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Exposure to Energy Risk in an Energy-Based Growth Model*

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Abstract

The note introduces a simple GDP accounting model, whereby the total GDP at a point in time is the sum of the various GDP sectors contributing to the production of added value. Among these sectors, the energy contribution takes a special place. Energy is viewed as the primary contributing one and the other sectors are treated as secondary, whose production derives from and are dependent on the energy production. The link between energy and the secondary production is established in a simple yet realistic linear-exponential manner, allowing for energy efficiency to be taken care of. In particular, an energy efficiency rate is introduced and translates the ability for example to output more secondary added value with as much primary energy input. The model could thereby be used to measure the sensitivity of the GDP growth to sudden changes in the primary energy prices or volumes.

Keywords

Production, Growth, Energy, Energy Efficiency, Energy Price Index, Ex-Energy Price Index

1. Introduction

Since the introduction of the first production function by Cobb and Douglas (1928), modelling the output (Y) of an economy from some input factors has been the subject of lots of debates. The Cobb-Douglas function is still used by economists as a reference, because it has the advantage of simplicity and tractability. In its original formulation, the function has only two inputs, namely capital (K) and labour (L), and its main limitation is to impose an arbitrary unit level

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of substitution between them (Arrow et al., 1961). Many efforts have been made ever since to improve the Cobb-Douglas model, by augmenting the number of factor inputs or by developing more general classes of functional forms, allowing flexible levels of substitution between inputs.

The key contribution of the Solow-Swan model (Solow, 1956; Swan, 1956) is the addition of a time variable to the Cobb-Douglas model. Formally, the time t is introduced in the production function to account for consumption and capital choice, for natural growth of labour and for exogenous technical advances, that is, output variations not attributable to input factor variations. Crucially, the time variable allows the derivation of the general equilibrium conditions of an economy. The factor representing the accumulation of technological advances (A) is supposed to be positive and neutral in terms of capital-labour substitution: under the assumption of "constant returns to scale", only the capital-labour factor elasticities need summing up to 1 (hereby, δ and $1-\delta$). Thus, in its three-factor capital-labour-advances (KLA) form, the production function is

$$Y(t) = A(t) K^{\delta}(t) L^{1-\delta}(t), \tag{1}$$

with $0 \le \delta \le 1$, in order to satisfy the requirements on the shape of the production function $Y(t)$ for $K(t), L(t) > 0$, and $\mathrm{d}A(t) = \alpha(t) A(t)\mathrm{d}t$, where $\alpha(t)$ represents a residual appreciation rate of the production capability. Alternative fomulations associate the technological advances with the labour input, in which case, the factor A can be interpreted as the skill level of the labour force:

$$Y(t) = K^{\delta}(t)\big(A(t)L(t)\big)^{1-\delta}. \tag{2}$$

Arrow et al. (1961) introduce a production function with a "constant elasticity of substitution" (CES) between the production factors and constant returns to scale. In its capital-labour (KL) form, the production function is

$$Y(t) = \gamma\left[\delta K^{\rho}(t) + (1-\delta) L^{\rho}(t)\right]^{1/\rho},$$

where γ is a scale parameter and where $0 \le \delta \le 1$ and $\rho \le 1$. In the litterature, δ is known as the "share" parameter and ρ is known as the "substitution" parameter. In the CES functional case, the elasticity of substitution is simply $\sigma = 1/(1-\rho)$ and, equivalently, the substitution is $\rho = (\sigma - 1)/\sigma$ (Arrow, et al., 1961).

The CES model has the advantage of generalising three classes of production functions: the linear function that implies an infinite elasticity of substitution (for "perfect substitutes"), the Leontief function (Leontief, 1936) that implies a zero elasticity of substitution (for "perfect complements") and the Cobb-Douglas function that implies a unit elasticity of substitution. Indeed, this CES setup embeds the following limiting cases:

$$Y(t)=\begin{cases}\gamma\left[\delta K(t)+(1-\delta)L(t)\right] & \sigma\to\infty,\ \rho\to 1\\ \gamma\min\left\{K(t),L(t)\right\} & \sigma\to 0,\ \rho\to-\infty\\ \gamma K^{\delta}(t)L(t)^{1-\delta} & \sigma\to 1,\ \rho\to 0\end{cases}\tag{3}$$

However, Uzawa (1962) and McFadden (1963) demonstrated the presence of an intrinsic limitation in the CES function formulation: essentially, the generalisation of this class of function to more than two factors necessarily imposes a common elasticity of substitution between all of them. To allow for different elasticities of substitution between inputs, Sato (1967) proposes a two-level CES production function, which could actually be generalised to any number of levels and is now referred to as the "nested CES function" formulation. Having the advantage of being both flexible and tractable, it has proved very successful in econometric studies.

In spite of the advent of these many new production functional forms, the place of energy and, more specifically, of any exhaustible resources in the factoring of production has been overlooked by economists for a long time. We have to wait until the 1970s, when the oil crises arise, to see the attention on the role of energy and other resources in production being triggered. The events provide an opportunity for economists to add energy (E), resources (R) or other materials (M) to their production model inputs.

Solow (1974) and Stiglitz (1974) work on a similar approach. They propose to extend the Solow-Swan model to four factors, under a capital-labour-advances-resources (KLAR) form:

$$Y(t)=K^{\delta}(t)\left(A(t)L(t)\right)^{\delta'}R^{1-\delta-\delta'}(t),\tag{4}$$

with $0\le\delta,\delta'\le 1$ and $\delta+\delta'\le 1$. They discuss the feasability of a general production-consumption equilibrium in the case of limited exhaustible resources and limited or unlimited labour growth and technical advances.

Berndt and Wood (1975) work on a capital-labour-energy-materials (KLEM) production function, which they derive from an approximation of a CES production function by a second-order Taylor expansion in the variable ρ about zero, that is, a CES function close to the Cobb-Douglas one,

$$Y(t)=K^{\delta}(t)L^{\delta'}(t)E^{\delta''}(t)M^{1-\delta-\delta'-\delta''}(t),\tag{5}$$

with $0\le\delta,\delta',\delta''\le 1$ and $\delta+\delta'+\delta''\le 1$. This leads to the so-called "transcendental logarithmic" production function, originally introduced by Diewert (1971). However, the formulation becomes quite complex, with the presence of cross-products of logarithms, and is, rightly or wrongly, not very much used in practice.

On the other hand, the introduction of the nested CES function makes possible the analysis of the role of energy in production and the substitution between energy and other inputs, such as capital, labour, materials etc., in a simple and flexible manner. A three-factor standard CES function using the capital-labour-energy factors (KLE) need to place all factors on equal foot (Sato, 1967; Uzawa, 1962):

$$Y(t)=\gamma\left[\delta K^{\rho}(t)+\delta' L^{\rho}(t)+(1-\delta-\delta')E^{\rho}(t)\right]^{1/\rho}, \tag{6}$$

where δ and δ', such that $0\le\delta,\delta'\le 1$ and $\delta+\delta'\le 1$, control the share of each factor and $\rho\le 1$ act as the common substitution parameter. Alternatively, the nesting approach can combine inner-to-outer nests of the three factors in different ways, generally denoted KL(E), KE(L) and LE(K), where the inner nest share-substitution is driven by (δ',ρ') and the outer nest share-substitution is driven by (δ,ρ):

$$Y(t)=\begin{cases}\gamma\left[\delta\left[\delta' K^{\rho'}(t)+(1-\delta')L^{\rho'}(t)\right]^{\rho/\rho'}+(1-\delta)E^{\rho}(t)\right]^{1/\rho}\\ \gamma\left[\delta\left[\delta' K^{\rho'}(t)+(1-\delta')E^{\rho'}(t)\right]^{\rho/\rho'}+(1-\delta)L^{\rho}(t)\right]^{1/\rho}\\ \gamma\left[\delta\left[\delta' L^{\rho'}(t)+(1-\delta')E^{\rho'}(t)\right]^{\rho/\rho'}+(1-\delta)K^{\rho}(t)\right]^{1/\rho}\end{cases} \tag{7}$$

In the first work of Kümmel et al. (1985), energy is integrated in the production model together with some limiting elasticity constraints. The model is a three-factor capital-labour-energy (KLE) one. In the second work of Kümmel et al. (2007), the model is amended to leave some room for a time factor. The important result is that, in order to satisfy the usual assumption of constant returns to scale, any production function, in the instance, a KLE model, is necessarily of the form

$$\begin{aligned}\frac{\mathrm{d}Y(t)}{Y(t)}&=\delta\big(K(t),L(t),E(t)\big)\frac{\mathrm{d}K(t)}{K(t)}+\delta'\big(K(t),L(t),E(t)\big)\frac{\mathrm{d}L(t)}{L(t)}\\&\quad+\Big(1-\delta\big(K(t),L(t),E(t)\big)-\delta'\big(K(t),L(t),E(t)\big)\Big)\frac{\mathrm{d}E(t)}{E(t)}+\alpha(t)\mathrm{d}t.\end{aligned} \tag{8}$$

For all $K(t)$, $L(t)$, $E(t)$, the coefficients δ and δ', with $0\le\delta,\delta'\le 1$ and $\delta+\delta'\le 1$, are the elasticities of capital and labour and their complement to 1 is the elasticity of energy. The coefficient $\alpha(t)$ represents improvements of the production organisation and efficiency due to human intelligence. As long as the influence of human creativity is negligible, capital, labour and energy uniquely determine the output performance. A non-zero $\alpha(t)$, on the other hand, represents influences like human ideas, innovations and inventions, which, in principle, cannot be measured in physical terms.

Under the requirement that the dynamics of the KLE production $\mathrm{d}Y(t)$ is a total derivative, this imposes equality constraints on the second partial cross-derivatives of the function, namely,
$\partial^2 Y(t)/\big(\partial K(t)\partial L(t)\big)=\partial^2 Y(t)/\big(\partial K(t)\partial E(t)\big)=\partial^2 Y(t)/\big(\partial L(t)\partial E(t)\big)$. Then, the characteristics of the production function are derived by setting the limiting boundary conditions of the elasticities. They are expressed in terms of the labour-to-capital and energy-to-capital ratios, as $\delta\big(\ell(t),e(t)\big)$ and $\delta'\big(\ell(t),e(t)\big)$,

with $\ell(t)=L(t)/K(t)$ and $e(t)=E(t)/K(t)$. This leads to the so-called "linear-exponential" (in short, "linex") model, which has the form

$$Y(t)=\gamma(\ell(t),e(t),t)E(t), \tag{9}$$

where the function $\gamma(\ell(t),e(t),t)$ is the exponential of a combination of the factors $\ell(t)$ and $e(t)$, together with an exogeneous time component. The final formulation of the production function is actually given by

$$Y(t)=E(t)\exp\left(a(t)\left(2-\ell(t)-e(t)\right)+a(t)b(t)\left(\frac{\ell(t)}{e(t)}-1\right)\right), \tag{10}$$

which depends linearly on energy and exponentially on the factor quotients. The arguments $a(t)$ and $b(t)$ are defined from the elasticity equations $\delta(\ell(t),e(t))=a(t)(\ell(t)+e(t))$ and $\delta'(\ell(t),e(t))=a(t)\ell(t)(b(t)/e(t)-1)$. These equations satisfy the asymptotic boundary conditions $\delta(\ell(t),e(t))\to 0$, if $\ell(t)+e(t)\to 0$, and $\delta'(\ell(t),e(t))\to 0$, if $K(t)\to K_m$ and $e(t)\to b(t)K_m$. The variable $b(t)K_m$ is the maximum energy that can be input into a capital K_m working at maximum automation (with minimum labour contribution) and at full capacity.

The asymptotic boundary condition for $\delta(\ell(t),e(t))$ indicates that capital cannot run without energy and labour. The asymptotic boundary condition for $\delta'(\ell(t),e(t))$ describes the effect of energy-to-labour and capital-to-labour substitutions. The function effectively contains a technology parameter $a(t)$ (capital efficiency) and an energy parameter $b(t)$ (energy efficiency of the fully utilised capital). The possible time dependency on the parameters $a(t)$ and $b(t)$ translates the potential creativity factor that improves production efficiencies. These parameters eventually show up in the variable $\alpha(t)$ of Equation 8.

Taking up on these foundations, we shall specifically work on a KLE type of production model. Taking the logarithmic derivative of the production function, we will obtain a model for the economic growth that will naturally exhibit a dependency on the capital and labour component, on the one hand, and on the energy volume and price, on the other hand. Modelling the dependency of the economic growth in such a way makes it possible to measure the exposure of an economy to the energy supply and, in particular, to assess the subsequent risk that an economy would be subjected to, should it be faced with volume or price shocks on energy. Such a model effectively emphasises the need for modern economies to lower their energy risk. Several avenues to tackle this challenge can be envisaged: keeping a relentless effort to improve their energy efficiency, as shown for example in (Jovanović et al., 2022) or (Xiao & Mei, 2019), or striving to diversify their energy supply, as shown for example in (Moudène et al., 2023).

The aim of the study is triple. First, we want to establish the most relevant place of energy in the production process and thus its sensible expression in an economic growth model. Beyond the question of exhaustibility of some of the energetic resources, such as fossil fuels, which one could argue is still a longer-term issue rather than a very short-term one, the objective is to know whether energy is an input that can be substituted against any of the other production factors, capital, labour etc. Our answer is radical: the modern economies would not be able to cope with a sudden drying-out of energy supply. Our model actually proposes to depict economic production as a simple one-factor energy-leveraging machine. Having said that, we do not postulate that economic growth is entirely related to energy consumption growth. We introduce an "energy efficiency rate" that essentially allows for more production with, possibly, the same or a lesser energy use and that works as a mitigant to energy dependency. We also introduce the energy input from the point of view of an energy-importing economy and the total "energy cost share" it creates in the whole production process, in terms of gross domestic product (GDP).

Once the model set, the second step is to analyse the interaction between production and energy in the case of the 27 countries of the European Union as of 2020 (EU27), with a specific focus on Germany (DE), France (FR) and Italy (IT), whose dependency on energy imports can be very substantial. We do so by analysing their GDP growth in terms of their energy supply cost. In practice, we input in the proposed production and growth model the energy volumes used by the EU27, DE, FR or IT, the market prices of the main energy benchmark resources, namely, coal, oil, gas and electricity, and relate these data to the nominal and real GDP production and growth.

Finally, in the last step, we analyse the current situation of the EU27, DE, FR and IT economies and compare it to the one of the United States of America (US). The model allows the derivation of the implied energy efficiency rate, which favours GDP growth, and also, by comparing nominal and real GDP growths, of the implied "energy inflation rate" and "ex-energy inflation rate", which together potentially work against GDP growth. This creates various exposures of these economies to a possible energy crisis.

2. Model Framework, Assumptions and Derivation

2.1. Framework

Among the production models that are discussed in the introduction, energy has always been treated as a factor to add on top of the existing ones. With all its shortcomings, the Cobb-Douglas model has a fundamental characteristic that still makes it appealing: production factors are multiplicative. It means that, if any one element of the input equation is set to zero, the production falls to zero. On the other hand, in the CES models, production factors are additive (albeit in a weighted-average way). It means that, if any one element of the input equation is set to zero, the production still continues.

The linex model sets energy at the core of the production function: without energy, production is nil. However, we go further than Kümmel et al. (1985, 2007), by taking capital and labour entirely out of the equation. Capital and labour accumulation and productivity proceeds from the storage of part of the output value, through investment into equipment and machinery and into health and education. We postulate that production is the consequence of energy transformation. Thus, capital and labour accumulation and productivity are in essence a storage facility for some of the transformed energy.

Indeed, the use of energy allowed the population growth, in both number and well-being. Feeding is the first need of the population in general and of the labour force in particular, since it provides people with their daily energetic intake. Energy enabled the passage from the pre-economic hunting-gathering epoch to the early economic era of subsistance farming and, finally, to the modern economic time of intensive agricultural production, with mechanical and artificial ploughing, fertilising, harvesting, conditioning and merchandising. The labour force went from a time of direct access to food, through hunting-gathering, to a time of partly distanced access to food, through subsistance farming, and, finally, to a totally indirect access to food, through energy-mediated production. Over this brief economic history, the only thing that changed is the energy mix that is used: wood and horse power in the old subsistance farming era and fossil fuels and electricity in the modern intensive farming time. The ensuing production development allowed a greater labour force specialisation and triggerred the industrial revolution. Finally, the population in general and the labour force in particular expressed a second need for health and education. Health and education favour the population growth and, ultimately, benefit the labour force, by developping its number, knowledge and productivity. This was only made possible by labour specialisation that is the result of industrialisation and a consequence of energy use.

Therefore, considering the criticality of energy supply in any modern economic production process, we postulate a model of the time-t production $Y(t)$ that factors out the energy input flow $E(t)$ and that embeds the energy input stock in the following way:

$$Y(t) = \gamma(E(t), t) E(t). \tag{11}$$

The function $\gamma(E(t), t)$ represents the partial accumulation of transformed energy flow into capital and labour over time. The multiplication of $\gamma(E(t), t)$ by $E(t)$ acknowledges that energy is a primary input that conditions the very possibility of any output creation. The production sub-function $\gamma(E(t), t)$ can be viewed as an energy-leveraging engine fueled by the energy input $E(t)$ and the production output is simply the result of the transformation of energy into goods and services.

Capital and labour stocks are supposed to increase through investments. The source of capital-labour investments may arise from either corporation-driven, state-driven or household-driven investments. Corporation investments are funded from current or future earnings (before interest, tax, depreciation and amortisation). Earnings are the production of added value expressed in monetary terms. Fractions of added value may be directed into material investments, through equipment acquisition, or into human investments, through staff training, benefiting both capital and labour. State investments are funded from current of future taxes. Presumably, we can ultimately relate any tax, either direct (on added value) or indirect (on corporation profits and household incomes), as a fraction of added value. Further fractions of levied taxations may be directed into material investments, through state-owned companies or state infrastructure projects, or into human investments, through health and education, benefiting both capital and labour. Finally, household investments are funded from current of future wages. Presumably, we can also ultimately relate any wage, as a fraction of added value. Likewise, further fractions of wages may be directed to private health, teaching and schooling, benefiting labour. Capital is also impaired by depreciation, whilst, labour is generally improved by natural increase in force and skill productivity. This accumulation process can be modelled as

$$\begin{aligned} \mathrm{d}\gamma\left(E(t),t\right) &= \left(b(t)Y(t) + a(t)\gamma\left(E(t),t\right)\right)\mathrm{d}t \\ &= \left(b(t)E(t) + a(t)\right)\gamma\left(E(t),t\right)\mathrm{d}t. \end{aligned} \tag{12}$$

The amount $b(t)Y(t)$ represents the fraction of total output not consumed but rather invested (directly or indirectly) into capital and labour at a total rate $b(t)$, made by corporation, state and household alike, where $Y(t)$ can be replaced by its defined value $\gamma\left(E(t),t\right)E(t)$. The amount $a(t)\gamma\left(E(t),t\right)$ combines, on the one hand, the potential capital stock depreciation (amortisation and disinvestment) and, on the other hand, the potential labour stock appreciation (growth and productivity) at a net rate $a(t)$. This differential equation lends itself to a very simple solution

$$\gamma\left(E(t),t\right) = \gamma(\cdot)\exp\left(\int_{\cdot}^{t}\left(b(s)E(s) + a(s)\right)\mathrm{d}s\right), \tag{13}$$

where $\gamma(\cdot)$ denote the starting value of the energy-leveraging function at an unspecified starting integration point. Under this dynamics, the production function takes a linear-exponential form, since

$$Y(t) = \gamma(\cdot)E(t)\exp\left(\int_{\cdot}^{t}\left(b(s)E(s) + a(s)\right)\mathrm{d}s\right). \tag{14}$$

This gives a total log-derivative of the output of the following form:

$$\frac{\mathrm{d}Y(t)}{Y(t)} = \frac{\mathrm{d}E(t)}{E(t)} + \left(b(t)E(t) + a(t)\right)\mathrm{d}t. \tag{15}$$

In this format, the function $\gamma\left(E(t),t\right)$ acts as a common pot of stored energy for both capital and labour. Alternatively, we may want to split the function $\gamma\left(E(t),t\right)$ into two implicit hidden pockets, namely, capital and labour. We can postulate that capital $K(E(t),t)$ appreciates with investment at a rate $b_K(t)$ of output $Y(t)$ (capital investment) and amortises with depreciation at a rate $a_K(t)$ of existing capital $K(E(t),t)$ (capital depreciation), whereas labour $L(E(t),t)$ benefits from investment at a rate $b_L(t)$ of output $Y(t)$ (labour investment) and increases in a natural way at a rate $a_L(t)$ of existing labour $L(E(t),t)$ (labour productivity). The notations $K(E(t),t)$ and $L(E(t),t)$ emphasise that capital and labour do not stand on their own but result from the accumulation of transformed energy. Thus, we have

$$\begin{aligned} \mathrm{d}K(E(t),t) &= \left(b_K(t)Y(t) - a_K(t)K(E(t),t)\right)\mathrm{d}t \\ &= \left(b_K(t)\gamma(E(t),t)E(t) - a_K(t)K(E(t),t)\right)\mathrm{d}t \end{aligned} \tag{16}$$

and

$$\begin{aligned} \mathrm{d}L(E(t),t) &= \left(b_L(t)Y(t) + a_L(t)L(E(t),t)\right)\mathrm{d}t \\ &= \left(b_L(t)\gamma(E(t),t)E(t) + a_L(t)L(E(t),t)\right)\mathrm{d}t. \end{aligned} \tag{17}$$

Now, the amount $(b_K(t)+b_L(t))Y(t)$ represents the fraction of total output not consumed but rather invested (directly or indirectly) into capital and labour from corporation, state and household alike. The resulting impact on the output of the capital-labour dynamics will effectively depend on how capital and labour are recombined into the leverage function $\gamma(E(t),t)$.

However, as will become clear shortly hereafter (c.f. Section 2.3), because of the multiplicative form of the production function $Y(t)=\gamma(E(t),t)E(t)$, there is actually no need to specify the actual form of $\gamma(E(t),t)$. Thus, the impact of the flow of energy immobilised into $\gamma(E(t),t)$ and its hidden split between capital and labour will not be further analysed. Therefore, in what follows, the energy leveraging function will be simply written $\gamma(t)$ and its dynamics $\mathrm{d}\gamma(t)=\alpha(t)\gamma(t)\mathrm{d}t$.

2.2. Assumptions

We shall work on an ouput model that refers to the added value per sector, that is, a model expressed in terms of GDP. Futhermore, the concept of energy that will be used in the study, also known as "gross energy supply", is the volume of energy made available in the economy through local production or imports and that we shall refer to as the "primary energy volume". More precisely, since all the analysis will be made using energy volumes converted into tons oil-equivalent (tOE) and volumes of added value calculated in constant currency terms, that is, in terms of GDP volumes, the unit of the function $\gamma(t)$ is the GDP volume per tOE (c.f. Appendix B). Once the production function gets multiplied by the current price per GDP volume applied to the output, we can obtain a current currency value.

We continue the model presentation by introducing some notations and by outlining some further assumptions underlying the proposed GDP production model. We look at the production from a GDP accounting point of view, that is, by summing up the GDP outputs across all sectors.

1) The economy is constituted of two sectors, a "primary sector" that produces energy and a "secondary sector" that produces the rest of the goods and services. The time-t nominal GDP $Y_n(t)$ is the addition of the added values output by the primary and secondary sectors, $Y_n'(t)$ and $Y_n''(t)$. We ignore value-added taxes, custom duties and state subsidies.

$$Y_n(t) = Y_n'(t) + Y_n''(t).$$

2) The total energy supply $X'(t)$ into the economy actually comprises the time-t fraction $\beta(t)$ of the energy volumes that are imported, with $\beta(t) \le 1$. Thus, the time-t nominal GDP of the primary sector is defined as the time-t price $P'(t)$ per unit of added value[1], multiplied by the time-t volume of added value (energy is supposed to be free before being extracted), from which the imported part $\beta(t)$ is subtracted, that is, $X'(t)(1-\beta(t))$:

$$Y_n'(t) = P'(t) X'(t)(1-\beta(t)).$$

3) The time-t nominal GDP of the secondary sector is defined as the time-t price $P''(t)$ per unit of added value, multiplied by the time-t volume of added value $X''(t)$:

$$Y_n''(t) = P''(t) X''(t).$$

4) The production of the secondary volume of goods and services is dependent upon the energy supply, from which the time-t fraction $\eta(t)$ of energy that is consumed by the households (and thus actually distracted from the production system) must be subtracted. Thus, the energy input in the production system is effectively $E(t) = X'(t)(1-\eta(t))$. The transformation is proportional to the primary energy used for production, with an energy-leveraging function denoted $\gamma(t)$:

$$X''(t) = \gamma(t) X'(t)(1-\eta(t)),$$

where

$$\mathrm{d}\gamma(t) = \alpha(t)\gamma(t)\mathrm{d}t.$$

The variable $\alpha(t)$ can be construed as the "energy efficiency rate" and thus translates the ability to produce more added value (if $\alpha(t) > 0$) over time with the same volume of primary energy. As outlined in Section 2.1, the content of this parameter is actually more complex than it looks, since it effectively embeds many other factors, such as capital investment and depreciation, labour invest-

[1]Notice the implicit approximation, when using a common price for the total energy supply into the domestic product account of the economy. Actually, the price of the locally produced energy may be different from the price of the imported energy. Nevertheless, later on in the study, the energy supply mix will be looked through in terms of coal, oil, gas or electricity components, for which international market prices will be the benchmarks.

ment and productivity etc.

5) The time-t "energy cost share" $c(t)$ is the cost of the whole primary energy supply value as a fraction of the total GDP:

$$c(t)=\frac{Y_n'(t)}{Y_n(t)}=\frac{P'(t)X'(t)}{Y_n(t)}.$$

Thus, the cost share of the secondary sector becomes

$$\frac{Y_n''(t)}{Y_n(t)}=\frac{Y_n(t)-P'(t)X'(t)\left(1-\beta(t)\right)}{Y_n(t)}=1-c(t)\left(1-\beta(t)\right).$$

6) The inflation rate[2] $i(t)$ aggregates the time-t inflation on the primary sector $i'(t)$ ("energy inflation rate") and the time-t inflation on the secondary sector $i''(t)$ ("ex-energy inflation rate"):

$$i(t)=c(t)i'(t)+\left(1-c(t)\right)i''(t),$$

with

$$\frac{\mathrm{d}P'(t)}{P'(t)}=i'(t)\mathrm{d}t \text{ and } \frac{\mathrm{d}P''(t)}{P''(t)}=i''(t)\mathrm{d}t.$$

7) At time t, the real GDP growth is the nominal GDP growth minus the time-t inflation rate $i(t)$:

$$\begin{aligned}\frac{\mathrm{d}Y_r(t)}{Y_r(t)}&=\frac{\mathrm{d}Y_n(t)}{Y_n(t)}-i(t)\mathrm{d}t\\&=\frac{\mathrm{d}Y_n(t)}{Y_n(t)}-c(t)\frac{\mathrm{d}P'(t)}{P'(t)}-\left(1-c(t)\right)i''(t)\mathrm{d}t.\end{aligned}$$

8) The secondary value-added price $P''(t)$ and the fraction of household energy consumption $\eta(t)$ are assumed to be insensitive to the volume of primary energy supply $X'(t)$. Thus, for all t,

$$\frac{\mathrm{d}P''(t)}{\mathrm{d}X'(t)}=\frac{\mathrm{d}\eta(t)}{\mathrm{d}X'(t)}=0.$$

The second assumption of independence between household consumption and energy supply is certainly debatable. Presuming that the household consumption $\eta(t)$ and its productive consumption counterpart $\left(1-\eta(t)\right)$ are stable with respect to energy supply changes is not necessarily obvious. In particular, if faced with any energy shortage, the relevant questions would be about the behaviour of consumers and producers and also about the choice of governments, in the way to split any rationing between households and corporations.

9) However, the volumes of secondary added values are sensitive to the vol-

[2]In what follows, the term "inflation rate" is actually used rather loosely. Price variations on the primary and secondary sectors are hereby treated as approximations of changes in the energy and ex-energy consumer price indices but are not equivalent to inflation rates, in the strict monetarist sense.

ume of primary energy supply, since $X''(t)=\gamma(t)X'(t)(1-\eta(t))$. The relative change is given by (c.f. Appendix A.1)

$$\frac{\mathrm{d}X''(t)}{\mathrm{d}X'(t)}=\frac{X''(t)}{X'(t)}+\alpha(t)X''(t)\frac{\mathrm{d}t}{\mathrm{d}X'(t)}.$$

2.3. Derivation

With

$$Y_n(t)=P'(t)X'(t)(1-\beta(t))+P''(t)X''(t), \tag{18}$$

The nominal GDP growth is given by

$$\frac{\mathrm{d}Y_n(t)}{Y_n(t)}=\frac{\mathrm{d}Y_n(t)}{\mathrm{d}X'(t)}\frac{\mathrm{d}X'(t)}{Y_n(t)}.$$

Therefore, after some simple algebra (c.f. Appendix A.2), we get

$$\begin{aligned}\frac{\mathrm{d}Y_n(t)}{Y_n(t)}=&\frac{\mathrm{d}X'(t)}{X'(t)}+c(t)(1-\beta(t))\frac{\mathrm{d}P'(t)}{P'(t)}\\&-c(t)\mathrm{d}\beta(t)+(1-c(t)(1-\beta(t)))\alpha(t)\mathrm{d}t.\end{aligned} \tag{19}$$

Here, we have built up a nominal GDP growth model that is now solely expressed in terms of primary energy quantities, namely, its volume and price relative changes, its cost share in the economy, its imported fraction level and variation and its efficiency rate. It is also completely independent from the allocation between capital, labour and any other production factors.

The real GDP growth is easily derived (c.f. Appendix A.3) as

$$\begin{aligned}\frac{\mathrm{d}Y_r(t)}{Y_r(t)}=&\frac{\mathrm{d}X'(t)}{X'(t)}-c(t)\beta(t)\frac{\mathrm{d}P'(t)}{P'(t)}-(1-c(t))i''(t)\mathrm{d}t\\&-c(t)\mathrm{d}\beta(t)+(1-c(t)(1-\beta(t)))\alpha(t)\mathrm{d}t.\end{aligned} \tag{20}$$

As for the real GDP growth, it is also expressed in terms of primary energy quantities (volume and price relative changes, cost share in the economy, imported fraction level and variation, efficiency rate) and, on top, in terms of the inflation fraction due to the secondary added value sector.

In the two equations, the striking feature is the one-to-one relationship between GDP growth and the relative change in the primary energy volume supply, namely, $\mathrm{d}X'(t)/X'(t)$.

The energy import fraction $\beta(t)$ is deemed positive, as far as net importing economies are concerned, and the primary energy cost share $c(t)$ is positive by construction. Consequently, the real GDP growth $\mathrm{d}Y_r(t)/Y_r(t)$ is a negative function of the relative change in primary energy price $\mathrm{d}P'(t)/P'(t)$; the impact of the primary energy price change is adjusted by the product between the imported energy fraction $\beta(t)$ and the energy cost share $c(t)$ and thus affects the GDP growth only to the extent that the imported energy cost share $c(t)\beta(t)$ is concerned.

Likewise, the real GDP growth is a negative function of the inflation on the secondary added value output $i''(t)$; the impact of inflation on the real GDP growth is adjusted by the cost share of the ex-energy output $(1-c(t))$, as expected from its definition.

Moreover, the real GDP growth is a negative function of increases in primary energy imports $\mathrm{d}\beta(t)$; the impact of an energy import variation on the real GDP growth is adjusted by the primary energy cost share $c(t)$.

Finally, in this construction, the main driver of the real GDP growth is the energy efficiency rate (everything else equal), applied to the cost share of secondary added value outputs $(1-c(t)(1-\beta(t)))$. The composite return $(1-c(t)(1-\beta(t)))\alpha(t)$ thus constructed reflects the ability of an economy to transform primary energy efficiency into real growth of secondary added value production.

We could have devised a model looking through individual primary energy sub-sectors (coal, oil, gas, electricity etc.) and secondary goods and services sub-sectors. It would allow a more refined analysis of each production component, however without fundamentally changing the results (c.f. Appendix A.4).

3. Analysis

3.1. Model Setup

Taking yearly time steps $\Delta t = 1$ and dropping out the references to the running time *t*, the year-on-year nominal and real GDP growths are described by:

$$\frac{\Delta Y_n}{Y_n} = \frac{\Delta X'}{X'} + c(1-\beta)\frac{\Delta P'}{P'} - c\Delta\beta + (1-c(1-\beta))\alpha \tag{21}$$

$$\frac{\Delta Y_r}{Y_r} = \frac{\Delta X'}{X'} - c\beta\frac{\Delta P'}{P'} - (1-c)i'' - c\Delta\beta + (1-c(1-\beta))\alpha, \tag{22}$$

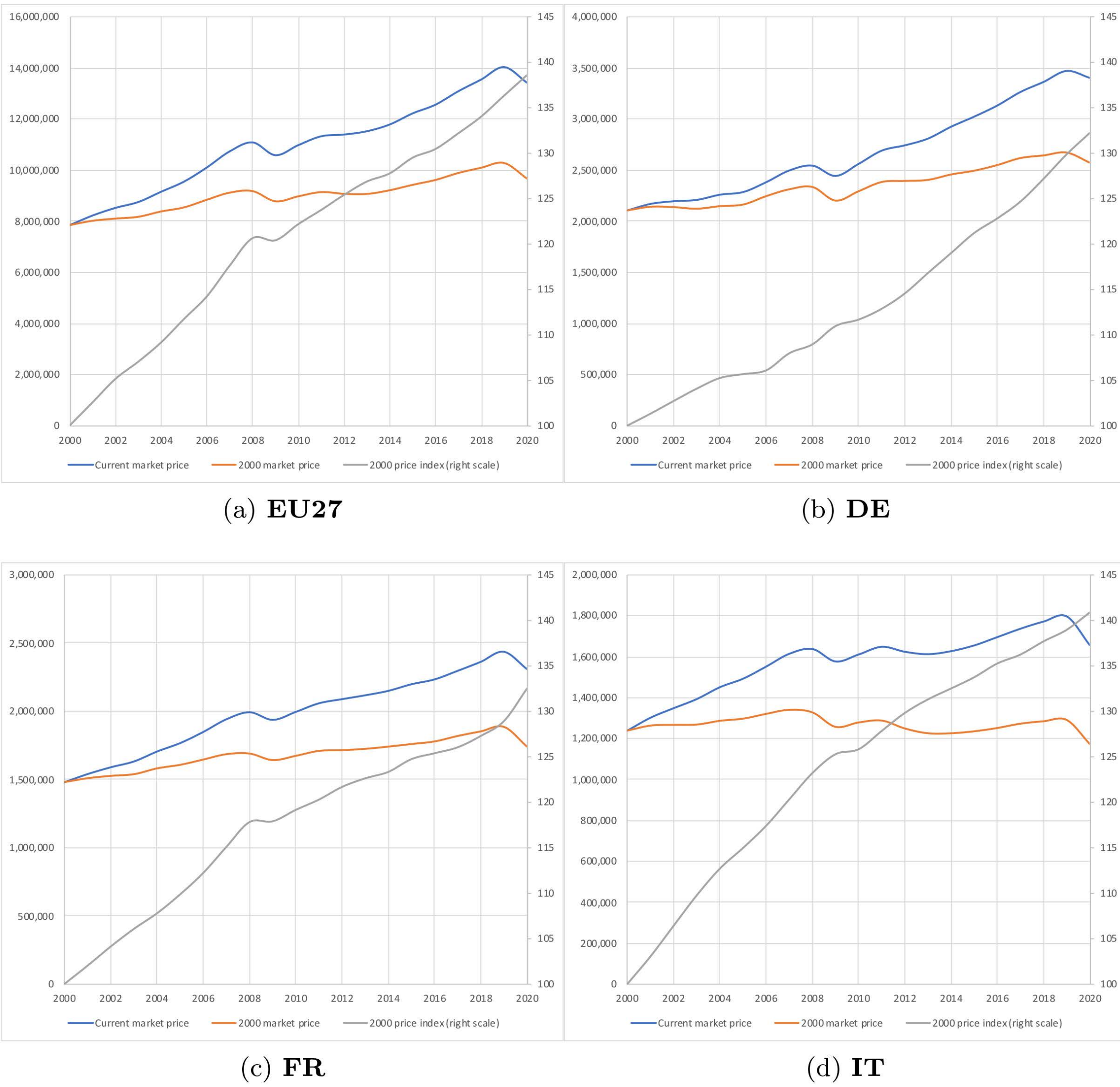
where all variables are deemed to be observed yearly. The estimation of the parameters of the model shall be performed on the EU27, DE, FR and IT.

The analysis of the model requires macroeconomic data, energy data and market data. They are taken from several sources, as summarised in **Table A6** (c.f. Appendix C.1). In the nominal and real growth equations, some of the variables are obervable whilst some others will require an estimation. In particular, on the one hand, the GDP (Y_n and Y_r) and inflation quantities (i' and i''), the energy volumes (X') and energy import fractions (β) are known, because they are reported by international statistics institutions. On the other hand, the effective energy prices (P') impacting the GDP growth equations require an estimation: it will be done from a selection of market-obervable coal, gas, oil and electricity prices, chosen so that the dynamics of the energy prices fits as best as possible the energy inflation rate (i') reported by these institutions. With the energy volume and price in hand, the calculation of the energy cost share (c) becomes straightforward. At the end of this process, what is left unknown is the energy efficiency rate (α). However, since the equations close the right-hand side elements on the left-hand side GDP growth element, the calculation of this value can eventually be done implicitly, which effectively provides a very neat new piece of information.

3.2. Data Analysis

3.2.1. GDP and Inflation

Data for the nominal and real GDP (Y_n, Y_r) and GDP growths ($\Delta Y_n / Y_n$, $\Delta Y_r / Y_r$) are readily available from numerous sources (Eurostat, IMF[3], World Bank). They are reported over the 2000-2020 period on Figure 1.



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

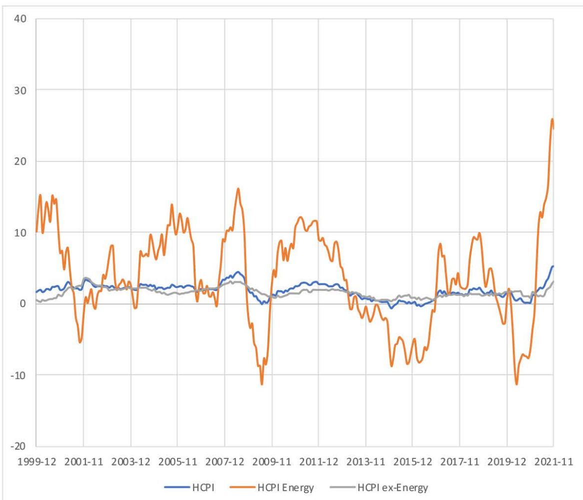
Source: Eurostat.

Figure 1. GDP (M€).

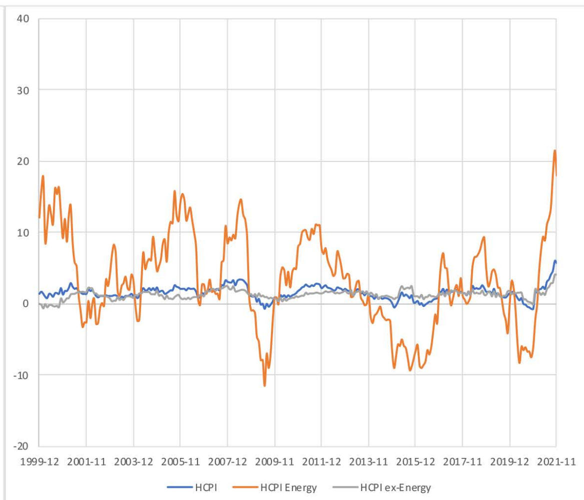
[3]International Monetary Fund.

Likewise, the inflation rate i'' on non-energy sector products (ex-energy inflation) is provided by many international or national institutions through the consumer price index (CPI) and should give a good-enough approximation of the inflation on the secondary added-value section of the GDP. However, at the same time, as will be shown hereafter, the ex-energy inflation rate can also be implied from the GDP growth model.

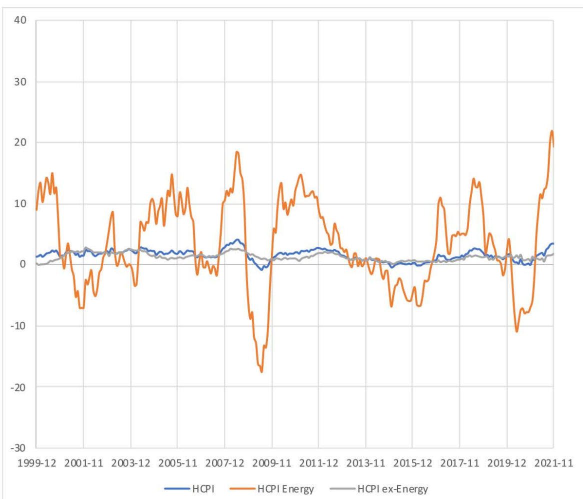
Note that Eurostat does not hold reconstructed inflation data on the EU27 before November 2001. Therefore, between December 1999 and November 2001, the time series charted on the next two figures are patched up with inflation data on the euro area (EA) with its 12 members of the time (c.f. Figure 2).



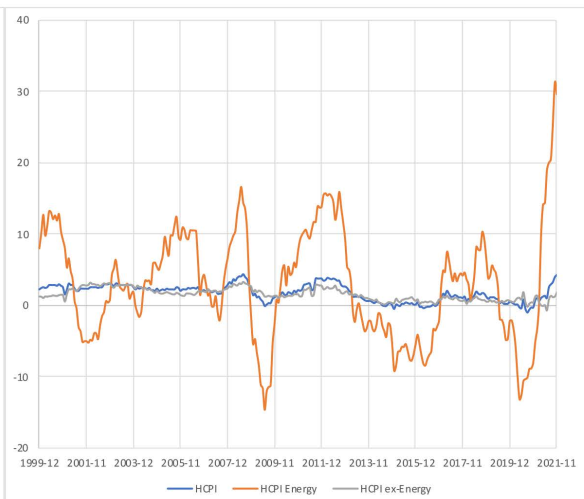
(a) **EU27**



(b) **DE**



(c) **FR**



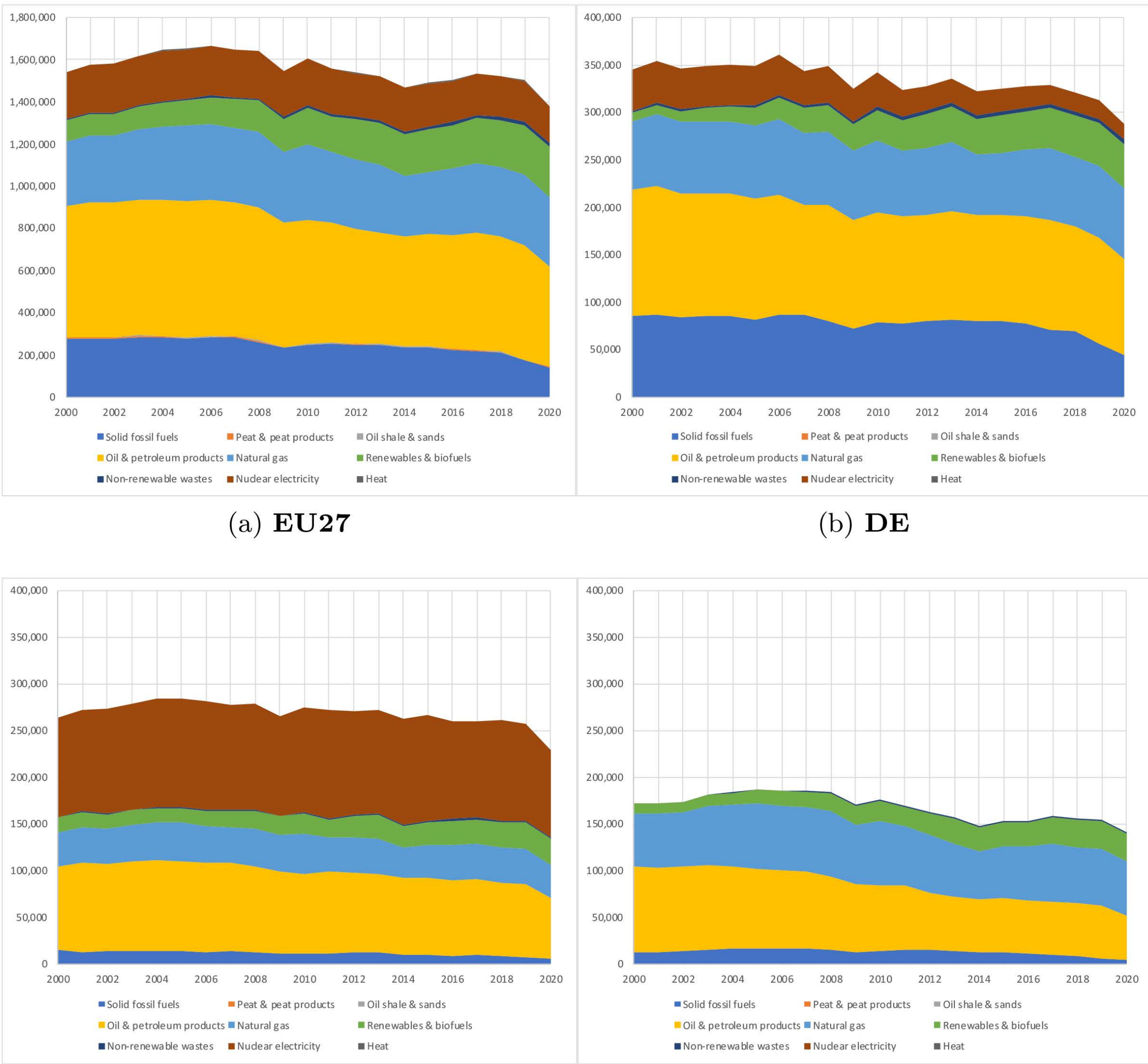
(d) **IT**

Source: Eurostat.

Figure 2. Annual inflation rate (%).

3.2.2. Energy Supply and Imports

The primary energy volumes (X') and volume changes ($\Delta X'/X'$) can be obtained as a whole, which requires the aggregation of each primary energy source (coal, oil, gas, electricity etc.) across a unique energy unit (c.f. Appendix B). This information is indeed provided by some international institutes (Eurostat, IEA[4]), as part of their energy balance reports by country and region (c.f. Figure 3).



(a) **EU27** (b) **DE**

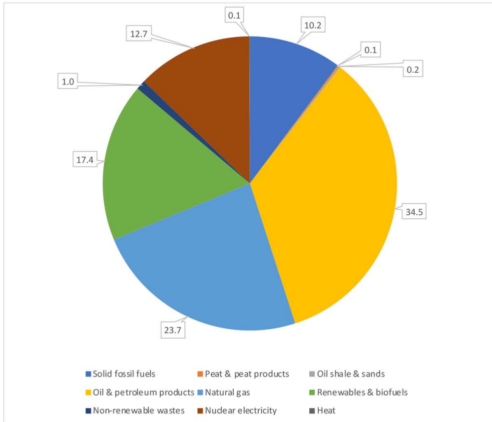
(c) **FR** (d) **IT**

Source: Eurostat.

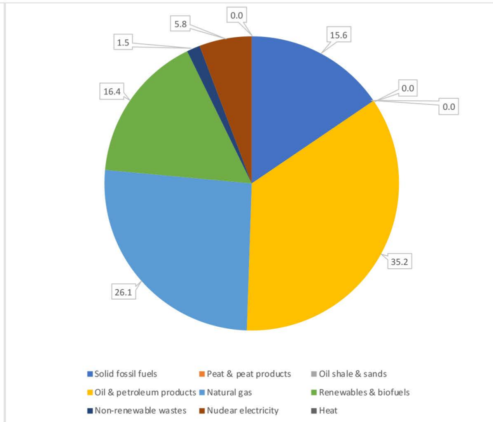
Figure 3. Gross energy supply breakdown volume (ktOE).

It allows getting the detailed gross energy supply percentage breakdown on the latest available year 2020 (c.f. Figure 4).

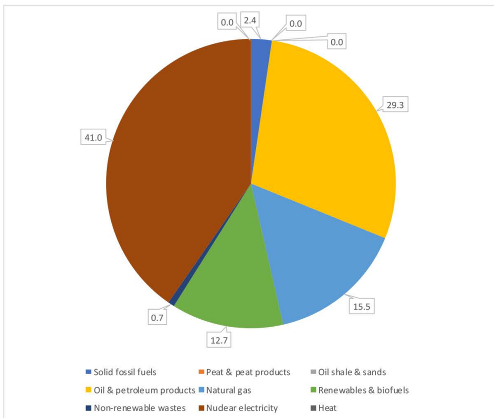
[4]International Energy Agency.



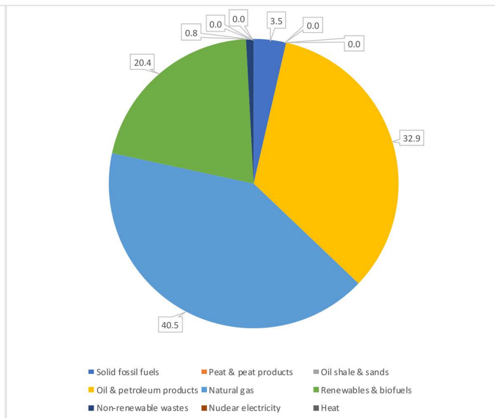
(a) **EU27**



(b) **DE**



(c) **FR**

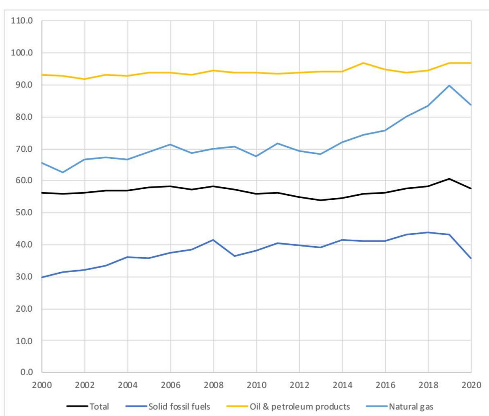


(d) **IT**

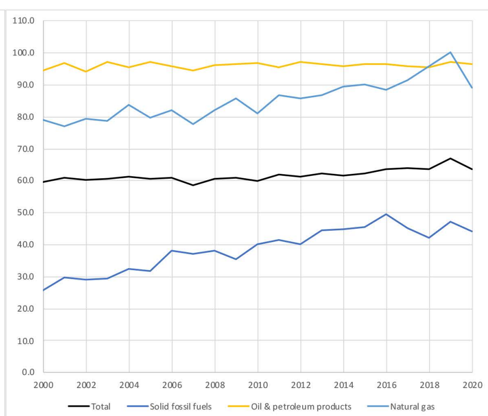
Source: Eurostat.

Figure 4. Gross energy supply breakdown 2020 (%).

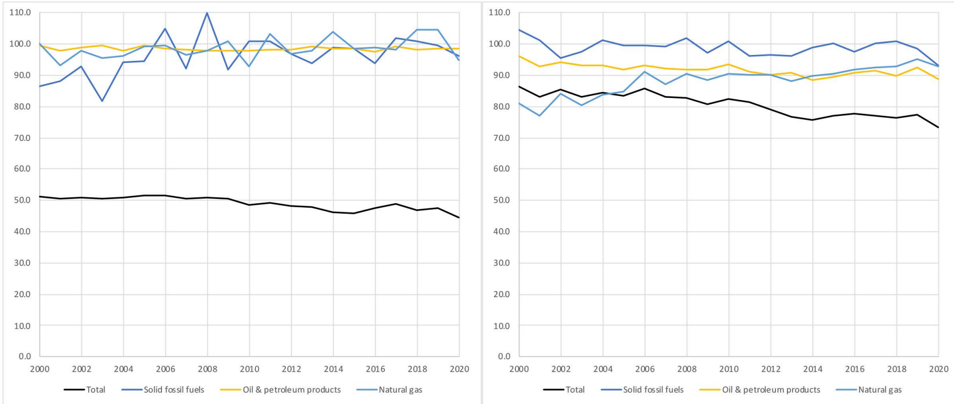
It also includes data on energy volume imports that allow getting the parameter β (c.f. **Figure 5**). When a country happens to import more than 100% of its supply of fossil fuels, it actually indicates storage increases.



(a) **EU27**



(b) **DE**

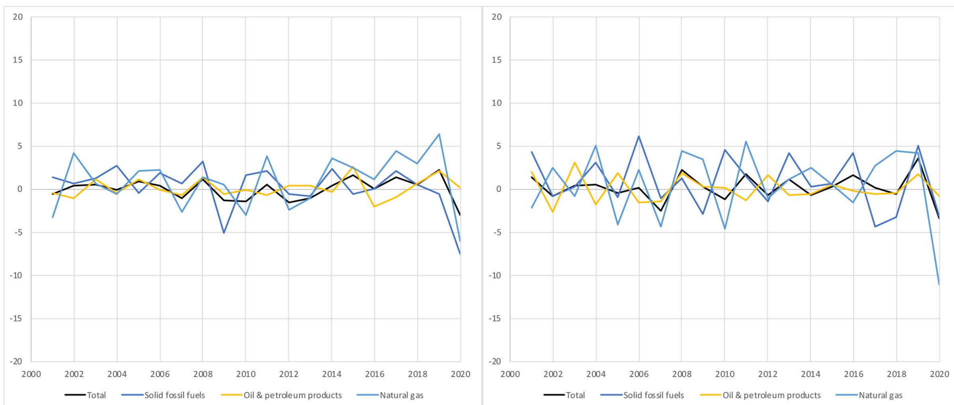


(c) **FR** (d) **IT**

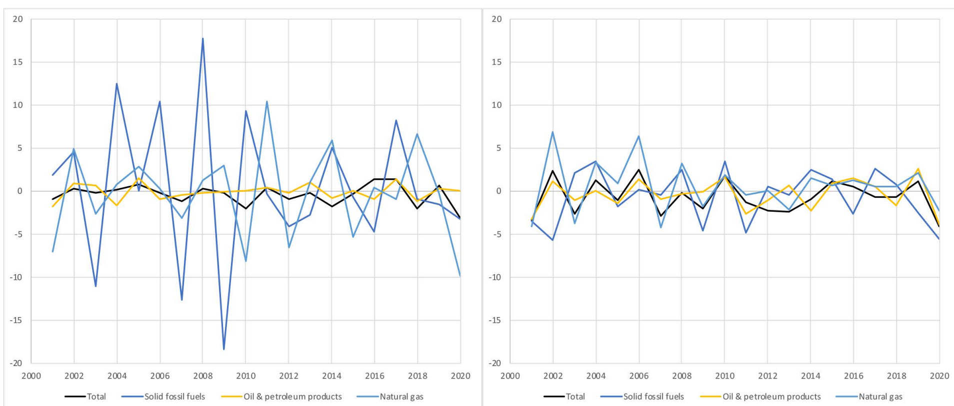
Source: Eurostat.

Figure 5. Gross energy import volume breakdown level (%).

From the parameter β, it is straightforward to derive the year-on-year import variation, $\Delta\beta$ (c.f. Figure 6).



(a) **EU27** (b) **DE**



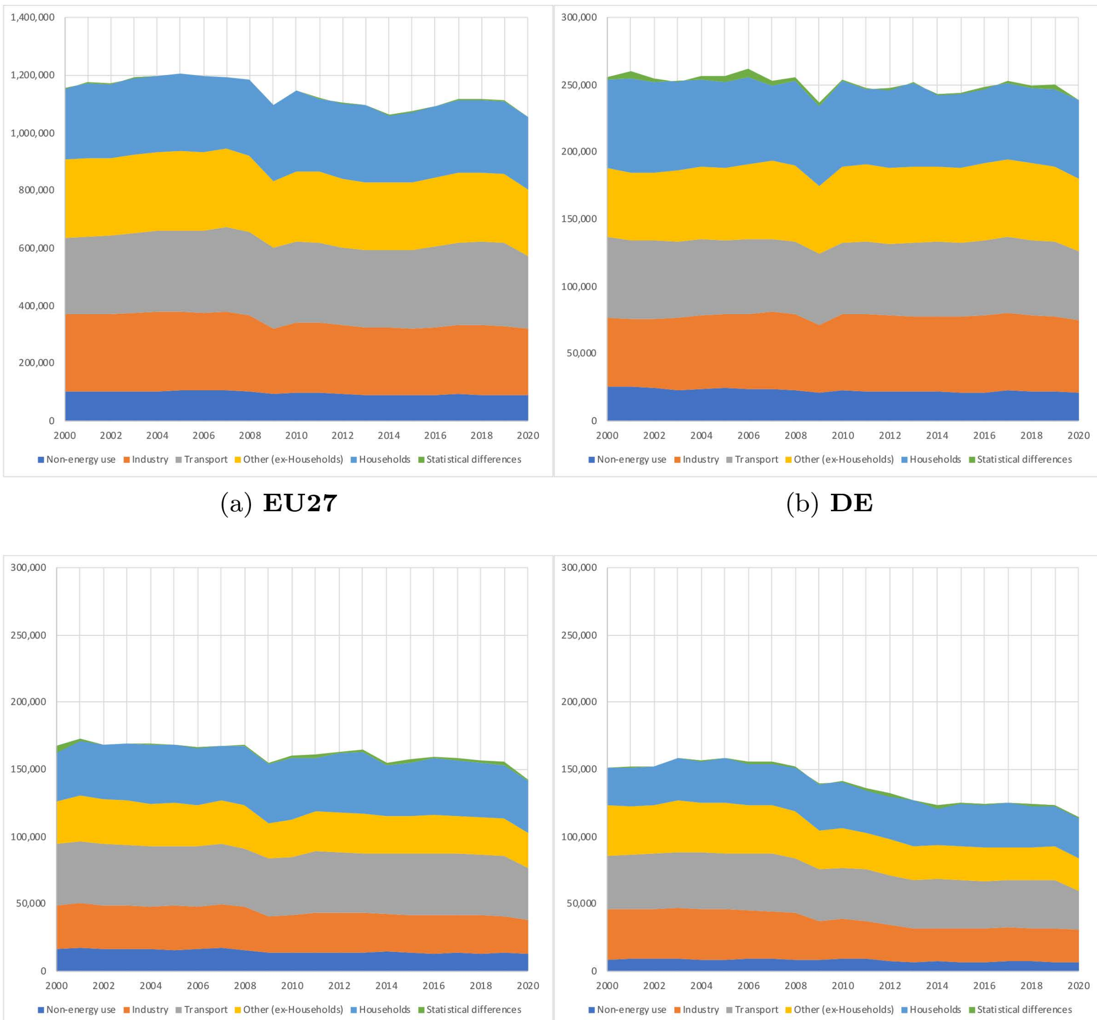
(c) **FR** (d) **IT**

Source: Eurostat.

Figure 6. Gross energy import volume breakdown variation (%).

3.2.3. Energy Benchmark Prices

Accessing yearly time series of primary energy prices (P') and price changes ($\Delta P'/P'$) can be more challenging. It generally requires some indirect estimation or approximation. An indirect estimation method may consist in calculating the energy expenditures, that is, an approximation of the product $P'X'$, as a fraction of the nominal or real GDP. It is the approach proposed by Bashmakov (2007), King (2015), King et al. (2015a, 2015b). However, as opposed to energy supply, their approach is based on energy consumption, broken down by industry, transport and service businesses, by fishing and farming activities and by households (c.f. Figure 7).



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

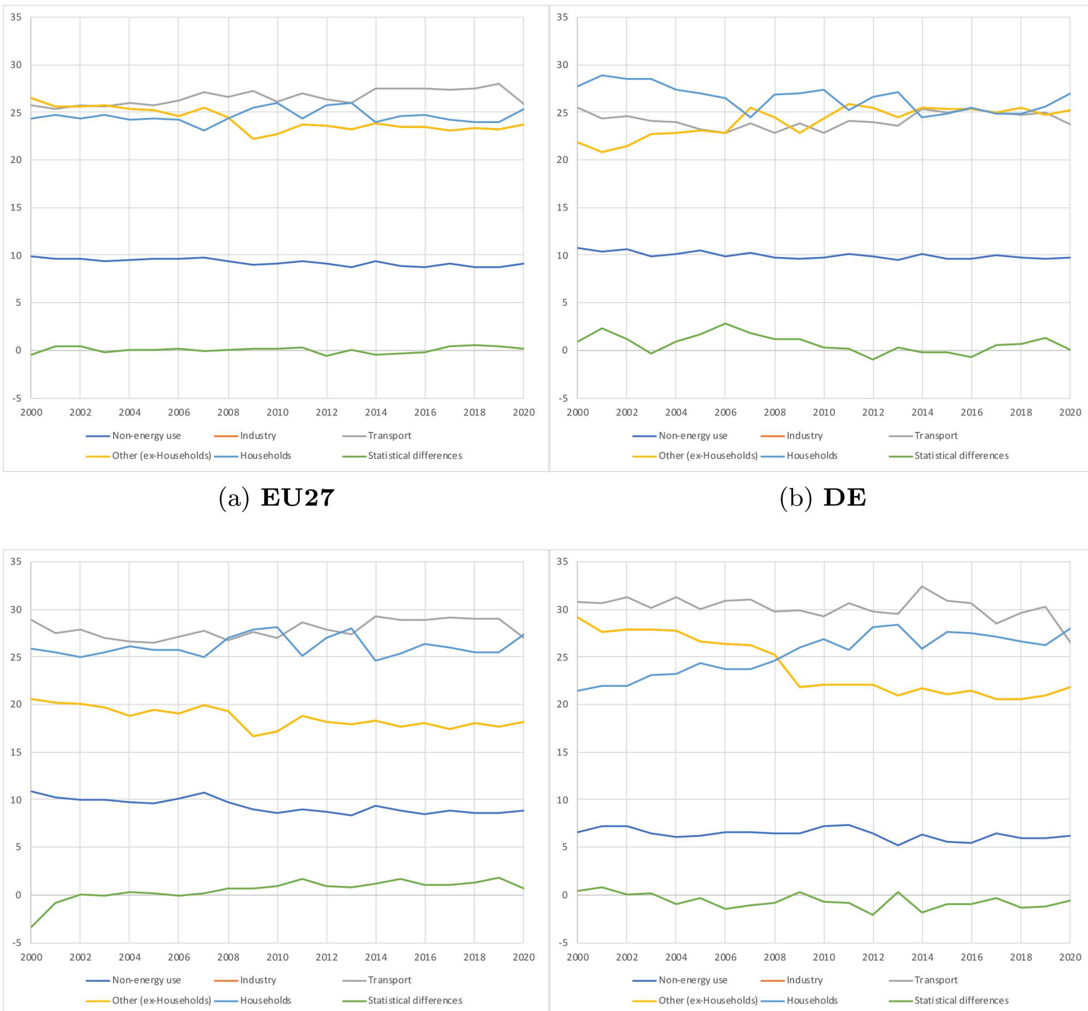
Source: Eurostat.

Figure 7. Net energy consumption breakdown (ktOE).

This information also allows measuring the consumption percentage per category of users, in particular, by the households and the production system (c.f.

Figure 8), and hence to gauge the assumption of invariance of the share of household consumption to the total energy volume, namely, $\mathrm{d}\eta(t)/\mathrm{d}X'(t)=0$.

One can observe that the assumption of a stability of the household consumption fraction is rather reasonable for the EU27, DE and FR but not for IT: the sharp decline in energy usage there was the result of a consumption reduction by the production system and not by the households. Consequently, the share of household consumption in the total substantially increased.



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

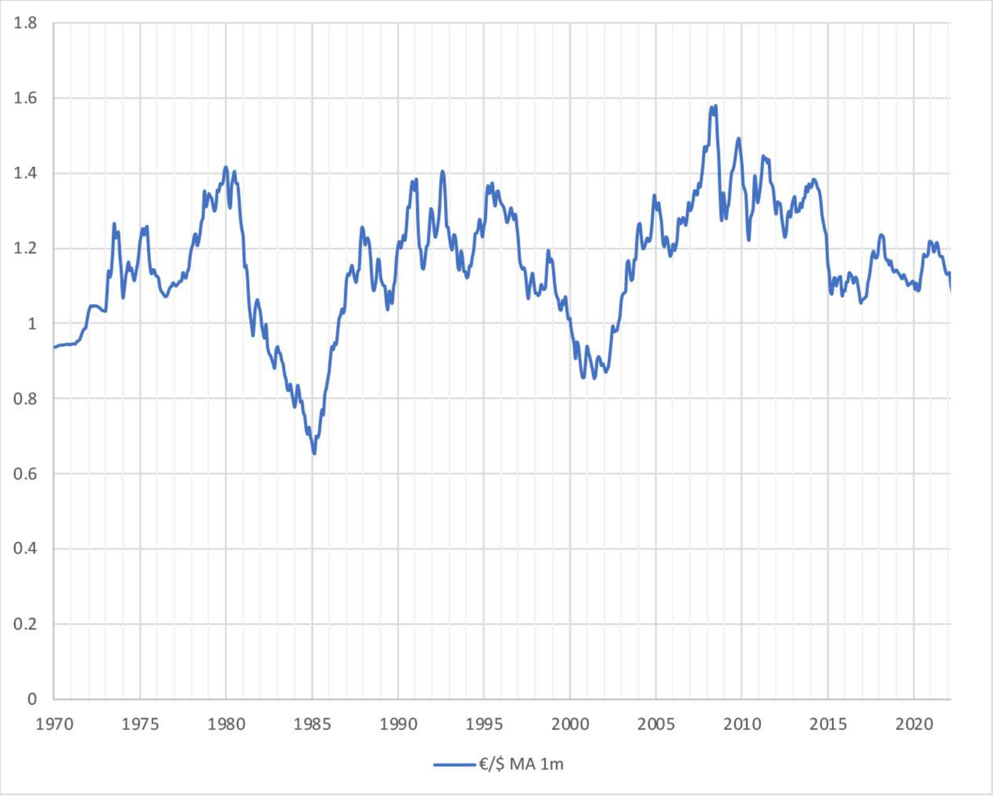
Source: Eurostat.

Figure 8. Net energy consumption breakdown (%).

Nevertheless, gross energy supply and net energy consumption are slightly different quantities: in particular, energy consumption takes into account secondary energy resources that are partly the result of the transformation of primary energy inputs (for example, crude oil after transformed into consumable refined products, coal, crude oil or gas after transformed into consumable electricity outputs) and, thus, carry an additional value-added layer that is locally produced.

The objective being to establish the cost of primary energy, it seems more appropriate to get an estimation of this cost by focussing on the primary energy supply. The approximation consists in breaking down the energy price into four broad constituents (coal, oil, gas and electricity) and, thereby, grouping the primary energy supply into four corresponding categories. The energy categories that are used in the energy balance reports follow the SIEC[5] standard classification and the simplified mapping onto the four constituents is detailed in Appendix C.2.

The breakdown of primary energy supply is available in the Eurostat or IEA data. Therefore, we start with the known decomposition $X' = \sum_e X'_e$, where X'_e is the energy supply in the category *e*. Then, we work with P'_e as the corresponding approximate benchmark price assigned to the energy category *e*. To be precise, coal, oil and gas prices are usually quoted in dollars, whilst electricity prices can be obtained in euros. Thus, all dollar-denominated monthly average commodity prices are converted beforehand into euros at the monthly average spot exchange rate (c.f. **Figure 9**).



Source: Banque de France.

Figure 9. Forex monthly average spot rate (€/$).

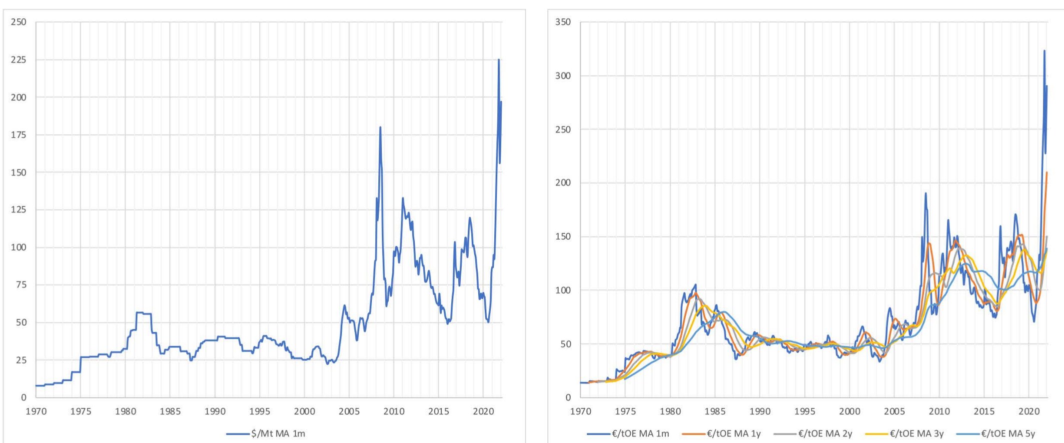
[5]Standard International Energy Classification.

Having said that, since the supply of primary energy is carried out along an entire GDP production year, it is reasonable to take, as an approximation, the annual average of the spot prices $\overline{P}_{0,e}$ instead of, say, the final spot price of the year. However, one can also consider that most of the supplies are in effect partly hedged through the use of short-term to long-term forward or future contracts and that, in the end, what is actually paid for in a given year results from using a mix of spot, forward and future prices in the previous years. The same can be said about hedging the exchange rate risk. To approximate such a pricing mechanism dealt on the commodity markets, one can thus extend the averaging of the euro-denominated commodity spot monthly price series over more than a year. The analysis will therefore look at one-year, two-year, three-year and five-year moving averages (MA) of the monthly average benchmark prices.

$$P' \approx \frac{1}{X'} \sum_{e} \overline{P}'_e X'_e.$$

The benchmarks used in the study are as follows (c.f. Appendix B). For coal, the Newcastle spot price quoted in \$/Mt and converted into €/tOE (c.f. **Figure 10**). For crude oil, the Brent spot price quoted in \$/b and converted into €/tOE (c.f. **Figure 11**). For natural gas, the Amsterdam spot price quoted in \$/MBTU and converted into €/tOE (c.f. **Figure 12**). For electricity, the Platts PEP electricity index (followed by the EPB electricity index) of the wholesale day-ahead spot price across most EU27 countries (extrapolated prior to 2002 by the electricity price regressed against the coal, oil and gas spot prices) quoted in €/MWh and converted into €/tOE (c.f. **Figure 13**).

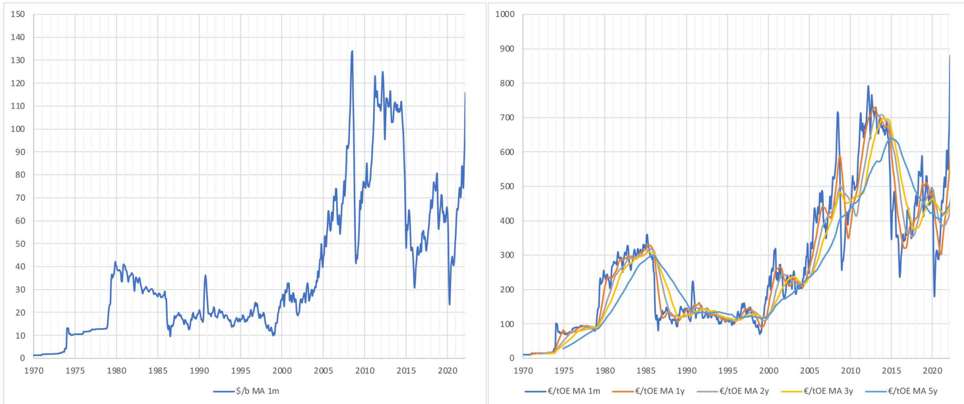
Combining the coal, oil, gas and electricity benchmark prices that are used allows approximating the average price of the primary energy supply in the EU27 in €/tOE terms. This exercise is carried out for different MA lengths (c.f. **Figure 14**).



(a) **Monthly average price (\$/Mt)** (b) **Monthly average & MA price (€/tOE)**

Source: World Bank.

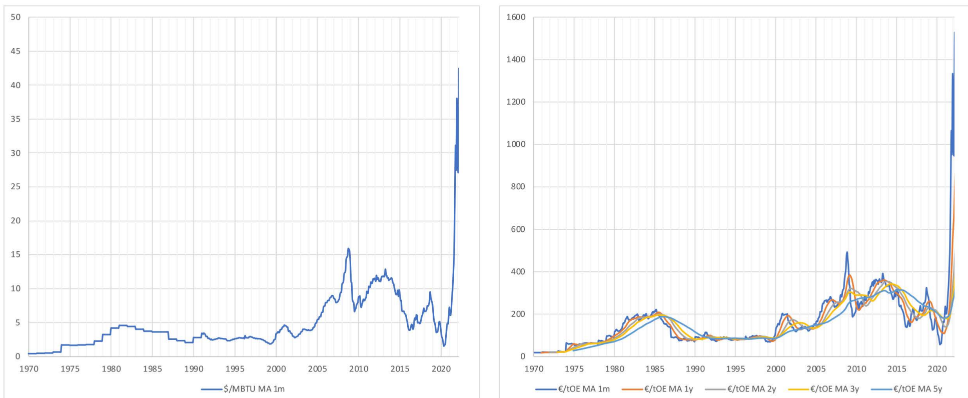
Figure 10. Newcastle coal spot price.



(a) **Monthly average price (\$/b)** (b) **Monthly average & MA price (€/tOE)**

Source: World Bank.

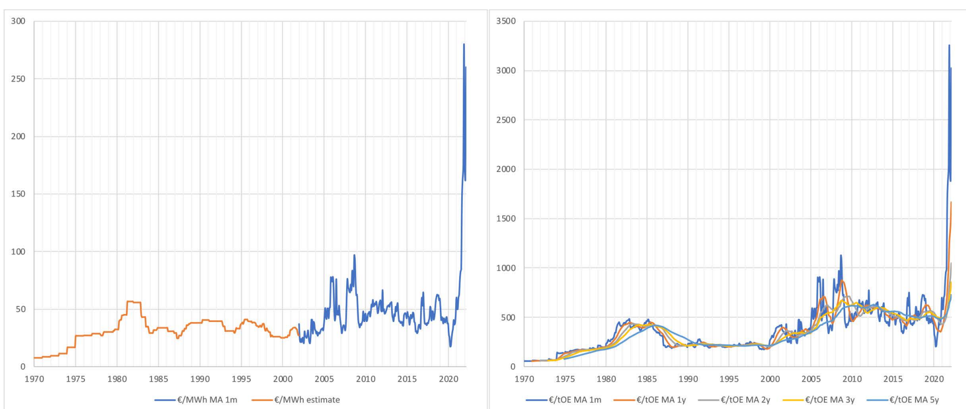
Figure 11. Brent oil spot price.



(a) **Monthly average price (\$/MBTU)** (b) **Monthly average & MA price (€/tOE)**

Source: World Bank.

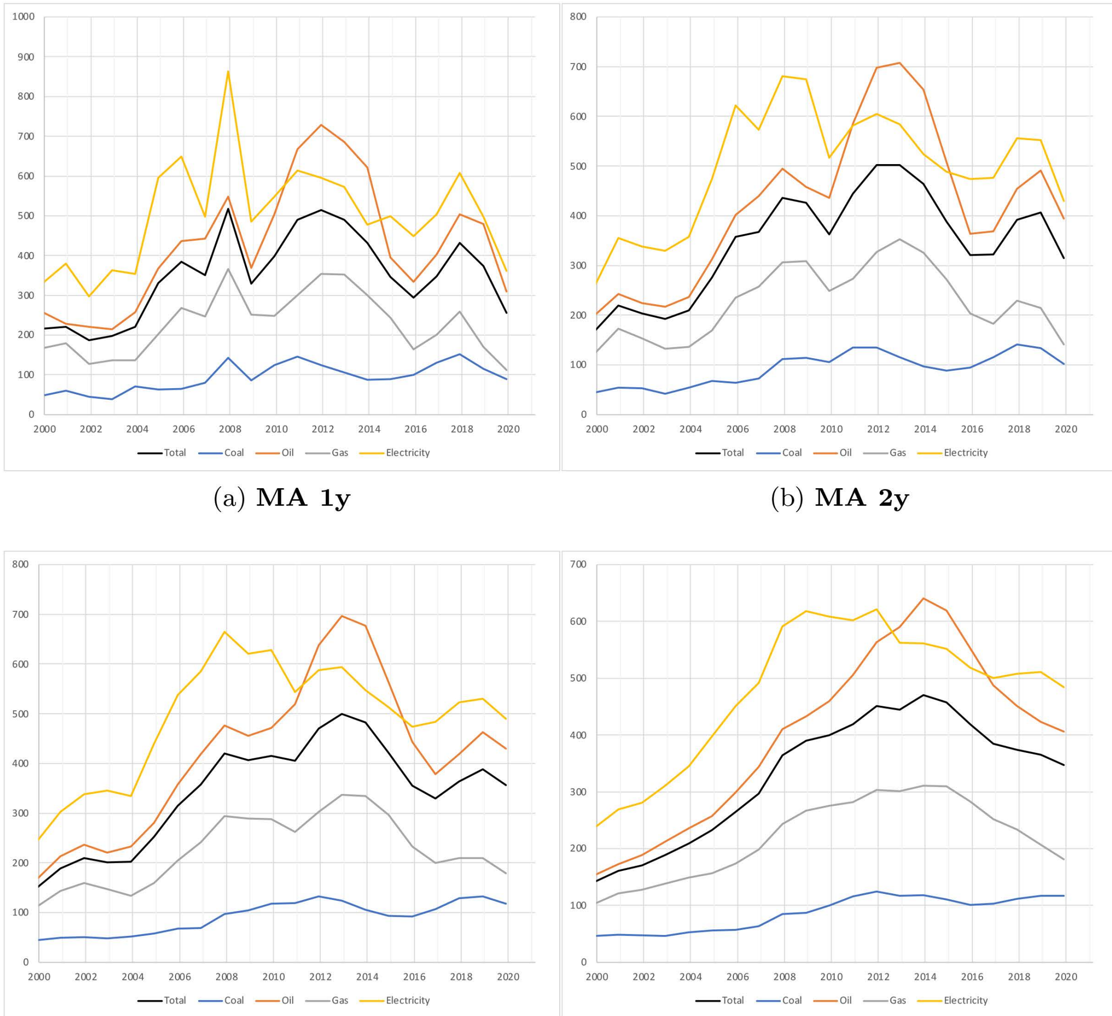
Figure 12. Amsterdam natural gas spot price.



(a) **Monthly average price (€/MWh)** (b) **Monthly average & MA price (€/tOE)**

Source: S&P Global Platts, European Commission and estimated extrapolation.

Figure 13. Platts PEP and EPB electricity spot price.



(a) **MA 1y** (b) **MA 2y**

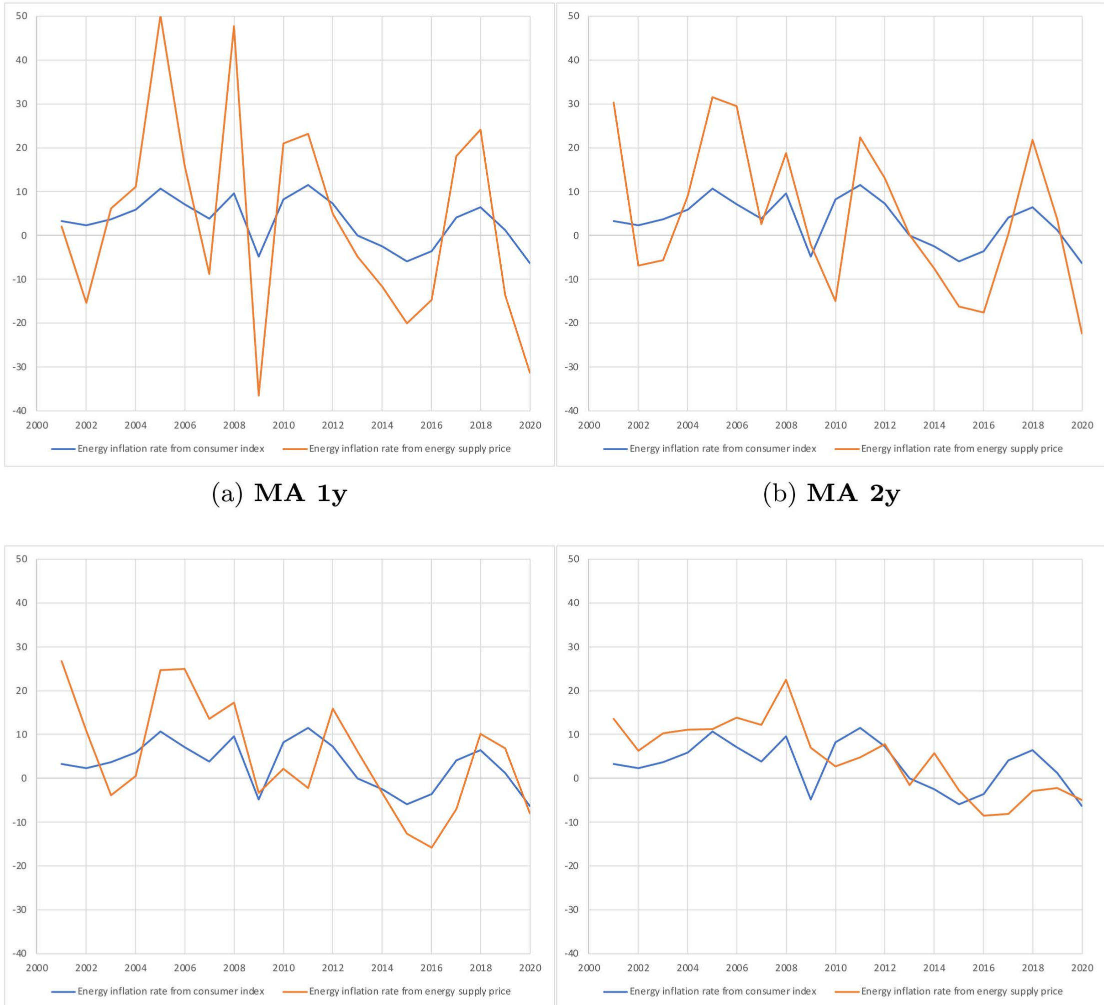
(c) **MA 3y** (d) **MA 5y**

Figure 14. Energy supply price (€/tOE).

The main observation is that the EU27 countries have generally benefited from a cheapening in the energy prices since the financial crisis of 2008, a phenomenon that continued until 2020. In particular, the overall energy cost decrease started with the electricity prices in 2008 and was followed by the crude oil and gas prices in 2014. This favourable era obviously ended in 2022 and these new circumstances will probably stay so for some years to come.

3.3. Energy Pricing Mechanism

Now, to assess which MA produces the best approximation for the pricing of energy, one can have a look at the estimated energy inflation rate $i' = \Delta P'/P'$ given by the average energy (coal, oil, gas, electricity) price change and compare its value to the energy inflation rate obtained from the energy CPI (c.f. **Figure 15**). The analysis is carried out for the whole EU27, used as a benchmark for their members and, in particular, for DE, FR and IT.



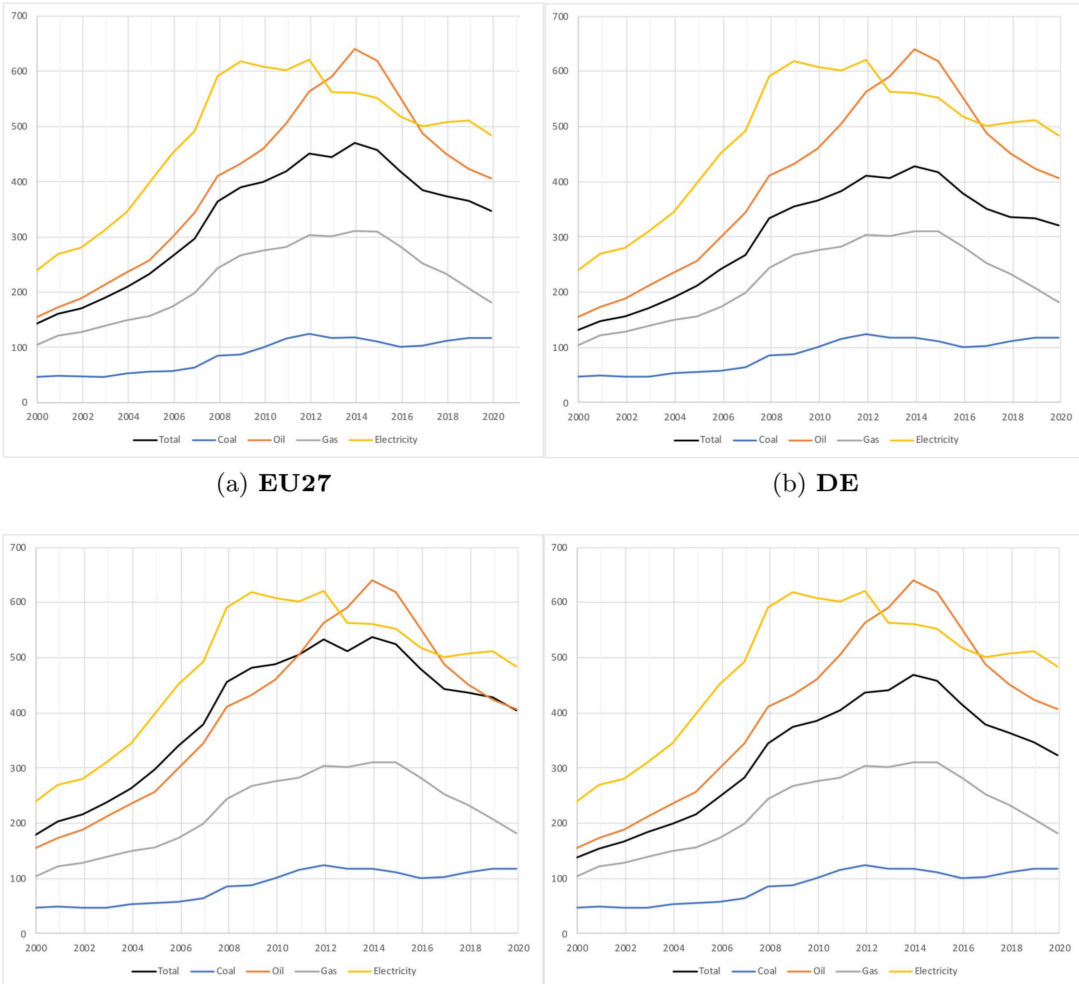
(a) **MA 1y** (b) **MA 2y**

(c) **MA 3y** (d) **MA 5y**

Figure 15. Estimated energy inflation rate (%).

This analysis seems to indicate that averaging the energy prices over five years allows a better alignment of the phases and amplitudes of the estimated ex-energy inflation rates, $i' = \Delta P'/P'$, derived from the energy benchmark prices, with the official ex-energy inflation rates, derived from the CPI. Thus, this same five-year MA methodology shall be used for DE, FR, IT in the rest of the note.

The combined coal, oil, gas and electricity benchmark prices are used to approximate the average price of the primary energy supply in the EU27, DE, FR and IT in €/tOE terms (c.f. Figure 16). Individual constituents (coal, oil, gas and electricity) are assumed to have the same prices in the four studied economies. However, the average prices borne by a given economy depends on its particular energy mix. For example, FR have experienced a higher average energy price for the last twenty years, because they make a larger use of electricity in their mix and electricity has generally been more expensive in tOE terms than fossil fuels.



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

Figure 16. Energy supply price (€/tOE).

3.4. Parameter Estimation

3.4.1. Energy Cost Share

The knowledge of the primary energy product $P'X'$ in a given year, albeit approximate, allows the calculation of the yearly energy cost share $c = P'X'/Y_n$ (c.f. **Figure 17**), which is a key component of the GDP equation that was laid out.

The energy cost share peaks around the financial crisis of 2008 and has tended to decline since 2010-2012. The decrease follows a reduction trend in the energy supply and the energy consumption in the EU27 that has been observed since 2006, as can be seen in **Figure 3** and **Figure 7**. It was also accentuated by the decline in the average energy prices that has been experienced since 2008 (c.f. **Figure 16**). Over the twenty years between 2001 and 2020, compared to developed economies, the energy cost share averages and standard deviations in the EU27, DE, FR and IT, on a five-year MA basis, as reported in **Table 1**, is in line with the findings of similar studies (Kümmel et al., 2007).

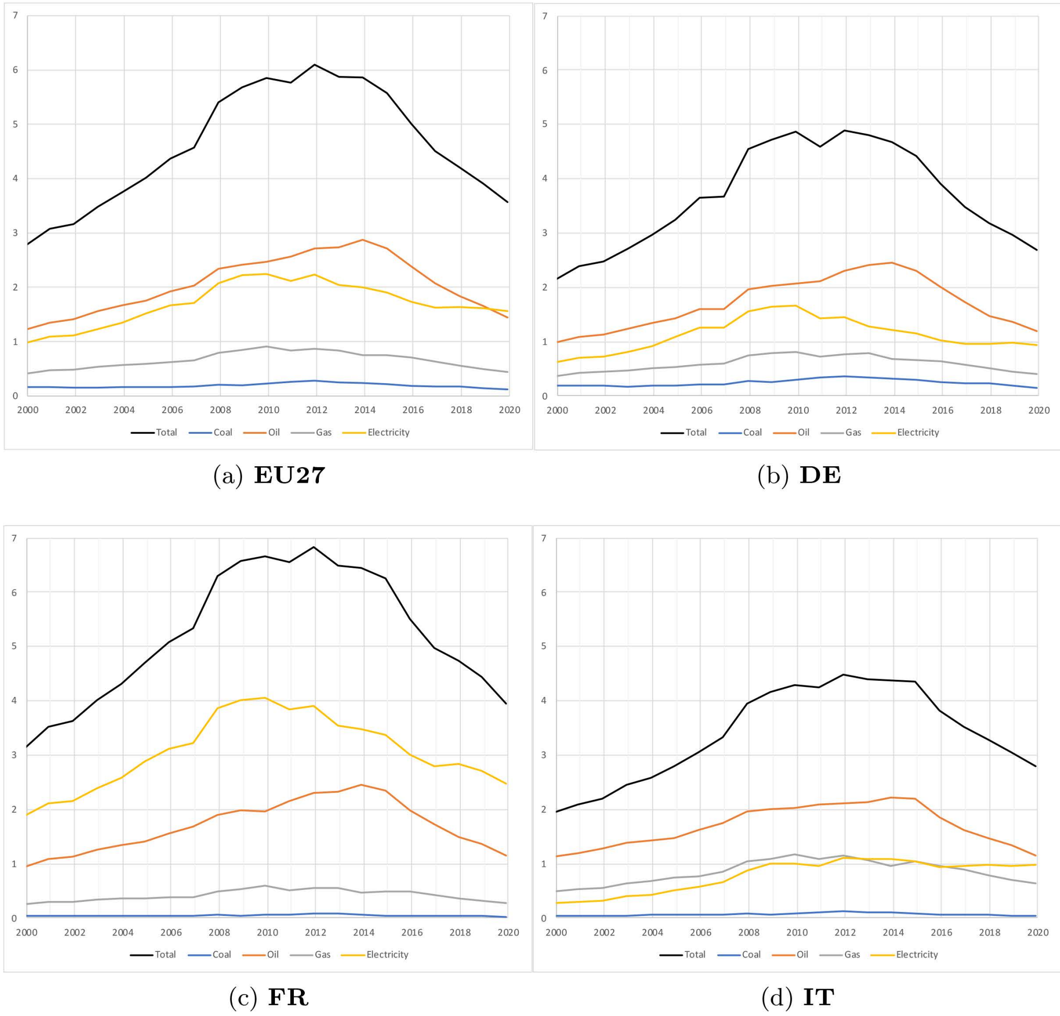


Figure 17. Energy cost share (%).

Table 1. Energy cost share average and standard deviation (%) 2001-2020.

	Average	Standard Deviation
EU27	4.6	1.1
DE	3.7	0.9
FR	5.2	1.2
IT	3.4	0.9

Notably, DE and IT have a lower overall energy cost share than the EU27 average, whereas FR have a higher overall energy cost share than the EU27 average, because of their higher share of electricity in their mix.

3.4.2. Energy Efficiency Rate

In the model, the only variable that is neither formally known nor observed, is the energy efficiency rate α. From the nominal GDP change formulation, it can be inferred as

$$\alpha_n = \frac{1}{1-c(1-\beta)}\left[\frac{\Delta Y_n}{Y_n} - \frac{\Delta X'}{X'} - c(1-\beta)\frac{\Delta P'}{P'} + c\Delta\beta\right]. \quad (23)$$

The implied values of the energy efficiency rate are charted on Figure 18.

Between 2001 and 2020, the efficiency rate α_n implied from the nominal GDP equations of the four economies tends to show very similar cycles. All show a drop in energy efficiency in 2010. Furthermore, the EU27, FR and IT have an additional drop in 2003. However, the IT economy has a marked negative efficiency rate in 2015 that follows a four-year period in 2011-2014 of positive energy efficiency, contrasting with the other economies (Table 2).

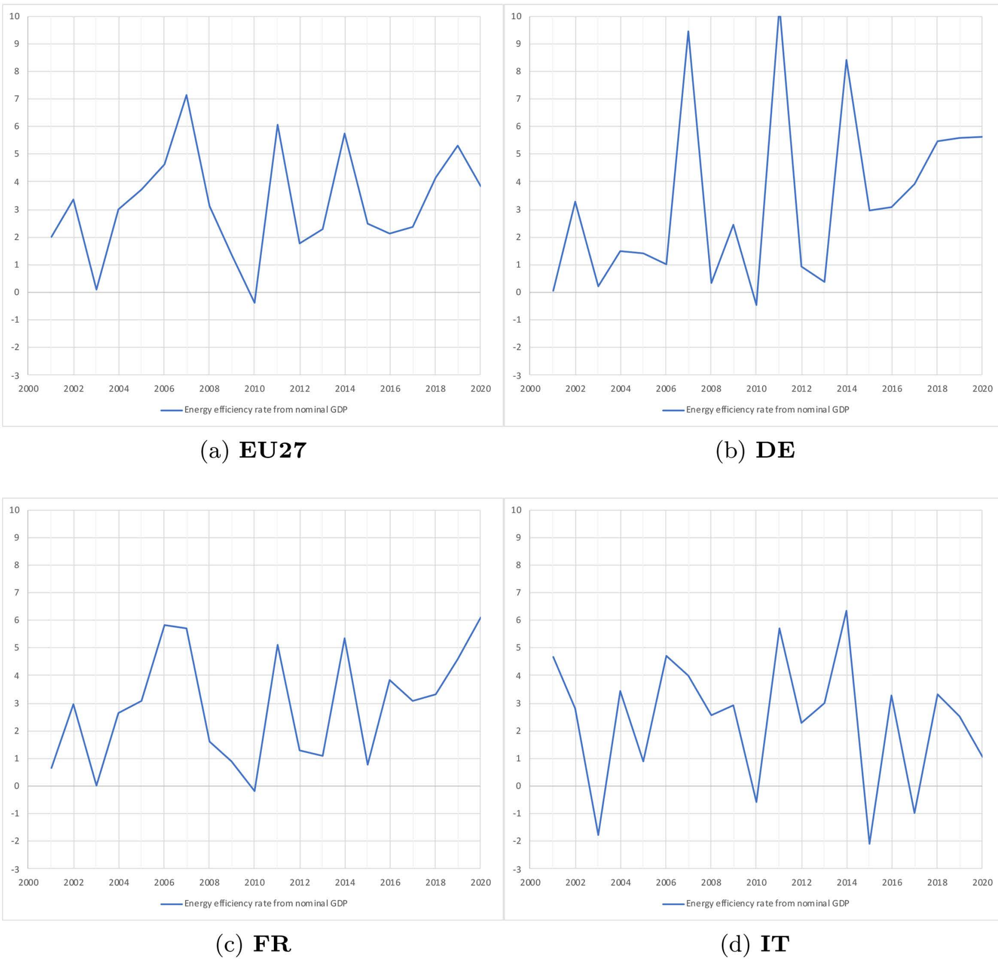
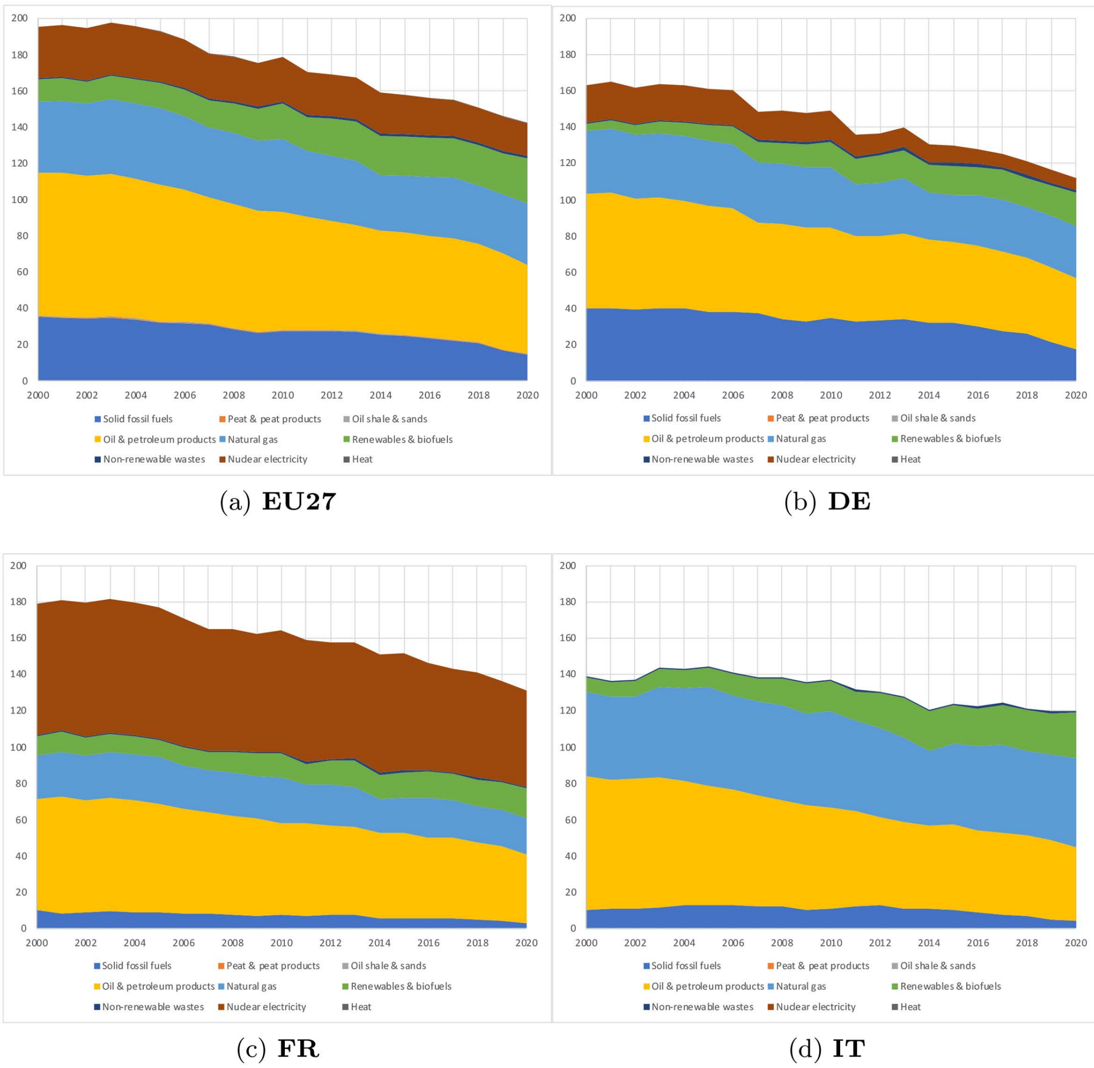


Figure 18. Implied energy efficiency rate (%).

Table 2. Implied energy efficiency rate average and standard deviation (%) 2001-2020.

	Average	Standard Deviation
EU27	3.3	1.9
DE	3.5	3.2
FR	3.0	2.1
IT	2.3	2.3

In general, the energy efficiency rate is positive, showing a tendency of the EU27 economy to be able to produce more secondary added-value with less primary energy. This is reflected in the energy intensity, which is the ratio of the energy volume over the GDP volume, that is, X'/Y_r, and is measured in tOE/€ in constant € 2000 terms. The energy intensity has a tendency to decrease almost steadily between 2000 and 2020 (c.f. Figure 19).



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

Source: Eurostat.

Figure 19. Energy intensity breakdown (tOE/€).

Interestingly, the year 2010 shows a negative energy efficiency rate (c.f. Figure 18). It follows the recession of 2009 and coincides with an increase in the energy intensity in 2010. On the reported time series (c.f. Figure 19), only in 2003 and 2010 does the energy intensity in the EU27 increase, caused, in particular, by an increase in gas supply and, to a lesser extent, in crude oil supply. In DE, an additional energy intensity increase is observed in 2013. In IT, additional increases are also observed in 2015 and 2017. In each instance, the increase in the energy intensity translates into a drop in the efficiency rate that reaches zero or negative levels.

3.4.3. Inflation Rate

From the energy efficiency rate α_n implied by the nominal GDP formulation, one can in turn infer an estimated ex-energy inflation rate i''_r from the real GDP formulation (c.f. Figure 20).

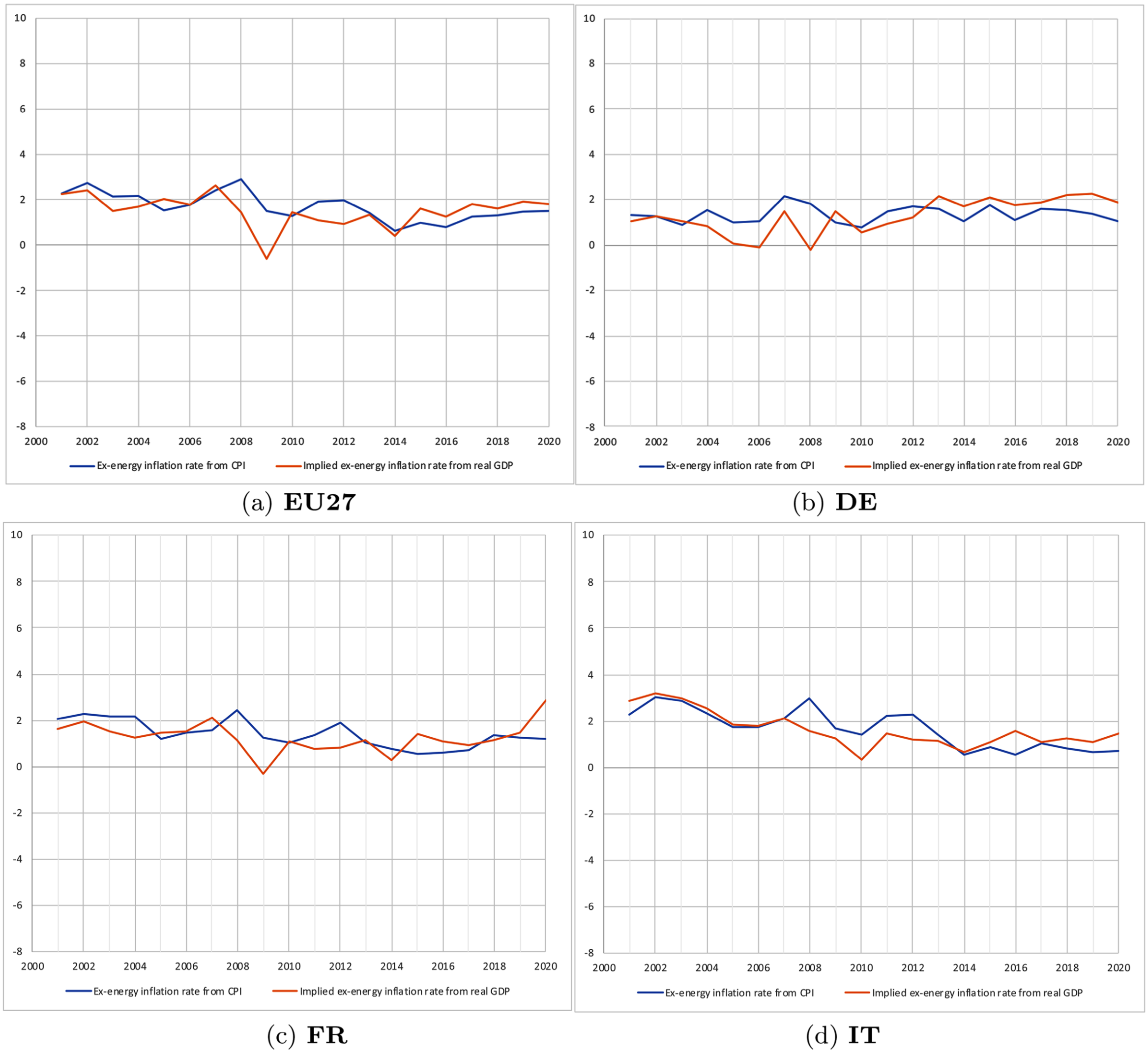
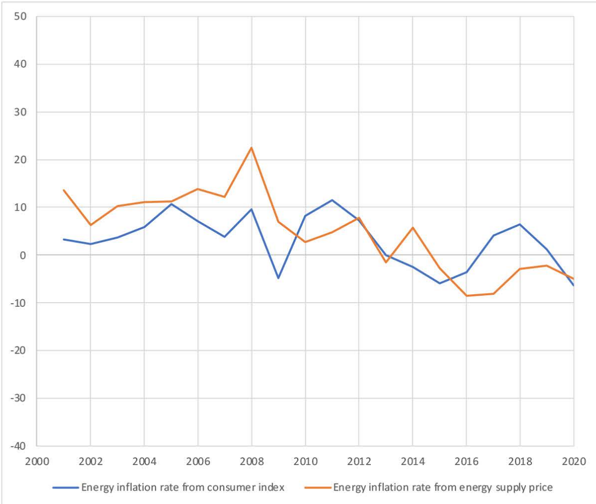


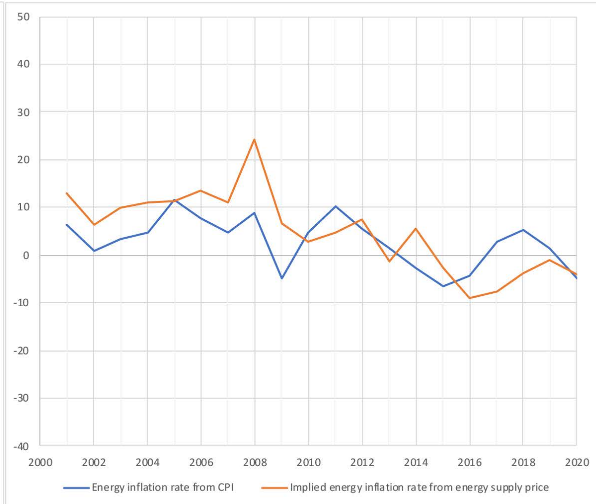
Figure 20. Implied ex-energy inflation rate (%).

$$i''_r = \frac{1}{1-c}\left[-\frac{\Delta Y_r}{Y_r} + \frac{\Delta X'}{X'} - c\beta\frac{\Delta P'}{P'} - c\Delta\beta + \left(1-c\left(1-\beta\right)\right)\alpha_n\right]. \tag{24}$$

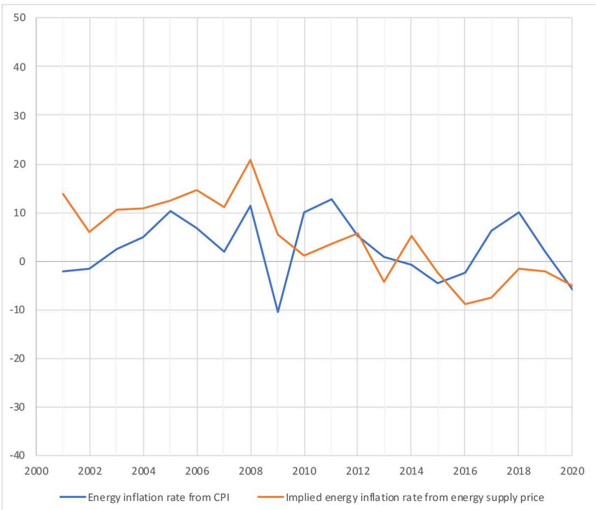
Now, one can also have a look at the estimated energy inflation rate i' given by the five-year MA energy (coal, oil, gas, electricity) price change, that is, the calculated $i' = \Delta P'/P'$, and compare its value to the energy inflation rate obtained from the energy CPI (c.f. Figure 21).



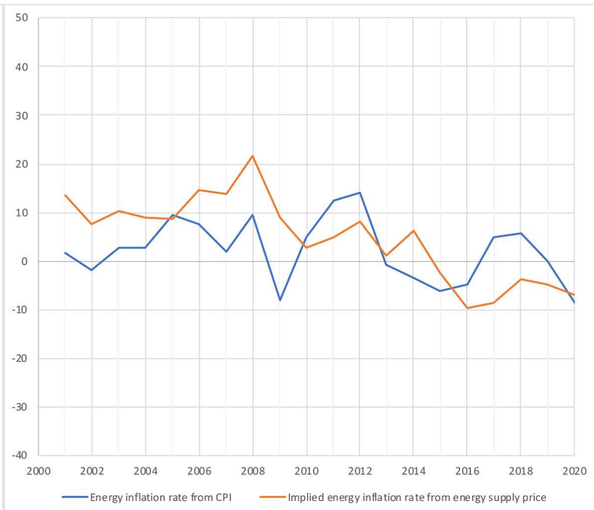
(a) **EU27**



(b) **DE**



(c) **FR**

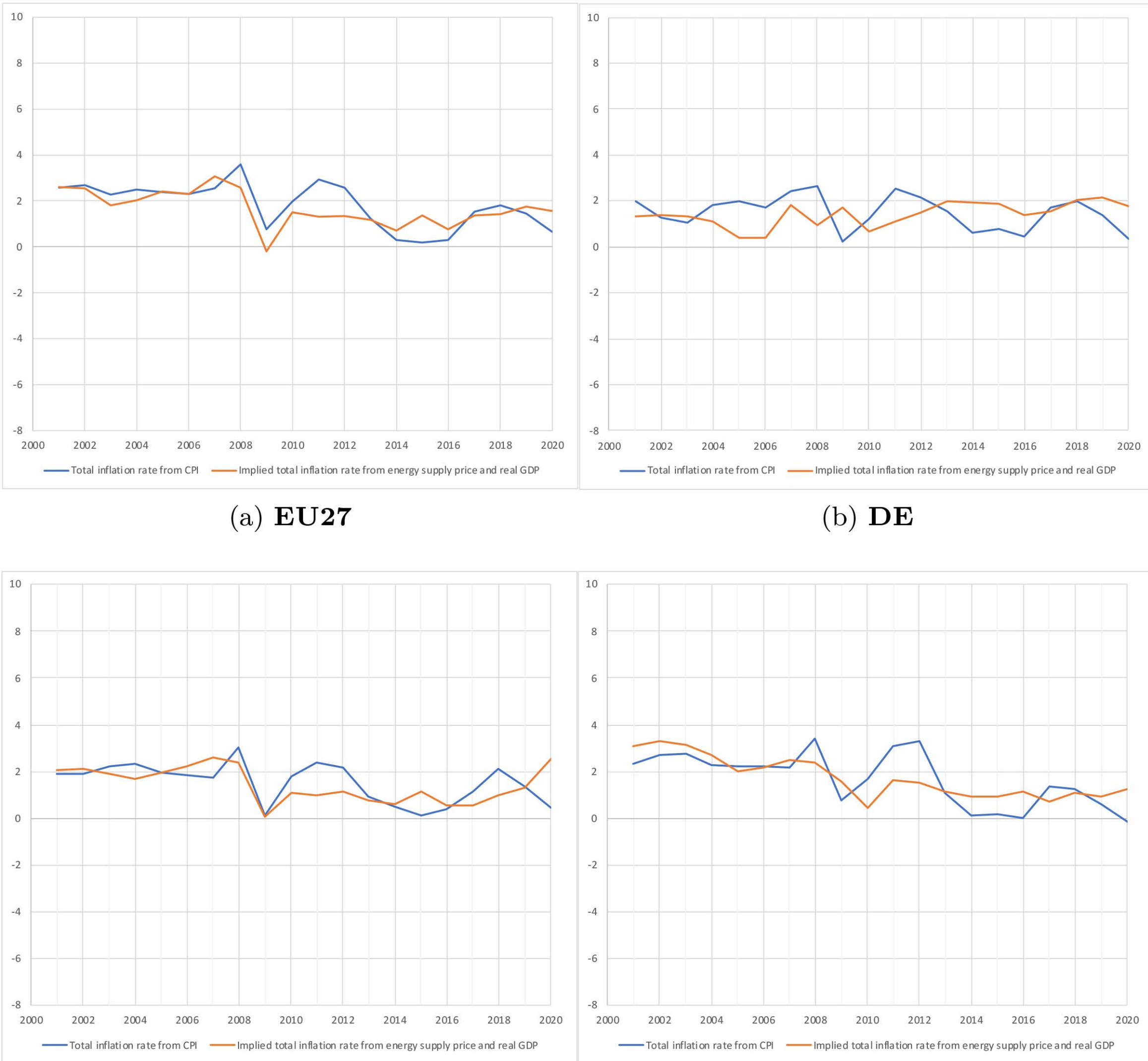


(d) **IT**

Figure 21. Implied energy inflation rate (%).

Finally, using the calculated energy cost share c, the two implied energy and ex-energy inflation rates i_r' and i_r'' can be re-combined to return the implied total inflation rate, as $i_r = ci_r' + (1-c)i_r''$. This value can in turn be compared to the official annual inflation rate, as calculated from the CPI (c.f. Figure 22).

Note that the choice of the window length for the moving averaging of the energy prices (here, five years) does not change the total inflation rate, because it only affects the breakdown between the ex-energy inflation rate and the energy inflation rate. Furthermore, the synthesised total inflation rate $i_r = ci_r' + (1-c)i_r''$ actually converges to the GDP deflator (which is used for the calculation of the real GDP from the nominal GDP) and thus structurally differs from the CPI definition and calculation.



(a) **EU27** (b) **DE**

(c) **FR** (d) **IT**

Figure 22. Implied total inflation rate (%).

4. Situation Analysis

4.1. Past Average Behaviour

To have a better idea and understanding of the model parameters and their impacts, we first work out the average model output values of the nominal and real GDP growths from the parameter values calculated over the last twenty years. For this purpose, we use the energy efficiency rate implied by the nominal GDP growth equation and the ex-energy inflation rate implied by the real GDP growth equation, given the implied energy efficiency rate.

Introducing the variables

$$f_0 = +c(1-\beta)$$

$$f_1 = -c\beta$$

$$f_2 = -(1-c)$$
$$f_3 = +(1-c(1-\beta))$$
$$f_4 = -c,$$

these two GDP growth equations (real and nominal) can be recast as

$$\frac{\Delta Y_n}{Y_n} = \frac{\Delta X'}{X'} + f_0 \frac{\Delta P'}{P'} + f_3\alpha + f_4\Delta\beta$$

$$\frac{\Delta Y_r}{Y_r} = \frac{\Delta X'}{X'} + f_1 \frac{\Delta P'}{P'} + f_2 i'' + f_3\alpha + f_4\Delta\beta.$$

The various factors translate the growth exposure to the energy price risk, ex-energy price risk, energy efficiency risk and energy import change risk.

In Table 3, the reported values are calculated over the twenty-year period. The argument values are the averages (columns "value"), whilst the factor weights are the argument-weighted averages (columns "weight"), so that the growth contributions (columns "impacts") sum up to the average year-on-year nominal and real GDP growths actually observed over the period.

Table 3. GDP equation argument (%) and factor average (%) 2001-2020.

Argument	Value				Factor	Weight				Impact			
	EU27	DE	FR	IT		EU27	DE	FR	IT	EU27	DE	FR	IT
$\Delta X_0/X_0$	−0.66	−1.07	−0.85	−1.00	+1	+100	+100	+100	+100	−0.66	−1.07	−0.85	−1.00
$\Delta P_0/P_0$	+4.44	+4.50	+4.00	+4.28	f_0	+2.04	+1.58	+2.61	+0.52	+0.09	+0.07	+0.10	+0.02
					f_1	−2.72	−2.35	−2.78	−2.91	−0.12	−0.11	−0.11	−0.12
i''	+1.48	+1.29	+1.24	+1.56	f_2	−95.51	−96.23	−94.96	−96.73	−1.41	−1.24	−1.18	−1.51
α	+3.28	+3.47	+3.01	+2.29	f_3	+97.97	+98.57	+97.33	+99.28	+3.21	+3.42	+2.93	+2.27
$\Delta\beta$	+0.09	+0.15	−0.31	−0.52	f_4	−2.60	−5.28	−5.70	−3.90	−0.00	−0.00	+0.02	+0.02
				Total nominal GDP (%)						+2.63	+2.42	+2.19	+1.31
				Total real GDP (%)						+1.01	+1.00	+0.80	−0.35

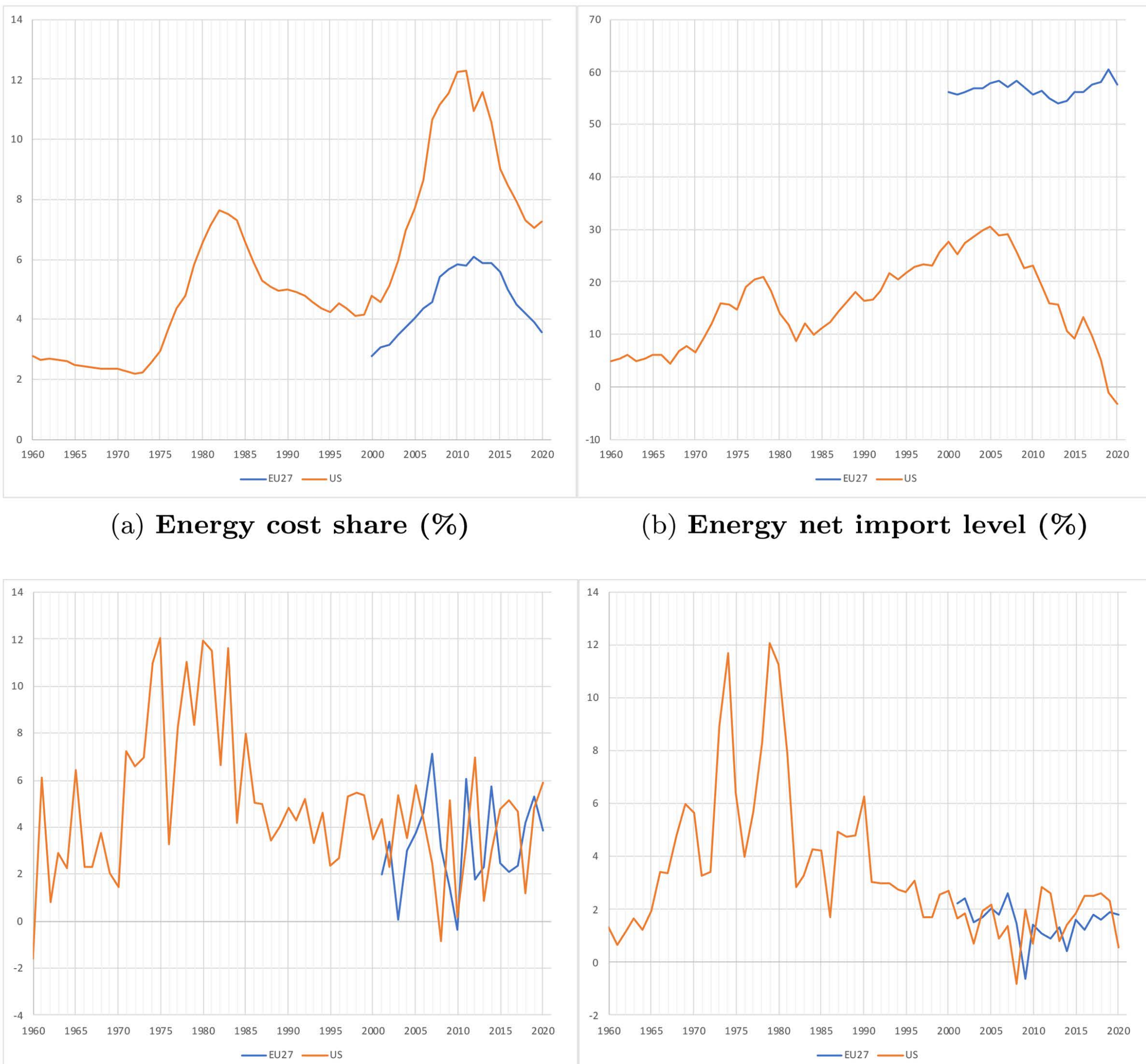
The GDP decomposition that the model implies clearly shows that the key drivers of the economic growth are the primary energy volume and the energy efficiency rate, whereas the prime source of an economic recession (in real terms) is the ex-energy inflation rate.

The main difference between the different economies is in their ability to turn around their energy efficiency rate α factored by $f_3 = +(1-c(1-\beta))$ into growth. From this point of view, FR has the lowest turning factor combined with a below-average efficiency rate, whereas, IT has the highest turning factor (thanks to their very low energy cost share) however combined with the lowest efficiency rate. In the end, the two countries exhibit below-average growth rates over the period.

4.2. Current Situation Comparison

Obviously, the GDP growth decomposition is very dependent on the situation of an economy in terms of its energy fraction of expenditures c and its energy import dependency β. Based on the year 2020 situation, it is easy to derive the factor weights impacting the potential GDP growth of each economy. The same analytical exercise can be carried out for the US. Conveniently, longer time series are available that allows getting some interesting information.

Figure 23 illustrates the striking differences between the US and the EU27 situations. If the US had much higher energy cost shares than the EU27 (almost double) but similar efficiency rates and ex-energy inflation rates between 2000 and 2020, their competitive advantage has come from their ability to become independent from energy imports.



(a) **Energy cost share (%)** (b) **Energy net import level (%)**

(c) **Implied energy efficiency rate (%)** (d) **Implied ex-energy inflation rate (%)**

Figure 23. Key GDP energy components US vs. EU27 (%).

Based on the year 2020 situation, it is easy to derive the factor weights impacting the potential GDP growth of each economy (c.f. Table 4).

Table 4. GDP factor weights (%) 2020.

Factor	US	EU27	DE	FR	IT
f_0	+7.5	+1.5	+1.0	+2.2	+0.7
f_1	+0.2	−2.1	−1.7	−1.8	−2.1
f_2	−92.7	−96.4	−97.3	−96.1	−97.2
f_3	+92.5	98.5	+99.0	+97.8	+99.3
f_4	−7.3	−3.6	−2.7	−3.9	−2.8

The key factor $f_1 = -c\beta$ translates the sensitivity of the real growth with respect to changes in primary energy prices. Unlike the EU27 economy, the US economy is about immune to price spikes. In the EU27, the DE and FR economies are the better placed to mitigate the arrival of energy price spikes. The key factor $f_3 = +\left(1 - c\left(1 - \beta\right)\right)$ translates the sensitivity of the real growth with respect to the energy efficiency levels. In other words, it measures the ability of an economy to capitalise on the energy process improvement by transforming any efficiency gain into secondary goods and services. In that respect, clearly, the EU27 economy is much better placed than the US one and the DE and IT economies are better placed than the FR one.

The two terms c and β are detrimental to economic growth. With a low β, the US are in a good shape. With a high c, caused by high volumes of energy consumption, some of their advantage is wasted. With a high β, the EU27 are in an economic conundrum. With a low c, they improve somewhat their situation. Having said that, is there still some margin for the EU27 to lower their c and β factors? The c factor of the DE and IT economies is already at a remarkably low level. The β factor of DE, FR and IT seems to be still out of their hands, at least, in the short term.

The model also says that energy efficiency rate α is a variable that is a key source of growth. This last factor also deserves a special attention, since it will probably shape the medium-term future of the EU27: either a successful economic model based on energy sobriety or a possible collapse. From this point of view, it seems that the FR and IT economies are still lagging behind DE. Combining these factors, the economy that currently seems to be the more exposed and the least capable to respond to an energy crisis is IT.

As far as the US are concerned, between 1960 and 2020, this factor exhibited a high variability of 3% per year, given the diversity of its origins, however with clear cycles and trends (c.f. Figure 23(c)). A substantial improvement following the oil shocks of the 1970s took place with the productivity then increasing from about 3% per annum to more than 10%. It was followed by a return to a stable regime for about 30 years, with an efficiency rate of around 3% - 4% per annum. Things must have been very similar in the EU27.

5. Conclusion

In the proposed model, economic production is seen as energy transformation. Using actual observations as well as reasonable approximations, namely, production growths, energy volume variations and energy price variations, it becomes easy to imply the unknown overall productivity factor of the production system, which, in the context, can be interpreted as an energy efficiency rate. The analysis of the past pattern of the energy efficiency rate shows a certain ability of the analysed economies to adjust to changes in energy market shocks. However, under potential primary energy price spikes and volume shortages, unlike the US, the production structure of the EU27 and DE, FR and IT alike could suffer a major shock, not on one growth factor but on three: primary energy inflation, secondary ex-energy inflation and access to primary energy volume. Amongst the analysed countries and regions, the economy having the highest exposure to a sudden energy dryout or major energy crisis would be IT.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

A. Nominal and Real GDP Growths

A.1. Output-to-Input Product Change

Since $X''(t)=\gamma(t)X'(t)(1-\eta(t))$, by simple total time derivation,

$$\mathrm{d}X''(t)=\mathrm{d}\gamma(t)X'(t)(1-\eta(t))+\gamma(t)\mathrm{d}X'(t)(1-\eta(t))-\gamma(t)X'(t)\mathrm{d}\eta(t).$$

Now, using the dynamics of the energy lever (c.f. Section 2.2-4), namely, $\mathrm{d}\gamma(t)=\alpha(t)\gamma(t)\mathrm{d}t$, this can be written

$$\mathrm{d}X''(t)=\alpha(t)\gamma(t)X'(t)(1-\eta(t))\mathrm{d}t+\gamma(t)\mathrm{d}X'(t)(1-\eta(t))-\gamma(t)X'(t)\mathrm{d}\eta(t).$$

Dividing by $\mathrm{d}X'(t)$ we get

$$\frac{\mathrm{d}X''(t)}{\mathrm{d}X'(t)}=\alpha(t)\gamma(t)X'(t)(1-\eta(t))\frac{\mathrm{d}t}{\mathrm{d}X'(t)}+\gamma(t)(1-\eta(t))-\gamma(t)X'(t)\frac{\mathrm{d}\eta(t)}{\mathrm{d}X'(t)}.$$

From the zero relative change of the secondary sector energy fraction to the energy supply (c.f. Section 2.2-8), that is, $\mathrm{d}\eta(t)/\mathrm{d}X'(t)=0$, we have the simpler formulation

$$\frac{\mathrm{d}X''(t)}{\mathrm{d}X'(t)}=\gamma(t)(1-\eta(t))\left(\alpha(t)X'(t)\frac{\mathrm{d}t}{\mathrm{d}X'(t)}+1\right).$$

Finally, from the definition of $X''(t)$, using that $X''(t)/X'(t)=\gamma(t)(1-\eta(t))$, we obtain

$$\frac{\mathrm{d}X''(t)}{\mathrm{d}X'(t)}=\alpha(t)X''(t)\frac{\mathrm{d}t}{\mathrm{d}X'(t)}+\frac{X''(t)}{X'(t)}.$$

A.2. Nominal GDP Change

The nominal GDP growth is derived starting from the GDP definition (c.f. Sections 2.2-1, 2.2-2, 2.2-3)

$$Y_n(t)=P'(t)X'(t)(1-\beta(t))+P''(t)X''(t)$$

and the relation

$$\frac{\mathrm{d}Y_n(t)}{Y_n(t)}=\frac{\mathrm{d}Y_n(t)}{\mathrm{d}X'(t)}\frac{\mathrm{d}X'(t)}{Y_n(t)}.$$

We then obtain

$$\frac{\mathrm{d}Y_n(t)}{Y_n(t)}=\left[\left(\frac{\mathrm{d}P'(t)}{\mathrm{d}X'(t)}X'(t)(1-\beta(t))+P'(t)(1-\beta(t))-P'(t)X'(t)\frac{\mathrm{d}\beta(t)}{\mathrm{d}X'(t)}\right)+\frac{\mathrm{d}P''(t)}{\mathrm{d}X'(t)}X''(t)+P''(t)\frac{\mathrm{d}X''(t)}{\mathrm{d}X'(t)}\right]\frac{\mathrm{d}X'(t)}{Y_n(t)}.$$

Therefore, using the assumption that $\mathrm{d}P''(t)/\mathrm{d}X'(t)=0$ (c.f. Section 2.2-8), replacing $\mathrm{d}X''(t)/\mathrm{d}X'(t)$ by its value, as posited in Section 2.2-9, and factoring in the term $\mathrm{d}X'(t)/Y_n(t)$,

$$\begin{aligned}\frac{\mathrm{d}Y_n(t)}{Y_n(t)} = &\left(\frac{\mathrm{d}X'(t)}{Y_n(t)}\frac{\mathrm{d}P'(t)}{\mathrm{d}X'(t)}X'(t)(1-\beta(t)) + \frac{\mathrm{d}X'(t)}{Y_n(t)}P'(t)(1-\beta(t))\right.\\ &\left. - \frac{\mathrm{d}X'(t)}{Y_n(t)}P'(t)X'(t)\frac{\mathrm{d}\beta(t)}{\mathrm{d}X'(t)}\right)\\ &+ \frac{\mathrm{d}X'(t)}{Y_n(t)}P''(t)\left(\alpha(t)X''(t)\frac{\mathrm{d}t}{\mathrm{d}X'(t)} + \frac{X''(t)}{X'(t)}\right).\end{aligned}$$

Now, using the definition given for the cost share $c(t)$ of the primary sector and for the cost share $(1-c(t)(1-\beta(t)))$ of the secondary sector, as posited in Section 2.2-5, and simplifying, we have

$$\begin{aligned}\frac{\mathrm{d}Y_n(t)}{Y_n(t)} = &\left(c(t)(1-\beta(t))\left(\frac{\mathrm{d}P'(t)}{P'(t)} + \frac{\mathrm{d}X'(t)}{X'(t)}\right) - c(t)\mathrm{d}\beta(t)\right)\\ &+ (1-c(t)(1-\beta(t)))\left(\alpha(t)\mathrm{d}t + \frac{\mathrm{d}X'(t)}{X'(t)}\right),\end{aligned}$$

That is, after re-ordering,

$$\begin{aligned}\frac{\mathrm{d}Y_n(t)}{Y_n(t)} = &\frac{\mathrm{d}X'(t)}{X'(t)} + c(t)(1-\beta(t))\frac{\mathrm{d}P'(t)}{P'(t)}\\ &- c(t)\mathrm{d}\beta(t) + (1-c(t)(1-\beta(t)))\alpha(t)\mathrm{d}t.\end{aligned}$$

A.3. Real GDP Growth

From its definition (c.f. Section 2.2-7)

$$\frac{\mathrm{d}Y_r(t)}{Y_r(t)} = \frac{\mathrm{d}Y_n(t)}{Y_n(t)} - c(t)\frac{\mathrm{d}P'(t)}{P'(t)} - (1-c(t))i''(t)\mathrm{d}t,$$

The real GDP growth is easily derived from the nominal GDP growth formulation as

$$\begin{aligned}\frac{\mathrm{d}Y_r(t)}{Y_r(t)} = &\frac{\mathrm{d}X'(t)}{X'(t)} + c(t)(1-\beta(t))\frac{\mathrm{d}P'(t)}{P'(t)} - c(t)\mathrm{d}\beta(t)\\ &+ (1-c(t)(1-\beta(t)))\alpha(t)\mathrm{d}t - c(t)\frac{\mathrm{d}P'(t)}{P'(t)} - (1-c(t))i''(t)\mathrm{d}t,\end{aligned}$$

which, after re-ordering, yields

$$\begin{aligned}\frac{\mathrm{d}Y_r(t)}{Y_r(t)} = &\frac{\mathrm{d}X'(t)}{X'(t)} - c(t)\beta(t)\frac{\mathrm{d}P'(t)}{P'(t)} - (1-c(t))i''(t)\mathrm{d}t\\ &- c(t)d\beta(t) + (1-c(t)(1-\beta(t)))\alpha(t)\mathrm{d}t.\end{aligned}$$

A.4. Producer Formulation

A more detailed analysis could be carried out by looking through the various primary energy and secondary goods and services sub-sectors or producers.

Let $X'(t)$ be defined as the sum of the energy supplies across the energy sub-sectors or producers (coal, oil, gas, electricity etc.) $X'_e(t)$, that is $X'(t)=\sum_e X'_e(t)$, and the total imported part $\beta(t)$, as the average imported part per energy sub-sector or producer $\beta_e(t)$, from the relation $X'(t)\beta(t)=\sum_e X'_e(t)\beta_e(t)$. Moreover, let $X''(t)$ be defined as the sum of the volumes of added value across the goods and services sub-sectors or producers $X''_k(t)$, that is $X''(t)=\sum_k X''_k(t)$.

The total average primary energy price $P'(t)$ of the total primary sector nominal production $Y'_n(t)$ is then defined from the price per energy sub-sector or producer $P'_e(t)$, from the relation $P'(t)X'(t)=\sum_e P'_e(t)X'_e(t)$, and the total average secondary added value price $P''(t)$ of the total secondary sector nominal production $Y''_n(t)$ is then defined from the price per goods and services sub-sector or producer $P''_k(t)$, from the relation $P''(t)X''(t)=\sum_k P''_k(t)X''_k(t)$.

The production of the secondary volumes of goods and services for the *k*-th sub-sector or producer is in turn defined from the fraction $\phi_k(t)$ of the total energy supply used, with $0\le\phi_k(t)\le 1$ and $\sum_k \phi_k(t)=1$, and for a specific energy-leveraging function $\gamma_k(t)$ leading to an output $X''_k(t)$ of the form

$$X''_k(t)=\gamma_k(t)X'(t)\phi_k(t)(1-\eta(t)).$$

We add the assumptions that $\mathrm{d}P''_k(t)/\mathrm{d}X'(t)=0$ and $\mathrm{d}\phi_k(t)/\mathrm{d}X'(t)=0$. It is consistent with the original assumptions that $\mathrm{d}P''(t)/\mathrm{d}X'(t)=0$ and $\mathrm{d}\eta(t)/\mathrm{d}X'(t)=0$. It says that the secondary prices $P''_k(t)$ made and energy proportions $\phi_k(t)$ used by individual sub-sectors or producers do not change (at least markedly on the short term), when the volume of energy supply $X'(t)$ changes.

If we postulate an energy efficient rate per production unit of $\alpha_k(t)$, that is a dynamics of the energy lever $\mathrm{d}\gamma_k(t)=\alpha(t)\gamma_k(t)\mathrm{d}t$, then, we have

$$\begin{aligned}\mathrm{d}X''_k(t)=&\,\alpha(t)\gamma_k(t)\phi_k(t)(1-\eta(t))X'_k(t)\mathrm{d}t\\&+\gamma_k(t)\mathrm{d}\phi_k(t)(1-\eta(t))X'_k(t)-\gamma_k(t)\mathrm{d}\eta(t)X'_k(t)\\&+\gamma_k(t)\phi_k(t)(1-\eta(t))\mathrm{d}X'_k(t).\end{aligned}$$

Then, we can divide by $\mathrm{d}X'(t)$ and obtain

$$\frac{\mathrm{d}X''_k(t)}{\mathrm{d}X'(t)}=\alpha_k(t)X''_k(t)\frac{\mathrm{d}t}{\mathrm{d}X'(t)}+\frac{X''_k(t)}{X'(t)}.$$

The partial growth of the secondary sector to the total GDP,

$$\frac{\mathrm{d}Y''_n(t)}{Y_n(t)}=\frac{\mathrm{d}Y''_n(t)}{\mathrm{d}X'(t)}\frac{\mathrm{d}X'(t)}{Y_n(t)},$$

can be written

$$\frac{\mathrm{d}Y_n''(t)}{Y_n(t)} = \sum_k \left(\frac{\mathrm{d}P_k''(t)}{\mathrm{d}X'(t)} X_k''(t) + P_k''(t) \frac{\mathrm{d}X_k''(t)}{\mathrm{d}X'(t)} \right) \frac{\mathrm{d}X'(t)}{Y_n(t)}$$
$$= \sum_k P_k''(t) \left(\alpha_k(t) X_k''(t) \frac{\mathrm{d}t}{\mathrm{d}X'(t)} + \frac{X_k''(t)}{X'(t)} \right) \frac{\mathrm{d}X'(t)}{Y_n(t)},$$

after using the assumption that $\mathrm{d}P_k''(t)/\mathrm{d}X'(t) = 0$ and replacing $\mathrm{d}X_k''(t)/\mathrm{d}X'(t)$ by its value. Now, we can also define the price-volume-weighted average energy efficiency rate $\alpha(t)$ by

$$\alpha(t) = \frac{\sum_k P_k''(t) X_k''(t) \alpha_k(t)}{\sum_k P_k''(t) X_k''(t)}.$$

Whence, using the definition given for the cost share $(1 - c(t)(1 - \beta(t)))$ of the secondary sector, we finally have

$$\frac{\mathrm{d}Y_n''(t)}{Y_n(t)} = (1 - c(t)(1 - \beta(t))) \left(\frac{\mathrm{d}X'(t)}{X'(t)} + \alpha(t) \mathrm{d}t \right).$$

Furthermore, we can split the cost share per energy sub-sector or producer $c_e(t)$, with $c_e(t) = P_e'(t) X_e'(t)/Y_n(t)$ and $c(t) = \sum_e c_e(t)$. Then, the partial growth of the primary sector to the total GDP,

$$\frac{dY_n'(t)}{Y_n(t)} = \frac{dY_n'(t)}{\mathrm{d}X'(t)} \frac{\mathrm{d}X'(t)}{Y_n(t)},$$

can be written

$$\frac{\mathrm{d}Y_n''(t)}{Y_n(t)} = \sum_e \left(\frac{\mathrm{d}P_e'(t)}{\mathrm{d}X'(t)} X_e'(t)(1 - \beta_e(t)) + P_e'(t) \frac{\mathrm{d}X_e'(t)}{\mathrm{d}X'(t)} (1 - \beta_e(t)) \right.$$
$$\left. - P_e'(t) X_e'(t) \frac{d\beta_e(t)}{\mathrm{d}X'(t)} \right) \frac{\mathrm{d}X'(t)}{Y_n(t)}.$$

Using the definition given for the cost share $c_e(t)$ of the secondary sub-sectors, we finally have

$$\frac{\mathrm{d}Y_n''(t)}{Y_n(t)} = \sum_e \left(c_e(t)(1 - \beta_e(t)) \left(\frac{\mathrm{d}P_e'(t)}{P_e'(t)} + \frac{\mathrm{d}X_e'(t)}{X_e'(t)} \right) - c_e(t) \mathrm{d}\beta_e(t) \right).$$

Therefore, the total nominal GDP growth is

$$\frac{\mathrm{d}Y_n(t)}{Y_n(t)} = \sum_e \left(c_e(t)(1 - \beta_e(t)) \left(\frac{\mathrm{d}P_e'(t)}{P_e'(t)} + \frac{\mathrm{d}X_e'(t)}{X_e'(t)} \right) - c_e(t) \mathrm{d}\beta_e(t) \right)$$
$$+ (1 - c(t)(1 - \beta(t))) \left(\frac{\mathrm{d}X'(t)}{X'(t)} + \alpha(t) \mathrm{d}t \right)$$
$$= \frac{\mathrm{d}X'(t)}{X'(t)} + \sum_e \left(c_e(t)(1 - \beta_e(t)) \frac{\mathrm{d}X_e'(t)}{X_e'(t)} \right) - c(t)(1 - \beta(t)) \frac{\mathrm{d}X'(t)}{X'(t)}$$
$$+ \sum_e \left(c_e(t)(1 - \beta_e(t)) \frac{\mathrm{d}P_e'(t)}{P_e'(t)} - c_e(t) \mathrm{d}\beta_e(t) \right)$$
$$+ (1 - c(t)(1 - \beta(t))) \alpha(t) \mathrm{d}t.$$

We could also crack down the energy volume mix by defining $v_e(t)=X'_e(t)/X'(t)$. Noticing that $\sum_e\left(c_e(t)(1-\beta_e(t))-v_e(t)c(t)(1-\beta(t))\right)=0$, by construction, it yields a fully decomposed formulation of the nominal growth equation in terms of energy volume and price by sub-sector or producer:

$$\frac{\mathrm{d}Y_n(t)}{Y_n(t)}=\sum_e v_e(t)\frac{\mathrm{d}X'_e(t)}{X'_e(t)}+\sum_e\left(c_e(t)(1-\beta_e(t))\frac{\mathrm{d}P'_e(t)}{P'_e(t)}-c_e(t)\mathrm{d}\beta_e(t)\right)+\left(1-c(t)(1-\beta(t))\right)\alpha(t)\mathrm{d}t.$$

It has a similar formulation as the two-sector only approach, provided a weighted average energy efficiency rate $\alpha(t)$ is suitably defined across the secondary sub-sectors. The particular advantage of this detailed formulation is that the energy inflation rate can be decomposed by energy sub-sector price in a more analytical way, with

$$\begin{aligned}i(t)&=c(t)i'(t)+(1-c(t))i''(t)\\&=\sum_e c_e(t)i'_e(t)+(1-c(t))i''(t)\end{aligned}$$

and

$$\frac{\mathrm{d}P'_e(t)}{P'_e(t)}=i'_e(t)\mathrm{d}t.$$

B. Energy Units and Conversions

There are several ways to approach the concept of energy in physics and, subsequently, several ways to measure it. Energy is essentially a measure of the amount of work that is required:

- to move an electric charge of one coulomb through an electric potential of one volt and measured in "coulomb-volt" (C·V);
- to accelerate a mass of one kilogram at one metre per squared second through a distance of one metre and measured in "kilogram-squared-metre per squared second" ($\mathrm{kg{\cdot}m^2/s^2}$);
- to displace a mass of one kilogram by a force of one newton through a distance of one metre and measured in "newton-metre" (N·m);
- to exert a pressure of one pascal in a volume of one cubic metre and measured in "pascal-cubic-metre" ($\mathrm{Pa{\cdot}m^3}$);
- to produce one watt of power for one second and measured in "watt-second" (W·s).

All these physical measurements are strictly equivalent between one another and equally relate to one unit of "joule" (J).

Now, in practice, the "watt-hour" is the energy unit that is commonly used in the production of electricity and the "British thermal unit" is an energy unit that uses the old Anglo-Norman avoirdupois system and is still commonly used in the trading of gas (c.f. Table A1). The commodity market also refers to many weight-to-energy or volume-to-energy equivalence units, in particular, the "ton oil-equivalent", "ton coal-equivalent" or the "barrel oil-equivalent", "cubic-metre natural-gas-equivalent", among others (c.f. Table A2).

Table A1. Standard energy units.

Physics	
Joule	J
Watt-hour	W·h
British thermal unit	BTU
Commodities	
Barrel oil-equivalent	bOE
Ton oil-equivalent	tOE
Ton coal-equivalent	tCE
Cubic-metre natural-gas-equivalent	m^3NGE

Table A2. Energy unit conversions.

	J	W·h	BTU	bOE	tOE	tCE	m^3NGE
J	1	2.777778×10^{-4}	9.470863×10^{-4}	1.633987×10^{-10}	2.388459×10^{-11}	3.412084×10^{-11}	2.653843×10^{-8}
W·h	3,600	1	3.409511	5.882353×10^{-7}	8.598452×10^{-8}	1.228350×10^{-7}	9.553836×10^{-5}
BTU	1,056	2.932972×10^{-1}	1	1.725278×10^{-7}	2.521902×10^{-8}	3.602717×10^{-8}	2.802114×10^{-5}
bOE	6,120,000,000	1,700,000	5,796,168	1	1.461737×10^{-1}	2.088196×10^{-1}	1.624152×10^{2}
tOE	41,868,000,000	11,630,000	39,652,609	6.8411765	1	1.428571	1.111111×10^{3}
tCE	29,307,600,000	8,141,000	27,756,826	12.4675329	0.70	1	7.777778×10^{2}
m^3NGE	37,681,200	10,467	35,687	0.0061570588	0.0009	0.00128571429	1

However, the commodity energy-equivalence conversions (in particular, ton oil-equivalent, ton coal-equivalent or the barrel oil-equivalent), critically depend on the weight and the heating value of the commodity in question.

For example, a ton coal-equivalent is the reference unit of a ton of a so-called "hard gas-flame coking coal" (the highest ranked type of so-called "bituminous coal"), whereas a ton of a so-called "hard anthracite coal" has more heating value than that of a ton coal-equivalent and a ton of crude lignite coal may have down to 0.19 times the heating value of a ton coal-equivalent. Table A3 reports the standard classification of coals according to their calorific value, volatile matter and agglomerating character, as established by ASTM[6] International. Note that caution must be exercised when analysing coal energy equivalence, because the coal ranking may overlap. The so-called "FOB Newcastle 6000 NAR" coal that is used as the benchmark for the analysis is ranked as a bituminous coal with a ca. 25.1 mega-joule per kilogram energy content.

[6]American Society for Testing and Materials.

Table A3. Coal weight-to-energy conversions

Rank	MJ/kg		tCE/t	
	Min	Max	Min	Max
Anthracite	30.0		1.02	
Bituminous	18.8	29.3	0.64	1.00
Sub-bituminous	8.3	25.0	0.28	0.85
Lignite	5.5	14.3	0.19	0.49
FOB Newcastle 6000 NAR	25.1		0.89	

Source: ASTM D288-19: standard classification of coals by rank.

Also, crude oil is generally traded and priced in dollar per barrel. A barrel is a unit of volume that is often converted into metric ton for convenience. The US IRS[7] have established a volume-to-weight (barrel-to-ton) and volume-to-energy (barrel-to-joule) conversion standard for a reference crude oil of a certain average quality and weight. However, the benchmark crude oils traded in the world (such as the so-called "Brent", "WTI"[8] or "Dubai") have different weights and qualities for the same given volume. Table A4 reports the volume-to-weight and weight-to-energy conversions that can be applied to these different oil types. The Brent crude oil that is mostly traded in Europe is used as the benchmark for the analysis.

Table A4. Oil volume-to-weight and weight-to-energy conversions.

Type	MBTU/b	MJ/b	t/b	MJ/t	tOE/t
US IRS	5.80	6120	6.84	41,868	1.00
Brent	5.87	6198	7.52	46,609	1.11
WTI	5.83	6156	7.33	45,121	1.08
Dubai	6.09	6430	7.20	46,298	1.11

Source: US IRS and BP plc.

Conveniently, natural gas is most often traded in dollar per British thermal unit, which leaves no uncertainty for conversion, apart from the euro-dollar exchange rate. The so-called "Amsterdam natural gas TTF"[9] contract is the main reference in Europe and is used as the benchmark for this analysis.

Likewise, electricity in Europe is priced in euro per watt-hour and the transformation into euro per ton oil-equivalent is unambiguous. However, the difficulty is to get a price that reflects the European market as a whole. The S&P Global Platts "pan-European" price index (PEP) selects various European electricity wholesale market exchanges[10]. The Platts PEP index was started in 2002

[7]United States Internal Revenue Service.

[8]Western Texas Intermediate.

[9]Title Transfer Facility.

[10]Nord Pool (DK, NO, SE, FI, EE, LV, LT and DE), European Energy Exchange Group (EU), Energy Exchange Austria (AT), Belpex (BE), Operátor trhu s Elektřinou (CZ), Mercado de Electricidad (ES), Powernext (FR), Anexartètos Diaxeiristès Metaphoras Elektrikès Energeias (GR), Hungarian Power Exchange (HU), Gestore del Mercato Elettrico (IT), Amsterdam Power Exchange (NL), Towarowa Giełda Energii (PL), Societatea Operatorul Pietei de Energie Electrică si Gaze Naturale din România (RO) and, finally, to complement the continental market with the British one, Amsterdam Power Exchange UK (GB).

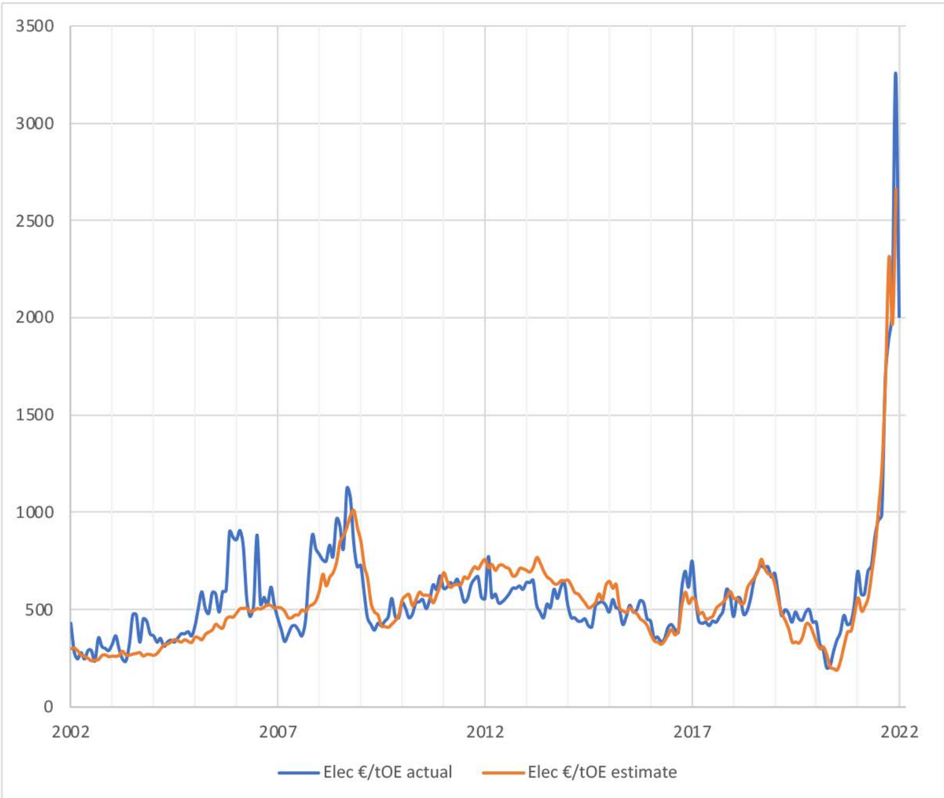
but has been discontinued since January 2017 and replaced by the "European Power Benchmark" index (EPB) calculated by the European Commission. These indices aim at representing the wholesale demand-weighted spot price on the so-called "day-ahead market".

To complete the index before 2002, we performed a linear regression of the electricity wholesale price against the coal, crude oil and gas prices. All the input prices are the monthly averages expressed in euro per ton oil-equivalent.

Table A5. Electricity vs. coal, oil and gas regression coefficients.

	Coal	Oil	Gas
Coefficient	1.8190	−0.1685	1.7219
t-stat	6.7480	−3.1620	20.9567
R2	0.9599		
F-stat/df[11]	1900.10	238	

The regression results are statistically and visually quite satisfactory (c.f. Table A5 and Figure A1).



Source: S&P Global Platts, European Commission, World Bank.

Figure A1. Electricity vs. coal, oil and gas regression estimates.

[11]Student t-statistics (t-stat), ratio of explained variation over total variation (R2), Fisher-Snedecor F-statistics (F-stat) and degrees of freedom (df).

C. Data analysis

C.1. Data Sources

Table A6 summarises the various sources for the time series that are used in the analysis.

Table A6. Data sources.

Series		Period	Source
GDP	EU27	12/2000-12/2020	Eurostat
	DE	12/2000-12/2020	Eurostat
	FR	12/2000-12/2020	Eurostat
	IT	12/2000-12/2020	Eurostat
Inflation	EU27	12/2001-12/2020	Eurostat
	EA[12]	12/1999-11/2001	Eurostat
	DE	12/1999-12/2020	Eurostat
	FR	12/1999-12/2020	Eurostat
	IT	12/1999-12/2020	Eurostat
Gross energy supply	EU27	12/2000-12/2020	Eurostat
	DE	12/2000-12/2020	Eurostat
	FR	12/2000-12/2020	Eurostat
	IT	12/2000-12/2020	Eurostat
Gross energy import	EU27	12/2000-12/2020	Eurostat
	DE	12/2000-12/2020	Eurostat
	FR	12/2000-12/2020	Eurostat
	IT	12/2000-12/2020	Eurostat
Net energy consumption	EU27	12/2000-12/2020	Eurostat
	DE	12/2000-12/2020	Eurostat
	FR	12/2000-12/2020	Eurostat
	IT	12/2000-12/2020	Eurostat
EUR/USD spot		01/1970-03/2021	Banque de France
Newcastle coal spot		01/1970-03/2021	World Bank
Brent oil spot		01/1970-03/2021	World Bank
Amsterdam natural gas spot		01/1970-03/2021	World Bank
Pan-European price		05/2002-01/2017	S&P Global Platts
European power benchmark		02/2002-03/2021	European Commission

C.2. Energy Supply Categories

Table A7 reports the broad energy supply categories, as per the SIEC nomenclature, and the price mapping that is used in the study to establish an approximation of the primary energy average price.

[12]Euro area (EA) with its 12 original members: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain.

Table A7. Energy supply categories and price mapping.

Price index	Energy category
Coal	Solid fossil fuels
	Peat and peat products
Oil	Oil shale and sands
	Oil and petroleum products
Gas	Natural gas
Electricity	Renewables and biofuels
	Non-renewable waste
	Nuclear heat
	Heat
	Electricity