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On the Global Economic Exploitation Process: Steady States and Stability Aspects for economic Networks with Scale-Free and Limited (Cut-Off) Transitions

—The Concrete Case of the European Union

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Abstract

The main objective of this Article is to determine the Steady States of two probable economic Scenarios between national economies within the global economic exploitation Process. A mathematical tool extracting the Steady States on the specific Dynamics could be the modeling of appropriate Differential Equations. As Steady States represent solutions that remain unchanged in time evolution, we define these Dynamics Constellations as "Metastability". However, Metastability can not be identified unconditionally with Economic Stability. We demonstrate in this Article that unrelated Aspects and Criteria could lead to meaningful Concepts for economic Stability (for example, in analyzing the Concept of the small world Networks). Additionally, we point out that Criteria for economic Criticality are often contained in the Metastability Phase in the form of hidden Information. In this Article, we observe two important and probable economic Constellations, which could appear within the global economic Exploitation Process. As we observe and focus on the economic Relations of national economies it is necessary to define at this stage the concept of economic Antagonism. An antagonistic economic environment means that the economic exploitation of the players depends only on their economic strength (power) tending to reach economic Dominance. This economic environment can be compared with a fair sports Discipline while the players have generally different Levels of Ability (competitiveness, Competence) to reach Superiority. In this sense, we look for Steady States using the Lotka-Volterra Model as it seems to be appropriate to simulate such Dynamics. In a further Step, we proceed to consider additively time-fractal Aspects in the classical Lotka-Volterra Differential Equations. In this context, two possible Scenarios within the global economic exploitation

Process (Globalization) could be established. The first one is an antagonistic environment in which the exchange of Goods and services is not characterized by any Trade Restrictions. Specifically, the national economies do not impose Customs and one single currency is valid for trade activities. This economic Constellation could be described as a specific economic Alliance between national economies and, in an extreme case, could take the form of a single national economy worldwide, while the word "national" degenerates as not meaningful. Further, the second Scenario within the global exploitation economic Process could consist of a strong protective economic environment in which the Players impose Customs and establish Exchange-currency Strategies or other Trade restrictions in order to control and monitor their Export-Import Relations. Using Concepts of the Econophysics we characterize the first Scenario as a transition-free Environment and the second one as a protective economic Frame. We extract within the first antagonistic Scenario the Steady States with the Help of the Lotka-Volterra Model as the unequal Competitiveness Level of the Players leads to Predator-Pray Dynamics. According to this aim, we define the Competitiveness Level of a national economy by constructing a Rang Scala with appropriate normalized Parameters. In a further Step, we detect economic Instability by analyzing specific Relations within the economic "Small World Networks" concept. Finally, we extract mathematically Steady States considering both Scenarios with the Help of the "topological induced Diffusion" Concept, assuming that the exchange activities of economic Players depend on the environment in which they evolve. We proceed using the fractional Master Equation, which provides Solutions in a purely stochastic sense opening a big window for the interpretation of Steady States in Dynamics. Using the Concept of the topological induced Diffusion we identify the transition-free Scenario as a scale-free Scenario as the economic transitions are assumed to be unlimited. In this case, we assume that the economic Exploitation Process takes place in a superdiffusive environment (Levy Flights). Within the second Scenario, we assume the Existence of Cut-Offs in the Dynamics caused by the diverse trade restrictions. At this stage, it is of interest to observe the Experiences between the single economies of the European Union. The Exchange of Goods and services takes place without important Trade Restrictions representing a very important particular Paradigm in the Globalization Process. However, the E.U. promises economic Cohäsion between the single economies establishing Compensation Funds and other Balance Strategies. This Fact has been respected in this Article by incorporating a specific parameter in the Lotka-Volterra Model. Thus the E.U. economic evolution could stand as a realistic and paradigmatic Concept within the global economic exploitation Process.

Keywords

Steady States, Economic Stability, Lotka-Volterra Differential Equations, Time-Fractality, Topological Induced Diffusion, Small World Networks

1. Introduction

The global economic exploitation between single national Economies is a catalytic factor for World Freedom, having a high impact on Conflicts in a quickly changing World. An important fact (Dimension) of Global economic exploitation consists in the Differences in the welfare between the single national economies. Additively of high Interest is the question of economic Dominance within the economic evolution worldwide. Thus a possible economic Cohäsion or Divergence of the Players is an important factor in detecting Stability Aspects. At first, we begin the analysis using the classical Lotka-Volterra Model as we observe a transition-free antagonistic economic Environment in which the Competitiveness Grade of the Players is of fundamental meaning characterized by Predator-Pray Dynamics. The Question of Cohäsion has been respected introducing a specific parameter in the Model. In this case, the stability will be extracted with the Help of the Lotka-Volterra Differential equations. As we are interested in an extended Concept of stability we observe economic clusters in the form of Networks analyzing specific Relations. At last, we use the "topological induced Diffusion" Model to bother Steady States and critical Distributions in scale-free and Networks with limited transitions.

2. A General Ansatz of Economic Dynamics in the Global Economic Exploitation Process Following the Classical Predator-Prey Model of Lotka and Volterra (Lotka, 1925; Voltera, 1931)

The Exploitation of globalized Dynamics in an antagonistic and transition-free economic environment in view of Capital Concentration, Export Activities, or National Product (per Capita) between national Economies could be approached by the following Ansatz:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \Gamma x - \lambda xy \tag{1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -(\alpha + \mu) y + \sigma xy \tag{2}$$

The corresponding Variables do have the following meaning:

X = Square Displacement of the Export activities (per Capita) Distribution between National Economies;

y = Square Displacement of the Grand National Product per Capita Distribution.

The Coefficients (Parameters) $\gamma, \lambda, \alpha, \mu, \sigma$ have the following economic meanings:

Γ = Distance Mass between the Competitiveness Indexes γ_i of the i National Economies being under Consideration;

λ = Customs on Export Activities;

α = Balance Benefits to establish economic Cohesion;

μ = Technological Benefits;

σ = economic wealth benefits, entailed through local cost minimization.

The γ-Parameter arises naturally from a benchmark scaling of the following factors, which determine the competitiveness Ability (economic attractivity) of a national economy:

Technology Know-how by Firms, Organizations, and Education, Investment in Research and Development, Capital Concentration, Supporting Economic Activities through the State, political Stability, Ability to produce Goods and Services at low Costs by certain quality, and others. However, the above Coefficients $\Gamma, \lambda, \alpha, \mu, \sigma$ should be appropriately normalized (pro Capita). The above factors can count also for the Competitiveness between Firms and Organisations. "Γ" represents a Distance mass on the γ_i quantity. For example, "Γ" could be represented by the Identity $\Gamma = \sum_i |\gamma_i - \gamma_\mu|$, whereas $\gamma_\mu = \langle \gamma_i \rangle =$ Mean Value. In order to estimate the stationarity Conditions of the above system (1) we use the Jacobi Matrix, which leads to a Trace of Zero and $Det\ J = \Gamma(\alpha + \mu)$. The Eigenvalues $\lambda_{1,2}$ of the System can be estimated with the Help of

$$\lambda_{1,2} = \frac{Trace}{2} \pm \left(\frac{Trace^2}{4} - Det\ J \right)^{0.5} = \pm i \sqrt{\Gamma(\alpha + \mu)} \tag{3}$$

The evaluation of the Jacobi Matrix does not allow statements for the stability of no trivial stationary solutions of the Predator-Prey Model. To get some insight into the trajectories of the solutions we use the relation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\partial y}{\partial t} * \frac{\partial t}{\partial x} = \frac{\dot{y}}{\dot{x}} = \frac{g(x, y)}{f(x, y)} \frac{-((\alpha + \mu) y + \sigma x y)}{\Gamma x - \lambda x y} \tag{4}$$

With the help of Variable separation, and integration we get the time-invariant solution (Solution Trajectories)

$$\begin{aligned} F(x, y) = {} & \Gamma \ln(y) - \lambda y - \Gamma \ln(y_0) + \lambda y_0 - (\alpha + \mu) \ln(x) \\ & - \sigma x + (\alpha + \mu) \ln(x_0) + \sigma(x_0) \end{aligned} \tag{5}$$

with y_0, x_0 initial Values (see Aulbach's Gewöhnliche Differential Gleichungen (Aulbach, 2004)). From a physical Point of view, we can state that the classical Lottka-Volterra Model exhibits stationary solutions dependent on the y_0, x_0 Values. To get a partial view of (5) we use the following Relation.

$F(x, y) = 1.2 * \log[y] - 0.8y + 0.8 + 0.9 \log[x] - 0.4x + 0.4$ with $y_0, x_0 = 1$ in the area $\{x \in [0.1, 5]\}, \{y \in [0.1, 5]\}$ (**Figure 1**).

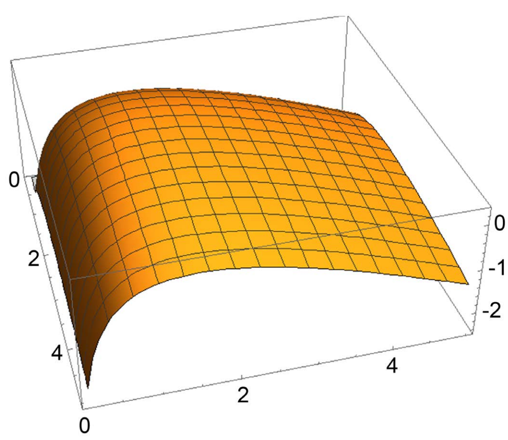


Figure 1. Time invariant Solution $F(x, y)$ versus x, y changes assuming certain quantities of the Model parameters and initial Values of x, y.

Important Remark: 1) According to the Theory of Stability for Systems of Differential Equations of integer Order the above-considered System exhibits Stationary Solutions. The *x*, *y* Variables move cyclically at a certain distance to each other. A Stability Criterion, in the economic sense, consists in the normalized mass of the Distance between the two Variables. The Coefficients of the considered Equation System are in reality no Constants and do have their own Dynamics. A good approximation for the solutions could be obtained, assuming and defining analytic functions for $\Gamma, \lambda, \alpha, \mu, \sigma$. However, the above classical Predator-Prey Model allows only a general insight into the Stability Aspects.

2) The case of well-adjusted Export-Import Balances between the *i* national economies, does not mean equality for the bulge of Export Volume in every economy. Thus the Distribution of the percentage share of Exports in the economies usually is not of Gauss character. Further must be noted that Exports and Imports in an economy are not of the same importance in view of their significance for economic development.

3. The Fractional Aspect of the Lotka-Volterra Model (Time-Fractionality)

It is of high interest in a mathematical sense to solve the above Predator-Prey Model regarding fractional Aspects, although the variability of the Coefficients remains a problem. In this context, we consider the system

$$\frac{\partial^\alpha x}{\partial t^\alpha} = \Gamma x - \lambda xy \tag{6}$$

$$\frac{\partial^\beta y}{\partial t^\beta} = -(\alpha + \mu) + \sigma xy \tag{7}$$

with $\Gamma, \lambda, \alpha, \mu, \sigma$ like above, and $\alpha, \beta \in (0,1)$ reals. Looking for stationarity we set

$$\frac{\partial^\alpha x}{\partial t^\alpha} = \Gamma x - \lambda xy = 0 \tag{8}$$

$$\frac{\partial^\beta y}{\partial t^\beta} = -(\alpha + \mu) y + \sigma xy = 0 \tag{9}$$

To solve the system (8) (9), we use the Homotopy Method (see "Solving Systems of Fractional Differential Equations by Homotopy-Perturbation Method" (Abdulaziz, Hashim, & Momani, 2008)).

We estimated for the general fractional Predator-Prey Model $\frac{\partial^\alpha x}{\partial t^\alpha} = ax - bxy$, $\frac{\partial^\beta y}{\partial t^\beta} = -cx + dxy$ following solutions:

$$\begin{aligned} X(t) = {} & \frac{(a-b)t^\beta}{\text{Gamma}[1+\beta]} + \frac{\left(a^2 - ab + b(c-d)\right)t^{2\beta}}{\beta \text{Gamma}[\beta]\text{Gamma}[1+\beta]} \\ & - \frac{\left(-a^3 + a^2 b + ab(-2c+d) + b\left(bc + d(-c+d)\right)\right)t^{3\beta}}{\beta^2 \text{Gamma}[\beta]^2 \text{Gamma}[1+\beta]} \end{aligned} \tag{10}$$

$$Y(t)=\frac{(-c+d)t^{\beta}}{\text{Gamma}[1+\beta]}-\frac{(ac-bc+(c-d)d)t^{2\beta}}{\beta\text{Gamma}[\beta]\text{Gamma}[1+\beta]}-\frac{(ac(a^2-ab+bc)-(a+(-1+a)b)cd-cd^2+d^3)t^{3\beta}}{\beta^2\text{Gamma}[\beta]^2\,\text{Gamma}[1+\beta]} \tag{11}$$

under the Conditions: $a,b,c,d>0$, $X(0)=1$, $Y(0)=1$, $\alpha,\beta\in(0,1)$.

The Definition for the stability of Differential Equations of integer Order reads as follows: For a given differential equation with integer Order of the form $\frac{\mathrm{d}x}{\mathrm{d}t}=f(t,x)$ with the solution $\mu(t)$ counts for Stability the following statement: For every $e>0$, and every $t_0>t^0$ does exist a $\delta=\delta(x,t_0)>0$ so that forevery initial Value ξ with $\|\xi-\mu(t)\|<\delta$ there is a Solution $\lambda(t;t_0,\xi)$ with $\|\lambda(t;t_0,\xi)-\mu(t)\|<e$ for every $t>t_0$ (see "Gewöhnliche Differentialgleichungen" (Aulbach, 2004)). However, the stability in the fractional sense differs from that of classical Interpretation (see for Details the Article "Fractional Stability" (Tarasov, 1997)). It is important to note here that the Concept of fractional Stability is much wider than the "Lyapunov" one. Especially "*a dynamical system, which is unstable with respect to first variations, can be stable with respect to fractional variation*" (Citation by V. Tarasov). We show here paradigmatically the Graphs of $f(t)=X(t),Y(t)$ with $a=1$, $b=0.8$, $c=0.9$, $d=0.4$, $\alpha,\beta=0.7$, and the Initial Conditions $X(0)=2$, $Y(0)=1$ (**Figure 2**), and $X(0)=1$, $Y(0)=2$ (**Figure 3**). The Function $d(t)$ could hold as a relative stability Criterion considering the convergence or divergence of $d(t)=X(t)-Y(t)$. For high importance in this sense is the behavior of the Variables for the time tending to infinity.

The Existence of Stationarity Values depends however on the Coefficients a,b,c,d, on the Initial Conditions, and the Fractality Order.

Important Remark: We note that the Distance between the Solutions of the above system becomes higher in Values versus time evolution. Therefore Stability in the sense of differential systems of integer order does not exist. Varying the fractal order we gain the curve of **Figure 4**, which exhibits a Zero.

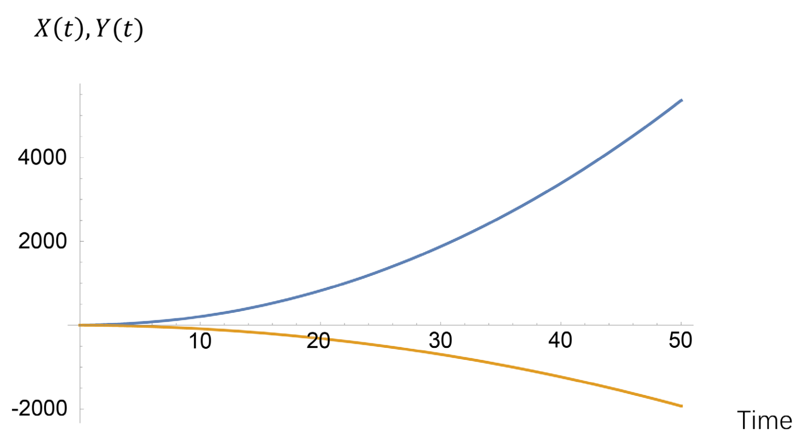


Figure 2. The functions $X(t)$, $Y(t)$ versus time. ($X(t)$ blue line, $Y(t)$ brown line) by certain initial conditions $X(0)=2$, $Y(0)=1$.

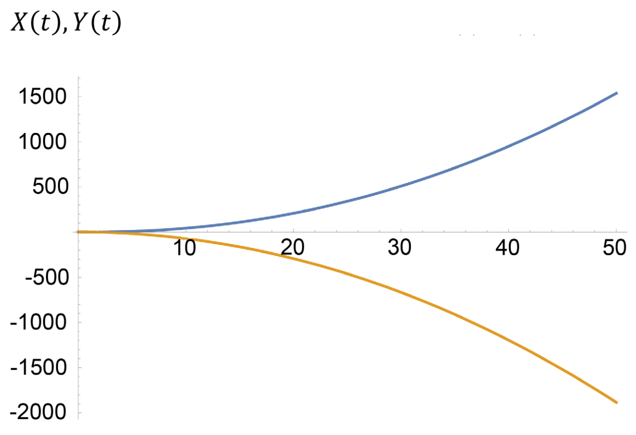


Figure 3. The function $X(t)$, $Y(t)$ versus time by certain initial conditions $X(0) = 1$, $Y(0) = 2$.

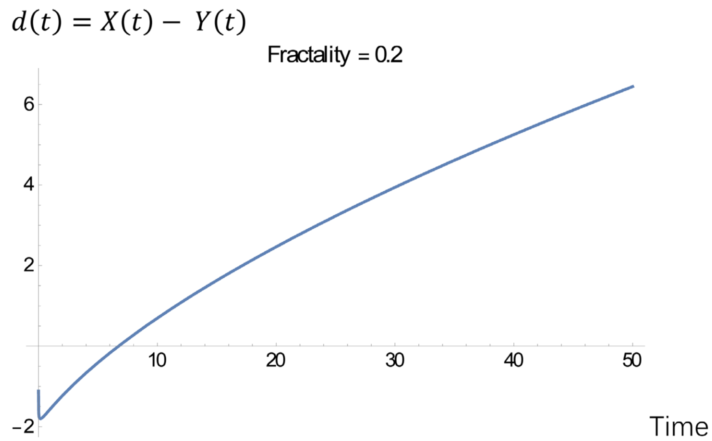


Figure 4. Distance between the solutions of the system variables by certain fractality order.

4. Extended Aspects of Stability in Coiled National Economies. The Small World Network Structure: Physical Aspects and Criticality

To get results about the stability of economic Clusters, which could be modeled with the Help of the small World Network dynamics, one should not restricted only to the Theory of Differential Equations and the corresponding stationarity. In order to guarantee scientific respectability it is first of major Importance to define the Term "Stability" as quite an operable Quantity. Especially in the case of E.U. the Special Conditions of the economic Activeness allow us to consider it as a network, particularly in view of the Import-Export Relations. Normally the Transport of Goods and Services in the network should not be referred to as Import-Export-Activities as Customs and related Restrictions fail. E.U. acts economically as a single national economy, although it is not. However, the taxis must pay Firms and Enterprises are connected to the national economy in which the firms are registered. This Hybrid Character allows us nonetheless to observe flows of Goods and services as a network identifying Production Source Points, and Consumer Areas (destination) formally as national economies. The network analysis allows especially the detection of Stability regarding the Probability Density Distribution Functions that arise naturally on the Connectivity structure of the Network. D.J Watts and D.H. Strogatz defined in their Article in "Nature" 1998, the concept of "Small-world networks" with the help of which we can bother Stability in economic relations.

Classification of small-world networks: Evidence of the occurrence of small-world network (see for Details in "Classes of the Behavior of Small-World Networks" (Amaral, Scala, Barthelemy, & Stanley, 2000)). According to the connectivity Distribution, we differ in three major classes: 1) Scale-free networks at which the Distribution decays as a power law. 2) Broad-scale networks characterized by a power law regime followed by a sharp cut-off. 3) single-scale networks characterized by a connectivity distribution with a fast-decaying tail. It should be here noted that for broad-scale and single-scale networks, the addition of new links underlies specific Conditions. The connectivity Conditions are the most significant controlling factor, which determines the emergence of diverse classes in the network topology (see also (Barabási, 2006)).

In the actual activeness of the E.U Network, we can state the extraordinary role of adding new members to the Union. From this point of view, we should categorize the E.U. as a scale-free network, referring to the Export-Import Relations within. One of the most significant factors in analyzing the emergence of network classes and their specific characteristics consists of the preferential connection of adding new members. Despite the Class categorization of the network, the structure of the decaying tail is the most critical factor for the dynamics of a network, from a physical point of view. Preferential Connection leads mathematically to power Law decay (see also (Barabási, 2006)). The newly-formed fellows generate Import-Export Activities to the Hubs, in fast 100% of the cases. The last are Nodes (Vertices) in a network that have the most connections to the other members of the network. This is but a criterion to define Stability/Criticality within a network. Power Law distributions fail to have a definite square displacement and under specific conditions a definite expected value. Avoiding describing details of the Emergence of economic Networks, and Political Events, which could have an unexpected impact on a network structure, we state at this stage the following: The transition of the connection Probability Distribution from a LogNorm Form to a power Law one, defines a critical Event in economic networks. Adding new fellows (contacts) to a network that leads to Hubs has economic Consequences as Changes in the Hubs do have significant Impact on the Rest of the Members. The occurrences of Hubs mean a higher economic activation at these vertexes associated with possibly corresponding higher economic Divergence between the Members. The goal of economic Cohesion gets out of control even though the Import-Export Balances are well-adjusted. This Criticality aspect does not correspond with a higher probability of getting Stationarity of the physical point of view. To this problem, we will focus on details in the next chapters (**Figure 5**).

However, the Analysis of Network Stability could refer to additional Aspects like critical Diameter, the costs of adding links to vertices, or the limited capacity of a vertex (Amaral, Scala, Barthelemy, & Stanley, 2000) to which we do not obtain with details at this stage.

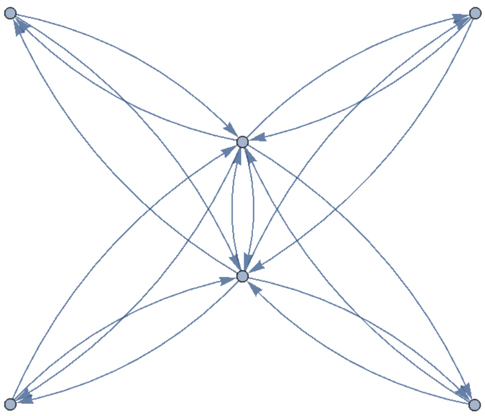


Figure 5. Network-graph with 2 hubs.

5. The Economic Cluster "European Union" as a Specific Economic Alliance

We define here as an economic Cluster the particular Alliance of national economies while the Export-Import Activities underlie specific Conditions and Treaties. A typical economic Cluster in this Context is the European Union. The following most significant Characteristics presuppose the Import-Export Activation and Investition Capital in the E.U.

a) It fails Customs between the Import-Export Activities of the Members of the Union.

b) One single currency is valid for all Payments (Economic Activities).

c) The Transport of Goods, Money, Technology, and Knowledge does not underlie specific Regulations (Restrictions) between the Union.

d) The E.U. tends to realize an economic Convergence (Cohesion) between the diverse national Economies with the Help of compensation payments and economic Balance Strategies.

The above characteristics define a transition-free antagonistic economic Frame. Therefore Costs in the single economies tend to be more transparent. The E.U. is not a national Economy but represents an economic Group with strong national Properties. Economic Clusters like the E.U. could play an important role in the Globalization Process. Thus their economic exploitation especially the fulfillment of the "Cohäsion Criterion" could have an immense Impact on the global economic Exploitation Process.

Recommendations on the E.U. Economic Evolution

By observing and analyzing the economic Network E.U. we can state:

1) There are two strong Hubs (Germany and France) in the European Union Network caused by the preferential economic connection adding new Members.

2) While the Growth of the national Grand Product of the Hubs appears to be continuous, the other members are characterized by a significant high displacement in their NGP Development.

3) The Welfare Differences between the Members are in the E.U. relatively high. The corresponding Distribution is not of Gauss Character.

4) Economic Statistics do not indicate still an economic Cohesion Trend within the EU.

6. Detecting Steady States on Specific Global Economic Exploitation Scenarios with the Help of the Topologically Induced Diffusion Concept (Brockmann, 2003)

In this Chapter, we use the Concept of the "topologically induced Diffusion" to extract Steady States (Stability) for the above-described two Scenarios within the global economic Exploitation Process.

(The chapter "D" is a free interpretation of chapter "3" "Topological Diffusion" of the Dissertation of D. Brockmann)

The economic activities within a cluster in which the associated processes are running quite barrier-free establish an economic environment that can be characterized as quantum-mechanical. The complexity reduction and the simulation of dynamics is a difficult undertaken. Popular physical Concepts like the Langevin Ansatz fail usually to reproduce the Dynamics in this specific environment. A significant Results-promising Concept in this context is that of the "topological induced Diffusion" introduced by M. Brockman in his Promotion 2004…The Analysis of this concept and some corresponding Applications concerning the detection of stationarity is the centerpiece of our Article. Long-tailed spatial transitions in the sense of Random Walk Dynamics, are the result of the complex topology on which they evolve. Thus they differ significantly from generalized Langevin Dynamics where the existence of physical forces has to be assumed. Topological-induced Diffusion can be effectively simulated as a Random Walk on a hetero-polymer in the form of a small-world network. Of fundamental Importance are two characteristic features of the polymer. They are the conformation state and the Euclidean Distance. A thermally activated Random Walker performs a hopping process along the network. The conformation state in our concrete research case does have a chemical character and can be identified as the competitiveness state of the economic polymer. The polymer changes rapidly its conformational state. Therefore positions, which are far along the chemical axis may decrease their Distance in the Euclidean sense. Folding Polymers can be rigid, coiled, or flexible. Rigid chains make no long or intermediate-range contacts. Coiled polymers exhibit intermediate connections with a typical length σ. A flexible polymer does have conformational states with long-range connections on all scales. One could assume that Levy Flights, which can be interpreted as spatial Transitions of a Random Walk with an indefinite Displacement, could be the result of a fluctuating Drift (Force) and fractional Diffusion Term leading to a generalized Langevin Ansatz. Levy Flights can occur but in thermally inconspicuous systems like in economic networks. The Diffusion of such structures is caused by the topology on which they evolve. An appropriate Tool for simulating dynamics in the topologically induced diffusion is the conditional probability function (pdf) of finding the Random Walker at site x, and at time t starting by t_0, x_0. A potential $V(x)$ at each site x indicates the energetic state of the walker when it is attached to monomer *x*. The monomer *x* (Coordinate) quantifies chemical distance along the chain or network. The Walker jumps randomly along the chain with a typical waiting time τ. The Process can be characterized as the Markovian Jump Process at this stage, neglecting memory effects. The Propagator of such processes is known in the literature as the "Master Equation" and is of pure probabilistic character. The formula reads as

$$\partial_t(p,t) = \int \mathrm{d}y\left[w(x \mid y,t)\,p(y,t) - w(y \mid x,t)\,p(x,t)\right] \tag{12}$$

The time-dependent rate $w(x \mid y,t)$ means the Probability of making a jump from monomer y to monomer x at the instant time t within a short time interval Δt. The thermally conditioned (Gibs-Boltzmann Statistics) transition from *y* to *x* can be defined as probability quantity by the formula

$$y \to x : w(x \mid y,t) \propto \frac{1}{\tau}\mathrm{e}^{-\beta[V(x)-V(y)]/2} \tag{13}$$

Two basic Assumptions condition the Formula (13). 1) The probability of jumping to a target site x decreases as the Potential at that site $V(x)$ increases. 2) The hopping between monomers depends on the energetic difference concerning the Gibbs-Boltzmann energy scale $K_B T = \beta^{-1}$. Taking into account the geometrical complexity of the system, it must be said that transitions may occur if the corresponding sites are nearby in Euclidean space. This means the relation $\left[R(x,t) - R(y,t)\right] \le \alpha$ must hold, where $R(x,t), R(y,t)$ denote the Euclidean Coordinates of the monomers. In a nearly linear chain, the Walker jumps between neighboring sites, whereas in a more complex conformational state locations far apart along the chemical coordinates could converge in Euclidean space. Taking into account the thermal as well as the geometrical aspect, we can make the following ansatz for the rate:

$$w(x \mid y,t) = \frac{1}{\tau}\mathrm{e}^{-\frac{\beta[V(x)-V(y)]}{2}} * G(x,y;t) \tag{14}$$

for $G(x,y;t) = 1$ if $\left[R(x,t) - R(y,t)\right] \le \alpha$ and $G(x,y;t) = 0$ otherwise.

The definition of the geometrical constraint implies the symmetry $G(x,y;t) = G(y,x;t)$. Additionally, at any time, the condition of detailed balance is fulfilled:

$$w(x \mid y,t) * \mathrm{e}^{-\beta V(y)} = w(y \mid x,t)\,\mathrm{e}^{-\beta X(t)} \tag{15}$$

The parameter τ in Equation (14) is the typical waiting time in a given Location. At this stage, it must be noted that the geometrical constraint factor $G(x,y;t)$ is extremely complex, especially as a function of time.

6.1. The Case of Rigid Chains

In this case, the walker performs jumps only on nearest neighbor states. Rigidity and neighbor hopping provide a geometrical constraint factor $G(x,y;t)$, which is time-independent and homogenous in chemical space in the form

$$G(x,y;t]=g(x-y) \quad \text{with} \quad g(x)=\frac{1}{2}*(\delta(x-a)+\delta(x+a)) \tag{16}$$

On scales larger than the intermonomer space α and times larger than the typical waiting time τ, the process is asymptotically governed by the diffusion limit $\tau,\alpha \to 0$ with $D=\alpha^2/2\tau$ = Diffusion Coefficient of the system. Inserting (16) into (15), after algebraic manipulations, and assuming $D=1$, we get the following formula for the master equation:

$$\partial_t p = \mathrm{e}^{-\frac{\beta V}{2}} \Delta \mathrm{e}^{\frac{\beta V}{2}} * p - p\mathrm{e}^{\frac{\beta V}{2}} \Delta \mathrm{e}^{-\frac{\beta V}{2}} = \mathcal{L}_p \tag{17}$$

with Δ = Laplacian, $V=V(x)-V(y)$, and $p=p(x,t)$.

By expanding the Laplacians in (17) one gets the final Form

$$\mathrm{e}^{-\frac{\beta V}{2}} \Delta \mathrm{e}^{\frac{\beta V}{2}} * p - p\mathrm{e}^{\frac{\beta V}{2}} \Delta \mathrm{e}^{-\frac{\beta V}{2}} \propto -(\nabla F + \Delta) p \quad \text{with} \quad F(x) = -\beta * \frac{\mathrm{d}V(x)}{\mathrm{d}x} \tag{18}$$

The above Operator is an ordinary Fokker-Planck Operator, observing deterministic gradient dynamics in the potential V subjected to Gaussian white noise. Thus, on scales larger than the inter-monomer spacing, the process can be modeled by the associated Langevin Ansatz

$$\mathrm{d}X(t) = \beta * \frac{\mathrm{d}V(x)}{\mathrm{d}x}\mathrm{d}t + \mathrm{d}W(t) \tag{19}$$

In which $\mathrm{d}W(t)$ is the differential of the Wiener process. In the Limit, the Jump process becomes a diffusion process with continuous sample paths.

6.2. Applying the Mean Field Theory for Non-Local Transitions: The Case of Coiled Polymers

Observing the most interesting and relevant cases in our study of rapidly folding polymers it is meaningful to interpret the master equation by averaging the disorder as the geometrical constraint factor $G(x,y;t)$ isa complex stochastic process itself. We study the expression $\partial_t \|p\| = \|\mathcal{L}_m(t)p\|$, where $\|.\|$ denotes the disorder average. The disorder average $\|G(x,y;t)\|$ is just the probability, of finding monomers x and y at a distance from each other less than "α" in Euclidean space. That means $\|G(x,y;t)\| = P(|R(x,t)-R(y,t)|) \le \alpha$. If the conformational dynamics are equilibrated the above problem is time-independent. This leads to the assumption that the probability is a function "ρ", which depends on the chemical distance $|x-y|$. I.e. $P(|R(x,t)-R(y,t)|) \le \alpha \propto \rho(|x-y|)$. Thus the mean-field approximation yields a temporally homogenous master equation with time-independent rates

$$w(x \mid y) = \frac{1}{\rho}\mathrm{e}^{-\beta[V(x)-V(y)]/2} * \rho(|x-y|) \tag{20}$$

Detailed Balance is on the above function fulfilled, and can be assumed that it decreases with increased distance.

__Coiled Polymers:__ The above-described folding polymer can be characterized as a coiled polymer, which performs changes by a typical scale σ. This length scale σ represents a natural cutoff, above which long-range transitions are highly unlikely to occur. Proceeding like in the case of the rigid polymer the dynamics in the diffusion limit $\sigma, \rho \to 0$ with $D_c = \frac{\sigma^2}{2\tau}$ = Diffusion Coefficient of the system, with the help of Kramers-Moyal expansion, the governed formula reads as

$$\partial_t p(x,t) = D_c \left(-2 \frac{\mathrm{d}}{\mathrm{d}x} \mathrm{e}^{\frac{\beta V(X)}{2}} \left[\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{e}^{-\frac{\beta V(x)}{2}} \right] + \frac{\mathrm{d}^2}{\mathrm{d}x^2} \right) \propto D_c \left(-\frac{\mathrm{d}}{\mathrm{d}x} F(x) + \frac{\mathrm{d}^2}{\mathrm{d}x^2} \right) P(x,t) \quad (21)$$

The disordered averaged master equation leads like in the case of rigid polymers to a Fokker-Planck Ansatz with limited long-range transitions (cutoff). In general, it holds the relation $\frac{D_c}{D} = \frac{\sigma^2}{\alpha^2}$. This means that the diffusive character of the dispersion is not changed when long- range transitions of finite variance come into play.

6.3. The Case of the Topological Superdiffusion

The dynamics change significantly when scale-free transitions occur between monomers. The function $\rho(l)$ is now in general terms an inverse power law as $\rho(l) \propto |l|^{-(1+\mu)}$ for $|l| \gg \alpha$. The exponent μ depends on the polymer configuration, and is typically less unity. In this case holds the following relation

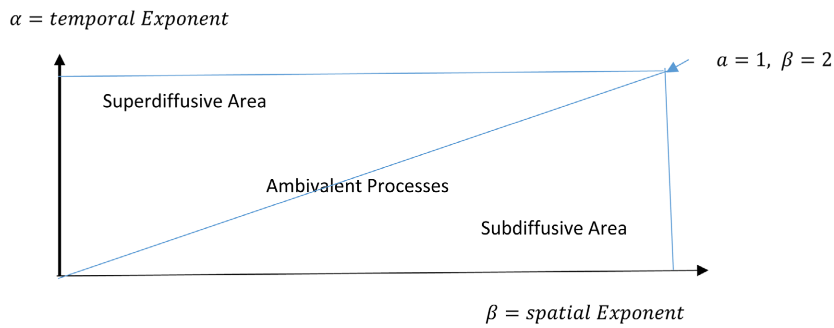
$$\rho(l) = \frac{\mu \alpha^\mu}{2} * |l|^{-(1+\mu)} \text{ for } |l| > \alpha, 0 < \mu \le 2 \text{; otherwise } \rho(l) = 0 \quad (22)$$

If the exponent μ lies in the area $(0,2]$ the density $\rho(l)$ lacks a finite variance. In the case of $\mu \le 1$ especially the expected value diverges. The statistics of jump lengths along a random polymer became scale-free. Inserting the relation (22) into the master equation and performing diverse algebraic manipulations we get in the limit $\tau, \alpha \to 0$, with $D = \frac{\pi\mu}{2\Gamma(1+\mu)\sin\left(\frac{\pi\mu}{2}\right)} \left(\frac{\alpha^\mu}{\tau}\right)$ the fractional Fokker-Planck Equation for superdiffusion

$$\partial_t P = \mathcal{L}_M P = \mathrm{e}^{-\beta V/2} \nabla^{\mu/2} P - P \mathrm{e}^{\beta V/2} \Delta^{\mu/2} \mathrm{e}^{-\beta V/2} \quad (23)$$

with $\left(\Delta^{\frac{\mu}{2}} f\right)(x)$ = fractional Laplacian = $C_\mu * \int \mathrm{d}y \frac{f(y) - f(x)}{|x-y|^{1+\mu}}$, and $C_\mu = \Gamma(1+\mu)\sin\left(\frac{\pi\mu}{2}\right)$. For $\mu = 2$, $\mathcal{L}_M P$ gets out in an ordinary Fokker-Planck operator. In the case of $\beta V = 0$, which takes place if $V = 0$ or $\beta \to$ ultra high temperature all the exponentials in (23) are unity and the fractional Laplacian degenerates to the formula $\mathcal{L}_\mu = \Delta^{\mu/2}$ (free superdiffusion). In this case, the dynamics coincides with the FFPE describing additive Levy stable Noise or associated with highly fluctuating temporal behavior in the next chapter. The following Graphic delivers an overview of fractal Dynamics in Space and Time.

Overview of Fractal Dynamics in Space, and Time



⟶ Temporal Area $\rightarrow [0,1]$, Spatial Area $\rightarrow [0,2]$.

By $2\alpha > \beta$, we refer to the "superdiffusive" Area. By $2\alpha < \beta$, we refer to the "subdiffusive" Area.

Superdiffusive Area: Divergent Spatial Moments, Scale-free Displacements, Discontinous sample Paths, and Temporal Moments exist. The square Displacement relation reads: $X(t) = t^{1/\beta}$.

Subdiffusive Area: (fractional Brownian Motion): Spatial Moments exist, continuous sample paths, and divergent Temporal Moments exist. The square Displacement relation reads: $X(t) = t^{a/2}$.

Gauss Diffusion: $a = 1, \beta = 2$. Markovian, Spatial Moments exist, Temporal Moments exist, Continuous sample Paths. The square Displacement relation reads $X(t) = t^{1/2}$.

Ambivalent Processes: Non-Markovian, divergent Spatial Moments, Scale-free Displacements, Divergent temporal Moments. The square Displacement relation reads $X(t) = t^{a/\beta}$.

Fractal Time Processes: We refer to the subdiffusive area according to the above Graph, while the waiting time distribution leaves the Gaussian Area. These Dynamics are called Fractal Time Processes in the Superdiffusive Domain. That means it holds $2\alpha > \beta$. In this subordination Scenario, the Process evolves with an operational time $\tau(t)$, which is a strongly fluctuating function of the physical time. This type of Dynamics is met by temporal fluctuations near phase transitions, superdiffusion by anomalous drift, and phenomena with scale-free memories. The Process $X(t)$ is superdiffusive if the Increments of the operational time are generated according to an inverse Power Law $\rho(\Delta_\tau) = \frac{1}{\Delta_\tau^{1+\mu/2}}$, with $\Delta_\tau \gg \Delta_t$, $0 < \mu \le 2$.

7. Applying the Topologically Induced Diffusion Concept to Detect Steady States on two probable Scenarios within the global Economic Exploitation Process. Implications to Small World Networks

There are a lot of Arguments to favor "topologically induced Diffusion" as an effective tool for analyzing Dynamics in the Globalization Process. An important Argument consists in the fact that the Dynamics are caused by the underlying topology in which they evolve. In this sense, pure probabilistic modeling seems to be more appropriate as physical Forces can not be defined, and the Ansatz can not be separated into a deterministic and stochastic component, excluding some specific constellations in the Limit. We focus on two basic Scenarios, which can be characterized as probable within the evolution of the Globalization Process. The first of them assumes a transition-free economic environment in which the transport of goods and services does not underlie significant conditions. In this context, the Cluster Europa Union serves as an efficient experiment predicting possible economic emergence in a future global Constellation. The major questions in this Context are the Existence of Steady States and the economic Convergence of the members. The classical Analysis of this problem with the Help of a Predator-Prey Model confirms the Existence of Stationarity in which the Protagonists perform a cyclical behavior at a specific distance from each other. Indeed using deterministic Differential equations for such problems is quite disputable Undertaken. More light comes on the Complexity of observing Dynamics concerning relations in specific prepared economic Networks. In this case, we detect rather economic Stability/Instability than Stationarity. A criterion in this Context can be defined by the "preferring attachment" of new members in the network that tends to establish Hubs in it, and therefore lead to Contact Distribution of Power law Form. From high interest is the fact that the Existence of Hubs in an economic Network can be linked while performing the probability analysis of a jump within the "topologically induced diffusion" framework to which we will refer in the next chapters. The other second basic scenario refers to an economic constellation in which economic activities are broken through Barriers of Clusters that support their own Interests. The development and analysis of the dynamics with the Help of the topologically induced Diffusion indicate the Existence of a specific Fokker-Planck Ansatz when the Transitions in the chain or Network are of a limited length scale, or rigid. This Model differs from the Ansatz delivered when the Transitions are scale-free.

7.1. Szenario I: Steady States on the Scale-Free Model

The Existence of Steady States can be derived setting

$$\partial_t P = \mathcal{L}_M P = \mathrm{e}^{-\frac{\beta V}{2}} \nabla^{\frac{\mu}{2}} P - P\mathrm{e}^{\frac{\beta V}{2}} \Delta^{\frac{\mu}{2}} \mathrm{e}^{-\frac{\beta V}{2}} = 0\,. \tag{24}$$

The solution of the above Equation results in $P_{st}(x) = \mathrm{e}^{-\beta V(x)}$, and does not depend on the Fractality-Order Factor μ. This is not a surprising factum as the evolution of the master equation takes place under Gibbs-Boltzmann thermal conditions. In this case holds the normalization Condition $\int P_{st}(x)\mathrm{d}x = 1$ on the Maxwell Spectrum. In the case of rigid chains or transitions that are limited in scale length α, the government reads as $\mathrm{e}^{-\frac{\beta V}{2}} \nabla^{\frac{1}{2}} P - P\mathrm{e}^{\frac{\beta V}{2}} \Delta^{\frac{1}{2}} \mathrm{e}^{-\frac{\beta V}{2}} = \partial_t P$, which results like above in

$$P_{st}(x) = \mathrm{e}^{-\beta V(x)} \tag{25}$$

Interestingly the operator $\mathcal{L}_{1/2}P$ can be interpreted as an ordinary Fokker-Planck operator operating in gradient dynamics subjected to Gaussian white noise. It must be denoted that the master equation describing topologically induced Diffusion does have stationary states according to Gibbs-Boltzmann Equilibrium thermodynamics.

7.2. Szenario II: Steady States on Coiled Polymers Characterized by a Typical Length Scale (Cutoff)

This economic Szenario (Constellation) corresponds to a global economic constellation in which diverse economic clusters exist. The inter-cluster economic connections are relatively rare, while each cluster exhibits its self-scale-free behavior. The corresponding steady states are of Gibbs-Boltzmann Character of the Forrm $P_{st}(x) = \mathrm{e}^{-\beta V(x)}$ like in the scale-free case.

7.3. Szenario III: Levy Flights in the Superdiffusivity Area Evolving as a Fractal Time-Process

Remaining in the superdiffusive area dynamics can occur, while the time-fractional Order becomes more and more lower values (scale-free Memory). The process is superdiffusive in space and time. The operational time $\tau(t)$ is a stochastic process itself. This is a specific subordination process in the superdiffusive area. The corresponding conditional pdf $\partial_t P(x,t \mid x_0,t_0)$ can be represented by fractional Fokker-Planck Equation for symmetric Levy stable processes. It holds $\partial_t P(x,t \mid x_0,t_0) = \Delta_x^{\mu/2} P(x,t \mid x_0,t_0)$. For non-vanishing Force the correct Equation reads: $\partial_t P(x,t \mid x_0,t_0) = -(-L_{x,FP})^{\mu/2} P(x,t \mid x_0,t_0)$ with $L_{x,FP} = \nabla_x F(x,t) + \frac{1}{2} * \Delta_x D(x,t)$. As the Operator $L_{x,FP}$, and the term $-(-L_{x,FP})^{\mu/2}$ commute with each other we can state that superdiffusive subordinated processes evolve under Gibbs-Boltzmann thermal Conditions. Thus if $D(x,t) = 1$, and $F(x,t) = -\beta \mathrm{d}V(x)/\mathrm{d}x$ the stationary solution reads $P_{st} = \mathrm{e}^{-\beta V}$.

<u>**Summary**</u>: Assuming dynamics that can be analyzed by the topologically induced Diffusion, we can state the existence of Gibbs-Boltzmann Stationary Solutions of the form $P_{st} = \mathrm{e}^{-\beta V}$. This result can be derived by simulating the Dynamics by the Master Equation in the Superdiffusiv Area. It must be noted that the topologically induced Diffusion exhibits Levy Flights with stationary Solutions as opposed to the generalized Langevin ansatz where the Levy Flight dynamics fail to generate Stationarity.

7.4. A Generalization of the Master Equation. What Does It Mean for Economic Networks with Hubs?

The transition rate $w(x \mid y) = \frac{1}{\rho} \mathrm{e}^{-\beta[V(x)-V(y)]/2} * \rho(|x-y|)$, which has been used in the master equation ansatz to get concrete solutions, can be generalized, as the Potentials contribute in the above equation only by their Difference. The Term

$\Delta V = \frac{1}{2}\left[V(x) - V(y)\right]$ can be generalized assuming that a contribution of the potentials by their offset is more appropriate. Inserting in the above relation $\Delta V = cV(x) - (1-c)V(y)$ with $c \in [0,1]$, we obtain a more general rate in the form

$$w(x \mid y) = \frac{1}{\rho} \mathrm{e}^{-\beta c V(x)} \rho(|x-y|) * \mathrm{e}^{\beta(1-c)V(y)} \tag{26}$$

In the case of economic preferential attachment where Hubs occur by adding new members to the network, we can assume that the entire potential increases by a constant V_c. That means $c \neq \frac{1}{2}$ and $c < 0.5$. in a physical sense, the influence of the source location increases, while an increase in the rate takes place. This is however a general problem in networks with hubs, while the latter have a catalytic and accelerated influence on the further dynamics of the network. In this context ($c < 0.5$) if the entire potential increases the algebraic structure of the rate remains the same but the waiting time constant τ changes effectively. That means

$$V \to V + V_C \Rightarrow \tau = \frac{\tau}{\mathrm{e}^{\beta V_c (2c-1)}} \tag{27}$$

Equation (27) indicates that high temperatures and abrupt changes of V_c could transfer the dynamics from the superdiffusive area to that of ambivalent processes. It should be emphasized here that ambivalent processes exhibit different Steady State solutions. However, we do not handle this topic notwithstanding that could represent dynamics that are probable within the general Globalization evolution. It is important to say that we observe specific economic Constellations in the global network, which can be effectively approximated by the topologically induced Diffusion.

7.5. Analyzing the Coiled Polymer Dynamics Assuming the Fractal Order Value of 2

The coiled Polymer Dynamics could be characterized as the most probable real scenario affecting the evolution of the general Globalization process. This assumption relies on the actual economic relations and trends worldwide. It consists of diverse economic clusters with intercluster connections characterized by a typical scale (cutoff), while the dynamics in every single cluster are scale-free dependent naturally on the specific conditions and agreements needed to establish these as discrete (i.e. European Union). In the case of ordinary Diffusion in the rate $w(x \mid y) = \frac{1}{\rho} \mathrm{e}^{-\beta c V(x)} \rho(|x-y|) * \mathrm{e}^{\beta(1-c)V(y)}$, the factor $\rho(|x-y|)$ must be replaced by $\delta(x-y)\Delta_y$. The master Equation consequently results in

$$\partial_t p = \mathrm{e}^{-\beta c V} \Delta e^{1-c} \mathrm{e}^{\beta(1-c)V} p - p \mathrm{e}^{\beta(1-c)V} \Delta \mathrm{e}^{-c\beta V} \tag{28}$$

<u>Important Note:</u> The generalization of the probability rate $w(x \mid y)$ does have only in the case of $c = 1/2$ a quantum-mechanical Equivalent and <u>can not</u> be recast in the Fokker-Planck Ansatz

$$\partial_t p = \{-\nabla F + 0.5\Delta D\} \tag{29}$$

like In the case of $\rho(l) = \frac{\mu\alpha}{2} * |l|^{-(1+\mu)}$, which induces scale-free long-range transitions. Concerning Coiled Polymer Dynamics and varying the Value of $c \in [0,1]$ we could use Equation (29) to gain results that are most probably in real Dynamics. The more the value c tends to 1 the more the Influence of the target location is higher. In the opposite case a low value of c in the area $0 \le c \le 0.5$ indicates a higher influence of the source location (Brockmann, 2003). Drift and diffusion coefficients can be represented by the potential $V(x)$ as

$$F(x) = -c\beta \frac{\mathrm{d}V}{\mathrm{d}x} * D(x), \quad D(x) = 2\mathrm{e}^{-\beta(2c-1)V(x)} \tag{30}$$

Setting $c = 0.5$ we obtain the usual diffusion equation in a gradient force field. In the case of $c = 0$, which corresponds to a situation where the transitional rates are determined by the potential at the source location, we obtain an equation without drift term. The environment inhomogeneity is incorporated in the diffusion coefficient. We get the following Identities assuming $c = 0$: $F = 0$,

$$D = 2\mathrm{e}^{\beta V}, \quad \partial_t p = \frac{1}{2}\Delta D p \tag{31}$$

From the Relation in (31) we can assert that the dynamics in this case are caused only by the Laplacian of the Diffusion coefficient. In the extremal case of $c = 1$ (Target location determines the transition rate) we gain the identity $\partial_t p = \mathrm{e}^{-\beta V} p - p\Delta\mathrm{e}^{-\beta V}$. In this case, only the Laplacian of the Term $\mathrm{e}^{-\beta V}$ occurs in the master equation, while the probability density is respected with the prefactor $\mathrm{e}^{-\beta V}$.

8. Summary Conclusions: From Metastability to Criticality

The Globalization Process is a very complex Dynamics, which does not allow us to define an exact Ansatz that could concern it getting solutions about Steady States. Remaining in the Domain of economic events and excluding political and hegemonical Trends, we focus on two important Dynamics (Scenarios) that could form Stability and Steady States in this Process. The first case observes a scale-free scenario of economic activities (Exchange of Goods and Services). The second case considers a global economic scenario consisting of more clusters that hold economic contacts with each other within a limited scale. Within the first Scenario, we extract the Steady States with the help of the Lotka-Volterra Model. However Steady States from a mathematical point of view, refer to Solutions of the corresponding Differential Equations. Steady States but indicate Dynamic Constellations, which remain constant in the time evolution. This Dynamics-Phase, which we define here as “Metastability” does not mean economic Stability, which needs in the specific case an exact Definition. Thus Metastability solutions could contain hidden Signals (Information) for Criticality. This is, for example, the case referring to the Lotka-Volterra Model. The Steady States Solutions point out the Existence of a cyclical Behavior of Predator and Pray, while they hold a certain distance from each other for a time tending to infinity. However, the Distance Mass of the Players could represent a Criterion of Criticality, which depends on the Parameter Values of the considered Model. In the case of Network Analysis and in the case of the Stade State Solutions in certain differential Equations, the real economic Stability is hidden in the nature of important field-variable Distributions. From a thermodynamical Point of View, this is usually the Transition from a Distribution like the LogNorm to a Power Law. In this case, Expectation Value and Displacement get indefinite. Interestingly while the classical Model of Lotka-Volterra delivers periodical Solutions with the help of a time-invariant functional, it is not the fractional version of the Lotka-Volterra model gained by the Homotopy Method. A further Analysis of the fractional Ansatz indicates the Existence of Zeros regarding the distance of the system solutions under specific Conditions of the fractality Order. Thus, the concept of Stability in the Lotka-Volterra model, from the fractal point of view needs further research. However, analyzing scale-free dynamics on economic clusters and considering these as small-world networks, opens a new window regarding their economic stability or criticality. If the distribution of important field variables (Connection Distribution) fails to exhibit a definite square displacement and expectation value a concrete criterion for instability could be established. In a further step implementing the Analysis Method of topologically induced Diffusion, we are confronted with a tool that is rather appropriate for observing globalization Dynamics. This mathematical tool has been developed by D. Brockmann, in his doctoral Promotion to which we refer extensively in this article. In the case of scale-free network dynamics as well as in networks with a limited jump scale (cutoff) do exist Steady States according to the classical Maxwell Energy Spectrum. However, the Distribution of the Energy states represented by their Potentials can degenerate into a power Law Distribution, indicating Criticality.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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