

The Impact of Information Asymmetry on Strategic Inventory Considering Online Reviews

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Abstract

To minimize the losses of running out of stock after the appearance of online reviews and enhance their negotiating leverage with upstream manufacturers, retailers are motivated to adopt strategic inventory, which means to place large orders before the start of the sales season. Simultaneously, retailers may conceal the inventory information to maintain an information advantage compared to manufacturers. Therefore, based on the two aspects, we construct a theoretical model to examine how online reviews and information asymmetry influence strategic inventory. This study examines the intricate impacts of online reviews and information asymmetry on strategic inventories. We find that the retailer strategically maintains inventory under information symmetry (asymmetry) when consumers assign minimal (substantial) importance to online reviews. Counter-intuitively, information symmetry scenario dictates the existence of a holding cost threshold below which retailers are prohibited from maintaining strategic inventory. Besides, as online reviews improve, the retailer's propensity to maintain inventory is augmented (reduced) under information symmetry (asymmetry). Furthermore, in the context of online reviews, implementing strategic inventory could still lead to a decrease in future pricing when information is symmetric. Nevertheless, prices might increase or decrease under information asymmetry. Finally, the manufacturer could gain an advantage from information asymmetry.

Keywords

Strategic Inventory, Online Reviews, Information Asymmetry

1. Introduction

In recent years, there has been a notable increase in the popularity of ecommerce.

According to forecasts, the whole retail e-commerce sales are projected to increase by 20.1% in 2024 and maintain a steady growth trend until 2027¹. Despite the convenience and efficiency of online shopping, customers have reservations about the perceived value of products, particularly those that provide experiential value (Feng et al., 2023). Practically, numerous consumers read online reviews to reduce ambiguity before purchasing. According to the survey², a significant majority of customers, specifically 90%, check online reviews before they decide to visit a business; additionally, a high percentage of customers, 88%, attribute the same degree of confidence in online reviews as they do in personal recommendations. A newly published study reveals that the global quantity of online reviews doubled in the year after the start of COVID-19³, suggesting that businesses should heed consumer reviews. Online businesses typically make dynamic adjustments to not only prices but also inventory in response to the effects of online reviews. Within scholarly circles, a multitude of researchers have emphasized the significance of online consumer reviews in the context of inventory management (Yang & Zhang, 2019; Vahdani & Sazvar, 2022; Cheung et al., 2022). When retailers recognize that the subsequent appearance of online reviews can assist in alleviating consumers' uncertainty regarding the product, thereby increasing sales, they are equipped with a fresh opportunity to maintain strategic inventory early on. Strategic inventory refers to a business practice wherein retailers deliberately acquire a greater quantity of products than they can sell during the current period, and the excess inventory is then transferred to the subsequent sales period. Two possible explanations exist for why retailers may choose to maintain strategic inventory. On the one hand, strategic inventory can assist retailers in mitigating the risk of shortages. A lack of inventory results in financial losses for retailers. Upon its introduction to the market in 2023, Mate 60 was met with significant consumer enthusiasm, propelled mainly by online reviews praising its exceptional camera capabilities. As a result, retailers encountered challenges related to inventory shortages due to the increased level of demand, and those who proactively maintain strategic inventories have the potential to generate substantial financial gains. On the other hand, strategic inventory can improve the retailers' bargaining power concerning forthcoming wholesale prices. Researchers have shown that implementing strategic inventory can effectively motivate upstream manufacturers to decrease their future wholesale pricing (Anand et al., 2008). When manufacturers cannot provide an optimal price, retailers will instead choose to sell the pre-owned inventory. A case in point is CELSA, an iron and steel corporation that pre-accumulates significant volumes of raw materials, thereby compelling its suppliers to decrease their prices.

In practice, information asymmetries in the supply chain are pervasive. While downstream retailers and upstream manufacturers agree on the advantages of

¹<https://www.emarketer.com/content/worldwide-retail-e-commerce-forecast-2024>.

²<https://www.e-satisfaction.com/how-important-are-customer-reviews-to-consumers/>.

³<https://www.mckinsey.com/capabilities/operations/our-insights/why-business-must-heed-customer-reviews>.

information sharing, achieving information symmetry throughout the entire supply chain remains a challenge. Generally, retailers exhibit reluctance to disclose confidential information to manufacturers. An example is the home appliance industry, where manufacturers frequently face challenges in obtaining critical data, including negotiated retail pricing, sales figures, and inventory information. This is because, to protect their interests, retailers typically maintain the privacy of this information. Due to the inventory information asymmetry, manufacturers could not observe retailers' strategic inventory behavior. Hence, manufacturers encounter difficulties in effectively adjusting wholesale pricing due to the lack of precise knowledge regarding retailers' inventory levels and sales despite being provided with order quantity information. As a result, a question arises: in light of the subsequent consequences of online evaluations, is it advantageous for retailers to maintain strategic inventory levels if they conceal inventory information?

Typically, retailers of household appliances, cell phones, and vehicles tend to place large orders before the start of the sales season not only to minimize the possibility of running out of stock after the appearance of online reviews but also to enhance their negotiating leverage with upstream manufacturers. In contrast, retailers rarely disclose confidential inventory data to manufacturers. Expanding upon prior investigations concerning strategic inventory, we augment our analysis by integrating the effects of information asymmetry and online reviews. This research addresses the questions below: 1) How do online reviews affect the strategic inventory? What are the implications of information asymmetry on strategic inventory that is based on online reviews? Furthermore, what are the differences in strategic inventory between information symmetry and asymmetry? 2) How do online reviews affect the prices over the two periods under strategic inventory? How does information asymmetry affect the prices? 3) Does the manufacturer suffer from an information disadvantage?

To tackle these inquiries, we develop a two-period model including a manufacturer (he) and a retailer (she). The retailer can order an excessive number of items during the early period and maintain the surplus for future sales. Consumers who buy the products in Period 1 do not have access to online reviews because the products are newly launched, but they can post reviews after purchasing, which affects the purchasing decisions of consumers in Period 2. We first consider two benchmark cases where the retailer's strategic inventory can be witnessed and then pay close attention to the scenario of information asymmetry, namely when the manufacturer cannot witness the retailer's inventory. Analyzing the equilibrium outcomes, we deduce managerial insights for strategic inventory. The analysis is based on two effects under the impact of online reviews: the inventory expansion effect, which refers to the retailer's tendency to store items in advance, and the demand-expansion effect, which means that the potential market is expected to expand. By analyzing the tradeoff between the two effects, one can understand how information asymmetry and online reviews affect strategic inventory.

This research complements the existing knowledge on strategic inventory by

analyzing the intricate impacts of online reviews and information asymmetry. Our study focuses on the impact of review signals and consumers' reliance on online reviews on strategic inventory. Our objective is to elucidate the distinctions between information symmetry and asymmetry, an area that has yet to be explored in the academic literature despite some researchers examining strategic inventory with online reviews or under information asymmetry independently.

The subsequent sections are structured as follows. Section 2 summarizes the relevant literature to fill the existing gap and align our study accordingly. In Section 3, we provide a detailed explanation of our model and give the fundamental assumptions. In Section 4 and Section 5, we provide the equilibrium solutions for strategic inventory when online reviews are present in Period 2, considering both information symmetry and asymmetry scenarios. In Section 6, we compare the results. Section 7 further analyze the overall supply chain, and welfare. Ultimately, in Section 8, we bring this paper to a conclusion and delineate possible avenues for further investigation. Refer to **Appendix** for comprehensive information on certification procedures.

2. Literature Review

This paper researches strategic inventory considering online reviews and information asymmetry. Our work is intimately connected to three primary areas of research: online reviews, strategic inventory, and information asymmetry.

2.1. Online Reviews

Online reviews, also known as online comments or ratings, have garnered considerable interest during the past two decades. [Kuksov & Xie \(2010\)](#) examine price and frill tactics concerning user reviews. The study conducted by [Godes \(2017\)](#) examine how word-of-mouth communications impact product policy. [Sun \(2012\)](#) examines the informational function of ratings. Several studies examine pricing tactics in the setting of customer reviews or comments ([Zhao et al., 2022](#); [Liu et al., 2017](#); [Chen et al., 2011](#); [Li & Hitt, 2010](#); [Feng et al., 2019](#)). [Kim et al. \(2019\)](#) reveals the impact of third-party reviews on quality provision. Numerous academics research the business ramifications associated with online reviews throughout supply chains. [Kwark et al. \(2014\)](#) conduct a study on online product reviews, notably investigating the dominance of quality or fit dimension compared to competitors. [Yang et al. \(2021\)](#) investigate dual-channel with online customer reviews. [Shin & Zeevi \(2023\)](#) develop a two-stage model to examine the quality decisions made by manufacturers and the information-sharing choices made by retailers, taking into account user comments. In addition, the influence of comments and ratings on inventory management has recently garnered significant attention. [Yang & Zhang \(2019\)](#) devise a probabilistic model that integrates price and inventory management, considering customer feedback. The study done by [Cheung et al. \(2022\)](#) investigate the correlation between inventory balance and consumers'

online learning behavior. In addition, [Vahdani & Sazvar \(2022\)](#) conduct a study on the dynamic management of perishable commodities with social learning. These studies demonstrate that online reviews and ratings significantly impact not just customer behavior but also the operational management of organizations, such as their pricing strategy and inventory management.

2.2. Strategic Inventory

The concept of “strategic inventory” is proposed by [\(Anand et al., 2008\)](#). It reveals that retailers might employ strategic inventory to negotiate with manufacturers on forthcoming wholesale prices. Subsequently, [Desai et al. \(2010\)](#) examine the trade between manufacturers and strategic inventory, while [Hartwig et al. \(2015\)](#) validate the efficacy of cutting prices for strategic inventory by empirical methodologies. Literature on strategic inventory primarily concentrates on encroachment ([Li et al., 2021](#); [Guan et al., 2019](#)), rebates ([Arya & Mittendorf, 2013](#); [Chen et al., 2022b](#)), quality deteriorating products ([Mantin & Jiang, 2017](#); [Miao et al., 2022](#)), process improvement ([Mantin & Veldman, 2019](#)), cost misreporting ([Chen et al., 2022c](#)), Corporate Social Responsibility ([Wang et al., 2022b](#)) and so on. Nevertheless, there is a scarcity of studies related to online reviews. [Wang et al. \(2022a\)](#) initially explore strategic inventory with online consumer reviews. Two key differences separate our work from [Wang et al. \(2022a\)](#). Our primary objective is to examine the influence of online reviews on strategic inventory, whereas [Wang et al. \(2022a\)](#) primarily explore wholesale pricing with online reviews. Besides, our paper examines the effects of information asymmetry, a phenomenon universally observed in real-world situations.

2.3. Information Asymmetry

Decision-making among supply chain members becomes intricate when information asymmetry exists within a supply chain ([Vosooghizaji et al., 2020](#); [Pruthi & Koul, 2019](#)). It is commonly acknowledged that participants seek information from others while safeguarding their information. Prior research on asymmetric information has predominantly concentrated on cost ([Lei et al., 2015](#); [Chen et al., 2022c](#)), demand ([Gao et al., 2021](#); [Liu et al., 2019](#)) and inventory ([Zhang et al., 2010](#); [Song, 2020](#)). In strategic inventory, it is generally assumed that information sharing exists between retailers and manufacturers (i.e., retailers’ inventory decisions are observable to manufacturers). There is a scarcity of studies investigating the strategic inventory under information asymmetry. [Roy et al. \(2019\)](#) examine the issue that manufacturers cannot view retailers’ strategic inventory. [Miao et al. \(2022\)](#) conduct a more in-depth analysis of the strategic inventory management of degrading commodities, considering asymmetric information. However, there is a shortage of evidence about how online reviews affect strategic inventory under information asymmetry.

Our paper is most relevant to [Wang et al. \(2022a\)](#). The study conducted by [Wang et al. \(2022a\)](#) examine the supplier’s pricing strategy in the presence of consumer

reviews and the retailer's strategic inventory. Although Wang et al. (2022a) initially explore strategic inventory with online reviews, they assume that the retailer's strategic inventory behavior is observable. Hence, based on current literature, our research makes a valuable contribution by using the aforementioned three components to examine the impact of online reviews and information asymmetry on strategic inventory. The relevant literature is summarized in Table 1. It is easy to observe that there the impact of information asymmetry on strategic inventory is relatively unexplored.

Table 1. Related literature.

Papers	Online reviews	Strategic Inventory	Information Asymmetry		
			Demand	Cost	Inventory
Kwark et al. (2014)	√				
Shin & Zeevi (2023)	√				
Wang et al. (2022a)	√	√			
Anand et al. (2008)		√			
Arya & Mittendorf (2013)		√			
Gao et al. (2021)			√		
Liu et al. (2019)			√		
Ma et al. (2017)				√	
Chen et al. (2022c)		√		√	
Song (2020)					√
Zhang et al. (2010)					√
Miao et al. (2022)		√			√
Roy et al. (2019)		√			√
Our paper	√	√			√

3. Research Design and Methodology

We employ a two-period supply chain model, including a manufacturer (he) in charge of production and a retailer (she) responsible for procuring and reselling the products to end customers. As previously demonstrated, the retailer can procure additional products for sale in Period 2 in addition to those ordered for sale in Period 1. The main notations are shown in Table 2.

To simplify the process, we standardize the manufacturing and selling costs to zero. Following Li et al. (2021) and Guan et al. (2019), it is assumed that both participants in the supply chain possess market information and that the potential demand remains constant throughout both periods. Besides, we assume that all

consumers exhibit myopic behavior. In other words, we do not consider the case of consumers strategically comparing products from the two periods. This also ensures that the potential demand is identical in both periods, and we can focus more on the analysis of strategic inventory. The myopic consumer assumption is commonly adopted in prior scholarly works (Wang et al., 2022a; Liu et al., 2021).

Table 2. Notation list.

Parameters	Description
v	Consumers' prior belief of the product, uniformly distributed in $[0,1]$
v_0	Value of the product reflected by online reviews (ratings); $-1 < v_0 < 1$
r	The weight of online reviews; $0 < r < 1$.
h	Holding cost of the strategic inventory
Variables	
π_{it}^j	The profit of i in Period t , $t=1,2$, $j=\{\bar{B}, B, \bar{N}, N\}$; $i=\{R, M, SC\}$, denotes the retailer, the manufacturer, the overall supply chain, respectively
q_t	The retailer's order quantity in Period t
I	The retailer's inventory level
s_t	The selling quantity in Period t
p_t	The retail price in Period t
w_t	The wholesale price in Period t
CS/SW	Consumer surplus/Social welfare

Generally, there are no reviews accessible for a newly launched product. Therefore, in Period 1, customers assess the product only based on their initial belief v , which is uniformly distributed in $[0,1]$. These preconceived notions may be from the content seen on a website, commercial advertising the product, or a company's reputation. The customer's utility is represented by the function $u_1 = v - p_1$. Therefore, we can calculate the quantity sold in Period 1 as $s_1 = 1 - p_1$. The inverse demand function is $p_1 = 1 - s_1$.

Customers in Period 1 may post feedback about the product on the Internet after purchasing, allowing customers in Period 2 to browse and examine reviews online. Following Kwark et al. (2014) and Yang et al. (2021), we denote v_0 as the product's value indicated by reviews. More precisely, in Period 2, customers see a favorable signal from online reviews when $v_0 > 0$, and conversely, an unfavorable signal when $v_0 < 0$. Hence, the consumer's expected evaluation is $(1-r)v + rv_0$, with r denoting the customers' emphasis on the information derived from online reviews.

The utility function is defined as $u_2 = (1-r)v + rv_0 - p_2$. Therefore, the quantity being sold could be calculated by $s_2 = 1 - \frac{rv_0 - p_2}{r-1}$. The inverse demand function is $p_2 = (1-s_2)(1-r) + rv_0$.

Additional assumptions for our research include:

1) Studies have discovered that sellers can distort consumer reviews (Chen et al., 2022a; Wu & Qiu, 2023). For simplicity, we assume that reviews posted by customers online reviews provide complete and accurate information (Zhao et al., 2022).

2) To focus on the analysis of strategic inventory, following prior works regarding strategic inventory (Roy et al., 2019; Miao et al., 2022), we do not consider safety inventory, pipeline inventory, or other related considerations.

4. Benchmarks

To examine the effects of information asymmetry, two benchmarks are established first: no strategic inventory with information symmetry and strategic inventory with information symmetry, denoted by \bar{B} and B , respectively.

4.1. No Strategic Inventory with Information Symmetry (\bar{B})

Under Scenario \bar{B} , the retailer does not maintain surplus inventory in Period 1. Thus, in each period, the manufacturer sets the wholesale price, and then the retailer determines the order quantity, which equals the sale quantity in this period.

Lemma 1. The equilibrium results under Scenario \bar{B} are: $w_1^{\bar{B}^*} = \frac{1}{2}$,

$$w_2^{\bar{B}^*} = \frac{1-r(1-v_0)}{2}, \quad q_1^{\bar{B}^*} = s_1^{\bar{B}^*} = \frac{1}{4}, \quad q_2^{\bar{B}^*} = s_2^{\bar{B}^*} = \frac{1-r(1-v_0)}{4(1-r)}, \quad p_1^{\bar{B}^*} = \frac{3}{4},$$

$$p_2^{\bar{B}^*} = \frac{3(1+r(1-v_0))}{4}, \quad \pi_M^{\bar{B}^*} = \frac{1}{8} \left(1 + \frac{(1-r(1-v_0))^2}{1-r} \right) \text{ and}$$

$$\pi_R^{\bar{B}^*} = \frac{1}{16} \left(1 - \frac{(1-r(1-v_0))^2}{r-1} \right).$$

Lemma 1 reveals that unlike studies that neglect online reviews (Anand et al., 2008), the pricing and quantity decisions vary between the two periods. This distinction is because, in Period 2, customers assess the product by considering online ratings and their pre-existing opinions. However, in Period 1, they depend on their pre-existing beliefs. In other words, the two periods are never independent when online reviews are considered. This is in line with studies of online reviews (Zhao et al., 2002; Feng et al., 2023).

Furthermore, pricing and order/selling quantities are both experiencing an upward trend concerning the review signal v_0 during the two periods. This phenomenon is understandable, as positive ratings consistently lead to higher pricing and greater quantity. Nevertheless, prices are universally declining, although quantities may fluctuate with the weight of reviews. Consequently, as

customers attach greater importance to online evaluations, prices are expected to decrease, and there may be fluctuations in sales volumes based on the influence of these reviews.

Finally, the review signal positively affects the profitability of both participants in the supply chain, regardless of whether customers' emphasis on reviews results in a profit rise or reduction. Therefore, enhancing product reviews is advantageous for all participants in the supply chain, yet encouraging customers to prioritize online reviews may not provide positive outcomes.

4.2. Strategic Inventory with Information Symmetry (B)

Figure 1 depicts the decision-making sequence when the retailer maintains strategic inventory under information symmetry. In Period 1, the manufacturer establishes the wholesale price, denoted as w_1^B . Subsequently, the retailer selects the number of products to sell, represented as s_1^B , as well as the inventory to retain for the next period, denoted as I^B . Customers who purchase the product during Period 1 can write comments online; hence, the signal v_0 is realized at the end of Period 1. In Period 2, the manufacturer establishes the wholesale price w_2^B before the retailer determines the selling quantity s_2^B .

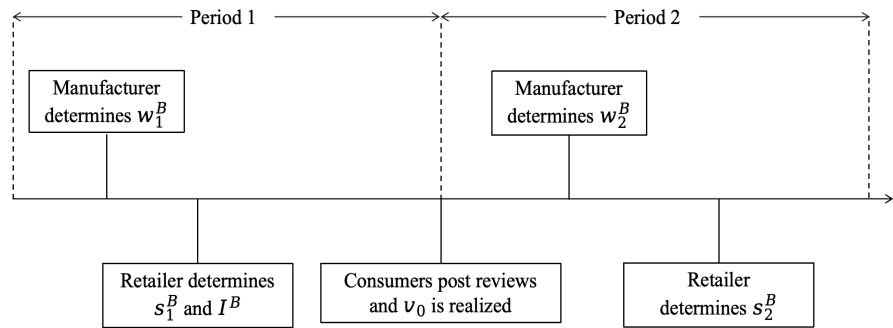


Figure 1. Decision sequence under strategic inventory with information symmetry (B).

Lemma 2. The equilibrium results under Scenario B are:

$$w_1^{B*} = \frac{4h - 9(r(v_0 - 2) + 2)}{18r - 34}, \quad w_2^{B*} = \frac{h(10 - 6r) + 3r(v_0 - 2) + 6}{17 - 9r},$$

$$q_1^{B*} = \frac{4h(5r - 9) - (9r - 13)(2 - r(2 - v_0))}{4(1 - r)(17 - 9r)},$$

$$q_2^{B*} = \frac{4h - 9(2 - r(2 - v_0))}{(34 - 18r)(1 - r)} + \frac{h + r(1 - v_0) - 1}{r - 1}, \quad s_1^{B*} = \frac{4h - 9rv_0 + 16}{68 - 36r},$$

$$s_2^{B*} = \frac{18 - 4h + 9r(v_0 - 2)}{6(9r - 17)(1 - r)} + \frac{3(1 - r) - 2h + 3rv_0}{6 - 6r},$$

$$I^{B*} = \frac{4h(3r - 5) + 9r^2(1 - v_0) + r(11v_0 - 14) + 5}{18r^2 - 52r + 34}, \quad p_1^{B*} = \frac{52 - 4h - 9r(4 - v_0)}{68 - 36r},$$

$$p_2^{B*} = \frac{1}{6} \left(3 + \frac{4h - 9(2 - r(2 - v_0))}{9r - 17} + 2h - 3r(1 - v_0) \right),$$

$$\pi_M^{B*} = \frac{9[2-r(2-v_0)]^2 - 8h[2-r(2-v_0)] + 16(2-r)h^2}{8(17-9r)(1-r)} \text{ and}$$

$$\pi_R^{B*} = \frac{z_1 v_0^2 + z_2 v_0 + z_3}{16(17-9r)^2 (r-1)}, \text{ where } z_1 = r^2(9(109-36r)r-733),$$

$$z_2 = 4r(6h(58-27(3-r)r) + (9r-5)(1-r)(31-18r)) \text{ and}$$

$$z_3 = 8h(-2h(76-r(91-27r)) + r(9r(29-9(r-(3-r)v_0)) + 174v_0 - 239) + 59) - 620$$

5. Strategic Inventory with Information Asymmetry

This section examines the strategic inventory with information asymmetry, considering the impact of online reviews (represented as N). **Figure 2** illustrates the decision sequence when information is asymmetric. Initially, the manufacturer establishes the wholesale pricing denoted as w_1^N , and the retailer determines the order quantity represented as q_1^N subsequently. The retailer then independently decides on the amount s_1^N to sell and the inventory I^N to hold for Period 2. Simultaneously, the manufacturer adjusts the wholesale price to w_2^N without witnessing her inventory decisions. At this stage, customers who bought the items in Period 1 write comments, resulting in the realization of the signal v_0 . In Period 2, the retailer determines the amount to orders (q_2^N) and sells (s_2^N).

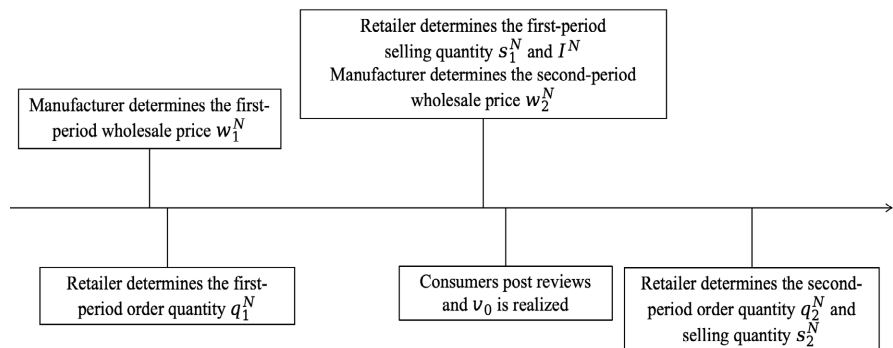


Figure 2. Sequence of decisions under strategic inventory with information asymmetry (N).

Following backward induction, the retailer’s sale in Period 2 is discussed first. Given the inventory of $I^N > 0$ carried over from Period 1 and knowledge of the wholesale price w_2^N , the retailer determines the optimal order quantity (q_2^N) to maximize her profit:

$$\pi_{R2}^N = p_2^N s_2^N - w_2^N q_2^N = \left[(1-s_2^N)(1-r) + rv_0 \right] s_2^N - w_2^N q_2^N, \tag{1}$$

$$\text{s.t. } s_2^N < q_2^N + I^N.$$

It can be easily confirmed that:

$$q_2^{N*} = \left[\frac{1-r(1-v_0)-w_2^N}{2(1-r)} - I^N \right]^+ \tag{2}$$

By substituting (2) to (1), we can derive her profit in Period 2 as:

$$\pi_{R2}^{N*} = \begin{cases} \frac{(1-r(1-v_0))^2 + w_2^N - 2w_2^N(1-2I^N(1-r)-r(1-v_0))}{4(1-r)} & \text{if } w_2^N \leq (1-r)(1-2I^N) + rv_0, \\ I^N((1-I^N)(1-r) + rv_0) & \text{otherwise.} \end{cases} \tag{3}$$

Thus, the manufacturer's profit in Period 2 is:

$$\pi_{M2}^{N*} = \begin{cases} w_2^N \left(\frac{1-r(1-v_0)-w_2^N}{2(1-r)} - I^N \right) & \text{if } w_2^N \leq (1-r)(1-2I^N) + rv_0, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Information asymmetry prevents the manufacturer from witnessing the retailer's inventory behavior while adjusting the wholesale price. Likewise, the retailer decides her order quantity q_1^N and inventory I^N with knowledge of this fact. Profit for the retailer upon procurement of q_1^N is:

$$\pi_{R1}^N = p_1^N q_1^N + \pi_{R2}^{N*} - hI^N. \tag{5}$$

Since both the manufacturer and the retailer could not observe each other's decisions, they are considered to have made decisions simultaneously (i.e., he determines w_2^N , and she determines I^N and s_1^N).

Lemma 3. A unique Nash equilibrium $(I^{N*}, s_1^{N*}, w_2^{N*})$ exists as:

$$(I^{N*}, s_1^{N*}, w_2^{N*}) = \begin{cases} \left(0, q_1^N, \frac{1}{2}(1+rv_0-r) \right) & \text{if } q_1^N < \underline{q}_1^N, \\ \left(\frac{4q_1^N + rv_0 - 2h - r - 1}{2(3-r)}, \frac{2h - 4q_1^N - rv_0 + r + 1}{2(3-r)} + q_1^N, \frac{(1-r)(h - 2q_1^N + 2) + rv_0}{3-r} \right) & \text{if } \underline{q}_1^N < q_1^N < \bar{q}_1^N, \\ \left(\frac{1-2h+rv_0-r}{2(1-r)}, \frac{1}{2}, h \right) & \text{if } q_1^N > \bar{q}_1^N. \end{cases} \tag{6}$$

where $\underline{q}_1^N = \frac{1}{4}(2h+1+r-rv_0)$ and $\bar{q}_1^N = \frac{2(1-h)+r(v_0-2)}{2(1-r)}$.

Lemma 3 illustrates that when $q_1^N < \underline{q}_1^N$, the retailer is unable to deliver the inventory and can only satisfy the customer demand in Period 1. When the value of q_1^N lies between the range of \underline{q}_1^N and \bar{q}_1^N , the retailer has the ability to maintain inventory. In other words, the items procured during Period 1 can be utilized to satisfy inventory needs for the ongoing period and sales in the

subsequent period. However, when $q_1^N > \bar{q}_1^N$, the retailer orders excessive goods in Period 1 (i.e., $q_1^N > I^N + s_1^N$), potentially leading to unnecessary wastage. Hence, the retailer must ensure that the amount of the order remains below the specified threshold of \bar{q}_1^N .

Then the retailer determines the order quantity (q_1^N) based on the wholesale pricing (w_1^N). The retailer's profit throughout the two periods is

$$\pi_R^N = \pi_{R1}^{N*} - w_1^N q_1^N. \tag{7}$$

By substituting (6), (5) and (3) into (7), we can derive:

$$\pi_R^{N*} = \begin{cases} \frac{(1-r(1-v_0))^2}{16(1-r)} + (1-q_1^N)q_1^N - w_1^N q_1^N & \text{if } q_1^N < \underline{q}_1^N, \\ \frac{h^2(r-5)(r-1) - 2h(r-1)(2q_1^N(r-5) + (r-4)r(v_0-1) + 1) - 4q_1^{N2}(1-r^2)(4-r)}{4(1-r)(3-r)^2} \\ + \frac{4q_1^N(r-4)(r-1)(r(v_0-2) + 2) + r(r((r-5)r+5)(1-v_0)^2 + 2v_0-3) + 2}{4(3-r)^2(1-r)} - w_1^N q_1^N & \text{if } \underline{q}_1^N < q_1^N < \bar{q}_1^N, \\ \frac{h^2 - 2h(r(v_0-1) + 1) + r(r(v_0-1)^2 + 2v_0-3) + 2}{4(1-r)} - w_1^N q_1^N & \text{if } q_1^N > \bar{q}_1^N. \end{cases} \tag{8}$$

From equation (8), π_R^{N*} is continuous and piece-wise concave or monotonically decreasing in q_1^N for a given w_1^N .

Lemma 4. The retailer determines q_1^N based on the manufacturer's wholesale pricing w_1^N as:

$$q_1^{N*} = \begin{cases} \frac{h(r-5) + (4-r)(r(v_0-2) + 2) - (r-3)^2 w_1^N}{2(4-r)(1-r)} & \text{if } 0 < w_1^N < W_1^N, \\ \frac{1-w_1^N}{2} & \text{if } W_1^N < w_1^N < 1. \end{cases} \tag{9}$$

The retailer will maintain the inventory when $w_1^N < W_1^N$, but will not maintain it when $w_1^N > W_1^N$.

Corollary 1. The retailer's order amount q_1^N experiences a discontinuity at the point when $w_1^N = W_1^N$ under information asymmetry. However, the manufacturer never selects W_1^N as w_1^N in equilibrium.

Corollary 1 demonstrates that when the retailer's inventory behavior is unobservable, the order quantity function becomes discontinuous at $w_1^N = W_1^N$ (see **Figure 3**). This finding aligns with previous studies on strategic inventory that also consider information asymmetry (Roy et al., 2019; Miao et al., 2022). However, the manufacturer never chooses the discontinuous point as the wholesale price as he would incur a loss of profit at $w_1^N = W_1^N$. Besides, by

comparing **Figure 3(a)** and **Figure 3(b)**, it is apparent that the disparity between A and B is more significant than the disparity between C and D. In other words, q_i^N experiences a larger jump at W_1^N with positive reviews. In addition, W_1^N under positive reviews is larger than that under negative reviews.

The manufacturer total profit can be derived as:

$$\pi_M^N = w_1^N q_1^N + \pi_{M2}^{N*} \tag{10}$$

By substituting (9) and (4) into (11), we can obtain the manufacturer's profit:

$$\pi_M^{N*} = \begin{cases} \frac{h^2(r-3)^2 + h((r-3)r-2)w_1^N + w_1^N((r-4)^2(r(v_0-2)+2) + (r-3)^3 w_1^N)}{2(r-4)^2(1-r)} & \text{if } 0 < w_1^N < W_1^N, \\ \frac{(r(v_0-1)+1)^2}{8(1-r)} - \frac{1}{2}(w_1^N-1)w_1^N & \text{if } W_1^N < w_1^N < 1. \end{cases} \tag{11}$$

The manufacturer determines w_1^N to maximize his total profit as in (11), which is summarized as follows.

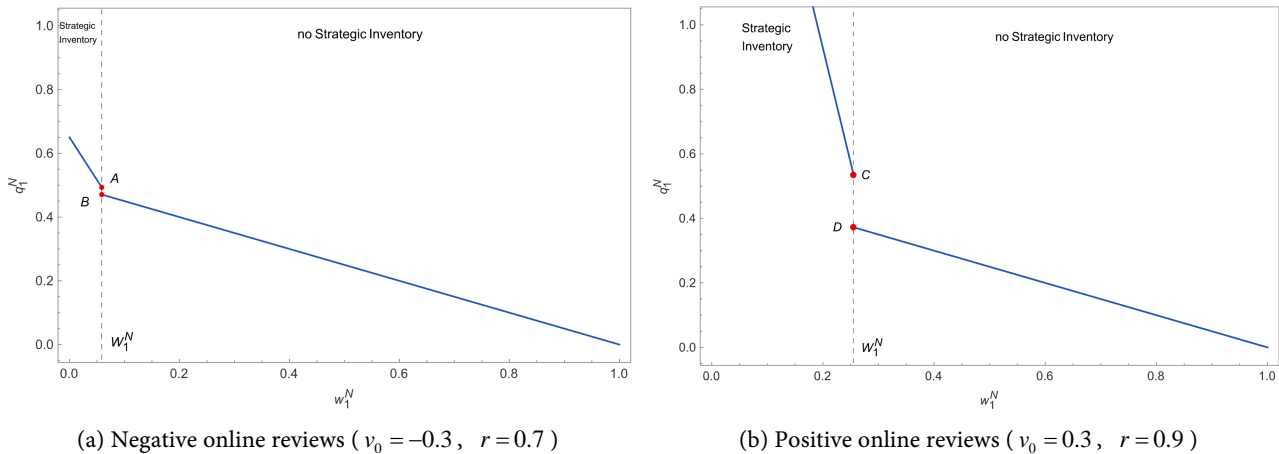


Figure 3. The retailer's ordering quantity q_i^N ($h = 0$).

Lemma 5. The manufacturer's decision on w_1^N is:

$$w_1^{N*} = \begin{cases} \frac{h(2-(r-3)r) - (r-4)^2(r(v_0-2)+2)}{2(r-3)^3} & \text{if } \max\{0, h_1\} < h < h_2, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{12}$$

For simplicity, we denote \bar{N} as the case where the retailer does not maintain strategic inventory under information asymmetry while N as the case where the retailer maintains the inventory under information asymmetry. Hence, we obtain the following Lemma.

Lemma 6. (1) When $\max\{0, h_1\} < h < h_2$ (Scenario N), the equilibrium

results are $w_1^{N^*} = \frac{h(2-(r-3)r)-(4-r)^2(2+r(v_0-2))}{2(r-3)^3}$,

$$w_2^{N^*} = \frac{h(r(2r-11)+13)+(4-r)(2-r(2-v_0))}{2(3-r)^2},$$

$$q_1^{N^*} = \frac{h(3r-7)-(r-2)(2-r(2-v_0))}{4(3-r)(1-r)},$$

$$q_2^{N^*} = \frac{h((11-2r)r-13)+(r-4)(2-r(2-v_0))}{4(3-r)^2(r-1)},$$

$$s_1^{N^*} = \frac{(r-4)rv_0-(h+2)(r-5)}{4(r-3)^2},$$

$$s_2^{N^*} = \frac{h(r(2r-11)+13)-(r-2)r(2r-7)v_0-2(1-r)(5+(r-5)r)}{4(r-1)(3-r)^2},$$

$$I^{N^*} = \frac{h[r(2r-11)+13]+r(1-v_0)((r-5)r+5)-1}{2(r-3)^2(r-1)},$$

$$p_1^{N^*} = \frac{h(r-5)+r(4v_0-r(v_0-4)-22)+26}{4(3-r)^2},$$

$$p_2^{N^*} = \frac{h(r(2r-11)+13)+rv_0(r(2r-13)+22)-2r((r-8)r+20)+26}{4(3-r)^2},$$

$$\pi_M^{N^*} = \frac{h^2(r((29-4r)r-70)+61)+2h((r-3)r-2)(2-r(2-v_0))+(4-r)^2(2-r(2-v_0))^2}{8(3-r)^3(1-r)}$$

and $\pi_R^{N^*} = \frac{z_4v_0^2+z_5v_0+z_6}{16(r-3)^4(r-1)}$, where

$$z_4 = r^2((r(184-r(45-4r))-320)r+196),$$

$$z_5 = 2h(r(73-r(31-4r))-50)(4-r)r$$

$$+ 4((r(76-r(21-2r))-104)r+34)(1-r)r \quad \text{and}$$

$$z_6 = h^2(r(r(r(4r-57)+278)-553)+376)$$

$$+ 4h(r-1)(r(r(r(2r-23)+89)-126)+38)$$

$$+ 4(r-1)(r(r((r-11)r+43)-72)+57)-34).$$

(2) Otherwise (Scenario \bar{N}), the equilibrium results are: $w_1^{\bar{N}^*} = \frac{1}{2}$,

$$w_2^{\bar{N}^*} = \frac{1}{2}(1+rv_0-r), \quad q_1^{\bar{N}^*} = s_1^{\bar{N}^*} = \frac{1}{4}, \quad q_2^{\bar{N}^*} = s_2^{\bar{N}^*} = \frac{1-r(1-v_0)}{4(1-r)}, \quad p_1^{\bar{N}^*} = \frac{3}{4},$$

$$p_2^{\bar{N}^*} = \frac{3}{4}(1-r(1-v_0)), \quad \pi_M^{\bar{N}^*} = \frac{(1+r(v_0-1))^2+1-r}{8(1-r)} \quad \text{and}$$

$$\pi_R^{\bar{N}^*} = \frac{r(r(1-v_0)^2+2v_0-3)+2}{16(1-r)}.$$

6. Comparable Analysis

Proposition 7. Under information symmetry,

- 1) $w_2^{\bar{B}^*} < w_1^{\bar{B}^*}$, $p_2^{\bar{B}^*} < p_1^{\bar{B}^*}$, $w_2^{B^*} < w_1^{B^*}$, $p_2^{B^*} < p_1^{B^*}$;
- 2) $w_1^{B^*} > w_1^{\bar{B}^*}$, $w_2^{B^*} < w_2^{\bar{B}^*}$.

Part (1) of Proposition 7 demonstrates the simultaneous decrease of both wholesale and retail prices under information symmetry. This implies that to attract potential consumers who may read online reviews, the retailer should reduce prices, and the manufacturer should decrease wholesale prices.

Part (2) of Proposition 7 demonstrates that under information symmetry, the wholesale price increases (decreases) when the retailer is (not) capable of maintaining strategic inventory. This aligns with Anand et al. (2008), which shows retailers employ strategic inventory to lower future wholesale pricing. As a response, manufacturers raise the first-period wholesale prices. Hence, part (2) of Proposition 7 establishes the continued validity of this assertion, considering the effects of online reviews. While the retailer may express concern about using strategic inventory in reaction to online ratings, she can utilize it to compel the manufacturer to lower future wholesale pricing, and he responds by raising the wholesale price during Period 1.

Proposition 8. Under information asymmetry,

- 1) $w_2^{\bar{N}^*} < w_1^{\bar{N}^*}$, $p_2^{\bar{N}^*} < p_1^{\bar{N}^*}$.
- 2) When $h_{w1}^N < h < h_{w2}^N$, $w_2^{N^*} > w_1^{N^*}$; otherwise, $w_2^{N^*} < w_1^{N^*}$.
- 3) When $h > h_p^N$, $p_2^{N^*} > p_1^{N^*}$; otherwise, $p_2^{N^*} < p_1^{N^*}$.

Part (1) of Proposition 8 demonstrates that under information asymmetry, if the retailer does not maintain strategic inventory, both the wholesale and retail prices decrease over the two periods, which aligns with Part (1) of Proposition 7. This suggests that with the effects of online reviews, the prices decrease under the case without strategic inventory, regardless of whether she reveals her inventory information or not.

Parts (2) and (3) of Proposition 8 demonstrate that prices exhibit complex fluctuations when the retailer maintains strategic inventory when concealing her inventory information. Specifically, the retail price rises when the inventory cost is high, while the wholesale price increases when the cost is moderate. The rationale behind this circumstance is that information asymmetry leads to a need for transparency in the supply chain. In other words, upstream manufacturers adjust wholesale prices with a need for more crucial information (i.e., sales quantity and inventory amount owned by downstream retailers). Consequently, the wholesale price may be unreasonably elevated or reduced.

Proposition 9. The retailer will maintain strategic inventory when $h_1^B < h < h_2^B$ under information symmetry and $\max\{0, h_1\} < h < h_2$ under information asymmetry.

Proposition 9 proposes that the retailer will maintain strategic inventory as long as the holding cost is below acceptable limits. The reason is simple: a high holding cost negatively impacts the retailer's profitability. Contrary to intuition,

information symmetry establishes a threshold \bar{h}_1^B below which the retailer rejects to maintain inventory. A plausible reason for this contradiction is that the retailer perceives the manufacturer as being responsible for predicting demand when the retailer's inventory level is readily apparent, and the minimal cost associated with holding inventory does not prompt the retailer to hold excess inventory in advance. As a result, the retailer tends to procure the quantity that aligns with the selling quantity in Period 1 rather than investing significant effort in pre-stocking. Nevertheless, an increase in holding costs signals the retailer that maintaining strategic inventory has the potential to generate profits. If this is the situation, the retailer will proceed with this action strategy.

Figure 4 visualize the conditions for strategic inventory in Proposition 9. It's easy to confirm that there is no common region under the two cases. Specifically, under information symmetry, the retailer will maintain the inventory when customers attach little significance to online ratings, while under information asymmetry, the retailer will maintain the inventory when customers attach much importance. This contrasts with Roy et al. (2019) suggesting that retailers directly disclose inventory levels to upstream manufacturers when the holding cost falls within certain thresholds.

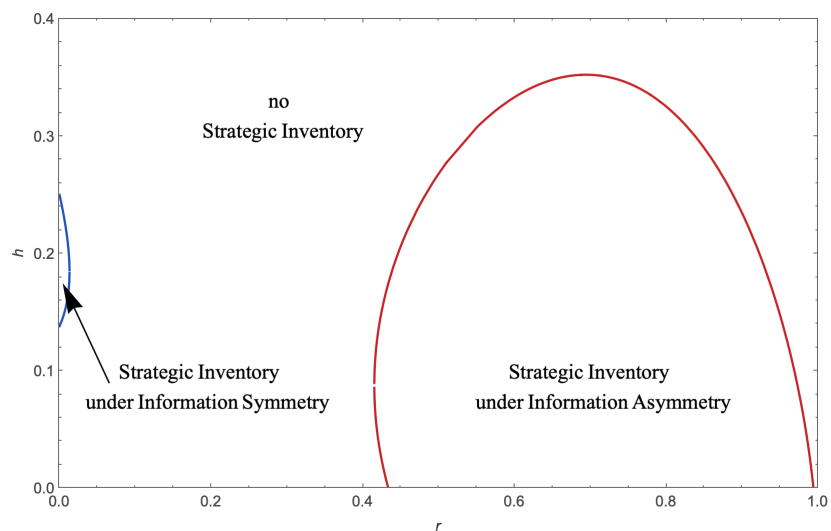


Figure 4. The retailer's decision on strategic inventory.

To further explore how online reviews affect strategic inventory under information symmetry (asymmetry), we analyze the range to hold inventory under the two cases. Specifically, we define the range of strategic inventory under information symmetry as $h_{range}^B = h_2^B - h_1^B$ and that under information asymmetry as $h_{range}^N = h_2 - h_1$. We obtain the following Proposition.

Proposition 10. $\frac{dh_{range}^B}{dv_0} > 0$; $\frac{dh_{range}^N}{dv_0} < 0$.

Proposition 10 demonstrates that h_{range}^D (h_{range}^N) increases (decreases) with v_0 under information symmetry (asymmetry). This is attributed to the tradeoff

between the *inventory-expansion effect* and the *demand-expansion effect*. The concept of the *inventory-expansion effect* claims that retailers tend to pre-stock items for future sales when online reviews become increasingly positive. By mitigating the risk of stock shortages, strategic inventory can assist the retailer in optimizing profits in this situation. On the other hand, the demand-expansion effect asserts that an improvement in online customer ratings will induce an expansion of the potential market.. A rise in the review signal corresponds to an enhanced propensity among consumers to buy the product; as a result, the retailer experiences a reduction in inventory and an upsurge in demand.

Figure 5(a) illustrates that when the retailer’s inventory is visible, the retailer is more likely to maintain the inventory as the review signal improves. Although the potential demand increases and hence accelerates inventory depletion, it is still more profitable for the retailer to hold some inventories in advance because she can have relatively strong bargaining power for future prices with the upstream manufacturer. As a result, the *inventory-expansion effect* dominates, and the retailer tends to store products in advance.

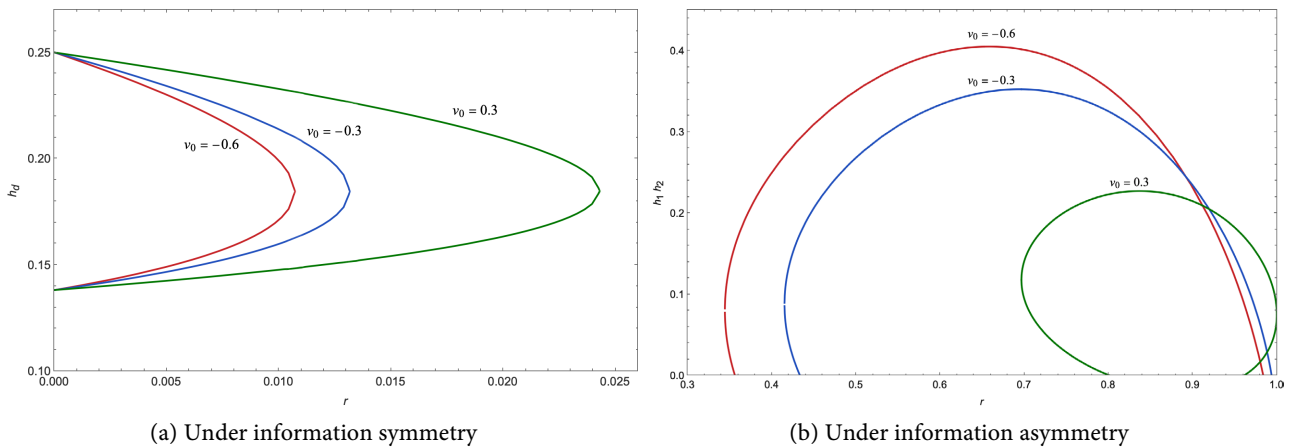


Figure 5. The range of strategic inventory with online reviews.

However, when the retailer conceals her inventory information, she is less likely to maintain the inventory as the reviews become more favorable, as shown in **Figure 5(b)**. This may appear paradoxical, given that retailers typically benefit from maintaining inventory in anticipation of favorable product evaluations. The rationale is that, under information asymmetry, the *demand-expansion effect* dominates the *inventory-expansion effect*. The prospective market demand is enhanced as the review signal improves. Nevertheless, when the retailer covertly organizes the stock for upcoming sales (i.e., the manufacturer cannot observe her inventory behavior), her bargaining power for future wholesale prices is weakened. In other words, the retailer’s ability to negotiate wholesale pricing is diminished when her inventory is not directly apparent. As a result, the *inventory-expansion effect* recedes under information asymmetry. According to Proposition 8, the wholesale price may increase when the retailer’s inventory is not visible. This further supports the notion that the *inventory-expansion effect* diminishes

under information asymmetry from the perspective of price fluctuations.

Corollary 2. $\pi_R^{\bar{N}^*} = \pi_R^{\bar{B}^*}$, $\pi_M^{\bar{N}^*} = \pi_M^{\bar{B}^*}$.

Corollary 2 indicates that in the absence of strategic inventory, the manufacturer’s and retailer’s profits remain unaffected by the disclosure of inventory information. This suggests that supply chain members are not interested in divulging inventory data if the retailer ignores strategic inventory practices.

Proposition 11. $\pi_M^{N^*} > \pi_M^{B^*}$.

Proposition 11 shows that information asymmetry confers a benefit on the manufacturer, which is consistent with the results presented by Roy et al. (2019) and Miao et al. (2022). The manufacturer anticipates a benefit from the retailer concealing its inventory behavior due to the sequence of decisions. As depicted in Figure 1, under information symmetry, the manufacturer possesses the opportunity to modify the wholesale price by utilizing sufficient knowledge (i.e., w_2^D is determined after I^D is set). In contrast, under information asymmetry (see Figure 2), both parties make decisions simultaneously (i.e., w_2^N , s_1^N and I^N are determined at the same time). Consequently, information asymmetry has led to the retailer relinquishing her leadership role in the subgame, a change that benefits the manufacturer.

7. Further Discussions

7.1. The Overall Supply Chain

The total supply chain profit and total quantity are calculated as $\pi_{SC}^{j^*} = \pi_M^{j^*} + \pi_R^{j^*}$ and $q_{SC}^{j^*} = q_1^{j^*} + q_2^{j^*}$, respectively. Figure 6(a) depicts the total supply chain profit under information symmetry. It’s easy to confirm that $\pi_{SC}^{B^*}$ initially experiences a decrease followed by a slight increase with h . Notably, $\pi_{SC}^{B^*}$ is lower than $\pi_{SC}^{\bar{B}^*}$ when h is high. This suggests that strategic inventory positively impacts the supply chain when the holding cost is small. Figure 6(b) shows that $q_{SC}^{B^*}$ is always higher than that $q_{SC}^{\bar{B}^*}$, though diminishing with h , which underscores that strategic inventory could enhance overall sales.

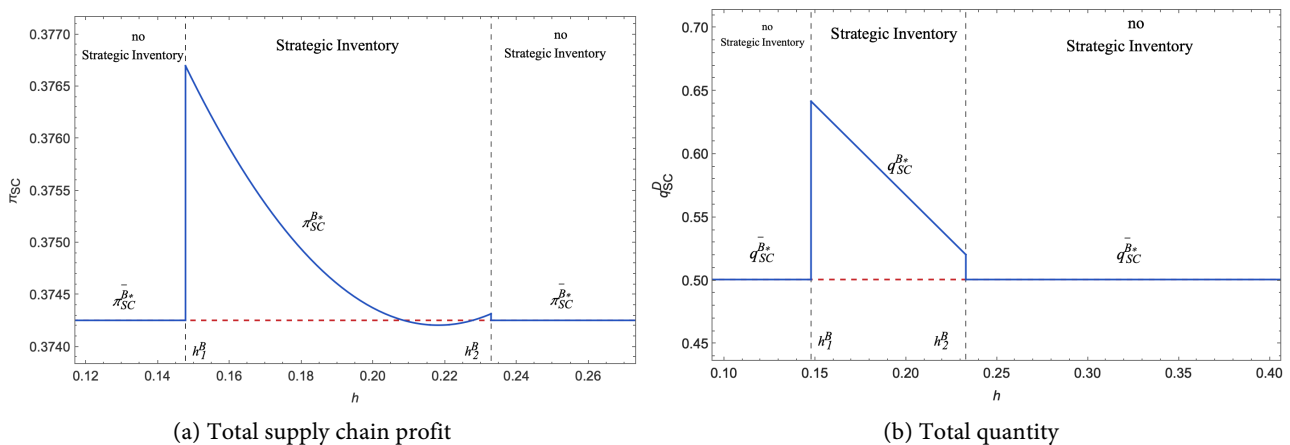


Figure 6. Total supply chain profit and quantity under information symmetry ($r = 0.01$, $v_0 = 0.3$).

Figure 7(a) shows that π_{SC}^{N*} is always higher than $\pi_{SC}^{\bar{N}*}$, which is in contrast with the case under information symmetry (see Figure 6(a)). This is because the manufacturer benefits from information asymmetry, as stated by Proposition 11. Consequently, the profitability of the supply chain rises. Figure 7(b) shows that q_{SC}^{N*} decreases with h and might even be lower than q_{SC}^{barN*} . This finding contrasts with Roy et al. (2019), which indicates that in the absence of inventory visibility, the total purchase is always higher compared with the case without the inventory. The rationale for this distinction arises from our consideration of consumer reviews. Specifically, the retailer holds the inventory only when $\max\{0, h_1\} < h < h_2$ while the thresholds (i.e., h_1 and h_2) exist only if r is sufficiently large (see Proposition 9). Therefore, influenced by the comprehensive effects of online reviews and the holding cost, the total quantity with strategic inventory might be lower than without strategic inventory.

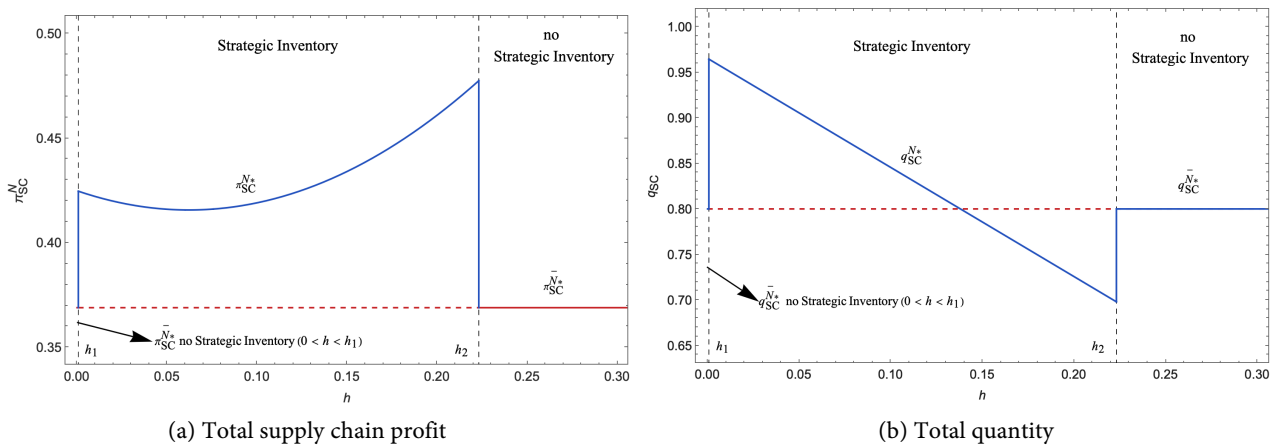


Figure 7. Total supply chain profit and quantity under information asymmetry ($r = 0.8$, $v_0 = 0.3$).

We further analyze how r and v_0 affect the total profit and quantities. Figure 8(a) shows that both π_{SC}^{B*} and $\pi_{SC}^{\bar{B}*}$ decrease with r but a better v_0 can mitigate this trend. Besides, $\pi_{SC}^{B*} > \pi_{SC}^{\bar{B}*}$ when $r > r_1$ ($r > r_2$) if $v_0 = -0.3$ ($v_0 = 0.3$). This implies that strategic inventory benefits the supply chain only when consumers weigh more on online reviews. In addition, Figure 8(a) also illustrates that the threshold under favorable reviews is higher than that under unfavorable ones ($r_1 < r_2$), which suggests that as online reviews become favorable, strategic inventory is not beneficial for the overall supply chain unless consumers put enough weight on online reviews. Figure 8(b) shows that both q_{SC}^{B*} and $q_{SC}^{\bar{B}*}$ decrease slightly with r . Besides, q_{SC}^{B*} is always higher than $q_{SC}^{\bar{B}*}$ no matter the review signal v_0 is favorable or not.

Intuitively, both q_{SC}^{B*} and $q_{SC}^{\bar{B}*}$ are higher when the review signal v_0 is better due to the expansion of the total purchase under the demand-expansion effect.

Figure 9(a) indicates that both π_{SC}^{N*} and $\pi_{SC}^{\bar{N}*}$ always increase with r . Besides, π_{SC}^{N*} is always higher than π_{SC}^{barN*} . This suggests that strategic inventory always

benefits the supply chain, which contrasts with the case under information symmetry (see **Figure 8**). In addition, π_{SC}^{N*} is higher when v_0 is better. **Figure 9(b)** shows that q_{SC}^{N*} increases (decreases) with r under positive (negative) review signal v_0 . This implies that as consumers put more weight on online reviews, positive reviews can boost total sales while negative ones have an opposite effect. Besides, $q_{SC}^{N*} > q_{SC}^{\bar{N}*}$ when $v_0 > 0$ and the result is opposite otherwise. This implies that favorable ratings have a significant impact on overall sales under information asymmetry.

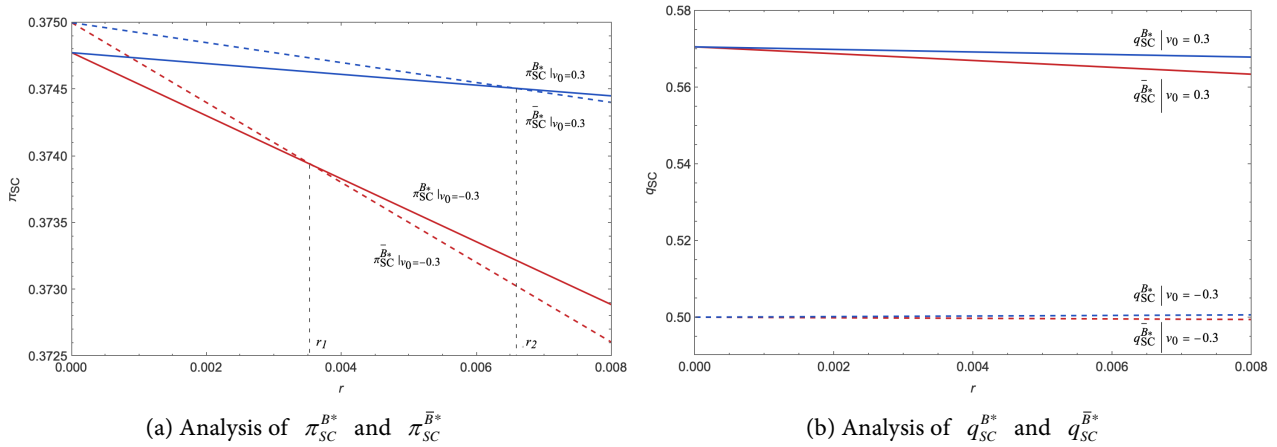


Figure 8. The impact of r and v_0 under information symmetry ($h = 0.2$).

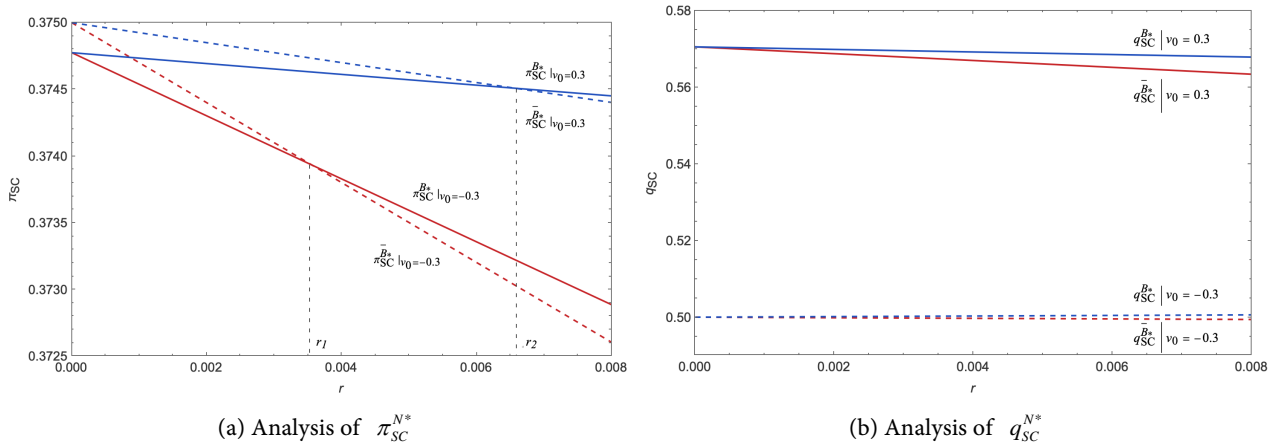


Figure 9. The impact of r and v_0 under information asymmetry ($h = 0.1$).

7.2. Consumer Surplus and Social Welfare

First, we calculate consumer surplus and social welfare as

$$CS = \int_{1-s_1}^1 v - p_1 dv + \int_{1-s_2}^1 (1-r)v + rv_0 - p_2 dv, \tag{13}$$

$$SW = CS + \pi_R^* + \pi_M^*. \tag{14}$$

By substituting the equilibrium results in Lemma 1, 2 and 6 into (13) and (14), respectively, we can obtain the consumer surplus and social welfare under each

scenario. The detailed expressions are summarized in **Appendix B**.

Figure 10(a) demonstrates that both CS^B and SW^B slightly decrease with h when strategic inventory is held. This is intuitive because strategic inventory entails costs that eventually pass to consumers, ultimately decreasing social welfare. **Figure 10(b)** shows that CS^N experiences a slight decrease with h , which is similar to CS^B observed in **Figure 10(a)**. However, SW^N demonstrates an increase with h . This is attributed to the profit gains achieved by the manufacturer. From the perspective of society, the loss caused by the holding cost can be offset by the profit gains from the manufacturer. Consequently, social welfare exhibits an increase under information asymmetry.

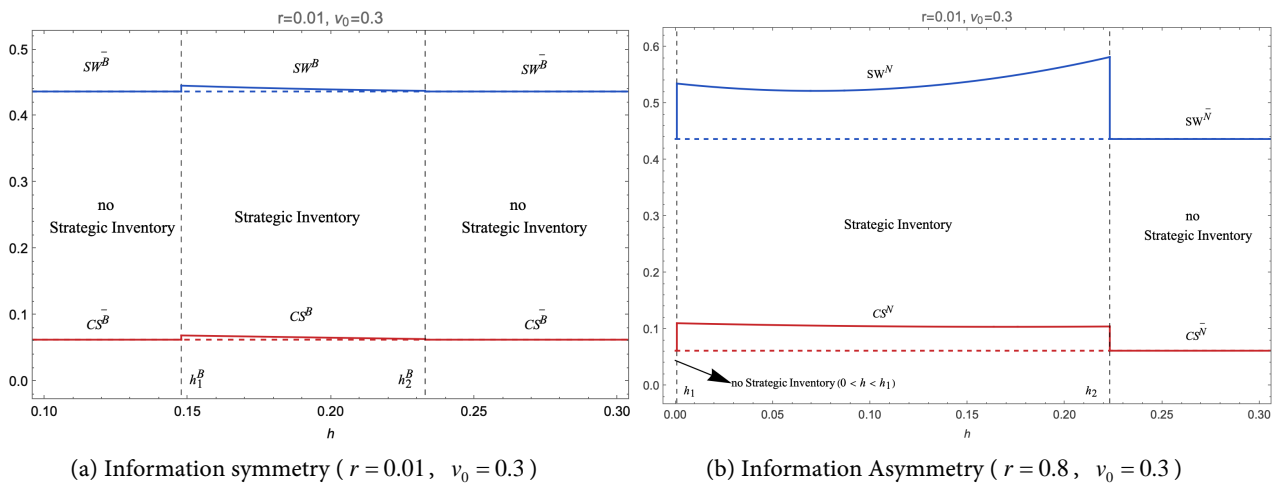


Figure 10. Consumer surplus and social welfare.

8. Conclusion

Retailers are motivated to place large orders for items before the start of the sales season to minimize the possibility of running out of stock after the appearance of online reviews and enhance their negotiating leverage with upstream manufacturers. Simultaneously, retailers may conceal the inventory information to maintain an information advantage. Therefore, based on the two aspects, we formulate a conceptual framework to analyze the impact of information asymmetry on strategic inventory with online reviews. The results show that the retailer strategically maintains inventory under information symmetry (asymmetry) when consumers assign minimal (substantial) importance to online reviews. Counter-intuitively, information symmetry dictates the existence of a holding cost threshold below which retailers are prohibited from maintaining strategic inventory. Besides, as online reviews improve, the retailer's propensity to maintain inventory is augmented (reduced) under information symmetry (asymmetry). Furthermore, in the context of online reviews, the implementation of strategic inventory could lead to a decrease in future pricing when information is symmetric. Nevertheless, prices might increase or decrease under information asymmetry. Finally, the manufacturer could gain an advantage from information asymmetry. Our research generates some

managerial insights as follows.

For the retailer, online reviews significantly influence her decisions regarding strategic inventory. Specifically, given suitable holding costs, the retailer can store some stocks in advance and disclose her inventory information to the manufacturer when consumers rarely care about online reviews; on the other hand, she can secretly hold the inventory when consumers attach great importance to online reviews. However, the retailer should avoid carrying the inventory when consumers weigh moderately on online reviews. In addition, as online reviews become more favorable, stocking inventories in advance can bring more (less) profits to the retailer when she discloses (privately owns) her inventory information.

For the manufacturer, he will gain more profits if he cannot observe the retailer's inventory. This underscores that when the manufacturer finds himself in an information-disadvantaged position, signing contracts with retailers may not be necessary to promote information sharing. Furthermore, if the retailer refuses to disclose information about her inventory, the manufacturer may consider raising the wholesale price.

There are some limitations in this paper. First, we simply assume that consumers are myopic in each period. However, anticipating future price reductions or online reviews, they may choose to delay their purchases, i.e., make decisions in the two-period setting. In addition, we solely focus on a straightforward two-echelon supply chain structure. Incorporating competition into the research would be an intriguing development.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

A. Proofs

A1. Proof of Lemma 1

Under Scenario (\bar{B}), since the retailer does not hold strategic inventory, the decisions are independent over the two periods. The retailer's profit sets $q_2^{\bar{B}}$ to maximize profit in Period 2 $\pi_2^{\bar{B}} = (p_2^{\bar{B}} - w_2^{\bar{B}})q_2^{\bar{B}}$, where $p_2^{\bar{B}} = (1 - s_2^{\bar{B}})(1 - r) + rv_0$ and $s_2^{\bar{B}} = q_2^{\bar{B}}$. It is derived that $\frac{\partial \pi_{R2}^{\bar{B}}}{\partial q_2^{\bar{B}}} = r(2q_2^{\bar{B}} + v_0 - 1) - 2q_2^{\bar{B}} - w_2^{\bar{B}} + 1$ and

$$\frac{\partial^2 \pi_{R2}^{\bar{B}}}{\partial (q_2^{\bar{B}})^2} = 2(r - 1) < 0. \text{ Let } \frac{\partial \pi_{R2}^{\bar{B}}}{\partial q_2^{\bar{B}}} = 0 \text{ we have } q_2^{\bar{B}*} = \frac{r(1 - v_0) + w_2^{\bar{B}} - 1}{2(r - 1)}. \text{ Then the}$$

manufacturer sets $w_2^{\bar{B}}$ to maximize profit in Period 2 $\pi_{M2}^{\bar{B}} = w_2^{\bar{B}} q_2^{\bar{B}*}$. It can be derived that $\frac{\partial \pi_{M2}^{\bar{B}}}{\partial w_2^{\bar{B}}} = \frac{r(-v_0) + r + w_2^{\bar{B}} - 1}{2(r - 1)}$ and $\frac{\partial^2 \pi_{M2}^{\bar{B}}}{\partial (w_2^{\bar{B}})^2} = \frac{1}{r - 1} < 0$. Let $\frac{\partial \pi_{M2}^{\bar{B}}}{\partial w_2^{\bar{B}}} = 0$

we have $w_2^{\bar{B}*} = \frac{1}{2}(rv_0 - r + 1)$. Hence, we can obtain $\pi_{R2}^{\bar{B}*} = \frac{(r(v_0 - 1) + 1)^2}{16(1 - r)}$ and

$$\pi_{M2}^{\bar{B}*} = \frac{(r(v_0 - 1) + 1)^2}{8(1 - r)}.$$

In Period 1, the retailer sets $q_1^{\bar{B}}$ to maximize the total profit $\pi_R^{\bar{B}} = (p_1^{\bar{B}} - w_1^{\bar{B}})q_1^{\bar{B}} + \pi_{R2}^{\bar{B}}$, where $p_1^{\bar{B}} = 1 - s_1^{\bar{B}}$ and $s_1^{\bar{B}} = q_1^{\bar{B}}$. It can be derived that

$$\frac{\partial \pi_R^{\bar{B}}}{\partial q_1^{\bar{B}}} = -2q_1^{\bar{B}} - w_1^{\bar{B}} + 1 \text{ and } \frac{\partial^2 \pi_R^{\bar{B}}}{\partial (q_1^{\bar{B}})^2} = -2 < 0. \text{ Let } \frac{\partial \pi_R^{\bar{B}}}{\partial q_1^{\bar{B}}} = 0 \text{ we have}$$

$q_1^{\bar{B}} = \frac{w_1^{\bar{B}} + 1}{2}$. Then the manufacturer sets $w_1^{\bar{B}}$ to maximize the total profit

$$\pi_M^{\bar{B}} = w_1^{\bar{B}} q_1^{\bar{B}*} + \pi_{M2}^{\bar{B}*}. \text{ It can be derived that } \frac{\partial \pi_M^{\bar{B}}}{\partial w_1^{\bar{B}}} = \frac{1}{2} - w_1^{\bar{B}} \text{ and } \frac{\partial^2 \pi_M^{\bar{B}}}{\partial (w_1^{\bar{B}})^2} = -1 < 0.$$

Let $\frac{\partial \pi_M^{\bar{B}}}{\partial w_1^{\bar{B}}} = 0$ we have $w_1^{\bar{B}*} = \frac{1}{2}$. Hence, we can obtain the equilibrium results

shown in Lemma 1.

A2. Proof of Lemma 2

Under strategic inventory with information symmetry (B), the retailer sets q_2^B to maximize profit in Period 2 $\pi_{R2}^B = p_{R2}^B s_2^B - w_2^B (s_2^B - I^B)$, where

$p_2^B = (1 - s_2^B)(1 - r) + rv_0$ and $s_2^B = q_2^B + I^B$. It can be derived that

$$\frac{\partial \pi_{R2}^B}{\partial q_2^B} = 2I^B (r - 1) + 2q_2^B (r - 1) + r(v_0 - 1) - w_2^B + 1 \text{ and } \frac{\partial^2 \pi_{R2}^B}{\partial (q_2^B)^2} = 2(r - 1) < 0.$$

Let $\frac{\partial \pi_{R2}^B}{\partial q_2^B} = 0$ we have $q_2^{B*} = \frac{2I^B (r - 1) - rv_0 + r + w_2^B - 1}{2(r - 1)}$. Then the manufacturer

sets w_2^B to maximize profit in Period 2 $\pi_{M2}^B = w_2^B q_2^{B*}$. It can be derived that

$$\frac{\partial \pi_{M2}^B}{\partial w_2^B} = \frac{2I^B r - 2i + rv_0 - r - 2w_2^B + 1}{2 - 2r} \quad \text{and} \quad \frac{\partial^2 \pi_{M2}^B}{\partial (w_2^B)^2} = \frac{1}{r-1} < 0. \quad \text{Let} \quad \frac{\partial \pi_{M2}^B}{\partial w_2^B} = 0$$

we have $w_2^{B*} = \frac{h(10-6r) + 3r(v_0 - 2) + 6}{17 - 9r}$. Hence, we can obtain

$$\pi_{R2}^{B*} = \frac{9(h(10-6r) + 3r(v_0 - 2) + 6)^2}{12(r-1)(17-9r)^2} - \frac{3(r(v_0 - 1) + 1)^2}{12(r-1)} \quad \text{and}$$

$$\pi_{M2}^{B*} = -\frac{(h(10-6r) + 3r(v_0 - 2) + 6)^2}{2(17-9r)^2(r-1)}.$$

In Period 1, the retailer sets q_1^B and the inventory level I^B to maximize the total profit $\pi_R^B = p_1^B s_1^B - w_1^B (s_1^B + I^B) - hI^B + \pi_{R2}^{B*}$, where $p_1^B = 1 - s_1^B$ and $s_1^B = q_1^B - I^B$. It can be derived that $\frac{\partial \pi_R^B}{\partial q_1^B} = 2I^B - 2q_1^B - w_1^B + 1$, $\frac{\partial^2 \pi_R^B}{\partial (q_1^B)^2} = -2 < 0$,

$$\frac{\partial \pi_R^B}{\partial I^B} = \frac{1}{4}(-4h + 2I^B(3r-7) + 8q_1^B + 3r(v_0 - 1) - 1) \quad \text{and} \quad \frac{\partial^2 \pi_R^B}{\partial (I^B)^2} = \frac{1}{2}(3r-7) < 0.$$

$$\text{Let} \quad \frac{\partial \pi_R^B}{\partial q_1^B} = 0 \quad \text{and} \quad \frac{\partial \pi_R^B}{\partial I^B} = 0 \quad \text{we have} \quad q_1^{B*} = \frac{4h - 3r(v_0 + w_1^B - 2) + 7w_1^B - 6}{6(r-1)}$$

and $I^{B*} = \frac{-4h + 3rv_0 - 3r - 4w_1^B + 3}{6 - 6r}$. Then the manufacturer sets w_1^B to

maximize the total profit $\pi_M^B = w_1^B q_1^B + \pi_{M2}^{B*}$. It can be derived that $\frac{\partial \pi_M^B}{\partial w_1^B} = \frac{4h - 9r(v_0 + 2w_1^B - 2) + 34w_1^B - 18}{18(r-1)}$ and $\frac{\partial^2 \pi_M^B}{\partial (w_1^B)^2} = \frac{8}{9(r-1)} - 1 < 0$. Let

$$\frac{\partial \pi_M^B}{\partial w_1^B} = 0 \quad \text{we have} \quad w_1^{B*} = \frac{4h - 9(r(v_0 - 2) + 2)}{18r - 34}.$$

Hence, we can obtain the equilibrium results shown in Lemma 2.

A3. Proof of Lemma 3

First, the retailer determines the selling quantity s_2^N in Period 2:

$$\text{Max}_{s_2^N} \pi_{R2}^N = p_2^N s_2^N - w_2^N q_2^N, \tag{15}$$

where $p_2^N = (1 - s_2^N)(1 - r) + rv_0$.

Since π_{R2}^N is concave in q_2^N , we can derive the retailer's best response:

$$q_2^N = \begin{cases} \frac{1 - r(1 - v_0) - w_2^N}{2(1 - r)} - I^N & \text{if } w_2^N \leq (1 - r)(1 - 2I^N) + rv_0, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Therefore, the profit of the manufacturer in Period 2 is:

$$\pi_{M2}^{N*} = \begin{cases} w_2^N \left(\frac{1 - r(1 - v_0) - w_2^N}{2(1 - r)} - I^N \right) & \text{if } w_2^N \leq (1 - r)(1 - 2I^N) + rv_0, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

By applying the first-order conditions on (17), we can derive the manufacturer's decision on w_2^N as:

$$w_2^N = \left[\frac{1}{2} [1 - r(1 - v_0)] + (1 + r)I^N \right]^+ \tag{18}$$

Similarly, the retailer maximizes her profit-to-go in (5), rewritten below:

$$\pi_{R1}^N = p_1^N q_1^N + \pi_{R2}^{N*} - hI^N, \text{ s.t. } s_1^N + I^N \leq q_1^N. \tag{19}$$

where

$$\pi_{R2}^{N*} = \begin{cases} \frac{[1 - r(1 - v_0)]^2 + w_2^{N*} - 2w_2^N [1 - 2I^N(1 - r) - r(1 - v_0)]}{4(1 - r)} & \text{if } w_2^N \leq (1 - r)(1 - 2I^N) + rv_0, \\ I^N [(1 - I^N)(1 - r) + rv_0] & \text{otherwise.} \end{cases}$$

When the constraint $s_1^N + I^N \leq q_1^N$ is binding, we can replace s_1^N by $q_1^N - I^N$. Thus, we can derive the retailer's decision on I^N :

$$I^N = \left[\frac{1}{2} (-h + 2q_1^N + w_2^N - 1) \right]^+ \tag{20}$$

When the the constraint $s_1^N + I^D \leq q_1^N$ is non-binding, the retailer maximizes π_{R1}^N by choosing s_1^N and q_1^N independently. This happens only when q_1^N is big enough. Applying the first-order condition (19) again, we can obtain $s_1^N = \frac{1}{2}$

when $s_1^N + I^D \geq q_1^N$. Observe that the lowest value for q_1^N at which $s_1^N = \frac{1}{2}$ is $q_1^N = \frac{2(1 - h) + r(v_0 - 2)}{2(1 - r)} \equiv \bar{q}_1^N$. Below this threshold, w_2^N and I^N from (18)

and (20) intersect at $I^{N*} = \frac{4q_1^N + rv_0 - 2h - r - 1}{2(3 - r)}$ and

$w_2^{N*} = \frac{(1 - r)(h - 2q_1^N + 2) + rv_0}{3 - r}$. It is easy to confirm that $I^{N*} = 0$ for

$q_1^N \leq \frac{1}{4}(2h + 1 + r - rv_0) \equiv \underline{q}_1^N$. Thus, we can obtain

$$(I^{D*}, s_1^{N*}, w_2^{N*}) = \begin{cases} \left(0, q_1^N, \frac{1}{2}(1 + rv_0 - r) \right) & \text{if } q_1^N < \underline{q}_1^N, \\ \left(\frac{4q_1^N + rv_0 - 2h - r - 1}{2(3 - r)}, \frac{2h - 4q_1^N - rv_0 + r + 1}{2(3 - r)} + q_1^N, \frac{(1 - r)(h - 2q_1^N + 2) + rv_0}{3 - r} \right) & \text{if } \underline{q}_1^N < q_1^N < \bar{q}_1^N, \\ \left(\frac{1 - 2h + rv_0 - r}{2(1 - r)}, \frac{1}{2}, h \right) & \text{if } q_1^N > \bar{q}_1^N. \end{cases} \tag{21}$$

where $\underline{q}_1^N = \frac{1}{4}(2h + 1 + r - rv_0)$ and $\bar{q}_1^N = \frac{2(1 - h) + r(v_0 - 2)}{2(1 - r)}$.

A4. Proof of Lemma 4

From (8) we can get three possible q_1^N : when $q_1 \in (0, \underline{q}_1^N]$, $q_1 \in (\underline{q}_1^N, \bar{q}_1^N]$ and $q_1 \in (\bar{q}_1^N, \infty)$. The slope of π_R^N is given by the derivative $\frac{\partial \pi_R^N}{\partial q_1^N}$.

$$\frac{\partial \pi_R^N}{\partial q_1^N} = \begin{cases} 1 - 2q_1^N - w_1^N & \text{if } q_1 \in (0, \underline{q}_1^N], \\ \frac{h(r-5) - (r-4)(2(q_1^N - 1)(r-1) + rv_0) - (r-3)^2 w_1^N}{(r-3)^2} & \text{if } q_1 \in (\underline{q}_1^N, \bar{q}_1^N], \\ -w_1^N & \text{if } q_1 \in (\bar{q}_1^N, \infty). \end{cases} \quad (22)$$

Taking the second derivative with respect to q_1^N yields:

$$\frac{\partial^2 \pi_R^N}{\partial q_1^{N2}} = \begin{cases} -2q_1^N & \text{if } q_1 \in (0, \underline{q}_1^N], \\ \frac{2(4-r)(1-r)}{(3-r)^2} & \text{if } q_1 \in (\underline{q}_1^N, \bar{q}_1^N], \\ 0 & \text{if } q_1 \in (\bar{q}_1^N, \infty). \end{cases} \quad (23)$$

Therefore, π_R^N is piece-wise concave in q_1^N in each of the three intervals $q_1 \in (0, \underline{q}_1^N]$, $q_1 \in (\underline{q}_1^N, \bar{q}_1^N]$ and $q_1 \in (\bar{q}_1^N, \infty)$.

Maximizing (8) with respect to q_1^N , the retailer is better off purchasing $q_1^N > \underline{q}_1^N$ and holding strategic inventory when

$$w_1^N < W_1^N = \frac{2h(r-5) - 2r(r(v_0 - 1) - 4v_0 + 5) - \sqrt{(1-r)(4-r)[1-r(1-v_0)]^2} + 8}{2(5-r)}$$

; and purchasing $q_1^N < \underline{q}_1^N$ and not hold strategic inventory when $w_1^N > W_1^N$. She is indifferent between purchasing a high quantity (holding inventory) and purchasing a much lower quantity (not holding inventory) at $w_1^N = W_1^N$. Mathematically,

$$q_1^{N*} = \begin{cases} \frac{h(r-5) + (4-r)(r(v_0 - 2) + 2) - (r-3)^2 w_1^N}{2(4-r)(1-r)} & \text{if } 0 < w_1^N < W_1^N, \\ \frac{1 - w_1^N}{2} & \text{if } W_1^N < w_1^N < 1. \end{cases} \quad (24)$$

A5. Proof of Corollary 1

Corollary 1 is summarized from A6.

A6. Proof of Lemma 5

Based on (11), the manufacturer sets w_1^N . However, π_M^N is discontinuous at $w_1^N = W_1^N$. By comparing the upper $w_1^N < W_1^N$ and lower $w_1^N > W_1^N$ branches of (11) at W_1^N , it can be seen that as $w_1^N \rightarrow W_1^N$,

$\lim_{w_1^N \rightarrow W_1^N(-)} \pi_M^N > \lim_{w_1^N \rightarrow W_1^N(+)} \pi_M^N$. By comparing the manufacturer's profits under different situations, we find that: for the lower branch of (11), the manufacturer

is better to set $w_1^N = \frac{1}{2}$; for the upper branch, the manufacturer is better to set $w_1^N = \frac{h(2-(r-3)r)-(r-4)^2(r(v_0-2)+2)}{2(r-3)^3}$. In other words, the manufacturer never chooses $w_1^N = W_1^N$.

Therefore, the manufacturer’s decision on w_1^N is as follows:

$$w_1^{N*} = \begin{cases} \frac{h(2-(r-3)r)-(r-4)^2(r(v_0-2)+2)}{2(r-3)^3} & \text{when } \max\{0, h_1\} < h < h_2, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \quad (25)$$

where

$$h_1 = \frac{r(r(r(rf_1-10f_1+v_0-2)+36f_1-3v_0+8)-2(27f_1+1+v_0))+27f_1-4}{r(r(4r-29)+70)-61},$$

$$h_2 = \frac{r(r(r(rf_1-10f_1-v_0+2)+36f_1+3v_0-8)-54f_1+2v_0+2)+27f_1+4}{61-r(r(4r-29)+70)} \text{ and}$$

$$f_1 = \sqrt{\frac{r(r(4r^2(1-v_0)^2-21r(1-v_0)^2+v_0(25v_0-44)+16)-22v_0+39)-22}{(3-r)^3(1-r)}}.$$

A7. Proof of Proposition 7

We obtain the formulas:

$$w_2^{\bar{B}^*} - w_1^{\bar{B}^*} = \frac{1}{2}r(v_0-1) < 0,$$

$$p_2^{\bar{B}^*} - p_1^{\bar{B}^*} = \frac{3}{4}r(v_0-1) < 0,$$

$$w_2^{B^*} - w_1^{B^*} = w_2^{D^*} - w_1^{D^*} = \frac{3(4h(r-2)+r(v_0-2)+2)}{18r-34} < 0,$$

$$p_{B2}^* - p_1^{B^*} = p_2^{B^*} - p_1^{B^*} = \frac{12h(r-2)+r(18r(v_0-1)-31v_0+28)+6}{36r-68} < 0,$$

$$w_1^{B^*} - w_1^{\bar{B}^*} = \frac{-4h+9r(v_0-1)+1}{34-18r} > 0 \text{ and}$$

$$w_2^{B^*} - w_2^{\bar{B}^*} = \frac{4h(3r-5)+r(-9r(v_0-1)+11v_0-14)+5}{18r-34} < 0.$$

A8. Proof of Proposition 5

A8.1. Without strategic inventory

Under information asymmetry, when the retailer does not hold strategic inventory, we have $w_2^{\bar{N}^*} - w_1^{\bar{N}^*} = \frac{1}{2}r(v_0-1) < 0$ and $p_2^{\bar{N}^*} - p_1^{\bar{N}^*} = \frac{3}{4}r(v_0-1) < 0$.

A8.2. With strategic inventory

Under information asymmetry, when the retailer holds strategic inventory, we

have $w_2^{N^*} - w_1^{N^*} = \frac{h(r(2(r-8)r+43)-41)-(r-4)(r(v_0-2)+2)}{2(r-3)^3}$. When

$$h_{w1}^N < h < h_{w2}^N, \quad w_2^{N^*} > w_1^{N^*}, \quad \text{where } h_{w1}^N = \frac{r^2v_0 - 2r^2 - 4rv_0 + 10r - 8}{2r^3 - 16r^2 + 43r - 41},$$

$$h_{w2}^N = (59 - 239r + 261r^2 - 81r^3 + 174rv_0 - 243r^2v_0 + 81r^3v_0) / (4(76 - 91r + 27r^2)) + f_3/4,$$

$$f_3 = \left(\frac{1}{(r-2)(27r^2-91r+76)^2} (6561r^7v_0^2 - 13122r^7v_0 + 6561r^7 - 52488r^6v_0^2 + 107892r^6v_0 - 55404r^6 + 165726r^5v_0^2 - 356076r^5v_0 + 190431r^5 - 257652r^4v_0^2) + \frac{1}{(r-2)(27r^2-91r+76)^2} (597528r^4v_0 - 340506r^4 + 196809r^3v_0^2 - 529002r^3v_0 + 334111r^3 - 58956r^2v_0^2 + 227460r^2v_0 - 171352r^2 - 34680rv_0 + 36737r - 578) \right)^{\frac{1}{2}}.$$

The retailer prices are $p_2^{N^*} - p_1^{N^*} = \frac{1}{2}(h + r(v_0 - 1))$. We find that when $h > h_p^N$, $p_2^{N^*} > p_1^{N^*}$, where $h_p^N = r(1 - v_0)$.

A9. Proof of Proposition 5

Under information symmetry, by setting $\pi_R^{B^*} - \pi_R^{\bar{B}^*} = 0$, we get two threshold h_1^B and h_2^B . We find that when $h_1^B < h < h_2^B$, $\pi_R^{B^*} > \pi_R^{\bar{B}^*}$, where

$$h_1^B = \frac{r(9r(9(r-3)v_0 - 9r + 29) + 174v_0 - 239) + 59 - f_2}{4(r(27r - 91) + 76)},$$

$$h_2^B = \frac{r(9r(9(r-3)v_0 - 9r + 29) + 174v_0 - 239) + 59 + f_2}{4(r(27r - 91) + 76)} \quad \text{and}$$

$$f_2 = \sqrt{(17 - 9r)^2 (r - 1) (r(12r(v_0^2 + v_0 - 2) - 60v_0 + 61) - 1)}.$$

Under information asymmetry, the condition for the retailer to hold strategic inventory is summarized from Lemma 3 to 5.

A10. Proof of Proposition 8

We can obtain h_{range}^B and h_{range}^N as $h_{range}^B = h_2^B - h_1^B = \frac{f_2}{2(r(27r - 91) + 76)}$ and

$$h_{range}^N = h_2 - h_1 = \frac{2(r-3)^3(1-r)f_1}{r(r(4r-29)+70)-61}. \text{ It's easy to confirm that}$$

$$\frac{dh_{range}^B}{dv_0} = \frac{3(17-9r)^2(r-1)r(2rv_0+r-5)}{(r(27r-91)+76)f_2} > 0 \quad \text{and}$$

$$\frac{dh_{range}^N}{dv_0} = \frac{2r(r(-r(4r-21)(v_0-1)-25v_0+22)+11)}{(r(r(4r-29)+70)-61)f_1} < 0.$$

A11. Proof of Corollary 2

From Lemma 1 and 6 it's easy to confirm that $\pi_R^{\bar{N}^*} = \pi_R^{\bar{B}^*}$ and $\pi_M^{\bar{N}^*} = \pi_M^{\bar{B}^*}$.

A12. Proof of Proposition 9

By comparing the manufacturer's profits we can obtain that:

$$\begin{aligned}\pi_M^{N^*} - \pi_M^{D^*} &= \frac{1}{8(r-3)^3(9r-17)} \left(h^2 \left(r((133-20r)r-270) + 173 \right) \right. \\ &\quad \left. + 2h(r(13r-67)+74)(r(v_0-2)+2) \right. \\ &\quad \left. - (8r-29)(r(v_0-2)+2)^2 \right) \\ &> 0.\end{aligned}$$

B. Consumer Surplus and Social Welfare

The consumer surplus and social welfare under each scenario are:

1) Scenario \bar{B}

$$CS^{\bar{B}} = \frac{r(r(1-v_0)^2 + 2v_0 - 3) + 2}{32(1-r)}, \quad SW^{\bar{B}} = \frac{7(2r(r(1-v_0)^2 + 2v_0 - 3))}{32(1-r)}$$

2) Scenario B

$$\begin{aligned}CS^B &= \frac{1}{32(17-9r)^2(r-1)} \left(-16h^2(r-2)(9r-13) \right. \\ &\quad \left. + 8(r(3r(18r(v_0-1)-61v_0+70)+149v_0-250)+94) \right. \\ &\quad \left. + r(r(-324r^2(1-v_0)^2 + 9r(v_0(121v_0-272)+160) \right. \\ &\quad \left. + v_0(2744-865v_0)-2392)-944v_0+2016) - 740 \right), \\ SW^B &= \frac{1}{32(17-9r)^2(r-1)} \left(-16h^2(r(99r-353)+314) \right. \\ &\quad \left. + 8hrv_0(3r(72r-235)+565) + 96h(1-r)(r(18r-49)+29) \right. \\ &\quad \left. + 3r(r(-324r^2(1-v_0)^2 + 9r(v_0(125v_0-296)+192) \right. \\ &\quad \left. + v_0(3560-981v_0)-3704) - 1544v_0 + 3776) - 4428 \right).\end{aligned}$$

3) Scenario N

$$\begin{aligned}CS^N &= \frac{1}{32(r-3)^4(r-1)} \left(h^2 \left(r(r((45-4r)r-184)+321) - 194 \right) \right. \\ &\quad \left. + 2hr(r(r(r(4r-45)+185)-326)+202)v_0 \right. \\ &\quad \left. - 4h(r-4)(r-1)(r(r(2r-13)+25)-10) \right. \\ &\quad \left. + r^2(r(r((45-4r)r-186)+332)-212)v_0^2 \right. \\ &\quad \left. + 4(r-1)r(r(r(r(2r-21)+78)-116)+50)v_0 \right. \\ &\quad \left. - 4(r-1)(r(r(r((r-11)r+45)-86)+85)-50) \right),\end{aligned}$$

$$\begin{aligned}
SW^N = & \frac{1}{32(r-3)^4(r-1)} \left(h^2 \left(r \left(r \left((323-28r)r - 1368 \right) + 2511 \right) - 1678 \right) \right. \\
& + 2hr \left(r \left(3r \left(r(4r-45) + 185 \right) - 982 \right) + 626 \right) v_0 \\
& - 4h(r-1)(2r-7) \left(3r \left((r-7)r + 14 \right) - 20 \right) \\
& + r \left(r \left(r \left((139-12r)r - 598 \right) + 1132 \right) - 796 \right) v_0^2 \\
& + 4(r-1)(2r-5) \left(r \left(r(3r-26) + 72 \right) - 62 \right) v_0 \\
& \left. - 4r \left(r \left(r(3r-40) + 216 \right) - 613 \right) + 985 \right) + 3444 \Big) - 1240 \Big).
\end{aligned}$$

4) Scenario \bar{N}

$$CS^{\bar{N}} = \frac{r \left(r(1-v_0)^2 + 2v_0 - 3 \right) + 2}{32(1-r)}, \quad SW^{\bar{N}} = \frac{7 \left(r \left(r(1-v_0)^2 + 2v_0 - 3 \right) + 2 \right)}{32(1-r)}.$$