{it} \end{aligned}$$

Step 4: Test the robustness of country-specific control of growth volatility.

The question here is: Does the introduction of different countries with specific effects affect the nature of the relationship tested here? In order to make this investigation, we will extract all the control variables that are statistically significant in the regression of volatility in terms of time and country and observe the impact of these variables on the new fixed-effects models in the time and space (country). This is done by introducing dummy variables for each country. To this end, we estimate country-specific equations for growth in government expenditures as follows:

Govexp = f (Log of GDP/head lagged by 2 periods, government expenditure log per capita

Delayed by 2 periods, a quadratic time trend, 4 dummy variables and a constant term)

The equations to be estimated have the following form:

$$grgdp_{it} = \lambda vol_{it} + \theta X_{it} + \varepsilon_{it} \tag{1a}$$

$$\varepsilon_{it} \sim N\left(0, \sigma_{it}^2\right), \quad vol_{it}^2 = a_0 + a_1 u_{it}^2 \tag{1b}$$

**Table 11.** Regression of the rate per capita growth on the trend of resources (control variable contributing to WTF) and on the dummy variables with countries and time fixed effects.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Grgdp | Income/head growth rate | −0.2634 | −18.49 | 0.14 | −0.291 | −0.235 |
| Vol1 | Standard deviation of growth rate (fluctuations) | 5.092 | 19.55 | 0.2604 | 4.58 | 5.6 |
| Govexplag2 | Loggovernment expenditure/head | −0.016 | −14.83 | 0.001 | −0.018 | −0.013 |
| Gdppccp | Log GNP/head Initial | −0.00007 | −21.440.87 | 0.00009 | 0 | 0 |
| Inv | Share of average investment in GNP | 0.08225 | 14.47 | 0.0056 | 0.071 | 0.093 |
| h-c | Initial human capital | −5.036 | | | | |
| aapgr | Average annual population growth rate | −0.1294 | −19.59 | 0.2571 | −5.54 | −4.53 |
| aapgr | Seasonal dummy variables | | −6.81 | 0.019 | −0.166 | −0.092 |
| q1t | | −0.06 | −4.09 | 0.01469 | −0.088 | −0.031 |
| q2t | | 0.0645 | 9.61 | 0.0067 | 0.051 | 0.077 |
| q3t | | 0.436 | −4.07 | 0.0107 | −0.064 | −0.22 |
| dot | | −0.0111 | −7.74 | 0.00144 | 0.013 | −0.083 |
| loggdplag2_Cons | Log of initial GNP/head delayed by 2 periods | 0.0008 | 14 | 0.00006 | 0 | 0..1 |
| Log likelihood = −691.47<br>Prob > chi$^2$ = 0.000 | Intercept | −1309 | −4.19 | 0.03123 | −0.192 | −0.069 |

**Table 12.** 108-country sample panel regression.

| Variable | Definition | Coefficient | T-Stats | Std. Dev. | [95% Conf. Interval] min | [95% Conf. Interval] max |
|---|---|---|---|---|---|---|
| Grgdp | Income/head growth rate | −0.00363 | −4.35 | 0.0083 | −0.052 | −0.019 |
| Vol1 | Standard deviation of growth rate (fluctuations) | 1.263 | 285.12 | 0.0044 | 1.254 | 1.271 |
| Govexplag2 | Loggovernment expenditure/head | 0 | 2.49 | 0 | 0 | 0 |
| Gdppccp | Log GNP/head Initial | −0.0026 | −1.81 | 0.0001 | 0.0005 | 0 |
| Inv | Share of average investment in GNP | 1.262 | 10.35 | 0.0002 | 0.0005 | 0 |
| h-c | Initial human capital | −0.0014 | −286.02 | 0.0044 | −1.27 | −1.25 |
| aapgr | Average annual population growth rate | 0 | 6.45 | 0.0002 | −0.00018 | 0 |
| q1t | Seasonal dummy variables | 0.1222 | 2.37 | 0 | 0 | 0 |
| q2t | Seasonal dummy variables | 0.0034 | 4.05 | 0.003 | 0.0063 | 0.018 |
| q3t | Seasonal dummy variables | 0.0028 | −1.31 | 0.0026 | −0.0086 | 0.0017 |
| dot | Seasonal dummy variables | 0.0012 | −2.35 | 0.0011 | −0.0051 | 0 |
| loggdplag2_Cons | Log of initial GNP/head delayed by 2 periods | 0.132 | −89 | 0.0013 | −0.0038 | 0.0014 |
| Log likelihood = −691.47<br>Prob > chi$^2$ = 0.000 | Intercept | −0.016 | 3.5 | 0.037 | 0.0058 | 0.00206 |

This regression gives us the forecast residuals of government spending. Then by regressing the variances of the innovations in growth on the squared forecast residuals of the government-spending (i.e. $vol_{it}^2 = a_0 + a_1\hat{u}_{it}^2$), we will obtain the measure of volatility as a function of both time and countries if the relationship estimated is statistically significant. The next and final step is to test if we have a positive or negative relationship between growth and volatility by introducing in other regressions time and countries fixed effects.

## 5. Conclusions and Policy Issues

How can the potential impact of the optimal policies of externalities trade to uncouple growth and volatility to generate sustainable growth and clean production be explored by means of agent-based modeling (ABM) or analogical econometric methods or through numerical econometrics. On the one hand, through the ABM test, we find that, at the equilibrium level, each productive factor and each good or service is far from having the same price in each generation and country. This cannot realize an efficient trade of externalities that decouples growth and volatility. The reliability of data and, above all, of econometric methods is one of the greatest challenges facing economics. In the 1970s, leading economists used the same statistical data to obtain contradictory results, bringing our science into great disrepute. For centuries, many economists have preferred positive analysis to normative analysis, as if human behavior were error-free, virtually neglecting the numerical econometric methods, which is the source of the glory of the great sciences. On the other hand, the aim was that if we could show that the theory of multidimensional trade is sufficiently similar to a biological mechanism, we could say of one or more factors that also apply to the other. We would then have more powerful analytical tools than the very approximate econometric methods do, and economic policy would become normative. New econometrics could rally move more confidently toward the status of an exact science. Using numerical evidence, through laboratory experiments, we find that the human organism's multidimensional exchange mechanism, consisting of more or less integral compensation processes for negative and positive externalities, is responsible for the volatility of human growth, the main determinant of life expectancy, analogous to the relationship between the processes of economic growth volatility and sustainable growth. Because the analogy is clearly established, the frontier of production (consumption) possibilities of neurotransmitter secretion centers appears to be the sole determinant of life expectancy (sustainability of economic growth).

The existence of several hormone or neurotransmitter secretion or synthesis centers and their distribution throughout the organism according to their specificity and, above all, their productivity in terms of carnal or spiritual satisfaction merit consideration. In multidimensional exchange, the existence of multiple generations of human beings succeeding one another in the same country, just as generations of cells are continually renewed during the life of the organism, on the one hand, and the existence of various separate centers of synthetization, just as countries are separated by borders so that the hypotheses of virtual immobility of neurotransmitters and real mobility of carnal and spiritual satisfactions can be retained in both systems of exchange, on the other hand, is truly intriguing. Moreover, multiple concepts and phenomena (volatility, stationary states, equilibria, exchange, time, information, marginal utility, productivity, marginal productivity, partial specialization, the frontier of production or consumption possibilities, efficiency, etc.) have been deemed relevant in both types of exchange.

The second component of this paper was to determine whether the theory of multidimensional trade is sufficiently similar to metabolic processes so that what can be said about or done with metabolic processes also applies to multidimensional exchange. Hence, the following research question was posed: Is there in the human organism a multidimensional exchange mechanism that is realized with more or less integral compensation between negative and positive externalities to determine either a volatility of human growth reducing life expectancy or an optimal life expectancy analogous to either the volatility of economic growth or the optimal or sustainable growth of an economy? More specifically, the aim was to investigate the impacts of non-Pareto-optimal Walrasian equilibria in the exchange of externalities between neurotransmitter secretion centers and/or between generations of neurotransmitter secretion centers as a fundamental mechanism of human growth volatility leading to disruptions in life expectancy. This would confirm the theory of multidimensional exchange, enabling policymakers to summon the same remedies as in modern medicine.

On the other hand, the analogy seems surprisingly well established. We find that the human organism's multidimensional exchange mechanism, consisting of more or less integral compensation processes for negative and positive externalities, is responsible for the volatility of human growth, the main determinant of life expectancy, analogous to the relationship between the processes of economic growth volatility and sustainable growth. Because the analogy is clearly established, the frontier of production (consumption) possibilities of neurotransmitter secretion centers appears to be the sole determinant of life expectancy (sustainability of economic growth). The analogy seems surprisingly well established. Several lessons can be drawn from this integration of the multidimensional exchange and mechanism of biological metabolism:

The assumption of multidimensional exchange (international mobility of final goods between countries and their immobility between generations and vice versa for factor goods, zero transport costs, absence of barriers to trade, etc.) becomes the optimal policy for managing ecosystems and biodiversity and is the only one to guarantee general equilibrium in all markets;

These same optimal policies are the only ones to guarantee the conservation or stability of natural and social ecosystems (the frontier of possibilities for neurotransmitter secretion and the frontier of possibilities for production) to ensure that each species can reproduce naturally in its stationary state (a necessary and sufficient condition for optimal life expectancy) through the international and intergenerational equalization of goods and factor prices, both in the behavioral sciences and in the natural sciences;

Any imbalance in social ecosystems is transmitted to natural ecosystems, affecting any efficient exchange system at all levels due to functional or conflicting interdependencies. Indeed, the most devastating damage seems to stem from the irresponsibility of bold or forged policies based on positive ecological modeling. The disruption of the production and consumption possibilities frontier affects the natural trajectories of life to induce instability and a depletion of biodiversity to spawn genetically modified organisms everywhere.

## Conflicts of Interest

There are no conflicts of interest to declare.

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