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Effects of CSR on Fresh E-Commerce Supply Chain and CSR Sharing Strategy under Demand Uncertainty

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Abstract

We examine a two-stage fresh e-commerce supply chain comprising a supplier and an e-tailer. It remains unclear how corporate social responsibility (CSR) levels affect supply chain performance or how the e-tailer's freshness-preserving effort is linked to CSR levels. This study investigates the impacts of a supplier's and an e-tailer's CSR levels on optimal pricing and freshness-preserving effort and how supply chain members should share CSR. A Stackelberg game is used to model four scenarios: 1) the supplier practices CSR under a wholesale price contract (WSN), 2) both the supplier and the e-tailer practice CSR under a wholesale price contract (WSO), 3) the supplier practices CSR under a revenue-sharing contract (RSN), and 4) both the supplier and the e-tailer practice CSR under a revenue-sharing contract (RSO). The results show that as CSR levels rise, freshness-preserving efforts increase, and retail prices fall. When the supplier and the e-tailer jointly practice CSR, the freshness-preserving effort level, order quantity, and consumer surplus are higher, and the retail price is lower than when only the supplier practices CSR. In Model WSN, the supply chain's profit grows as the supplier's CSR level increases. In Models WSO, RSN, and RSO, profits rise when the supplier's CSR level is low but fall when it is high. If the supplier's CSR level is low, the e-tailer should practice CSR but not excessively. However, if the supplier's CSR level is high, the e-tailer should refrain from engaging in CSR.

Keywords

Freshness-Keeping Effort, Corporate Social Responsibility, Fresh Produce, Wholesale Price Contract, Revenue-Sharing Contract

1. Introduction

With the increasing penetration of the internet into daily life and the evolving online consumption habits of consumers, the fresh produce e-commerce sector has experienced explosive growth (Kong et al., 2023; Liu et al., 2021). According to the 2023-2024 China Fresh E-Commerce Industry Research Report on Big Data and Development Prospects, the market size of the fresh food e-commerce industry reached 3637.5 billion in 2022, reflecting a 16.7 percent increase from 2021, and is projected to reach 6302.0 billion by 2026.[1] Beyond China, other countries have similarly witnessed significant growth in the fresh produce e-commerce sector (He et al., 2019). Weee!, a leading U.S. fresh produce e-commerce platform, successfully closed its Series E funding round in May 2022, achieving a post-money valuation of 4.1 billion.[2]

However, despite the rapid growth of fresh produce e-commerce, numerous cases demonstrate that the corporate social responsibility (CSR) of fresh produce enterprises is severely lacking, leading to various negative consequences. For instance, the "Dingdong" platform was investigated by the Haidian District Market Supervision Bureau for selling substandard goods at premium prices and unauthorized "repacking" of products for labeling.[3] "Miss Fresh" frequently received consumer complaints for selling expired food. Moreover, the consumption of spoiled food can result in disease outbreaks (Malka & Park, 2022). The World Health Organization estimates that 600 million people worldwide fall ill from eating contaminated food each year, resulting in 420,000 deaths and 33 million illnesses.[4] Consequently, enterprises must implement CSR practices to safeguard consumer food safety. Nevertheless, CSR initiatives may be either voluntary or mandated by law, and the pressure of regulatory obligations can sometimes stimulate voluntary activities (Lim & Prakash, 2014). As a result, governments worldwide are increasingly proactive in supporting and facilitating CSR efforts through public policy (Liu et al., 2016). China's 2021 revision of the Food Safety Law stipulates that food production and business enterprises must be accountable to consumers and assume social responsibility. Consequently, several leading fresh produce businesses have been impelled to incorporate CSR into their supply chains. For instance, Jingdong (JD) developed a "green channel for fresh agricultural products," while Tmall Fresh introduced "Bad Fruit Compensation" and "Seafood Death Compensation" services. These enterprises' CSR initiatives have been well-received by consumers, fostering strong customer relationships. Thus, the study of CSR is crucial for the long-term development of fresh produce enterprises and is also significant in promoting social harmony.

Notably, several uncertainties remain regarding the sale of fresh produce. As the economy grows rapidly, customer demands for fresh agricultural products are becoming increasingly personalized and diverse. This trend not only creates opportunities for fresh produce businesses but also increases market uncertainty. Additionally, as consumer income levels rise, there is greater focus on the freshness

[1] https://www.iimedia.cn/c400/92930.html

[2] https://36kr.com/p/1634946727481481

[3] https://news.cctv.com/2022/04/12/ARTI0a9eI988wOMgq4FuISn4220412.shtml

[4] https://www.who.int/

of agricultural products. However, due to the seasonal and regional nature of fresh produce, these products must undergo long-distance transportation, leading to significant circulation losses and supply delays. This not only compromises the freshness of produce but also exacerbates demand uncertainty. Uncertain demand results in a mismatch between market needs and business decisions, placing the entire supply chain at risk (Liu et al., 2023b; Mahar et al., 2012). According to China News, loquats were harvested in Huangshan (a city in Anhui province, China), but due to a sharp decline in consumer demand, a large portion of the excess loquats decayed rather than being sold, resulting in significant losses for farmers. The Food and Nutrition Development Institute of the Ministry of Agriculture and Rural Affairs conducted a recent study, revealing that the annual average loss and waste rate for seven food categories, including vegetables and fruits, amounts to 22.7 percent by weight.[5] Although some scholars have studied uncertain demand for fresh produce (Liu et al., 2021; Soto-Silva et al., 2016), they have not addressed the issue of CSR within the fresh produce supply chain. In practice, fresh produce enterprises must account for CSR in addition to managing uncertain demand. When fresh produce firms begin to prioritize CSR in the context of uncertain demand, the supply chain faces a variety of new challenges, including pricing strategies, preservation efforts, contractual arrangements, and more. Thus, examining CSR practices of fresh produce enterprises under conditions of uncertain demand holds significant theoretical and practical implications.

Building on the previous analysis, we investigate CSR in the fresh e-commerce supply chain under conditions of demand uncertainty. Our focus lies in addressing the following questions: 1) How can firms practicing CSR in the fresh produce supply chain determine the optimal freshness-preserving effort and pricing? 2) How can supply chain members practice CSR to enhance consumer surplus and overall supply chain performance? To further explore these questions, we must address a related issue: How do the CSR levels of the supplier and e-tailer impact the e-tailer's freshness-preserving effort decisions and the overall supply chain performance under varying supply chain structures and mechanisms?

To answer these research questions, we examined a two-stage supply chain consisting of a fresh produce supplier and an e-tailer, where both supply chain members consider CSR alongside profit maximization. We construct a Stackelberg game framework for four scenarios: The supplier practices CSR under a wholesale price contract (Model WSN), both the supplier and e-tailer jointly practice CSR under a wholesale price contract (Model WSO), the supplier practices CSR under a revenue-sharing contract (Model RSN), and both the supplier and e-tailer practice CSR under a revenue-sharing contract (Model RSO). Through equilibrium analysis, we seek to understand how CSR practices affect optimal decisions, profitability, and consumer surplus. We also compare and analyze the optimal decisions, profitability, and consumer surplus across the four models.

This paper makes three key contributions. First, to the best of the authors'

[5]https://news.sciencenet.cn/sbhtmlnews/2023/12/377628.shtm

knowledge, this paper represents the first attempt to explore the effects of CSR practices in a fresh produce supply chain through the use of game-theoretic models. The fresh produce supply chain exhibits unique characteristics due to the perishability of its products and the high demand uncertainty (Galal & El-Kilany, 2016). Second, this paper examines the scenario in which both the supplier and the e-tailer jointly practice CSR. Although prior literature (Panda et al., 2016) examines the scenario in which supply chain participants jointly engage in CSR, this research differs in the following ways: 1) We adopt a stochastic demand function in the form of an index, which is more widely used in fresh produce supply chains, while prior literature (Panda et al., 2016) adopts a linear deterministic demand function. 2) We focus on CSR sharing strategy selection when supply chain members jointly undertake CSR, whereas prior literature emphasizes channel coordination and profit distribution. 3) Unlike the retailer in (Panda et al., 2016), who only makes pricing decisions, the e-tailer in this paper makes decisions regarding both pricing and freshness-preserving efforts. Finally, our findings offer managerial insights into how CSR enterprises can make optimal freshness-preserving effort decisions and how fresh produce supply chain members can effectively undertake CSR.

The remainder of this paper is structured as follows: Section 2 reviews the relevant literature. Section 3 presents the model description. In Section 4, we calculate the equilibrium results and evaluate the role of CSR levels in the fresh produce supply chain, followed by a comparison of equilibrium values in Section 5. Section 6 contains the numerical analysis. Section 7 offers concluding remarks and recommendations for future research. The technical proofs supporting this study are provided in the Appendix.

2. Literature Review

Two streams of literature are closely related to our study: operations management of the fresh produce supply chain and supply chain management with CSR.

2.1. Operations Management of the Fresh Produce Supply Chain

Scholars have extensively studied the operational management of fresh produce supply chains, with research primarily focusing on inventory management (Shen et al., 2020; Wang et al., 2019), information sharing (Dan et al., 2023; Yang et al., 2022), pricing strategies (Fan et al., 2020; He et al., 2019), and supply chain coordination (Ma et al., 2020; Wang et al., 2019). Given the perishable nature of fresh produce, the existing literature primarily focuses on the effects of product freshness and quality. For example, (Cai et al., 2010) considered a scenario where distributors' freshness-preserving efforts improve both the survival rate and freshness of produce under decentralized and centralized decision-making. They examined optimal ordering, pricing, and coordination mechanisms for fresh food supply chains. (Song & He, 2019) developed the new models and characterized the optimal decisions in centralized and decentralized channels of a three-layer

FAP supply chain in an e-commerce environment. Based on this study, (Wu et al., 2023) investigated the strategies for adopting blockchain technology in the fresh product supply chain (FPSC). They found that the decision is related to the consumers' acceptance of the product without blockchain technology, the deterioration rate of the fresh product, and so on. (Li & Teng, 2018) explored how retailers optimize pricing and inventory decisions by measuring product freshness as it relates to expiration dates. (Liu et al., 2021) considered a supply chain consisting of a supplier responsible for ensuring freshness and an e-tailer providing value-added services, investigating information sharing in a fresh produce e-commerce supply chain. On this basis, (Dan et al., 2023) studied whether the platform chooses to share private requirement information and showed that the platform may benefit or suffer from information sharing, depending on freshness sensitivity and the choice of cooperation mode. (Ma et al., 2020) considered both the costs and benefits of cold chains and examined a coordination mechanism for the freshness-keeping effort of third-party logistics service providers. (Liu et al., 2023a) constructed an optimal control model to investigate how to incentivize logistics service providers' efforts to preserve product freshness during transport. Unlike previous studies, this study not only considers firms' freshness-preserving efforts but also incorporates CSR into the model. We integrate CSR levels of both suppliers and fresh e-commerce retailers into a Stackelberg game framework and consider the effects of CSR on freshness-preserving efforts, pricing, and consumer surplus under both wholesale price and revenue-sharing contracts.

2.2. Supply Chain Management with CSR

In recent years, scholarly interest in CSR issues has grown significantly. Early literature primarily focused on the intellectual content, business impact, and drivers of CSR from the perspective of a single company (Amaeshi et al., 2008; Falcone & Ridge, 2024; Gallear et al., 2012). In practice, CSR is no longer confined to a single core company. Other elements of the supply chain are also beginning to adopt CSR practices. As a result, over the past decade, academics have increasingly investigated CSR from a supply chain perspective. From this perspective, scholars have explored CSR from various angles. Numerous studies on corporate social responsibility (CSR) indicate that firms adopt CSR practices for various reasons related to their supply chain (Falcone & Ridge, 2024). Corporate social responsibility (CSR) is very important for society, and the public now monitors firms' CSR strategies (Zhu et al., 2023). Some scholars have approached CSR through the lens of consumer surplus (Mosanna et al., 2022; Panda et al., 2017). Specifically, (Bian et al., 2016) conducted a strategic analysis incorporating CSR factors into managerial incentive design. (Panda et al., 2017) examined channel coordination in a socially responsible manufacturer-retailer closed-loop supply chain and evaluated the effects of CSR. To analyze the issue of which type of private investor governments should sell public firms to, (Bcena-Ruiz et al., 2024) measured CSR concerns through consumer surplus. Meanwhile, other scholars have considered CSR to be a firm's investment strategy. For example, (Mosanna et al., 2022) considered corporate donations as a form of CSR and studied coordination in a dyadic supply chain. (Schinkel & Treuren, 2024) investigated which type of joint CSR agreement motivates competitors to increase CSR efforts within a model of oligopolistic competition featuring differentiated products. (Wei et al., 2023) developed a game-theoretic model for a two-sided platform and analyze the impacts of network externalities (including customer and provider network externalities) and risk aversion on CSR investment decision-making. In this paper, consumer surplus effectively captures the impact of CSR on customer satisfaction and loyalty, which are crucial in fresh produce markets due to the perishability of the products. This focus on consumer surplus allows us to emphasize freshness-preserving efforts, which is largely overlooked in existing CSR-related studies. However, related studies do not address freshness-preserving efforts. By examining this relationship, our study provides new insights into how CSR practices can enhance consumer surplus through improved freshness efforts, further distinguishing our contribution to the literature.

3. Model Description and Preliminaries

We consider a two-stage fresh E-commerce supply chain with a supplier and an e-tailer, in which the supplier and the e-tailer consider CSR in addition to profits. Following the literature (Goering, 2008; Panda, 2014; Panda et al., 2015) we use $\alpha_S \in [0,1]$ and $\alpha_O \in [0,1]$ to denote the CSR levels of the supplier and the e-tailer, where $\alpha_S + \alpha_O \in [0,1]$. When trading, supply chain members can choose between a wholesale price contract or a revenue-sharing contract. Under a wholesale price contract (denoted by W), the supplier provides fresh produce at a unit cost c and offers them to the e-tailer at the wholesale price w. The e-tailer purchases q units of fresh produce, which are then sold to the customers online at the unit retail price p. Under a revenue-sharing contract (denoted by R), the e-tailer pays the supplier a wholesale price for each unit purchased, plus $v \in [0,1]$ percentage of fresh produce sales. Thus, as shown in **Figure 1**, there are four alternative scenarios.

In model WSN, supply chain members trade through a wholesale price contract with the supplier practicing CSR. In model WSO, supply chain members trade through a wholesale price contract with both the supplier and the e-tailer jointly practicing CSR. In model RSN, supply chain members trade through a revenue-sharing contract with the supplier practicing CSR. In model RSO, supply chain members trade through a revenue-sharing contract where both the supplier and the e-tailer jointly practice CSR. The sequence of events in each model is as follows: First, as the game's leader, the supplier determines its optimal wholesale price w. Then, the e-tailer decides the retail price p, freshness-preserving effort level τ, and order quantity q independently.

Unlike traditional supply chains, which depend on fresh produce suppliers to maintain product quality, the e-commerce ecosystem relies on e-tailers for direct

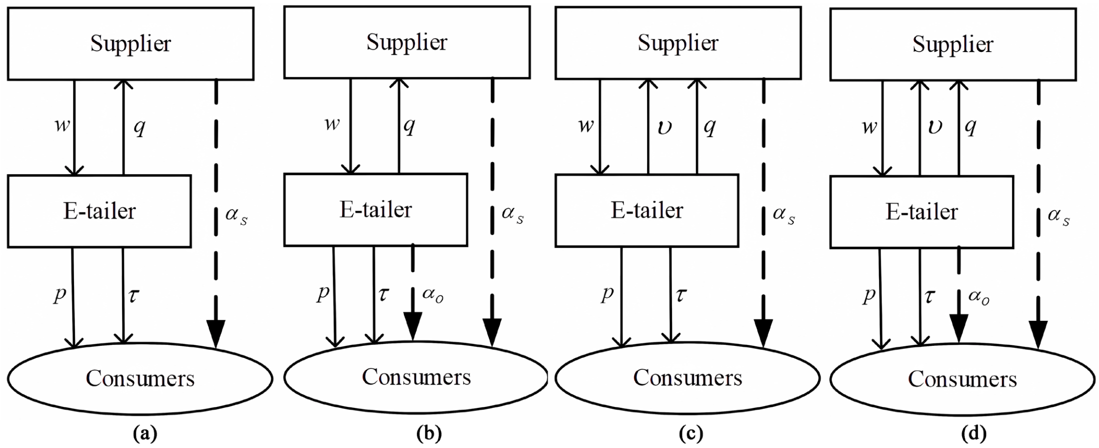

Figure 1. The CSR E-tailing supply chain structures for fresh produce. (a) Model WSN; (b) Model WSO; (c) Model RSN; (d) Model RSO.

sourcing and cold-chain preservation expenses. Drawing from JD Fresh's investments in cold-chain logistics and fresh delivery systems, we infer that the e-tailer invests in cold storage, cold-chain logistics, and refrigerated vehicle configurations. Therefore, we conclude that the e-tailer assumes responsibility for both the delivery and preservation of fresh produce. Let the freshness-preserving effort level be denoted by τ. The freshness of fresh produce θ is affected by the freshness-keeping effort level τ, denoted as $\theta(\tau)$. We adopt $\theta(\tau)=\tau\theta_0$, where $\tau\theta_0$ is the initial freshness of fresh produce (Cai et al., 2010). We further assume that $\theta_0=1$ (Liu et al., 2021), which does not affect the optimal decisions. Regarding the e-tailer's freshness-keeping cost $c_f(\tau)$, following the pieces of literature (Wu et al., 2015), we consider that the marginal cost of freshness-keeping is strictly increasing, we use $c_f=\frac{\xi\tau^2}{2}$ to capture freshness-keeping cost, where ξ is the freshness-keeping cost coefficient.

Market demand is inherently uncertain and influenced by both the retail price and the level of product freshness. We assume the market is composed of heterogeneous customers, each with a varying willingness y to pay for fresh produce, which is influenced by product freshness and price. It follows the cumulative distribution function $\Phi(y)=1-\alpha y^{-k}\theta(\tau)$ (Xue et al., 2014). Let the corresponding density function be $\Phi(y)$. Given the retail price p, consumers are willing to purchase fresh produce only when $y-p>0$. Therefore, the deterministic fraction of the market that is willing to pay as much as p for the fresh produce is $d(p,\tau)=\Pr(y\geq p)=\int_p^{+\infty}\phi(y)\mathrm{d}y=ap^{-k}\theta(\tau)$. Assuming that the uncertain size of the market demand is denoted by ε, then the total market demand function is normalized as follows

$$D(p,\tau,\varepsilon)=ap^{-k}\theta(\tau)\varepsilon \tag{1}$$

In Equation (1), parameter α is the base demand scale ($\alpha>1$), and the parameter k measures the price elasticity of expected demand and affected by the product type. Here we focus on the case of $k>1$ which corresponds to priceelastic products (like fresh produce) and also ensures that profit functions are bounded (Xue et al., 2014). The parameter $\varepsilon \in (0,+\infty)$ is a continuously distributed random factor independent of retail price and freshness, and $E(\varepsilon)=1$. The parameter ε has a probability density function $h(x)$, a cumulative distribution function $H(x)$ and the generalized failure rate of ε is $g(x)=\frac{xh(x)}{\bar{H}(x)}$, here $\bar{H}(x)=1-H(x)$. In addition, ε has an increasing generalized failure rate (IGFR).

When enterprises engage in CSR, their goodwill improves, and consumers become willing to pay a higher price for the firm's products (Panda et al., 2015). Thus, it is reasonable to consider consumer surplus as an indicator of the impact of CSR practices. According to (Panda, 2014; Xue et al., 2014), consumer surplus is defined as follows:

$$CS = E\left[\int_{p}^{+\infty}(y-p)\phi(y)\varepsilon \mathrm{d}y \cdot \min\left\{1,\frac{q}{d(p,\tau)\varepsilon}\right\}\right] \tag{2}$$

Throughout this paper, we use subscripts $i \in \{S,O,SC\}$ to denote the supplier, e-tailer and the supply chain, respectively and superscripts $j \in \{WSN,WSO,RSN,RSO\}$ to denote the Model *j*. For example, Π_S^{WSN} is the supplier's profits in model WSN.

4. Equilibrium Analysis

The primary objective of this section is to formulate the four models and derive the corresponding equilibrium results. Next, the effects of the firm's CSR practices on optimal decisions, profits, and consumer surplus are analyzed. Finally, the optimal decisions, profits, and consumer surplus across the four models are compared and evaluated.

4.1. Supplier Practices CSR under a Wholesale Price Contract (Model WSN)

In model WSN, the supplier and the e-tailer make independent decisions. The profit functions for the supplier and the e-tailer are given by:

$$\Pi_S^{WSN} = (w-c)q \tag{3}$$

$$\Pi_O^{WSN} = pE\left[\min\{q, D(p,\tau,\varepsilon)\}\right] - wq - \frac{\xi\tau^2}{2} \tag{4}$$

A firm engaging in CSR makes decisions based on the combined value of its profits and consumer surplus (Benjaafar et al., 2019; Panda et al., 2017). When the supplier practices CSR, the objective function for the supplier is defined as:

$$V_S^{WSN} = (w-c)q + \alpha_s CS \tag{5}$$

Following (Lariviere, 2006; Petruzzi & Dada, 1999), we definite the stocking factor, $z = q/ap^{-\kappa}\theta(\tau)$, and the decision variables of the e-tailer are transformed from (p,q,τ) to (z,q,τ). The e-tailer's profit function is derived as follows:

Substitute $p=\left(\frac{q}{a\tau z}\right)^{-\frac{1}{k}}$ into Equation (4), we obtain:

$$\begin{aligned}\Pi_O^{WS} &= pE\left[\min\left\{q,D(p,\tau,\varepsilon)\right\}\right]-wq-\frac{\xi\tau^2}{2}\\ &=\left(\frac{q}{a\tau z}\right)^{-\frac{1}{k}}E\left[\min\left\{q,a\left(\left(\frac{q}{a\tau z}\right)^{-\frac{1}{k}}\right)^{-k}\tau\varepsilon\right\}\right]-wq-\frac{\xi\tau^2}{2}\\ &=\left(\frac{q}{a\tau z}\right)^{-\frac{1}{k}}E\left[\min\left\{q,q\cdot\frac{\varepsilon}{z}\right\}\right]-wq-\frac{\xi\tau^2}{2}\\ &=\left(\frac{q}{\alpha\tau z}\right)^{-\frac{1}{k}}qE\left[\min\left\{1,\frac{\varepsilon}{z}\right\}\right]-wq-\frac{\xi\tau^2}{2}\\ &=\left(\frac{q}{\alpha\tau z}\right)^{-\frac{1}{k}}q\left[1-\int_0^z\left(1-\frac{x}{z}\right)h(x)\mathrm{d}x\right]-wq-\frac{\xi\tau^2}{2}\end{aligned}$$

The profit function of the supplier is derived as follows:

$$\begin{aligned}V_S^{WSN} &=(w-c)q+\alpha_s CS\\ &=(w-c)q+\alpha_s E\left[\int_p^{+\infty}(y-p)\phi(y)\varepsilon\mathrm{d}y\cdot\min\left\{1,\frac{q}{d(p,\tau)\varepsilon}\right\}\right]\\ &=(w-c)q+\alpha_s\int_p^{+\infty}(y-p)\phi(y)\mathrm{d}y\cdot\varepsilon E\left[\min\left\{1,\frac{q}{d(p,\tau)\varepsilon}\right\}\right]\\ &=(w-c)q+\alpha_s\cdot\left(p^{1-k}\frac{\alpha\tau}{k-1}\cdot\varepsilon E\left[\min\left\{1,\frac{q}{ap^{-k}\tau\varepsilon}\right\}\right]\right)\\ &=p^{1-k}\frac{\alpha\tau}{k-1}\cdot E\left[\min\{\varepsilon,z\}\right]\\ &=(w-c)q+\alpha_s\cdot\left(p^{1-k}\frac{a\tau}{k-1}\cdot zE\left[\min\left\{\frac{\varepsilon}{z},1\right\}\right]\right)\\ &=(w-c)q+\alpha_s\cdot\left(\frac{1}{k-1}\left(\frac{a\tau z}{q}\right)^{\frac{1}{k}}\left[1-\int_0^z\left(1-\frac{x}{z}\right)h(x)\mathrm{d}x\right]\right)\end{aligned}$$

The combination of the analysis of $\frac{\partial V_S^{WS}}{\partial z}$ and the property of generalized failure rate of results in Lemma 1.

(6)

Theorem 1. *The supplier's and the e-tailer's optimal decisions are as follows:*

$$p^{WSN^*}=\frac{\left(2k^2-3k+1\right)c}{2\left(k^2+k\alpha_s-2k+1\right)\left[1-H\left(z^*\right)\right]},$$

$$w^{WSN^*}=\frac{(2k-1)(k-1)c}{2\left(k^2+k\alpha_s-2k+1\right)},$$

$$q^{WSN^*}=\frac{\left(p^{WSN^*}\right)^{-2k}c(2k-1)a^2z^{*2}}{2\xi\left(k^2+k\alpha_s-2k+1\right)},$$

$$\tau^{WSN^*}=\frac{\left(p^{WSN^*}\right)^{-k}c(2k-1)\alpha z^*}{2\xi\left(k^2+k\alpha_s-2k+1\right)}.$$

The consumer surplus, along with the optimal profits of the supplier, the e-tailer, and the supply chain, are as follows:

$$CS^{WSN^*}=\frac{kz^2\alpha^2c^2(2k-1)^2\left(p^{WSN^*}\right)^{-2k}}{4\xi(k-1)\left(k^2+k\alpha_S-2k+1\right)^2},$$

$$\Pi_S^{WSN^*}=-\frac{\left(p^{WSN^*}\right)^{-2k}(2k-1)z^{*2}\alpha^2(2k\alpha_S-k+1)c^2}{4\left(k^2+k\alpha_S-2k+1\right)^2\xi},$$

$$\Pi_O^{WSN^*}=\frac{c^2(2k-1)^2z^{*2}\alpha^2\left(p^{WSN^*}\right)^{-2k}}{8\left(k^2+k\alpha_S-2k+1\right)^2\xi},$$

$$\Pi_{SC}^{WSN^*}=\frac{\left(p^{WSN^*}\right)^{-2k}(1-2k)z^2\alpha^2c^2(4k\alpha_S-4k+3)}{8\left(k^2+k\alpha_S-2k+1\right)^2\xi}.$$

Note that $\alpha_S<\frac{k-1}{2k}$ can ensure that the optimal profit of the supplier is always positive under the model WSN.

4.2. Supplier and E-Tailer Practice CSR under a Wholesale Price Contract (Model WSO)

In model WSO, the supplier and the e-tailer make independent decisions. The profit functions of the supplier and the e-tailer are given by:

$$\Pi_S^{WSO}=(w-c)q \tag{7}$$

$$\Pi_O^{WSO}=pE\left[\min\left\{q,D(p,\tau,\varepsilon)\right\}\right]-wq-\frac{\xi\tau^2}{2} \tag{8}$$

Since both the supplier and the e-tailer practice CSR, the objective functions of the supplier and the e-tailer are defined as follows:

$$V_S^{WSO}=(w-c)q+\alpha_S CS \tag{9}$$

$$V_O^{WSO}=pE\left[\min\left\{q,D(p,\tau,\varepsilon)\right\}\right]-wq-\frac{\xi\tau^2}{2}+\alpha_O CS \tag{10}$$

Theorem 2. *The supplier's and the e-tailer's optimal decisions are as follows:*

$$p^{WSO^*}=\frac{\left(2k^2+2\alpha_o k-3k-\alpha_o+1\right)c(k-1)}{2\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)\left(k+\alpha_o-1\right)\left[1-H\left(z^*\right)\right]},$$

$$w^{WSO^*}=\frac{\left(2k^2+2\alpha_o k-3k-\alpha_o+1\right)c}{2\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)},$$

$$q^{WSO^*}=\frac{a^2z^{*2}\left(2k^2+2\alpha_o k-3k-\alpha_o+1\right)c\left(p^{WSO^*}\right)^{-2k}}{2\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)\xi\left(k-1\right)},$$

$$\tau^{WSO^*}=\frac{\alpha z^*\left(2k^2+2\alpha_o k-3k-\alpha_o+1\right)c\left(p^{WSO^*}\right)^{-k}}{2\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)\xi\left(k-1\right)}.$$

The consumer surplus, along with the optimal profits of the supplier, the e-tailer, and the supply chain, are as follows:

$$CS^{WSO^*}=\frac{\left(2k-1\right)^2c^2a^2z^2\left(k+\alpha_o-1\right)k\left(p^{WSO^*}\right)^{-2k}}{4\left(k-1\right)^2\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)^2\xi},$$

$$\Pi_S^{WSO^*}=\frac{c^2\left(2k\alpha_s-k-\alpha_o+1\right)a^2z^2z^2\left(2k-1\right)\left(k+\alpha_o-1\right)\left(p^{WSO^*}\right)^{-2k}}{4\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)^2\xi\left(1-k\right)},$$

$$\Pi_O^{WSO^*}=\frac{\left(2k-1\right)^2c^2a^2z^2\left(1-k-\alpha_o\right)\left(2\alpha_o k-k-\alpha_o+1\right)\left(p^{WSO^*}\right)^{-2k}}{8\xi\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)^2\left(k-1\right)^2},$$

$$\Pi_{SC}^{WSO^*}$$
$$=\frac{c^2a^2z^{*2}\left(1-2k\right)\left(k+\alpha_o-1\right)\left(p^{WSO^*}\right)^{-2k}\left(4k^2\alpha_o+4k^2\alpha_s-4k^2-6\alpha_o k-4k\alpha_s+7k+3\alpha_o-3\right)}{8\xi\left(k^2+\alpha_o k+k\alpha_s-2k-\alpha_o+1\right)^2\left(k-1\right)^2}.$$

Note that $\alpha_S<\frac{k+\alpha_O-1}{2k}$ and $\alpha_O<\frac{k-1}{2k-1}$ ensure that the optimal profits of the supplier and the e-tailer remain positive under Model WSO. This implies that both the e-tailer and the supplier should avoid excessively high levels of CSR. This sets an upper limit for CSR levels.

4.3. Supplier Practices CSR under a Revenue-Sharing Contract (Model RSN)

In model RSN, the e-tailer pays the supplier a wholesale price for each unit purchased, plus $\upsilon\in\left[0,1\right]$ percentage of fresh produce sales. Thus, the profit functions of the supplier and the e-tailer under a revenue-sharing contract, where only the supplier practices CSR, are as follows:

$$\Pi_S^{RSN}=\left(w-c\right)q+\upsilon pE\left[\min\left\{q,D\left(p,\tau,\varepsilon\right)\right\}\right]\tag{11}$$

$$\Pi_O^{RSN}=\left(1-\upsilon\right)pE\left[\min\left\{q,D\left(p,\tau,\varepsilon\right)\right\}\right]-wq-\frac{\xi\tau^2}{2}\tag{12}$$

When the supplier practices CSR under a revenue-sharing contract, the objective function for the supplier is defined as:

$$V_S^{RSN} = (w-c)q + \upsilon pE\left[\min\left\{q, D(p,\tau,\varepsilon)\right\}\right] + \alpha_S CS \tag{13}$$

Theorem 3. *The supplier's and the e-tailer's optimal decisions are as follows:*

$$p^{RSN^*} = \frac{c(2k-1)(k-1)}{2\left(1-H\left(z^*\right)\right)\left(k^2+k\upsilon+k\alpha_s-2k-\upsilon+1\right)},$$

$$w^{RSN^*} = \frac{(2k-1)(1-k)(\upsilon-1)c}{2\left(k^2+k\upsilon+k\alpha_s-2k-\upsilon+1\right)},$$

$$q^{RSN^*} = \frac{\left(p^{RSN^*}\right)^{-2k} c(1-\upsilon)(2k-1)z^{*2}a^2}{2\xi\left(k^2+k\upsilon+k\alpha_s-2k-\upsilon+1\right)},$$

$$\tau^{RSN^*} = \frac{\left(p^{RSN^*}\right)^{-k} c(1-\upsilon)(2k-1)z^{*}a}{2\xi\left(k^2+k\upsilon+k\alpha_s-2k-\upsilon+1\right)}.$$

The consumer surplus, along with the optimal profits of the supplier, the e-tailer, and the supply chain, are as follows:

$$CS^{RSN^*} = \frac{c^2(2k-1)^2(1-\upsilon)z^2a^2k\left(p^{RSN^*}\right)^{-2k}}{4(k-1)\left(k^2+k\upsilon+k\alpha_S-2k-\upsilon+1\right)^2\xi},$$

$$\Pi_S^{RSN^*} = \frac{c^2(\upsilon-1)(2k-1)z^{*2}a^2(2k\alpha_S-k-\upsilon+1)\left(p^{RSN^*}\right)^{-2k}}{4\left(k^2+k\upsilon+k\alpha_S-2k-\upsilon+1\right)^2\xi},$$

$$\Pi_O^{RSN^*} = \frac{(2k-1)^2(\upsilon-1)^2c^2\left(p^{RSN^*}\right)^{-2k}z^{*2}a^2}{8\left(k^2+k\upsilon+k\alpha_S-2k-\upsilon+1\right)^2\xi},$$

$$\Pi_{SC}^{RSN^*} = \frac{c^2(\upsilon-1)(2k-1)z^{*2}a^2\left(p^{RSN^*}\right)^{-2k}(2k\upsilon+4k\alpha_S-4k-3\upsilon+3)}{8\left(k^2+k\upsilon+k\alpha_S-2k-\upsilon+1\right)^2\xi}.$$

Note that $\alpha_s < \frac{k+\upsilon-1}{2k}$ ensures that the supplier's optimal profits remain positive under Model RSO.

4.4. Supplier and E-Tailer Practice CSR under a Revenue-Sharing Contract (Model RSO)

In model RSO, the supplier and the e-tailer make independent decisions. The profit functions for the supplier and the e-tailer are as follows:

$$\Pi_S^{RSO} = (w-c)q + \upsilon pE\left[\min\left\{q, D(p,\tau,\varepsilon)\right\}\right] \tag{14}$$

$$\Pi_O^{RSO} = (1-\upsilon)pE\left[\min\left\{q, D(p,\tau,\varepsilon)\right\}\right] - wq - \frac{\xi\tau^2}{2} \tag{15}$$

When both the supplier and the e-tailer practice CSR, the objective functions for the supplier and the e-tailer are defined as follows:

$$V_S^{RSO} = (w-c)q + \upsilon pE\left[\min\left\{q, D(p,\tau,\varepsilon)\right\}\right] + \alpha_S CS \tag{16}$$

$$V_O^{RSO} = (1-\upsilon)pE\left[\min\left\{q, D(p,\tau,\varepsilon)\right\}\right] - wq - \frac{\xi\tau^2}{2} + \alpha_O CS \tag{17}$$

Theorem 4. *The supplier's and the e-tailer's optimal decisions are as follows:*

$$p^{RSO^*} = \frac{(k-1)(2k-1)c}{2\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)\left(1-H\left(z^*\right)\right)},$$

$$w^{RSO^*} = \frac{(1-2k)(k\upsilon-k-\upsilon-\alpha_O+1)c}{2\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)},$$

$$q^{RSO^*} = \frac{a^2z^{*2}(1-2k)(k\upsilon-k-\upsilon-\alpha_O+1)c\left(p^{RSO^*}\right)^{-2k}}{2\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)\xi(k-1)},$$

$$\tau^{RSO^*} = \frac{az^*(1-2k)(k\upsilon-k-\upsilon-\alpha_O+1)c\left(p^{RSO^*}\right)^{-k}}{2\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)\xi(k-1)}.$$

The consumer surplus, along with the optimal profits of the supplier, the e-tailer, and the supply chain, are as follows:

$$CS^{RSO^*} = \frac{(2k-1)^2c^2\alpha^2z^2(k\upsilon-k-\upsilon-\alpha_O+1)k\left(p^{RSO^*}\right)^{-2k}}{4(k-1)^2\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)^2\xi},$$

$$\Pi_S^{RSO^*} = c^2\alpha^2z^{*2}(2k-1)(k\upsilon-k-\upsilon-\alpha_O+1) \cdot \frac{(2k\alpha_S-k-\upsilon-\alpha_O+1)\left(p^{RSO^*}\right)^{-2k}}{4\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)^2\xi(k-1)},$$

$$\Pi_O^{RSO^*} = \alpha^2z^{*2}(2k-1)^2(k\upsilon-k-\upsilon-\alpha_O+1)c^2 \cdot \frac{(k\upsilon+2\alpha_O k-k-\upsilon-\alpha_O+1)\left(p^{RSO^*}\right)^{-2k}}{8\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)^2\xi(k-1)^2},$$

$$\Pi_{SC}^{RSO^*} = \frac{c^2\alpha^2z^{*2}(2k-1)(k\upsilon-k-\upsilon-\alpha_O+1)\left[2k^2(\upsilon+2\alpha_O+2\alpha_S-2)\right.}{8\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)^2\xi(k-1)^2} - \frac{\left.(5\upsilon+6\alpha_O+4\alpha_S-7)k+3(\upsilon+\alpha_O-1)\right]\left(p^{RSO^*}\right)^{-2k}}{8\left(k^2+k\upsilon+\alpha_O k+k\alpha_S-2k-\upsilon-\alpha_O+1\right)^2\xi(k-1)^2},$$

$$\Pi_{SC}^{WSO^*} = c^2\alpha^2z^{*2}(1-2k)(k+\alpha_O-1)\left(p^{WSO^*}\right)^{-2k} \cdot \frac{\left(4k^2\alpha_O+4k^2\alpha_S-4k^2-6\alpha_O k-4k\alpha_S+7k+3\alpha_O-3\right)}{8\xi\left(k^2+\alpha_O k+k\alpha_S-2k-\alpha_O+1\right)^2(k-1)^2}.$$

Note that $\alpha_S < \frac{k+\upsilon+\alpha_O-1}{2k}$ and $\alpha_O < \frac{k\upsilon-k-\upsilon+1}{1-2k}$ can ensure that the optimal profits of the supplier and the e-tailer are always positive under the Model RSO. In other words, a higher value of v corresponds to a lower level of social responsibility the retailer is willing to take on.

4.5. Effects of CSR Level on Optimal Decisions and Profits

In the above four models, WSO, WSN, RSN, and RSO, we first investigate the supply member's CSR level effects on retail price, wholesale price, order quantity, freshness-keeping effort level, consumer surplus, and profits, respectively, and obtain Proposition 1-4.

Proposition 1. $\frac{\partial p^{j^*}}{\partial \alpha_s} < 0$, $\frac{\partial p^{l^*}}{\partial \alpha_o} < 0$; $\frac{\partial q^{j^*}}{\partial \alpha_s} > 0$, $\frac{\partial q^{l^*}}{\partial \alpha_o} > 0$; $\frac{\partial \tau^{j^*}}{\partial \alpha_s} > 0$, $\frac{\partial \tau^{l^*}}{\partial \alpha_o} > 0$; $\frac{\partial CS^{j^*}}{\partial \alpha_s} > 0$, $\frac{\partial CS^{l^*}}{\partial \alpha_o} > 0$, $\frac{\partial w^{j*}}{\partial \alpha_s} < 0$, $\frac{\partial w^{l*}}{\partial \alpha_o} > 0$; *where* $i \in \{WSN, WSO, RSN, RSO\}$, $l \in \{WSO, RSO\}$.

Proposition 1 demonstrates that as the CSR level of supply chain members increases, the freshness-preserving effort level, order quantity, and consumer surplus rise, while the retail price declines. This occurs because consumer surplus is influenced by both the retail price and the freshness-preserving effort level. A CSR-conscious e-tailer will prioritize consumer benefits by increasing investments in freshness preservation and reducing the retail price, thereby increasing consumer surplus. Conversely, a CSR-conscious supplier will reduce its wholesale price. The e-tailer then responds to the supplier's actions by reducing retail prices and increasing cold chain preservation investments, as customers are encouraged to purchase more. Consequently, the e-tailer's order quantity increases. The CSR practices of Chinese fresh e-commerce enterprises confirm this perspective. Both e-tailers and suppliers took the initiative to undertake CSR during the COVID-19 pandemic. Freshippo and JD Fresh strengthened their cooperation with suppliers to ensure a sufficient supply of fresh produce and stabilize prices. Additionally, fresh produce e-commerce companies also prioritized consumer safety by leveraging their capabilities to introduce "no-touch delivery" services, reducing the risk of epidemic spread and enhancing social welfare.

Proposition 2. $\frac{\partial \Pi_S^{j*}}{\partial \alpha_S} < 0$, $\frac{\partial \Pi_O^{j*}}{\partial \alpha_S} > 0$; $\frac{\partial \Pi_O^{l*}}{\partial \alpha_O} < 0$, $\frac{\partial \Pi_S^{l*}}{\partial \alpha_O} > 0$; where $i \in \{WSN, WSO, RSN, RSO\}$, $l \in \{WSO, RSO\}$.

Proposition 2 demonstrates that when the supplier engages in CSR, it reduces its own profits while increasing the e-tailer's profits. And when the e-tailer engages in CSR, it reduces its own profits while increasing the supplier's profits. This implies that supply chain members are not intrinsically motivated to enhance their own CSR practices, which is a fundamental reason for the lack of CSR in many businesses. Therefore, core enterprises, as the dominant players in the supply chain, must develop effective incentive mechanisms to enhance the economic interests of individual enterprises with CSR awareness. In this way, individual supply chain members will take the initiative to cultivate or improve their own

CSR awareness in the overall interest.

Proposition 3. 1. For all $0 < \alpha_S < \frac{k-1}{2k}$, $\frac{\partial \Pi_{SC}^{WSN^*}}{\partial \alpha_S} > 0$;

2. If $0 < \alpha_S < \frac{-2\alpha_O k + k + \alpha_O - 1}{2k}$, then $\frac{\partial \Pi_{SC}^{WSO^*}}{\partial \alpha_S} > 0$;

3. If $0 < \alpha_S < \frac{-k\upsilon + k + \upsilon - 1}{2k}$, then $\frac{\partial \Pi_{SC}^{RSN^*}}{\partial \alpha_s} > 0$; If $\frac{-k\upsilon + k + \upsilon - 1}{2k} \le \alpha_s < \frac{k + \upsilon - 1}{2k}$, then $\frac{\partial \Pi_{SC}^{RSN^*}}{\partial \alpha_s} \le 0$.

4. If $0 < \alpha_S < \frac{k\upsilon + 2\alpha_0 k - k - \upsilon - \alpha_0 + 1}{-2k}$, then $\frac{\partial \Pi_{SC}^{RSO^*}}{\partial \alpha_s} > 0$; If $\frac{k\upsilon + 2\alpha_O k - k - \upsilon - \alpha_O + 1}{-2k} \le \alpha_S < \frac{k + \upsilon + \alpha_O - 1}{2k}$, then $\frac{\partial \Pi_{SC}^{RSO^*}}{\partial \alpha_S} \le 0$

Proposition 3 analyzes the impact of the supplier's CSR level on the supply chain's overall profits. It demonstrates that in Model WSN, the supply chain's profits increase as the supplier's CSR level rises. Furthermore, it shows that as the supplier's CSR level increases, the supply chain's profits rise when the supplier's CSR level is relatively low (*i.e.*, $0 < \alpha_S < \frac{-2\alpha_0 k + k + \alpha_0 - 1}{2k}$) but decrease when the supplier's CSR level becomes relatively high (*i.e.*, $\frac{-2\alpha_0 k + k + \alpha_0 - 1}{2k} \le \alpha_S < \frac{k + \alpha_0 - 1}{2k}$). This is because when the supplier practices CSR, it reduces its own profits but increases the e-tailer's profits. When the supplier's CSR level is relatively low, the increase in the e-tailer's profits exceeds the decrease in the supplier's profits. As a result, the supply chain's overall profits increase. When the supplier's CSR level is relatively high, Proposition 1 suggests that as the supplier's CSR level increases, the e-tailer will raise its freshness-preserving effort level. However, the marginal cost of the freshness-preserving effort is high, resulting in the e-tailer's profit increase being smaller than the supplier's profit decrease, causing the supply chain's overall profits to decline. Similar analyses lead to the conclusions in Proposition 3(c) and Proposition 3(d).

Proposition 1 and Proposition 3 together reveal that while firms' CSR behavior can benefit consumers, the supplier should avoid overcommitting to CSR from the perspective of maximizing the total profit of the supply chain. Only a reasonable level of CSR commitment can optimize the total profit of the entire supply chain. This supports Vickers' (Vickers, 1985) perspective in the fresh produce field, which argues that non-economic profit maximization goals can enhance economic profits under certain conditions.

Proposition 4. 1. When $0 < \alpha_S < \alpha_S^1$, if $0 < \alpha_O < \alpha_{O1}$, $\frac{\partial \Pi_{SC}^{WSO^*}}{\partial \alpha_O} > 0$, if $\alpha_{O1} \le \alpha_O < \frac{k-1}{2k-1}$, $\frac{\partial \Pi_{SC}^{WSO^*}}{\partial \alpha_O} \le 0$.

2. When $0<\alpha_S<\alpha_S^2$, if $0<\alpha_O<\min\left(\alpha_O^1, \frac{-k\upsilon+k+\upsilon-1}{2k-1}\right)$, $\frac{\partial\Pi_{SC}^{RSO^*}}{\partial\alpha_O}>0$. Otherwise $\frac{\partial\Pi_{SC}^{RSO^*}}{\partial\alpha_o}\le 0$. When $\alpha_S^2\le\alpha_S<\frac{k+\upsilon+\alpha_O-1}{2k}$, $\frac{\partial\Pi_{SC}^{RSO^*}}{\partial\alpha_o}\le 0$.

Proposition 4 analyzes the impact of the e-tailer's CSR level on the overall profits of the supply chain. It demonstrates that when the supplier's CSR level is relatively low (*i.e.*, $0<\alpha_S<\alpha_S^1$), and the e-tailer's CSR level is also relatively low (*i.e.*, $0<\alpha_O<\alpha_O^1$), the supply chain's profits increase as the e-tailer's CSR level rises. In contrast, when the e-tailer's CSR level is relatively high (*i.e.*, $\alpha_{O1}\le\alpha_O<\frac{k-1}{2k-1}$), the supply chain's profits decrease as the CSR level of the e-tailer increases (in Model WSO). When the CSR level of the supplier is relatively high (*i.e.*, $\alpha_S^1\le\alpha_S<\frac{k+\alpha_O-1}{2k}$), the supply chain's profits increase with the increasing CSR level of the e-tailer. Similar analyses result in Proposition 4(b). Proposition 4 reveals that if the supplier's CSR level is relatively low, the e-tailer should engage in CSR, but not to an excessive extent. If the supplier has a high level of CSR, the e-tailer should refrain from engaging in CSR.

5. Comparative Analysis

This section compares the retail prices, order quantities, freshness-preserving effort levels, and consumer surplus across Models WSN, WSO, RSN, and RSO. We derive Propositions 5 and 6.

Proposition 5. *If* $0<\upsilon<\alpha_O$, *then* $p^{WSN^*}>p^{RSN^*}>p^{WSO^*}>p^{RSO^*}$; *If* $\alpha_O<\upsilon<1$, *then* $p^{WSN^*}>p^{WSO^*}>p^{RSN^*}>p^{RSO^*}$.

Proposition 5 demonstrates that the retail price is highest in Model WSN and lowest in Model RSO. The retail price under a wholesale price contract where both the supplier and the e-tailer jointly practice CSR (Model WSO) can be equivalent to that under a revenue-sharing contract where only the supplier practices CSR (Model RSN), provided that the revenue-sharing fraction and the e-tailer's CSR level are the same (*i.e.*, $\alpha_O=v$). When the revenue-sharing fraction is relatively low (*i.e.*, $0<v<\alpha_O$), a wholesale price contract can result in the e-tailer setting a lower retail price. Conversely, if the revenue-sharing fraction is relatively large (*i.e.*, $\alpha_O<v<1$), a revenue-sharing contract can result in the e-tailer setting a lower retail price.

Proposition 6. *Under a wholesale price contract, we have* $q^{WSO^*}>q^{WSN^*}$, $\tau^{WSO^*}>\tau^{WSN^*}$. *Under a revenue-sharing contract, we have* $q^{RSO^*}>q^{RSN^*}$, $\tau^{RSO^*}>\tau^{RSN^*}$.

Proposition 6 compares the order quantity and freshness-preserving effort levels between the wholesale price contract and the revenue-sharing contract. By combining Proposition 5 and Proposition 6, we find that the freshness-preserving effort level and order quantity are higher, while the retail price is lower, when both the supplier and the e-tailer practice CSR compared to when only the supplier practices CSR. Thus, to stimulate demand and increase consumer surplus, both the supplier and the e-tailer should practice CSR. However, an increase in market demand does not necessarily translate into higher corporate profits. It is difficult to compare the profits of the supply chain members under different contracts, and this will be further examined through numerical examples.

6. Numerical Analysis

To further show the impact of α_S and α_O on the supply chain, numerical examples are employed to visualize the key propositions mentioned above and to provide additional managerial insights. According to $D(p,\tau,\varepsilon)=ap^{-k}\theta(\tau)\varepsilon$. The following benchmark parameter settings are used: $a=100$, $\varepsilon\in U[0,2]$, $c=1$, $k=2$, $\xi=30$, $z^*=\frac{4}{k+1}$, $H(z^*)=\frac{z^*}{2}=\frac{2}{k+1}$.

6.1. Equilibrium Results under Different Models

Assuming that $\alpha_S=0.15,\alpha_O=0.12,\upsilon=0.1$, the equilibrium values for the four models can be calculated as given in Table 1.

Table 1. Optimal values of Models WSN, WSO, RSN, RSO.

	Model WSN	Model WSO	Model RSN	Model RSO
p^*	3.46	3.17	3.21	2.96
q^*	4.76	6.95	5.35	7.76
τ^*	0.43	0.52	0.41	0.51
w^*	1.15	1.18	0.96	1.00
CS^*	10.99	14.69	11.47	15.32
Π_S	0.73	1.27	0.96	1.58
Π_O	2.75	2.35	2.58	2.07
Π_{SC}	3.48	3.62	3.54	3.65

Table 1 reveals that the market price is the lowest, and the order quantity and consumer surplus are highest under Model RSO. This indicates that consumers benefit most when the supplier and the e-tailer jointly practice CSR under a revenue-sharing contract. Additionally, the supplier's profits are highest, the e-tailer's profits are lowest, and the supply chain's profits are maximized under Model RSO. Therefore, only when a reasonable profit distribution mechanism is established to increase the e-tailer's profits will the e-tailer be willing to engage in CSR and share sales proceeds with the supplier. When comparing the freshness-preserving effort levels between Models RSO and WSO, we find that the effort levels are lower in Model RSO. This suggests that when supply chain members collaborate on CSR practices, implementing a revenue-sharing contract reduces the e-tailer's freshness-preserving effort level.

The effects of α_S on equilibrium decisions and profits are then examined in Model RSO. Assume that $\alpha_O = 0.12$, $\upsilon = 0.1$, and $\alpha_S \in [0, 0.3]$ is required to assure that the supplier's profits are positive. The equilibrium decisions for Model RSO can be calculated, as presented in Table 2.

Table 2. Optimal decisions of Model RSO ($\alpha_s = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$; $\alpha_O = 0.12$; $\upsilon = 0.1$).

	$\alpha_S = 0.05$	$\alpha_S = 0.1$	$\alpha_S = 0.15$	$\alpha_S = 0.2$	$\alpha_S = 0.25$	$\alpha_S = 0.3$
p^*	3.69	3.17	2.96	2.78	2.62	2.47
q^*	4.01	6.33	7.76	9.40	11.25	13.33
τ^*	0.41	0.48	0.51	0.54	0.58	0.61
w^*	1.25	1.08	1.01	0.94	0.89	0.84
CS^*	9.87	13.38	15.33	17.41	19.62	21.97
Π_S	2.00	1.83	1.58	1.22	0.72	0.073
Π_O	1.33	1.81	2.07	2.35	2.65	2.96
Π_{SC}	3.34	3.63	3.65	3.57	3.37	3.03

Table 2 reveals that the freshness-preserving effort level, order quantity, and consumer surplus increase, while the market price and the supplier's profits decrease as the supplier's CSR level rises. As a result, the supply chain's profits initially increase with the supplier's CSR level but eventually decrease. Thus, the supplier must strike a balance between supply chain profits, consumer welfare, and its own profit. Table 2 also shows that if firms seek to maximize the supply chain's profits, the supplier should practice more CSR than the e-tailer, especially when the e-tailer adopts a strategy with a CSR level of 0.12. This is because the e-tailer incurs significant costs when taking responsibility for freshness investments.

6.2. Effects of α_O on the Freshness-Keeping Effort Level, Wholesale Price, Retail Price, Order Quantity

This subsection examines the effects of α_O on equilibrium decisions, consumer surplus, and profits. We revert to using the parameters $a = 100$, $\varepsilon \in U[0, 2]$, $c = 1$, $k = 2$, $\xi = 30$, $z^* = \frac{4}{3}$, $H(z^*) = \frac{2}{3}$, $\alpha_S = 0.15$, and $v = 0.1$. Figures 2-9 are then obtained.

Figure 2 illustrates the effects of α_O on the wholesale price. Figure 2 shows that the wholesale price in a wholesale price contract (Models WSN, WSO) is higher than that in a revenue-sharing contract (Models RSN, RSO). This is because the revenue-sharing contract requires the supplier to reduce its wholesale price. Figure 3 illustrates the effects of α_O on the freshness-preserving effort level, with the lowest level observed under Model RSN. If α_O is small

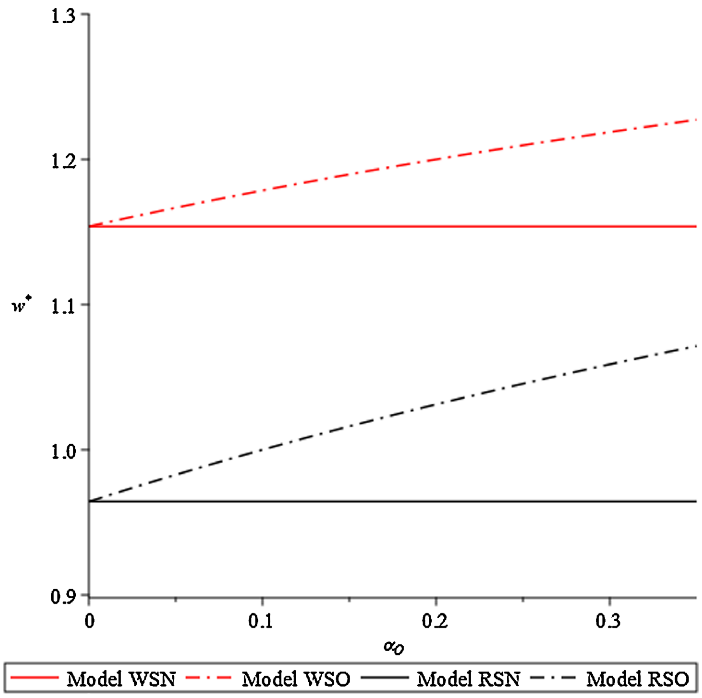

Figure 2. Effects of α_o on the wholesale price.

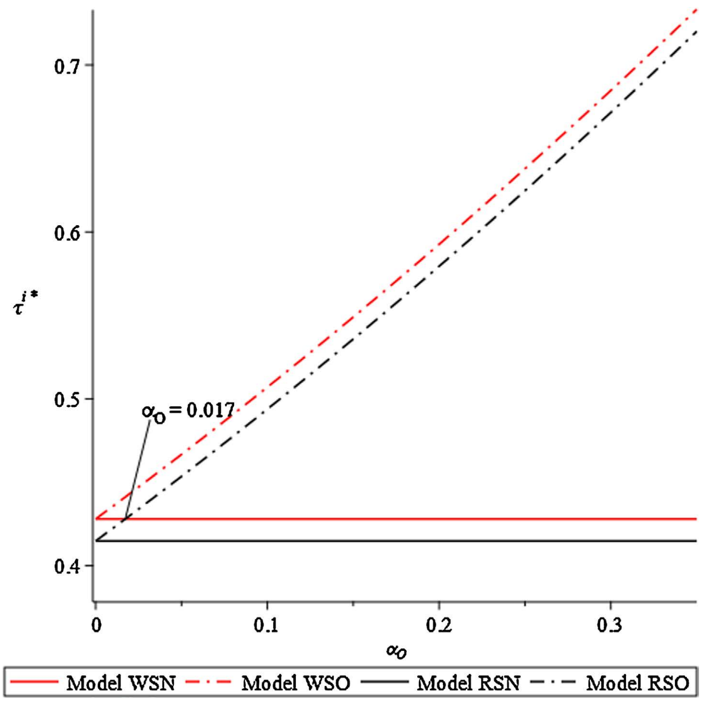

Figure 3. Effects of α_o on freshness-keeping effort.

($0 < \alpha_o < 0.017$), the freshness-preserving effort level under Model RSO is lower than that under Model WSN. As α_o increases, the freshness-preserving effort level under Model RSO becomes higher than that under Model WSN. Figure 4 illustrates the effects of α_o on the retail price. It demonstrates that the highest retail price is observed under Model WSN and the lowest under Model RSO. If

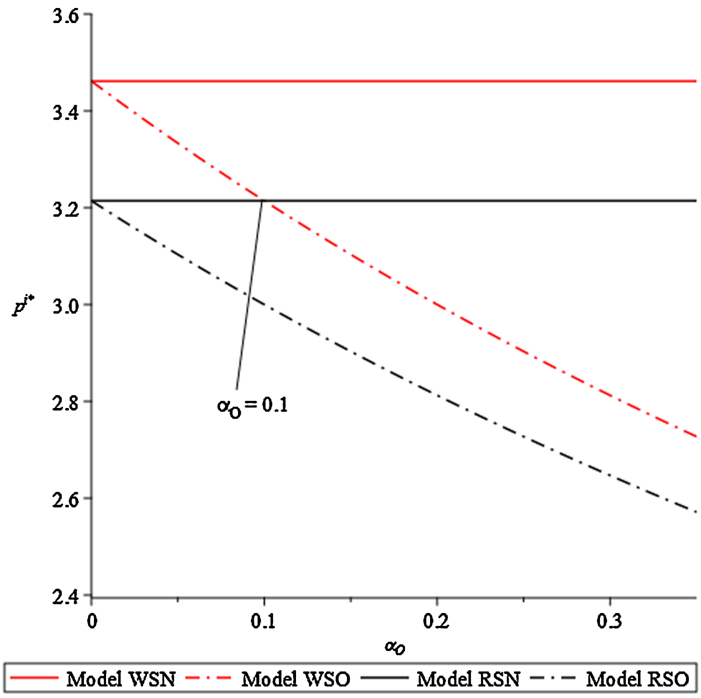

Figure 4. Effects of α_O on the retail price.

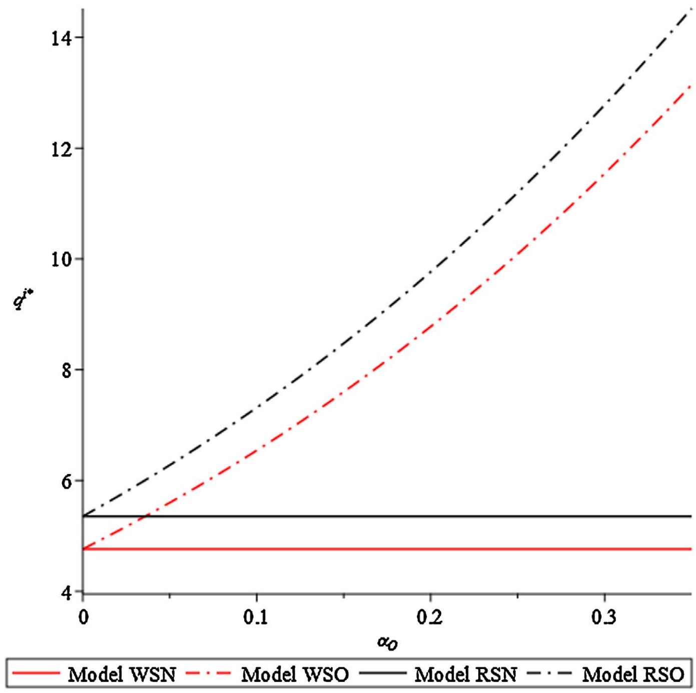

Figure 5. Effects of α_O on the order quantity.

α_O is small ($0 < \alpha_O < 0.1$), the retail price under Model RSN is lower than that under Model WSO. As α_O increases, the retail price under Model RSN becomes higher than that under Model WSO. By combining **Figure 3** and **Figure 4**, we observe that when supply chain members jointly practice CSR, the freshness-preserving effort level under Model RSO is lower than under Model RSN. However, since the retail price under Model RSO is lower than that under Model RSN, Model RSO generates a larger consumer surplus than Model RSN (see Figure 9), which results in a higher order quantity under Model RSO (see Figure 5).

Figure 6 illustrates the effects of α_o on the e-tailer's profit. It demonstrates that the e-tailer's profits increase with rising α_o under Models RSN and RSO.

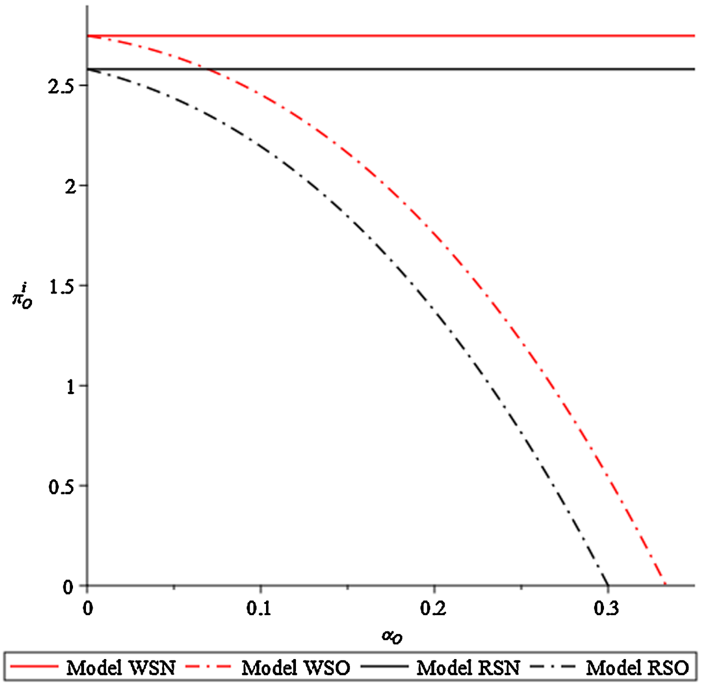

Figure 6. Effects of α_o on the e-tailer's profit.

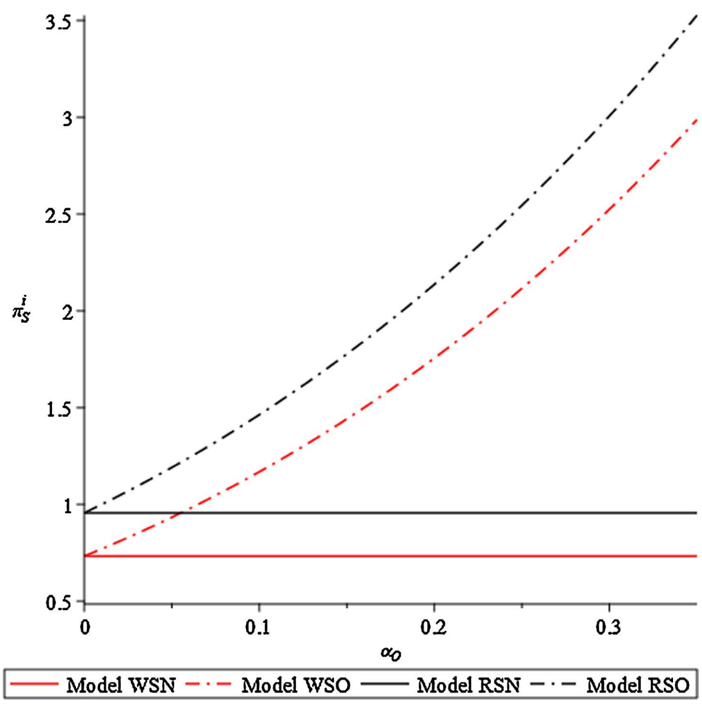

Figure 7. Effects of α_o on the supplier's profit.

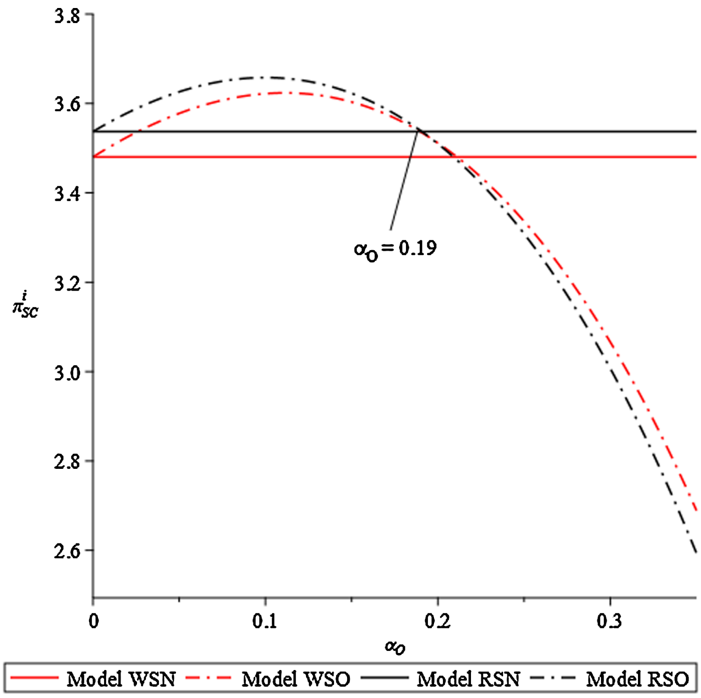

Figure 8. Effects of α_O on the supply chain's profit.

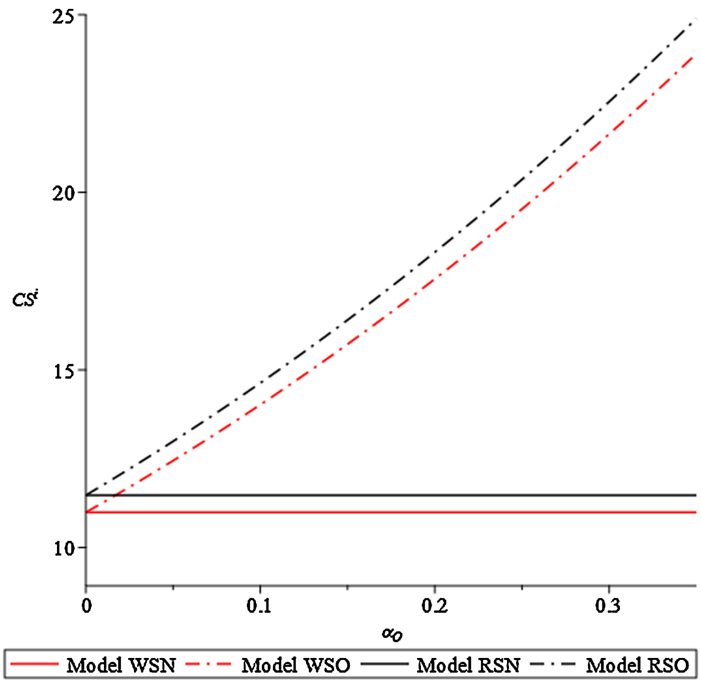

Figure 9. Effects of α_O on the consumer surplus.

This occurs because the freshness-preserving effort level and wholesale price increase, while the retail price decreases, as the e-tailer's CSR level rises under Models RSN and RSO (see Figures 2-4).

According to Figures 2-4, as α_O rises, the wholesale price increases at a slow rate, while the freshness-preserving effort level rises linearly and the market price decreases linearly. Thus, the decrease in retail price and the increase in freshness-preserving effort level are the main reasons for the decline in the e-tailer's profit. By combining Figure 6 and Figure 7, we find that Model RSO yields the lowest profit for the e-tailer and the highest profit for the supplier among the four models. This suggests that while the supplier prefers the e-tailer to engage in CSR and is willing to implement a revenue-sharing contract, the e-tailer lacks the motivation to practice CSR and adopt a revenue-sharing contract. Figure 8 shows that in Model RSO, the supply chain profit is maximized if $0 < \alpha_O < 0.19$. This suggests that if profits are reasonably distributed under certain conditions, the profit of an individual firm can be improved. When supply chain members jointly undertake CSR, they can not only enhance their own profits but also improve consumer surplus and product freshness, thereby achieving a win-win situation for both companies and consumers. Therefore, as the leader, the supplier should focus on motivating the e-tailer to undertake CSR and encouraging the e-tailer to adopt a revenue-sharing contract.

7. Conclusion

With the rapid development of fresh produce e-commerce, the issue of fresh food safety has become increasingly prominent, raising public concern regarding the social responsibility of fresh produce businesses. Given that the customer is the most important stakeholder in food safety, this paper uses consumer surplus to characterize the socially responsible behavior of firms. We construct Stackelberg games for four scenarios: a wholesale price contract where only the supplier practices CSR, a wholesale price contract where both the supplier and the e-tailer jointly practice CSR, a revenue-sharing contract where only the supplier practices CSR, and a revenue-sharing contract where both the supplier and the e-tailer jointly practice CSR. We then analyze the effects of CSR levels on the freshness-preserving effort level, wholesale price, retail price, order quantity, and the profits of the supplier, the e-tailer, and the supply chain. Additionally, we provide managerial implications for CSR undertakings and decision-making in fresh produce enterprises.

Our research yielded the following major findings: (a) The freshness-keeping effort level, order quantity, and consumer surplus increase, while the retail price decreases as the CSR level of supply chain members increases. This indicates that it is always beneficial for consumers when companies undertake CSR. (b) The wholesale price increases as the CSR level of the e-tailer rises but decreases as the CSR level of the supplier rises. (c) When the supplier engages in CSR, it reduces its own profits while increasing the e-tailer's profits. And when the e-tailer engages in CSR, it reduces its own profits while increasing the supplier's profits. Thus, core enterprises must develop effective incentive mechanisms to enhance the economic interests of individual enterprises with CSR awareness. This will motivate individual supply chain members to take the initiative to cultivate or enhance their own CSR awareness for the overall benefit of the supply chain. (d) In Model WSN, the supply chain's profits increase as the CSR level of the supplier rises. In Models WSO, RSN, and RSO, the supply chain's profits increase when the supplier's CSR level is relatively low but decrease when it is relatively high. (e) When both the supplier's and the e-tailer's CSR levels are relatively low, the supply chain's profits increase as the e-tailer's CSR level rises. However, when the e-tailer's CSR level is relatively high, the supply chain's profits decrease as the e-tailer's CSR level continues to rise. When the supplier's CSR level is relatively high, the supply chain's profits increase as the e-tailer's CSR level rises. Thus, if the supplier's CSR level is relatively low, the e-tailer should engage in CSR, but not excessively. Conversely, if the supplier has a high level of CSR, the e-tailer should refrain from engaging in CSR. (f) The freshness-preserving effort level, order quantity, and consumer surplus are higher, and the retail price is lower when both the supplier and the e-tailer jointly practice CSR compared to when only the supplier practices CSR. Therefore, from the perspective of stimulating demand and increasing consumer surplus, both the supplier and the e-tailer should jointly practice CSR.

There are a few limitations to our study. Here, we assume that the e-tailer is solely responsible for both the transportation and preservation of fresh produce. However, in real-world fresh produce supply chains, freshness preservation may involve coordination among multiple parties, such as suppliers, third-party logistics providers, and e-tailers. Consequently, the findings are primarily applicable to specific supply chain structures and may not fully capture the dynamics of decentralized or more complex supply chains. Future research could consider a more collaborative supply chain structure, involving detailed roles for suppliers, third-party logistics providers, and e-tailers. This would provide a more comprehensive view of how multi-stakeholder interactions influence CSR and freshness preservation in real-world supply chains. Subsequent studies could enhance the robustness of the model by incorporating empirical data or conducting case studies to validate the findings. For example, using operational data from firms could help assess the actual impact of CSR levels and freshness-preserving efforts on market demand and supply chain performance. We believe this is an avenue for future research.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

Proof of Lemma 1.

Taking the first derivative of V_S^{WSN} with respect to z, we can derive that $\frac{\partial V_S^{WSN}}{\partial z}=\frac{(a\tau)^{\frac{1}{k}}q^{-\frac{1}{k}}\int_0^z xh(x)\mathrm{d}x}{(k-1)kz^{2-\frac{1}{k}}}\cdot\left\{\frac{z[1-H(z)]}{\int_0^z xh(x)\mathrm{d}x}-(k-1)\right\}$. Let $G(x)=\frac{z\bar{H}(x)}{\int_0^z xh(x)\mathrm{d}x}$, we obtain $\frac{\mathrm{d}G(x)}{\mathrm{d}z}=\frac{\bar{H}(x)}{\left(\int_0^z xh(x)\mathrm{d}x\right)^2}\int_0^z\left(\frac{xh(x)}{\bar{H}(x)}-\frac{zh(z)}{\bar{H}(z)}\right)\bar{H}(x)\mathrm{d}x$. Since the generalized failure rate of ε is $g(x)=xh(x)/\bar{H}(x)$ in which $\bar{H}(x)=1-H(x)$.According the increasing generalized failure rate property of ε, we obtain $dG(x)/dz<0$. We further obtain $\lim_{x\to 0}\frac{\partial V_S^{ws}}{\partial z}>0$, $\lim_{\pi\to\infty}\frac{\partial V_S^{WS}}{\partial z}<0$. Thus V_S^{WSN} is a concave function of z on the interval $[0,+\infty]$. Therefore, when $\frac{\partial V_S^{wS}}{\partial z}=0$, there exists a unique optimal inventory factor z^* that yields $\int_0^z (k-1)xh(x)\mathrm{d}x=z[1-H(z)]$.

Proof of Theorem 1.

We use backward induction to solve the equilibrium results of the model WSN. According to Lemma 4.1, the objective function of the e-tailer in the second stage of the game is:

$$\Pi_O^{WSN}=pE[\min\{q,D(p,\tau,\varepsilon)\}]-wq-\frac{\xi\tau^2}{2}$$
$$=\frac{\left(\frac{a\tau z}{q}\right)^{\frac{1}{k}}qk(1-H(z))}{k-1}-wq-\frac{\xi\tau^2}{2}.$$

The e-tailer should then decide q and τ because z^* is already determined. The Hesse matrix of Pi_O^{WSN} is

$$H_1(q,\tau)=\begin{pmatrix}\frac{\partial^2\Pi_o^{WSN}}{\partial q^2} & \frac{\partial^2\Pi_o^{WSN}}{\partial q\partial\tau}\\ \frac{\partial^2\Pi_o^{WSN}}{\partial\tau\partial q} & \frac{\partial^2\Pi_o^{WSN}}{\partial\tau^2}\end{pmatrix}.$$

According to $0<H(z^*)<1$, we can see $\frac{\partial^2\Pi_o^{WSN}}{\partial q^2}=\frac{(-1+H(z^*))\left(\frac{a\tau z^*}{q}\right)^{\frac{1}{k}}}{kq}<0$, $|H_1(q,\tau)|=-\frac{(-1+H(z^*))\left(\frac{\alpha\tau z^*}{q}\right)^{\frac{1}{k}}\xi}{kq}>0$. Thus, the Hessian matrix of Pi_O^{WSN} is negative definite.

By jointly solving the equations $\frac{\partial \Pi_o^{WSN}}{\partial q} = \frac{\partial \Pi_o^{WSN}}{\partial \tau} = 0$, we obtain the reaction functions $q_O^{WSN}(w)$ and $\tau_O^{WSN}(w)$. In the first stage of the game, after substituting $q_O^{WSN}(w)$ and $\tau_O^{WSN}(w)$ into the objective function of the supplier. We can derive that

$$V_S^{WS}(w) = \frac{(w-c)a^2(z^*)^2 w\left(\frac{w}{1-H(z^*)}\right)^{-2k}}{\xi(k-1)} - \frac{\alpha_S w^2 a^2(z^*)^2\left(\frac{w}{1-H(z^*)}\right)^{-2k} k(1-H(z^*))}{(k-1)^3(-1+H(z^*))\xi}$$

If $\frac{\partial^2 V_S^{WSN}}{\partial w^2} < 0$, we have the constraint $w_1(w) = ck^2 - k^2 w - \alpha_S kw - ck + 2wk - w > 0$.

We then build the Lagrange function $L_1 = V_S^{WSN}(w) + g_1 w_1(w)$, with KKT conditions $\frac{\partial L_1}{\partial w} = 0$, $g_1 = 0$ and $w_1(w) > 0$. Solving the optimization problem, we can obtain w^{WSN*}. Substituting w^{WSN*} into $q_O^{WSN*}(w)$ and $\tau_O^{WSN*}(w)$, we can obtain q_O^{WSN*} and τ_O^{WSN*}. Substituting q_O^{WSN*} and τ_O^{WSN*} into $p = \left(\frac{q}{a\tau z^*}\right)^{-\frac{1}{k}}$, we can obtain p^{WSN*}.

Substituting w^{WSN*}, q^{WSN*}, τ^{WSN*} and p^{WSN*} into Equations (2)-(4), we can obtain CS^{WSN*}, Π_S^{WSN*}, Π_O^{WSN*} and $\Pi_{SC}^{WSN*} = \Pi_S^{WSN*} + \Pi_O^{WSN*}$. To ensure that Π_S^{WSN*} is positive, we have condition $\alpha_s < \frac{k-1}{2k}$.

Proof of Theorem 2.

We use backward induction to solve the equilibrium results of the model WSO. According to Lemma 4.1, the objective function of the e-tailer in the second stage of the game is

$$V_O^{WSO} = pE\left[\min\{q, D(p,\tau,\varepsilon)\}\right] - wq - \frac{\xi\tau^2}{2} + \alpha_O CS$$
$$= \frac{\left(\frac{a\tau z}{q}\right)^{\frac{1}{k}} qk(1-H(z))}{k-1} - wq - \frac{\xi\tau^2}{2} + \frac{\alpha_0\left(\frac{a\tau z}{q}\right)^{\frac{1}{k}} qk(1-H(z))}{(k-1)^2}.$$

The e-tailer should then decide q and τ because z^* is already determined.

The Hesse matrix of Π_o^{wso} is $H_z(q,\tau)=\begin{pmatrix}\frac{\partial^2\Pi_o^{wso}}{\partial q^2} & \frac{\partial^2\Pi_o^{wso}}{\partial q\partial\tau}\\ \frac{\partial^2\Pi_o^{wso}}{\partial\pi\partial q} & \frac{\partial^2\Pi_o^{wso}}{\partial\tau^2}\end{pmatrix}$.

According to $0<H(z^*)<1$ and $k>1$, we can see

$$\frac{\partial^2\Pi_O^{WSO}}{\partial q^2}=\frac{(k+\alpha_O-1)\left(-1+H(z^*)\right)\left(\frac{a\tau z^*}{q}\right)^{\frac{1}{k}}}{(k-1)kq}<0,$$

$$|H_2(q,\tau)|=\frac{\xi(k+\alpha_o-1)\left(1-H(z^*)\right)\left(\frac{a\tau z^*}{q}\right)^{\frac{1}{k}}}{kq(k-1)}>0$$. Thus, the Hessian matrix of pi_O^{WSO} is negative definite.

By jointly solving the equations $\frac{\partial\Pi_o^{wso}}{\partial q}=\frac{\partial\Pi_o^{wso}}{\partial\tau}=0$, we obtain the reaction functions $q_O^{WSO}(w)$ and $\tau_O^{WSO}(w)$. In the first stage of the game, after substituting $q_O^{WSO}(w)$ and $\tau_O^{WSO}(w)$ into the objective function of the supplier. We can derive that

$$V_S^{WSO}(w)=\frac{(w-c)a^2(z^*)^2 w\left(-\frac{w(k-1)}{(k+\alpha_0-1)\left(-1+H(z^*)\right)}\right)^{-2k}}{\xi(k-1)} -\frac{\alpha_s w^2a^2(z^*)^2\left(-\frac{w(k-1)}{(k+\alpha_0-1)\left(-1+H(z^*)\right)}\right)^{-2k}k\left(1-H(z^*)\right)}{(k-1)^2(k+\alpha_0-1)\left(-1+H(z^*)\right)\xi}$$

If $\frac{\partial^2V_S^{WSO}}{\partial w^2}<0$, we have the constraint

$w_2(w)=ck^2+ck\alpha_0-k^2w-kw\alpha_0-\alpha_s kw-ck+2wk+w\alpha_0-w>0$.

We then build the Lagrange function $L_2=V_S^{WSO}(w)+g_2w_2(w)$, with KKT conditions $\frac{\partial L_2}{\partial w}=0$, $g_2=0$ and $w_2(w)>0$. Solving the optimization problem, we can obtain w^{WSO*}. Substituting w^{WSO*} into $q_O^{WSO*}(w)$ and $\tau_O^{WSO*}(w)$, we can obtain q_O^{WSO*} and τ_O^{WSO*}. Substituting q_O^{WSO*} and τ_O^{WSO*} into $p=\left(\frac{q}{a\tau z^*}\right)^{-\frac{1}{k}}$, we can obtain p^{WSO*}. Substituting $w^{WSO^*}, q^{WSO^*}, \tau^{WSO^*}$ and p^{WSO*} into Equation (2) and Equations (7)-(8), we can obtain CS^{WSO*}, Π_S^{WSO*}, Π_O^{WSO*} and $\Pi_{SC}^{WSO*}=\Pi_S^{WSO*}+\Pi_O^{WSO*}$. To ensure that Π_S^{WSO*} is positive, we have condition $\alpha_s<\frac{k+\alpha_o-1}{2k}$ and $\alpha_O<\frac{k-1}{2k}$.

Proof of Theorem 3.

We use backward induction to solve the equilibrium results of the model RSN. According to Lemma 4.1, the objective function of the e-tailer in the second stage

$$\Pi_O^{RSN}=(1-\upsilon)pE\left[\min\left\{q,D(p,\tau,\varepsilon)\right\}\right]-wq-\frac{\xi\tau^2}{2}$$

of the game is $=\frac{(1-\upsilon)\left(\frac{\alpha\tau z}{q}\right)^{\frac{1}{k}}qk\left(1-H(z)\right)}{k-1}-wq-\frac{\xi\tau^2}{2}$. The e-tailer should then decide q and τ because z^* is already determined. The Hesse matrix of Π_O^{RSN} is $H_3(q,\tau)=\begin{pmatrix}\frac{\partial^2\Pi_o^{RSN}}{\partial q^2} & \frac{\partial^2\Pi_o^{RSN}}{\partial q\partial\tau}\\ \frac{\partial^2\Pi_o^{RSN}}{\partial\tau\partial q} & \frac{\partial^2\Pi_o^{RS}}{\partial\tau^2}\end{pmatrix}$. According to $0<H(z^*)<1$ and $0<\upsilon<1$, we can see

$$\frac{\partial^2\Pi_O^{RSN}}{\partial q^2}=-\frac{\left(-1+H(z^*)\right)\left(\frac{a\tau z^*}{q}\right)^{\frac{1}{k}}(-1+\upsilon)}{kq}<0,$$

$\left|H_3(q,\tau)\right|=\frac{\left(-1+H(z^*)\right)\left(\frac{a\tau z^*}{q}\right)^{\frac{1}{k}}(-1+\upsilon)\xi}{kq}>0$. Thus, the Hessian matrix of Π_O^{RSN} is negative definite.

By jointly solving the equations $\frac{\partial\Pi_O^{RSN}}{\partial q}=\frac{\partial\Pi_O^{RSN}}{\partial\tau}=0$, we obtain the reaction functions q_O^{RSN} and $\tau_O^{RSN}(w)$. In the first stage of the game, after substituting q_O^{RSN} and $\tau_O^{RSN}(w)$ into the objective function of the supplier. We can derive that

$$V_S^{RSN}(w)=\frac{(w-c)a^2\left(z^*\right)^2w\left(\frac{w}{(-1+\upsilon)\left(-1+H(z^*)\right)}\right)}{\xi(k-1)}$$

$$+\frac{\upsilon w^2a^2\left(z^*\right)^2\left(\frac{w}{(-1+\upsilon)\left(-1+H(z^*)\right)}\right)^{-2k}k\left(1-H(z^*)\right)}{(-1+\upsilon)\left(-1+H(z^*)\right)\xi(k-1)^2}$$

$$+\frac{\alpha_Sw^2a^2\left(z^*\right)^2\left(\frac{w}{(-1+\upsilon)\left(-1+H(z^*)\right)}\right)^{-2k}k\left(1-H(z^*)\right)}{(k-1)^3(-1+\upsilon)\left(-1+H(z^*)\right)\xi}$$

If $\frac{\partial^2V_S^{RSN}}{\partial w^2}<0$, we have the constraint

$w_3(w) = -\left(ck^2\upsilon - ck^2 - ck\upsilon + k^2w + \upsilon kw + \alpha_S kw + ck - 2wk - \upsilon w + w\right) > 0$.

We then build the Lagrange function $L_3 = V_S^{RSN}(w) + g_3 w_3(w)$, with KKT conditions $\frac{\partial L_3}{\partial w} = 0$, $g_3 = 0$ and $w_3(w) > 0$. Solving the optimization problem, we can obtain w^{RSN*}. Substituting w^{RSN*} into $q_O^{RSN*}(w)$ and $\tau_O^{RSN*}(w)$, we can obtain q_O^{RSN*} and τ_O^{RSN*}. Substituting q_O^{RSN*} and τ_O^{RSN*} into $p = \left(\frac{q}{\alpha\tau z^*}\right)^{-\frac{1}{k}}$, we can obtain p^{RSN*}. Substituting w^{RSN*}, q^{RSN*}, τ^{RSN*} and p^{RSN*} into Equations (2), (11) and (12), we can obtain CS^{RSN*}, Π_S^{RSN*}, Π_O^{RSN*} and $\Pi_{SC}^{RSN*} = \Pi_S^{RSN*} + \Pi_O^{RSN*}$. To ensure that Π_S^{RSN*} is positive, we have condition $\alpha_s < \frac{k+v-1}{2k}$.

Proof of Theorem 4.

We use backward induction to solve the equilibrium results of the model RSO. According to Lemma 4.1, the objective function of the e-tailer in the second stage of the game is

$$V_O^{RSO} = (1-\upsilon)pE\left[\min\{q, D(p,\tau,\varepsilon)\}\right] - wq - \frac{\xi\tau^2}{2} + \alpha_O CS$$

$$= \frac{(1-\upsilon)\left(\frac{\alpha\tau z}{q}\right)^{\frac{1}{k}} qk\left(1-H(z)\right)}{k-1} - wq - \frac{\xi\tau^2}{2} + \frac{\alpha_0\left(\frac{\alpha\tau z}{q}\right)^{\frac{1}{k}} qk\left(1-H(z)\right)}{(k-1)^2}$$

The e-tailer should then decide q and τ because z^* is already determined. The Hesse matrix of Π_O^{RSO} is $H_3(q,\tau) = \begin{pmatrix} \frac{\partial^2\Pi_o^{RSO}}{\partial q^2} & \frac{\partial^2\Pi_o^{RSO}}{\partial q\partial\tau} \\ \frac{\partial^2\Pi_o^{RSO}}{\partial\tau\partial q} & \frac{\partial^2\Pi_o^{RS}}{\partial\tau^2} \end{pmatrix}$.

According to $0 < H(z^*) < 1$ and $0 < \upsilon < 1$ and $k > 1$, we can see

$$\frac{\partial^2\Pi_O^{RSO}}{\partial q^2} = -\frac{\left(\frac{\alpha\tau z^*}{q}\right)^{\frac{1}{k}}\left((-1+\upsilon)k - \upsilon - \alpha_0 + 1\right)\left(-1+H(z^*)\right)}{kq(k-1)} < 0,$$

$$\left|H_4(q,\tau)\right| = \frac{\xi\left(\frac{\alpha\tau z^*}{q}\right)^{\frac{1}{k}}\left(k\upsilon - k - \upsilon - \alpha_0 + 1\right)\left(-1+H(z^*)\right)}{k(k-1)q} > 0.$$ Thus, the Hessian matrix of Π_O^{RSN} is negative definite.

By jointly solving the equations $\frac{\partial\Pi_O^{RSO}}{\partial q} = \frac{\partial\Pi_O^{RSO}}{\partial\tau} = 0$, we obtain the reaction functions q_O^{RSO} and $\tau_O^{RSO}(w)$. In the first stage of the game, after substituting q_O^{RSO} and $\tau_O^{RSO}(w)$ into the objective function of the supplier. We can derive that

$$V_S^{RSO}(w)=\frac{(w-c)a^2\left(z^*\right)^2 w\left(\frac{(k-1)w}{(ku-k-u-\alpha_0+1)\left(-1+H\left(z^*\right)\right)}\right)^{-2k}}{\xi(k-1)}$$
$$+\frac{vw^2a^2z^2\left(\frac{\binom{k-1}{1}w}{\binom{k\upsilon-k-\upsilon-\alpha_0+1}{1}\left(-1+H\left(z^*\right)\right)}\right)^{-2k} k\left(1-H\left(z^*\right)\right)}{\binom{k-1}{1}\binom{k\upsilon-k-\upsilon-\alpha_0+1}{1}\left(-1+H\left(z^*\right)\right)\xi}$$
$$+\frac{\alpha_S w^2a^2\left(z^*\right)^2\left(\frac{(k-1)w}{(k\upsilon-k-\upsilon-\alpha_0+1)\left(-1+H\left(z^*\right)\right)}\right)^{-2k} k\left(1-H\left(z^*\right)\right)}{(k-1)^2(k\upsilon-k-\upsilon-\alpha_0+1)\left(-1+H\left(z^*\right)\right)\xi}$$

If $\frac{\partial^2 V_S^{RSO}}{\partial w^2}<0$, we have the constrain $w_4(w)=-\left(ck^2u-ck^2-cku-ck\alpha_0+k^2w+uwk+kw\alpha_0+\alpha_S wk+ck-2kw-uw-w\alpha_0+w\right)>0$.

We then build the Lagrange function $L_4=V_S^{RSO}(w)+g_4w_4(w)$, with KKT conditions $\frac{\partial L_4}{\partial w}=0$, $g_4=0$ and $w_4(w)>0$. Solving the optimization problem, we can obtain w^{RSO*}. Substituting w^{RSO*} into $q_O^{RSO*}(w)$ and $\tau_O^{RSO*}(w)$, we can obtain q_O^{RSO*} and τ_O^{RSO*}. Substituting q_O^{RSO*} and τ_O^{RSO*} into $p=\left(\frac{q}{a\tau z^*}\right)^{-\frac{1}{k}}$, we can obtain p^{RSO*}. Substituting w^{RSO*}, q^{RSO*}, τ^{RSO*} and p^{RSO*} into Equations (2), (14) & (15), we can obtain CS^{RSO*}, Π_S^{RSO*}, Π_O^{RSO*} and $\Pi_{SC}^{RSO*}=\Pi_S^{RSO*}+\Pi_O^{RSO*}$. To ensure that Π_S^{RSO*} is positive, we have condition $\alpha_s<\frac{k+v+\alpha_O-1}{2k}$ and $\alpha_o<\frac{k\upsilon-k-\upsilon+1}{1-2k}$.

Proof of Proposition 1.

Taking the first derivatives of p^{j*}, q^{j*}, τ^{j*}, CS^{j*}, w^{j*} with respect to p^{l*}, q^{l*}, τ^{l*}, CS^{l*}, w^{l*} with respect to α_O, respectively. According to the condition $0<H\left(z^*\right)<1$, $k>1$ and $0<\upsilon<1$, Proposition 1 can be proved.

Proof of Proposition 2.

Taking the first derivatives of Π_S^{j*} and Π_O^{j*} with respect to α_S, respectively. According to the condition $0<H\left(z^*\right)<1$, $k>1$ and $0<\upsilon<1$, and the constraints that guarantee positive equilibrium outcomes under each model, $\frac{\partial \Pi_S^{j*}}{\partial \alpha_S}<0$ and $\frac{\partial \Pi_O^{j*}}{\partial \alpha_S}>0$ can be proved.

The below is proof for $\frac{\partial \Pi_S^{l*}}{\partial \alpha_O} > 0$.

Taking the first derivative of Π_S^{WSO*} with respect to α_O, we obtain

$$\frac{\partial \Pi_s^{WSO*}}{\partial \alpha_o} = \frac{n_1(\alpha_s) k \left(-\frac{(2k-1)(k-1)c}{2\left(-1+H\left(z^*\right)\right)\left(k^2+\alpha_0 k+k\alpha_s-2k-\alpha_0+1\right)}\right)^{-2k} (2k-1)a^2\left(z^*\right)^2 c^2}{2\left(k^2+\alpha_0 k+k\alpha_s-2k-\alpha_0+1\right)^3 \xi(k-1)} .$$

We can see $sign\left\{\frac{\partial \Pi_S^{WSO*}}{\partial \alpha_O}\right\} = sign\left\{n_1(\alpha_S)\right\}$, where $n_1(\alpha_S) = A_1\alpha_S^2 + B_1\alpha_S + C_1$, $A_1 = -k$, $B_1 = -2k^3 - 2k^2\alpha_0 + 5k^2 + 3k\alpha_0 - 3k$, $C_1 = (k-1)(k+\alpha_0-1)^2$. Since $k > 1$, we can see $A_1 < 0$, $C_1 > 0$. That is, the image of the quadratic function $n_1(\alpha_S)$ opens downward and $n_1(0) > 0$ because there are constraints $\alpha_S < \frac{k+\alpha_O-1}{2k}$ in the model WSO and $n_1\left(\frac{k+\alpha_0-1}{2k}\right) = \frac{(2k-1)(k+\alpha_0-1)^2}{4k} > 0$. Thus $\frac{\partial \Pi_S^{WSO*}}{\partial \alpha_O} > 0$ can be proved. A similar method can prove that $\frac{\partial \Pi_S^{RSO*}}{\partial \alpha_O} > 0$

The below is proof for $\frac{\partial \Pi_o^{l*}}{\partial \alpha_o} > 0$.

Taking the first derivative of Π_O^{RSO*} with respect to α_O, we obtain

$$\frac{\partial \Pi_s^{RSO*}}{\partial \alpha_o} = \frac{\left(z^*\right)^2 c^2 n_2(\alpha_O) \left(-\frac{c\left(2k^2-3k+1\right)}{2\left(-1+H\left(z^*\right)\right)\left(k^2+k\upsilon+k\alpha_O+k\alpha_O-2k-\upsilon-\alpha_O+1\right)}\right)^{-2k} (2k-1)^2 k\alpha^2}{4\left(k^2+k\upsilon+k\alpha_O+k\alpha_O-2k-\upsilon-\alpha_O+1\right)^3 \xi(k-1)^2} .$$

We can see $sign\left\{\frac{\partial \Pi_O^{RSO^*}}{\partial \alpha_O}\right\} = sign\left\{n_2(\alpha_O)\right\}$, where

$n_2(\alpha_O) = A_2\alpha_O^2 + B_2\alpha_O + C_2$, $A_2 = -2k^2 + 3k - 1$,

$B_2 = (2k-1)\left(\left(k^2-3k+2\right)\upsilon - (k-1)^2 - \alpha_s\right)$, $C_2 = (k-1)^2(u-1)(ku-u+\alpha_s)$.

Since $k > 1$, $0 < v < 1$ we can see $A_2 < 0$, $B_2 > 0$, $C_2 < 0$. Thus $\frac{\partial \Pi_O^{RSO*}}{\partial \alpha_O} < 0$ can be proved. A similar method can prove that $\frac{\partial \Pi_O^{WSO*}}{\partial \alpha_O} > 0$.

Proof of Proposition 3.

Proof of Proposition 3(1):

Taking the first derivative of Π_{SC}^{WSN*} with respect to α_S, we obtain

$$\frac{\partial \Pi_{SC}^{WSN*}}{\partial \alpha_S} = -\frac{k\alpha^2\left(-\frac{(2k^2-3k+1)c}{2(k^2+k\alpha_S-2k+1)(-1+H(z^*))}\right)^{-2k}(2k\alpha_S-k+1)(z^*)^2c^2(2k-1)^2}{4(k^2+k\alpha_S-2k+1)^3\xi}.$$

We can see $sign\left\{\frac{\partial \Pi_{SC}^{WSN*}}{\partial \alpha_S}\right\} = sign\{2k\alpha_S-k+1\}$. Since $\alpha_S < \frac{k-1}{2k}$ in model WSN, $2k\alpha_S-k+1>0$. Thus, $0<\alpha_S<\frac{k-1}{2k}$, then $\frac{\partial \Pi_{SC}^{WSN*}}{\partial \alpha_S}>0$.

Proof of Proposition 3(2):

Taking the first derivative of Π_{O}^{WSO*} with respect to α_S, we obtain

$$\frac{\partial \Pi_{SC}^{WNO*}}{\partial \alpha_S} = \frac{(2k-1)^2 n_3(\alpha_S)k\left(-\frac{(2k-1)(k-1)c}{2(-1+H(z))(k^2+\alpha_0k+k\alpha_S-2k-\alpha_0+1)}\right)^{-2k}a^2(k+\alpha_O-1)z^2c^2}{4(k^2+\alpha_Ok+k\alpha_S-2k-\alpha_O+1)^3\xi(k-1)}.$$

We can see $sign\left\{\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_S}\right\} = sign\{n_3(\alpha_S)\}$, where the root of equation $n_3(\alpha_S) = -2k\alpha_S-2\alpha_0k+k+\alpha_0-1=0$ is $\frac{-2\alpha_0k+k+\alpha_0-1}{2k}$. Since $\alpha_S<\frac{k+\alpha_O-1}{2k}$ in model WSO, if $0<\alpha_S<\frac{-2\alpha_0k+k+\alpha_0-1}{2k}$, $\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_S}>0$; if $\frac{-2\alpha_0k+k+\alpha_0-1}{2k}\le\alpha_S<\frac{k+\alpha_0-1}{2k}$, $\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_S}\le 0$.

Proof of Proposition 4.

Proof of Proposition 4(1):

Taking the first derivative of Π_{O}^{WSO*} with respect to α_S, we obtain

$$\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_O} = \frac{(2k-1)n_4(\alpha_O)k\left(-\frac{(2k-1)(k-1)c}{2(-1+H(z))(k^2+\alpha_0k+k\alpha_S-2k-\alpha_0+1)}\right)^{-2k}a^2z^2c^2}{4(k^2+\alpha_0k+k\alpha_S-2k-\alpha_0+1)^3\xi(k-1)^2}.$$

We can see $sign\left\{\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_O}\right\} = sign\{n_4(\alpha_O)\}$, where

$n_4(\alpha_O) = A_4\alpha_O^2+B_4\alpha_O+C_4$, $A_4=-4k^3+10k^2-9k+3$,

$B_4 = \left(-4k^3 + 6k^2 - 2k - 1\right)\alpha_S - 4k^4 + 16k^3 - 25k^2 + 18k - 5$,

$C_4 = -(k-1)\left(2k^2 - 4k + \alpha_S + 2\right)\left(2k\alpha_S - k + 1\right)$. According to $k > 1$, $0 < \upsilon < 1$, we can see $A_4 < 0$, $B_4 < 0$. We then determine the sign of C_4, $sign\{C_4\} = sign\{-2k\alpha_S + k - 1\}$. The root of equation $-2k\alpha_S + k - 1 = 0$ is $\alpha_S^1 = \frac{k-1}{2k}$. Since $\alpha_S < \frac{k+\alpha_O-1}{2k}$ in model WSO, if $0 < \alpha_S < \alpha_S^1$, $C_4 > 0$; if $\alpha_S^1 < \alpha_S < \frac{k+\alpha_O-1}{2k}$, $C_4 < 0$.

In summary, the image of the quadratic function $n_4(\alpha_O)$ opens downward, and the axis of symmetry is negative. When $0 < \alpha_S < \alpha_S^1$, $C_4 > 0$, the zero point of the function $n_4(\alpha_O)$ is α_{O1}. If $0 < \alpha_O < \alpha_{O1}$, $\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_O} > 0$; If $\alpha_{O1} \le \alpha_O < \frac{k-1}{2k-1}$, $\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_O} \le 0$. When $\alpha_S^1 \le \alpha_S < \frac{k+\alpha_O-1}{2k}$, $C_4 < 0$, we can see $n_4(\alpha_O) < 0$. Therefore, $\frac{\partial \Pi_{SC}^{WSO*}}{\partial \alpha_O} < 0$;

The proof of Proposition 4(2) is similar.

Proof of Proposition 5.

We can prove that

$$p^{WSO*} - p^{WS*} = \frac{(2k-1)(k-1)^2 \alpha_0 c}{2\left(k^2 + k\alpha_S - 2k + 1\right)\left(-1 + H\left(z^*\right)\right)\left(k^2 + k\alpha_0 + k\alpha_S - 2k - \alpha_0 + 1\right)} < 0,$$

$$p^{RSO*} - p^{RS*} = \frac{(2k-1)(k-1)^2 \alpha_0 c}{2\left(-1 + H\left(z^*\right)\right)\left(k^2 + k\upsilon + k\alpha_O + k\alpha_S - 2k - \upsilon - \alpha_O + 1\right) \times \left(k^2 + k\upsilon + k\alpha_S - 2k - \upsilon + 1\right)} < 0,$$

$$p^{RS*} - p^{WS*} = \frac{c\upsilon(2k-1)(k-1)^2}{2\left(k^2 + k\alpha_S - 2k + 1\right)\left(-1 + H(z)\right) \times \left(k^2 + k\upsilon + k\alpha_S - 2k - \upsilon + 1\right)} < 0,$$

$$p^{RSO*} - p^{WSO*} = \frac{c\upsilon(2k-1)(k-1)^2}{2\left(k^2 + k\upsilon + k\alpha_0 + k\alpha_S - 2k - \upsilon - \alpha_0 + 1\right) \times \left(-1 + H(z)\right)\left(k^2 + k\alpha_0 + k\alpha_S - 2k - \alpha_0 + 1\right)} < 0.$$

According to $\frac{p^{RS*}}{p^{WSO*}} - 1 = -\frac{(\upsilon - \alpha_O)(k-1)}{k^2 + k\upsilon + k\alpha_S - 2k - \upsilon + 1}$, we can derive that: if $0 < \upsilon < \alpha_O$, $p^{RS*} > p^{WSO*}$; if $\alpha_O < \upsilon < 1$, $p^{WSO*} > p^{RS*}$. Combining the above results, we complete the proof of Proposition 5.

Proof of Proposition 6.

$\frac{q^{WSO*}}{q^{WS*}}=\frac{(2k-1)(k+\alpha_O-1)\left(p^{WSO*}\right)^{-2k}\left(k^2+k\alpha_S-2k+1\right)}{\left(k^2+k\alpha_O+k\alpha_S-2k-\alpha_O+1\right)\left(2k^2-3k+1\right)\left(p^{WN}\right)^{-2k}}$. According to $p^{WSO*}-p^{WS*}<0$, we can see $\left(\frac{p^{WSO*}}{p^{WS*}}\right)^{-2k}>1$. We can obtain that

$$\frac{(2k-1)(k+\alpha_O-1)\left(k^2+k\alpha_S-2k+1\right)}{\left(k^2+k\alpha_O+k\alpha_S-2k-\alpha_O+1\right)\left(2k^2-3k+1\right)}-1=\frac{k\alpha_O\alpha_S}{(k-1)\left(k^2+k\alpha_O+k\alpha_S-2k-\alpha_O+1\right)}>0,$$

i.e., $\frac{(2k-1)(k+\alpha_0-1)\left(k^2+k\alpha_S-2k+1\right)}{\left(k^2+k\alpha_0+k\alpha_S-2k-\alpha_0+1\right)\left(2k^2-3k+1\right)}>1$. Combining the above results, we complete the proof of $\frac{q^{WSO*}}{q^{WS*}}>1$. The proofs of the remaining inequalities in Proposition 6 are similar.