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Multiple Contracts: The Case of the German System of Amortization in Compound Interest

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Abstract

In what appears to be a pioneering contribution of De-Losso et al., the multiple contracts scheme has been implemented in several amortization methods, such as constant installments, constant amortization and American. This paper will address the multiple contracts schema to the German amortization method, in compound capitalization. Since this method has the payments of interest at the beginning of each period, some adaptations were required in the De-Losso proposition. Additionally, a comparison with the French system is also presented.

Keywords

Amortization Systems, Multiple Contracts Scheme, Compound Interest Capitalization, German Amortization System, French Amortization System

1. Introduction

In what appears to be a pioneering contribution De-Losso et al. (2013), it is shown that if a single contract written in terms of the classical system of amortization with constant payments is substituted by multiple contracts, one for each payment of the single contract, the financial institution providing the loan may experience substantial gains in terms of the present value of tax deductions. With the amount of tax gains depending on the financial institution cost of capital.

Similarly, addressing the case of the system of periodic payments of interest only, de Faro (2021), the case of the system of constant amortization, de Faro (2022), and the case of two alternative versions of the SACRE, de Faro & Lachtermacher (2023a) and de Faro & Lachtermacher (2023b), the same results were observed when the original contracts were substituted by the corresponding multiple contracts.

With all the above-mentioned analysis addressing the more usual case, all the financing contracts have been written considering compound interest.

However, as compound interest implies the occurrence of anatocism, which means payment of interest upon interest, a more comprehensive analysis should also consider the possibility of the use of simple interest. As considered, for instance, in Lachtermacher & de Faro (2024), the case of SACRE-F. Since, by definition, simple interest does not imply in the occurrence of anatocism.

Focusing attention in the case of what has been called in Brazil as the German system of amortization, see Moraes (1967), Juer (2003) and de Faro & Lachtermacher (2012), a version of it being named in Italy as the "Tedesco" amortization system, see Palestini (2017), both of which are characterized by the payment of interest in advance, it will be shown that the financial institution granting the loan will also be better off if a single contract is substituted by multiple contracts.

Before proceeding it is appropriate to point out that the occurrence of anatocism when making use of compound interest, is a topic still not settled in Brazil. For instance, we have the recent opposing views of Pucinni (2023) and of DeLosso & Santos (2023). An issue that also not pacified on the Italian Judicial System; cf. Annibali et al. (2016).

2. The Case of a Single Contract—German Method

We will focus attention on the case where a loan in the amount of F units of capital must be repaid at the periodic rate i of compound interest, with the loan having to be repaid with a term of n periods, according to the German system of amortization.

The German amortization system is characterized by payments of interest in advance. That is, at the beginning, instead of at the end of each period, which is the usual procedure. With the first payment, at the beginning of the first period, being denoted as P_0, and equal to $i \times F$.

Therefore, as the n remaining periodic payments, P_k, for $k = 1, 2, \cdots, n$, are supposed to be constant and equal to P, we can imagine as if a loan in the amount $F \times (1 - i)$, must be repaid at the periodic rate of compound interest i^*, according to the more usual constant payments system also called French system.

It follows then that, considering the corresponding classical expression, see de Faro & Lachtermacher (2012):

$$P = \frac{F \times (1 - i) \times i^*}{1 - (1 + i^*)^{-n}} \tag{1}$$

An expression that is satisfied if:

$$i^* = i/(1 - i) \tag{2}$$

Noting that, by adding 1 to each side of equation (2), we have

$$1+i^{*}=1+\frac{i}{(1-i)}=\frac{1}{(1-i)}$$

$$\left(1+i^{*}\right)^{-n}=(1-i)^{n}$$

Therefore, we can also write equation (1) as:

$$P=\frac{F\times i}{1-(1-i)^{n}} \tag{3}$$

Denoting by S_k, for $k=0,1,\cdots,n$, the outstanding debt at time k, with $S_0 = F$, and $S_n = 0$, given that the debt has to be extinguished at the end of the term of n periods; denoting by J_k the parcel of interest that comprises the k^{th} payment; and taking into account that in the German system of amortization we have the payments of interest in advance, it follows that:

$$J_k=i\times S_k \quad \text{for } k=0,1,2,\cdots,n \tag{4}$$

On the other hand, denoting by A_k the parcel of amortization that comprises the k^{th} payment, we have, by definition, that:

$$A_k=P-J_k \quad \text{for } k=1,2,\cdots,n \tag{5}$$

with $A_0 = 0$, as the first payment is of interest only.

Furthermore, as the parcels of amortization must recompose the loan amount, we have:

$$\sum_{k=1}^{n}A_k=F \tag{6}$$

Thus, considering that

$$S_k=S_{k-1}-A_k \quad \text{for } k=1,2,\cdots,n \tag{7}$$

together with equation (5), we have:

$$A_1+J_1=A_2+J_2$$

$$A_1+(F-A_1)\times i=A_2+(F-A_1-A_2)\times i$$

Therefore:

$$A_2=\frac{A_1}{1-i}$$

Generalizing, it can be shown by induction, that:

$$A_k=\frac{A_{k-1}}{1-i} \quad \text{for } k=2,3,\cdots,n \tag{8}$$

Thus, the amortization sequence is a geometric progression with ratio $q=1/(1-i)$.

At this point, it is interesting to notice that, while in the case of the classical system of constant payments the sequence of the parcels of amortization follows a geometric progression with ratio $q'=1+i$, in the case of the German system the parcels of amortization follows a geometric sequence with ratio $q=(1-i)^{-1}$.

Consequently, considering the expression of the sum of the first n terms of a geometric progression, and equation (6), it follows that:

$$F = \frac{A_1 - (1-i)^{-1} \times (1-i)^{1-n} \times A_1}{1-(1-i)^{-1}}$$

Therefore, we have:

$$A_1 = \frac{i \times F \times (1-i)^n}{(1-i) \times \left[1-(1-i)^n\right]} \tag{9}$$

Being worth noting, considering equations (4) and (5), that $P = A_n$. Which is obvious, since $J_n = 0$.

As a simple numerical illustration, consider the case where F = $100,000.00 units of capital, the financing interest rate is i = 1% per period, and the number of periods is $n = 12$.

The first payment is P_0 = $1,000.00 and the constant payment is P = $ 8,801.64. Table 1 shows the evolution of the debt in this case.

Table 1. German amortization method—evolution of the debt.

Epoch (k)	J_k	A_k	P_k	S_k
0	1,000.00	0.00	1,000.00	100,000.00
1	921.20	7,880.45	8,801.64	92,119.55
2	841.60	7,960.05	8,801.64	84,159.50
3	761.19	8,040.45	8,801.64	76,119.05
4	697.97	8,121.67	8,801.64	67,997.38
5	597.94	8,203.71	8,801.64	59,793.67
6	515.07	8,286.57	8,801.64	51,507.09
7	431.37	8,370.28	8,801.64	43,136.82
8	346.82	8,454.82	8,801.64	34,681.99
9	261.42	8,540.23	8,801.64	26,141.77
10	175.15	8,626.49	8,801.64	17,515.27
11	88.02	8,713.63	8,801.64	8,801.64
12	0.00	8,801.64	8,801.64	0.00
Σ	6,619.74	100,000.00	106,619.74	

Financial Consistency of the German Method

Whatever the interest regime considered, whether simple interest or compound interest, and whatever amortization system has been stipulated, it is crucial, particularly in the event of early payment of one or more installments, that the debt status is appropriately calculated.

Focusing on the case of the compound interest regime, de Faro (2014) presents the concept of financial consistency, highlighting the need for financial equivalence among the three classic methods of determining the outstanding balance of a given method financing to be strictly observed.

In other words, the values determined by the retrospective, prospective and recurrence methods must be the same. As an illustration, let's calculate the outstanding balance of period $k = 6$, of our numerical example, using all three methods.

a) Retrospective Method

In the case of the compound interest regime, the outstanding balance at time k must be equal to the value F of the financing, subtracted from the sum of the k amortization parcels that have already been made.

In other words, you must have:

$$S_k = F - \sum_{\ell=1}^{k} A_k \tag{10}$$

So, the outstanding balance at period $k = 6$ is given by:

$$\begin{aligned} S_6 &= F - \sum_{\ell=1}^{6} A_\ell \\ &= 100000 - [7880.45 + 7960.05 + 8040.45 + 8121.67 + 8203.71 + 8286.57] \\ S_6 &= 100000 - 48492.90 = 51507.10 \end{aligned}$$

which is equal to S_6 in Table 1 (the difference between the values of 1 cent is due to rounding calculation).

b) Prospective Method

In the case of the compound interest regime, it must be considered equal to the present value, at time k, of the installments due. In other words, considering the compound interest rate i, it must be:

$$S_k = \sum_{\ell=k+1}^{n} \frac{P_\ell}{(1+i)^{\ell-k}} \tag{11}$$

However, in the case of the German method, the interest is paid in advance, at the beginning of the period. To account for this peculiarity, we must subtract the respective interest parcel of interest from each of the payments due. In other words, we must have:

$$S_k = \sum_{\ell=k+1}^{n} (P_\ell - J_\ell) \tag{12}$$

It should be noted that equations (11) and (12) are equivalent in the compound interest regime for methods with no interest paid in advance. And that equation (12) is also used in the calculation of the outstanding balance in simple interest regime, for all methods.

Thus, the outstanding balance at period $k = 6$ is given by:

$$\begin{aligned} S_6 &= \sum_{\ell=7}^{12} (P_\ell - J_\ell) \\ &= 6 \times 8801.64 - [431.34 + 346.82 + 261.42 + 175.15 + 88.02 + 0.00] \\ S_6 &= 52809.84 - 1302.75 = 51507.09 \end{aligned}$$

which is also equal to S_6 in **Table 1**.

c) Recurrence Method

As is well known, see de Faro & Lachtermacher (2012: p. 241), in the case of the compound interest regime at interest rate i, we have the following recurrence relationship:

$$S_k = (1+i) \times S_{k-1} - P_k \tag{13}$$

Therefore, generalizing what was also presented in the above reference, it follows that:

$$S_k = F \times (1+i)^k - \sum_{\ell=1}^{k} P_\ell \times (1+i)^{k-\ell} \tag{14}$$

That is, according to the recurrence method, we have the following financial interpretation for determining the outstanding balance.

What is owed at time k is equal to the value F of the financing, plus interest for k periods, subtracted from the sum of the values of the k installments already paid, including interest from their respective due dates.

However, in the case of the German Method, it should be considered that interest of a period is paid in advance. Which implies that we must adjust the installments by including the interest paid in advance and subtracting the interest included in the installments. So, for the German Method Equations (13) and (14) should be written as:

$$S_k = (1+i) \times S_{k-1} - (P_k - J_k + J_{k-1}) \tag{15}$$

$$S_k = F \times (1+i)^k - \sum_{\ell=1}^{k} \left[(P_\ell - J_\ell + J_{\ell-1}) \times (1+i)^{k-\ell} \right] \tag{16}$$

Thus, the outstanding balance at period $k = 6$ is given by:

$$\begin{aligned}
S_6 &= F \times (1+i)^6 - \sum_{\ell=1}^{6} \left[(P_\ell - J_\ell + J_{\ell-1}) \times (1+i)^{k-\ell} \right] \\
S_6 &= 100000 \times (1+0.01)^6 - \Big[(P_1 - J_1 + J_0) \times (1+i)^{6-1} \\
&\quad + (P_2 - J_2 + J_1) \times (1+i)^{6-2} + (P_3 - J_3 + J_2) \times (1+i)^{6-3} \\
&\quad + (P_4 - J_4 + J_3) \times (1+i)^{6-4} + (P_5 - J_5 + J_4) \times (1+i)^{6-5} \\
&\quad + (P_6 - J_6 + J_5) \times (1+i)^{6-6} \Big] \\
S_6 &= 106152.02 - \Big[8880.45 \times 1.01^5 + 8881.25 \times 1.01^4 + 8882.05 \times 1.01^3 \\
&\quad + 8882.86 \times 1.01^2 + 8883.68 \times 1.01^1 + 8884.51 \times 1.01^0 \Big] \\
S_6 &= 106152.02 - [9333.44 + 9241.86 + 9151.18 \\
&\quad + 9061.41 + 8972.52 + 8884.51] \\
S_6 &= 106152.02 - 54644.92 = 51507.10
\end{aligned}$$

which is also equal to S_6 in **Table 1** (the difference between the values, of 1 cent, is due to rounding calculation).

3. Comparison with the French System

Considering that, according to Annibali et al. (2020), the classical amortization system of constant payments is also named as the French System, it appears appropriated to make a comparison of these two somewhat similar amortization systems.

Denoting by F the value that is being financed, $\hat{S}_k$ the remaining debt at epoch k, consider a single contract with n constant periodic payments, and denote by i the periodic interest rate that is being charged.

If i is of compound interest, it is well known, cf. de Faro & Lachtermacher (2012: p. 241), that the value of the constant payment, denoted by $\hat{P}$, is:

$$\hat{P} = F \times \left[\frac{i \times (1+i)^n}{(1+i)^n - 1} \right] \tag{17}$$

the interest at epoch k is given by

$$\hat{J}_k = i \times \hat{S}_{k-1} \quad \text{for } k = 1, 2, \cdots, n \tag{18}$$

and the amortization term at epoch k is

$$\hat{A}_k = \hat{P} - \hat{J}_k \quad \text{for } k = 1, 2, \cdots, n \tag{19}$$

Considering our simple numerical example, **Table 2** presents the evolution of the debt if the French system is implemented.

Table 2. French amortization method—evolution of the debt.

Epoch (k)	$\hat{J}_k$	$\hat{A}_k$	$\hat{P}$	$\hat{S}_k$
0				100,000.00
1	1,000.00	7,884.88	8,884.88	92,115.12
2	921.15	7,963.73	8,884.88	84,151.39
3	841.51	8,043.36	8,884.88	76,108.03
4	761.08	8,123.80	8,884.88	67,984.23
5	679.84	8,205.04	8,884.88	59,779.19
6	597.79	8,287.09	8,884.88	51,492.11
7	514.92	8,369.96	8,884.88	43,122.15
8	431.22	8,453.66	8,884.88	34,668.49
9	346.68	8,538.19	8,884.88	26,130.30
10	261.30	8,623.58	8,884.88	17,506.72
11	175.03	8,709.81	8,884.88	8,796.91
12	87.97	8,796.91	8,884.88	0.00
Σ	6,618.19	100,000.00	106,618.55	

From Table 2, we see that the corresponding value of the constant payment is $\hat{P}$ = \$8,884.88 units of capital. A value that is only 0.95% ($\left[\left(P/\hat{P}\right)-1\right]\times 100$) greater than the corresponding one in the case of the German method.

Furthermore, from the strict accounting point of view, there are no significant differences in terms of the total interest payments. As the total of interest in the case of the German system is only 0.02% ($\left[\left(\sum J_k/\sum \hat{J}_k\right)-1\right]\times 100$) greater than the corresponding one in the case of the French system.

A result that is always observed. As confirmed in Table 3, for the cases where F = \$100000.00 units of capital, the financing interest i takes the values of 0.5%, 1% and 2% per period, and the number n of periods varies from 12 to 360.

Table 3. Percentage of the total of interest paid over the loan.

n	German Amortization System			French Amortization System		
	0.50%	1.00%	2.00%	0.50%	1.00%	2.00%
12	3.280	6.620	13.481	3.280	6.619	13.472
60	16.001	33.496	72.831	15.997	33.467	72.608
120	33.239	72.277	165.313	33.225	72.165	164.577
180	51.927	116.262	271.741	51.894	116.030	270.489
240	71.999	164.629	385.792	71.943	164.261	384.178
300	93.374	216.471	503.403	93.290	215.967	501.582
360	115.954	270.926	622.500	115.838	270.301	620.578

This confirms our previous finding that the total amount of interest of the German method is slightly greater than the French method.

However, a more relevant comparison must take into consideration the financial institution cost of capital. Which periodic value will be denoted as ρ.

That is, we must compare the present values, at the rate ρ, of the corresponding sequences of the parcels of interest payments. Respectively designated as $V_1(\rho)$, for the German method and $V_2(\rho)$, for the French method:

$$V_1(\rho)=\sum_{k=0}^{n} J_k \times (1+\rho)^{-k} \tag{20}$$

$$V_2(\rho)=\sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k} \tag{21}$$

where ρ is supposed be relative to the same period as the financing interest rate i.

For instance, if ρ_a is the financial institution cost of capital, in annual terms, is equal to 20%, which means that ρ = 1.531% per month, n = 120 periods, and the financing interest rate i = 1% per month, and F = 100,000.00, we have $V_1(\rho)$ = 41008.80 units of capital, and $V_2(\rho)$ = 40345.75 units of capital.

Which implies that the financial institution, in terms fiscal gains, will be better off, if the loan is implemented with the French method (smaller present value), in-stead of the German method.

This conclusion appears to be valid for every positive value of the rate ρ. Tables 4-6 show the results for $i = 1\%$, 1.5% and 2% per month, $F = 100,000.00$, $n = 120$, 240 and 360 months and ρ_a varying from 5% to 30% annually.

Table 4. Present value of the interest sequences for German and French method $n = 120$, $i = 1.0\%$ p.m, $F = 100,000$.

$n = 120$, $i = 1\%$ p.m, $F = 100,000$			
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$
5%	0.407%	61,018.52	60,684.85
10%	0.797%	52,576.29	52,092.54
15%	1.171%	46,093.48	45,505.20
20%	1.531%	41,008.80	40,345.75
25%	1.877%	36,944.76	36,226.88
30%	2.210%	33,641.33	32,882.32

Table 5. Present value of the interest sequences for German and French methods $n = 240$, $i = 1.5\%$ p.m, $F = 100,000$.

$n = 240$, $i = 1.5\%$ p.m, $F = 100,000$			
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$
5%	0.407%	188,050.40	186,766.77
10%	0.797%	139,699.33	138,284.42
15%	1.171%	109,635.43	108,169.49
20%	1.531%	89,789.99	88,304.19
25%	1.877%	76,000.63	74,507.35
30%	2.210%	65,994.10	64,498.30

Table 6. Present value of the interest sequences for German and French methods $n = 360$, $i = 2.0\%$ p.m, $F = 100,000$.

$n = 360$, $i = 2.0\%$ p.m, $F = 100,000$			
ρ_a	ρ	$V_1(\rho)$	$V_2(\rho)$
5%	0.407%	350,739.06	348,642.52
10%	0.797%	229,302.66	227,221.68
15%	1.171%	166,875.87	164,823.85
20%	1.531%	130,768.73	128,736.45
25%	1.877%	107,821.44	105,800.95
30%	2.210%	92,131.73	90,118.17

As shown in **Tables 4-6**, the values of every $V_1(\rho)$ is bigger than the corresponding $V_2(\rho)$, which means that the financial institution will be better off using the French Method.

4. The Case of Multiple Contracts

Instead of a single contract, the financial institution has the option of requiring the borrower to write $n+1$ subcontracts. One for each of the $n+1$ payments that would be associated with the case of a single contract. With the principal of the k^{th} subcontract being the present value, at the same considered interest rate i, of the k^{th} payment of the single contract.

That is, the principal of the k^{th} subcontract, denoted by $\tilde{F}_k$, is given by:

$$\tilde{F}_k = \tilde{P}_k \times (1+i)^{-k}, \quad k = 0,1,\cdots,n \tag{22}$$

where $\tilde{P}_k$ is equal to the corresponding installment of the single contract.

In this case, the parcel of amortization associated with the k^{th} payment, denoted by $\tilde{A}_k$, will be:

$$\tilde{A}_k = \tilde{F}_k = \tilde{P}_k \times (1+i)^{-k}, \quad k = 0,1,2,\cdots,n \tag{23}$$

On the other hand, from an accounting point of view, it follows that the parcel of interest associated with the k^{th} subcontract, which will be denoted by $\tilde{J}_k$, is given by:

$$\tilde{J}_k = \tilde{P}_k \times \left\{1-(1+i)^{-k}\right\} = \tilde{P}_k - \tilde{F}_k = \tilde{P}_k - \tilde{A}_k \quad \text{for } k = 1,2,\cdots,n \tag{24}$$

with $\tilde{J}_0 = 0$.

From a strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the subcontracts, the total interest payments are the same comparing a single contract with multiple contracts.

However, in terms of present values, and depending on the financial institution's opportunity cost, it is possible that the financial institution will be better off if it adopts the option of multiple contracts. As it will be shown.

Considering the same numerical example of section 2, **Table 7** replicates the sequence of payments in the single contract.

Additionally, **Table 7** also presents the sequence of the principals of the individual contracts, as well as the sequences of the corresponding components of amortization and interest. Furthermore, it also presents the sequence of differences, of the single contract and multiple contracts, for the German method.

The sequence of differences d_k, has only one change of sign. Thus, characterizing what is defined as a conventional financing project, see de Faro (1974). Which internal rate of return is known to be unique, and, in this case, is equal to zero.

Therefore, we are assured that:

$$V_{single}(\rho) = \sum_{k=0}^{n} J_k \times (1+\rho)^{-k} > V_{multiple}(\rho) = \sum_{k=0}^{n} \tilde{J}_k \times (1+\rho)^{-k} \tag{25}$$

for all $\rho > 0$, where ρ is the financial institution cost of capital per month.

Table 7. German amortization system—multiple contracts scheme.

k	$\tilde{F}_k = \tilde{A}_k$	$\tilde{J}_k$	$\tilde{P}_k = P_k$	J_k	$d_k = J_k - \tilde{J}_k$
0	1,000.00	0.00	1,000.00	1,000.00	1,000.00
1	8,713.63	88.02	8,801.64	921.20	833.18
2	8,626.49	175.15	8,801.64	841.60	666.44
3	8,540.23	261.42	8,801.64	761.19	499.77
4	8,454.82	346.82	8,801.64	679.97	333.15
5	8,370.28	431.37	8,801.64	597.94	166.57
6	8,286.57	515.07	8,801.64	515.07	0.00
7	8,203.71	597.94	8,801.64	431.37	−166.57
8	8,121.67	679.97	8,801.64	346.82	−333.15
9	8,040.45	761.19	8,801.64	261.42	−499.77
10	7,960.05	841.60	8,801.64	175.15	−666.44
11	7,880.45	921.20	8,801.64	88.02	−833.18
12	7,801.64	1,000.00	8,801.64	0.00	−1,000.00
Σ	100,000.00	6,619.74	106,619.74	6,619.74	0.00

Figure 1 outlines the evolution of $\delta(\%) = \left[V_{single}(\rho_a)/V_{Multiple}(\rho_a) - 1\right] \times 100$, for $0 \le \rho_a \le 30\%$ per year, for F = \$100,000 units of capital and n = 12 months. Where ρ_a denotes the cost of capital in annual terms. Additionally, we also have the evolution of δ (%), when the interest rate i is equal to 0.5%, 1%, 1.5%, 2%, 2.5%, and 3% per month.

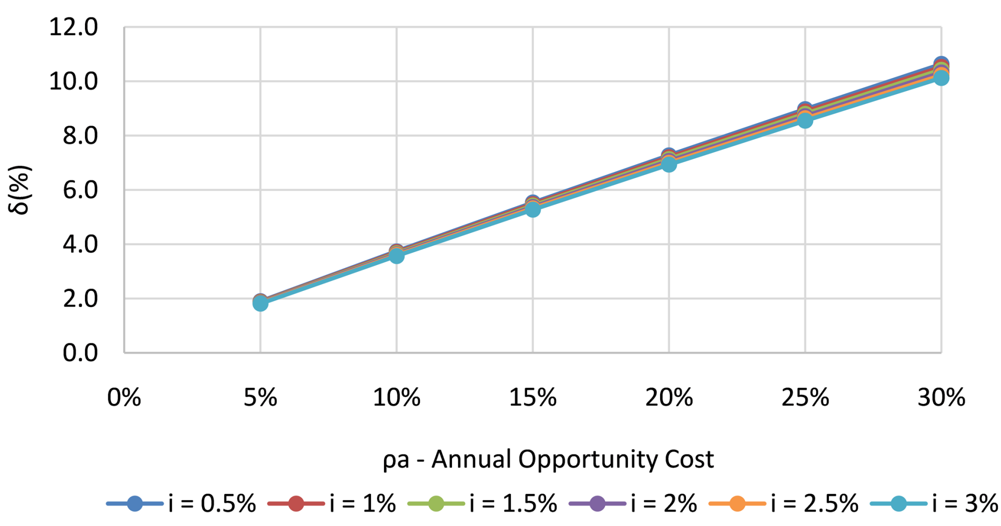

Figure 1. Evolution of δ (%).

Therefore, at least in the case of our simple numerical example, the financial institution granting the loan will be better off if it adopts the multiple contracts option.

5. A General Analysis

In the previous section, focusing attention on our simple numerical example, with only 12 periods, it was verified that the sequence of differences of the interest payments present just one change of sign. Thereby, it assures us of the uniqueness of the corresponding internal rate of return, which is known to be null.

Furthermore, this inference appears to always be true, as supported by the evidence provided in **Figure 2**. Which presents, the evolution of the difference of the interest sequence between the single and multiple contracts scheme, for the case where F = \$100,000.00 units of capital of a contract with 180 periods, and with the interest rate i being equal to 0.5%, 1%, 1.5%, 2%, 2.5%, and 3% per month, respectively.

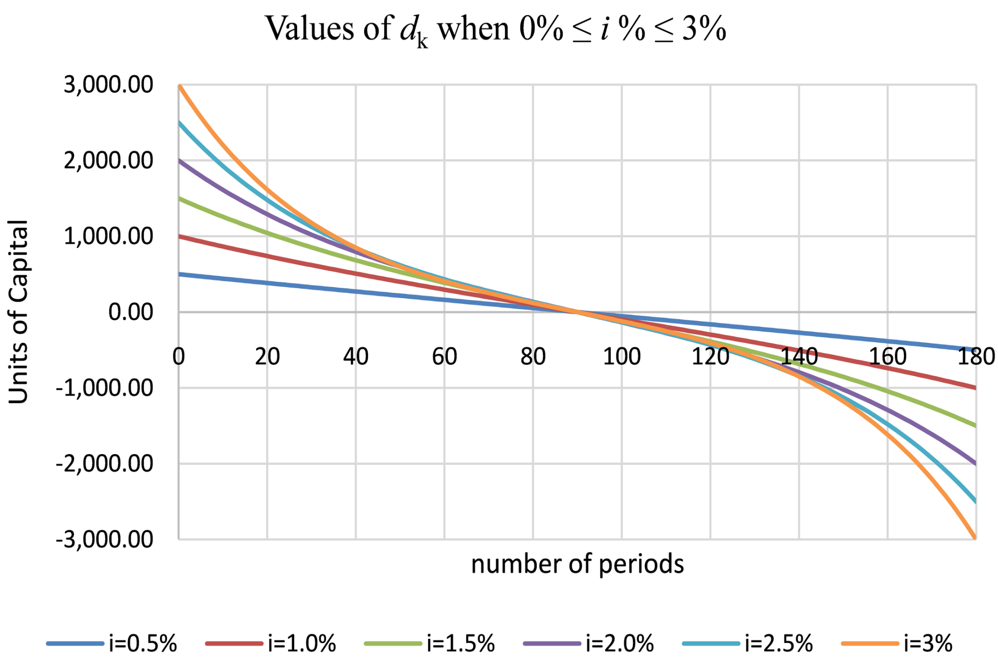

Figure 2. Difference of interest sequences—single and multiple contracts.

Consequently, it can be inferred that the financing institution is always better off if a single contract is substituted by multiple contracts. One for each one of the $n+1$ payments of the original single contract.

Taking into account that in Brazil the monthly interest rates charged do not exceed 3% per month, in real terms, we are going to analyze the behavior of the percentage increase of the fiscal gain $\delta(\%) = \left[V_{single}(\rho_a)/V_{Multiple}(\rho_a) - 1\right] \times 100$, for some values of the corresponding annual opportunity cost ρ_a, with each contract with a term of n_a years. This is depicted in **Tables 8-13**, for the case of the German Method.

Table 8. Fiscal gain $\delta(\%)$ – single x multiple contracts – $i = 0.5\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	8.3218	16.8779	25.6409	34.5840	43.6813	52.9080
10	16.1057	33.6779	52.5615	72.5717	93.5088	115.1708
15	23.5257	50.4046	80.1434	112.1081	145.6267	180.0730
20	30.5026	66.5340	106.8679	150.0062	194.5506	239.4142
25	36.9770	81.6062	131.4226	183.7750	236.6287	288.7533
30	42.9104	95.2737	152.9331	212.1373	270.6368	327.4510

Table 9. Fiscal gain $\delta(\%)$ – single x multiple contracts – $i = 1.0\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.9006	15.9928	24.2505	32.6486	41.1632	49.7716
10	14.4534	30.0040	46.4959	63.7577	81.6142	99.8968
15	19.9271	42.0551	65.9071	90.9556	116.6987	142.7090
20	24.3963	51.9428	81.5881	112.2822	143.1839	173.7246
25	27.9807	59.7241	93.4157	127.5277	161.1071	193.6977
30	30.8172	65.6301	101.8292	137.6561	172.3279	205.6435

Table 10. Fiscal gain $\delta(\%)$ – single x multiple contracts – $i = 1.5\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.4993	15.1525	22.9351	30.8239	38.7969	46.8335
10	12.9889	26.7905	41.2577	56.2371	71.5795	87.1470
15	16.9961	35.4322	54.8902	74.9523	95.2523	115.5015
20	19.8520	41.5187	64.1798	87.1193	109.8297	132.0107
25	21.8664	45.6441	70.0788	94.2787	117.7811	140.4209
30	23.2872	48.3655	73.6230	98.1819	121.7584	144.3496

Table 11. Fiscal gain $\delta(\%)$ − single x multiple contracts − $i = 2.0\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.1183	14.3575	21.6949	29.1091	36.5801	44.0895
10	11.7034	24.0029	36.7645	49.8544	63.1475	76.5325
15	14.6358	30.2135	46.3823	62.8172	79.2524	95.4924
20	16.4873	34.0356	52.0215	69.9532	87.5174	104.5524
25	17.6696	36.3418	55.1452	73.5363	91.2787	108.3201
30	18.4439	37.7270	56.8117	75.2215	92.8530	109.7498

Table 12. Fiscal gain $\delta(\%)$ − single x multiple contracts − $i = 2.5\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	6.7578	13.6079	20.5293	27.5024	34.5092	41.5332
10	10.5824	21.5974	32.9253	44.4512	56.0712	67.6958
15	12.7408	26.0968	39.7792	53.5359	67.1726	80.5559
20	13.9734	28.5731	43.3246	57.8832	72.0491	85.7342
25	14.7089	29.9509	45.1052	59.8234	73.9788	87.5639
30	15.1721	30.7362	45.9899	60.6531	74.6926	88.1584

Table 13. Fiscal gain $\delta(\%)$ − single x multiple contracts − $i = 3.0\%$ p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	6.4178	12.9031	19.4366	26.0006	32.5790	39.1570
10	9.6087	19.5269	29.6492	39.8776	50.1264	60.3237
15	11.2144	22.8279	34.6039	46.3459	57.9103	69.2047
20	12.0628	24.4935	36.9265	49.1140	60.9243	72.3093
25	12.5490	25.3749	38.0216	50.2553	62.0056	73.2828
30	12.8503	25.8652	38.5465	50.7187	62.3774	73.5692

So, as shown in the tables above, the fiscal gains decrease with the increase of the interest rate and increase with the increase of the cost of opportunity.

6. Conclusion

Focusing attention on the case of the German method, using compound interest capitalization, we have concluded that the financing institution granting the loan should always prefer the multiple contracts option since this can result in significant fiscal gains.

A conclusion confirms what was observed in the analysis of other systems of amortization, as can be seen in de Faro & Lachtermacher (2023a, 2023b) and Lachtermacher & de Faro (2024).

Comparing the German system of amortization with the French one, it was concluded that the financing institution providing the loan earns more interest with the German system, from the accounting point of view, since it results in greater payment of interest for the single contract option.

On the other hand, the French system will be a better option in terms of fiscal gains, since it presents a smaller present value of the interest sequence than the German system.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Annibali, A., Annibali, A., Barracchini, C., & Olivieri, F. (2020). *Ammortamento in capitalizzazione semplice di mutui "alla francese": Analisi e confronto dei modelli proposti o in uso.* http://www.attuariale.eu/Schede/Sito_Piamfr_20lug20.pdf

Annibali, A., Barracchini, C., & Annibali, A. (2016). *Anatocismo e Ammortamento di Mutui "Alla Francese" in Capitalzzazione Semplice*. Createspace Independent Publishing

de Faro, C. (1974). On the Internal Rate of Return Criterion. *The Engineering Economist, 19,* 165-194. https://doi.org/10.1080/00137917408902767

de Faro, C. (2014). Sistema de Amortização: O conceito de Consistência Financeira e suas implicações. https://repositorio.fgv.br/server/api/core/bitstreams/2968b8b2-6809-416e-86df-0741f95be35e/content

de Faro, C. (2021). Multiple Contracts: The Case of Periodic Payments Only. *American Research Journal of Business and Management, 7,* 1-6.

de Faro, C. (2022). The Constant Amortization Scheme with Multiple Contracts. *Revista Brasileira de Economia, 76,* 135-146. https://doi.org/10.5935/0034-7140.20220007

de Faro, C., & Lachtermacher, G. (2012). *Introdução à Matemática Financeira.* Editora FGV/Saraiva.

de Faro, C., & Lachtermacher, G. (2023a). An Alternative Multiple Contracts Version of the Sacre. *Journal of Economics and Management Sciences, 6,* 19-27. https://doi.org/10.30560/jems.v6n2p19

de Faro, C., & Lachtermacher, G. (2023b) A Multiple Contracts Version of the SACRE.

London Journal of Research in Management & Business, 23, 15-27. https://journalspress.uk/index.php/LJRMB/article/view/172

De-Losso, R., & Santos, J. (2023). *Autopsy of a Myth: Dissecting the Anatocism Fallacy in Amortization Systems.* Department of Economics-FEA/USP. http://www.repec.eae.fea.usp.br/documentos/De-Losso_Santos_09WP_V3.pdf

De-Losso, R., Giovannetti, B. C., & Rangel, A. S. (2013). Sistema de Amortização por Múltiplos Contratos: A Falácia do Sistema Francês. *Economic Analysis of Law Review, 4,* 160-180. https://doi.org/10.18836/2178-0587/ealr.v4n1p160-180

Juer, M. (2003). *Matemática Financeira: Praticando e aplicando.* Qualitymark Editora.

Lachtermacher, G., & Faro, C. D. (2024). Multiple Contracts with Simple Interest: The Case of SACRE-F. *Journal of Economics and Management Sciences, 7,* 17-27. https://doi.org/10.30560/jems.v7n1p17

Moraes, E. (9167) *Matemática Financeira.* Sulina.

Palestini, A. (2017). *Dispense di Matematica Finanziaria, MEMOTEF, Sapienza.* Università di Roma. https://web.uniroma1.it/memotef/sites/default/files/file%20lezioni/Slides%20MF%202017%20ammortamento.pdf

Pucinni, A., (2023). omo se livrar do anatocismo na Tabela Price para magistrados e advogados. *Conjuntura Econômica*, *77,* 32-34. https://periodicos.fgv.br/rce/article/view/89680/84224