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Amortization System in the Simple Interest Regime: Comparison between Two Proposals in the Case of Constant Installments

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Abstract

This article aims to compare methodologies for amortization systems with constant installments (French system), in simple interest, developed in Brazil (Forger and Lachtermacher & de Faro) and in Italy (Mari & Aretusi and Annibali et al.). This work considers the focal dates at the beginning and end of financing. In all cases, the methodologies proved to be financially consistent. At the end of the work, a study on possible tax gains for financing companies is carried out.

Keywords

Constant Installment Amortization Systems, Simple Interest Amortization Systems, Brazilian and Italian Amortizations Systems

1. Introduction

Motivated by an interpretation of an observation present in the classic treatise by Price (1772), Forger (2009) proposed a set of algorithms for repaying loans under the simple interest regime.

More recently, Mari and Aretusi (2018) also proposed procedures for the case of implementing the simple interest regime. Since, as explicitly expressed by Mari and Aretusi (2019), the purpose was to avoid the occurrence of anatocism, charging interest on interest.

Disregarding the issue of the occurrence, or not, of anatocism in the compound interest regime, which is a controversial issue in Brazil, see Puccini (2023) and De-Losso and Santos (2023), as well in the Italian literature, see Annibali et al. (2020), our objective is to compare the two distinct propositions, for the case of the constant payment system.

Subsidiarily, bearing in mind the concept of financial consistency, as proposed by de Faro (2014), and discussed in Lachtermacher and de Faro (2023), the issue relating to the calculation of the outstanding balance will also be addressed. This is essential in the case of early debt settlement.

2. An Idiosyncrasy of the Simple Interest Regime: The Concept of Focal Date

Consider a loan with value *F*, which must be amortized through *n* periodic installments. With the k^{th} installment being, generically, denoted by P_k, with $k = 1, 2, \cdots, n$.

In the case of adopting the compound interest regime, at the periodic rate *i*, the specification of the date of comparison between the value *F* and the sequence of *n* periodic payments, known as the focal date, see Ayres (1963), is not relevant. Because, whatever the focal date specified, the same condition of financial consistency will always be satisfied between the financing amount and the sequence of periodic payments.

On the other hand, if rate *i* is in simple interest regime, the choice of the focal date is essential. Because different focal dates lead to different results.

In what follows, as they appear to be the most relevant, we will consider two focal dates. The first, which appears to be the most natural, is the date on which the financing was granted. Date zero. This, according to Mari and Aretusi (2018), is the one that should be considered. Furthermore, as observed in De-Losso et al. (2020), it is what follows from the provisions of paragraph 1 of article 15-B of Brazilian Law 4380/64.

The second is the date of the last payment. Date *n*. With the latter being the proposal in the case of constant payment, see Nogueira (2013). And, in the case of constant amortization, see Rovina (2009) and Forger (2009), as well as in the case of the Italian literature, by Annibali et al. (2016, 2020).

3. Forger's Proposition

Forger (2009) stipulates that the *F* value of the loan must be divided into two distinct components. One, called capitalizable and denoted as F^C, and the other, called non-capitalizable, being denoted as F^N. That is:

$$F = F^C + F^N \tag{1}$$

Denoting as S_k the outstanding balance at time *k*, immediately after the payment of P_k, and A_k the amortization component at time *k*, which makes up the installment P_k, it is assumed that $S_k = S_k^C + S_k^N$, $P_k = P_k^C + P_k^N$, and $A_k = A_k^C + A_k^N$ for $k = 1, 2, \cdots, n$. Furthermore, also for $k = 1, 2, \cdots, n$, it is established that:

$$S_k^C = S_{k-1}^C - A_k^C = S_{k-1}^C - P_k^C \Leftrightarrow A_k^C = P_k^C \tag{2}$$

$$S_k^N = S_{k-1}^N - A_k^N = S_{k-1}^N + J_k - P_k^N \Leftrightarrow A_k^N = P_k^N - J_k \tag{3}$$

with the simple interest rate i being levied only on the capitalizable balance S_k^C. In other words, it is assumed that:

$$J_k = i \times S_{k-1}^C \tag{4}$$

where J_k denotes the interest component of the payment P_k. It should be noted that it is not subdivided.

At time *zero*, where the loan is granted, the introduction of a weighting factor f is considered, with $0 \le f \le 1$, such that:

$$F = F^C + F^N \Leftrightarrow S_0 = S_0^C + S_0^N \tag{5}$$

with $S_0^C = F \times f$, $S_0^N = F \times (1 - f)$ and $F = S_0$.

Additionally, as the capitalizable debt balance is supposed to decrease linearly, from its initial value $S_0^C = F \times f$, to the final value $S_n^C = 0$, and as, regardless of the particular amortization system that is adopted, it is established that $P^C = P_k^C = A_k^C = A^C$, whatever k, we have that:

$$P^C = A^C = \frac{F}{n} \times f \tag{6}$$

Consequently, using the recursion given by (2), we obtain:

$$S_k^C = F \times f - k \times P^C = F \times f \times \left(\frac{n-k}{n}\right) \tag{7}$$

Therefore, considering relationship (4), it follows that:

$$J_k = S_{k-1}^C \times i = F \times f \times i \times \left(\frac{n-k+1}{n}\right) \tag{8}$$

As we are focusing here on the case of constant payment, when $P_k = P = P^C + P^N$, with $P^C = C \times f / n$, for $k = 1, 2, \cdots, n$, considering, recursively, relation (3) and, as demonstrated in de Faro and Lachtermacher (2023a), we have:

$$S_k^N = F \times (1 - f) - \sum_{\ell=1}^{k} A_\ell^N \tag{9}$$

$$A_k^N = P^N - F \times f \times i + (k-1) \times i \times P^C \tag{10}$$

$$P^N = \left(\frac{F}{n}\right) \times \left\{1 - f + \frac{f \times i \times (n+1)}{2}\right\} \tag{11}$$

with value of the weighting factor f depending on the choice of the focal date.

As a numerical illustration, consider the case of F = 120000.00 units of capital, n = 12 and the periodic rate i of simple interest is established at 1% per period.

3.1. Focal Date in Time *Zero*

In this case, the financial equivalence between the value F of the loan and the sequence of n periodic payments implies that:

$$F=\sum_{k=1}^{n}\frac{P_k}{1+i\times k} \tag{12}$$

It follows that, in the case of a constant payment denoted by *P*, we have:

$$P=F\times\sum_{k=1}^{n}\left(1+i\times k\right)^{-1} \tag{13}$$

Therefore, in the case of our numerical example, we will have $P=10638.80$. Thus, making use of relations (6) and (11), it follows from the methodology presented in Lachtermacher and de Faro (2023), that the corresponding value of *f* is 0.92277415.

We will have the evolution of the debt as shown in **Table 1**.

Table 1. Evolution of the debt according to Forger, focal date time *zero*.

k	P	A_k	J_k	S_k
0	-		-	120000.00
1	10638.80	9459.48	1179.33	110540.52
2	10638.80	9557.75	1081.05	100982.77
3	10638.80	9656.03	982.77	91326.74
4	10638.80	9754.31	884.49	81572.43
5	10638.80	9852.58	786.22	71719.85
6	10638.80	9950.86	687.94	61768.99
7	10638.80	10049.14	589.66	51719.85
8	10638.80	10147.42	491.39	41572.43
9	10638.80	10245.69	393.11	31326.74
10	10638.80	10343.97	294.83	20982.77
11	10638.80	10442.25	196.55	10540.52
12	10638.80	10540.52	98.28	0.00
$\sum$	127665.60	120000.00	7665.60	

3.2. Focal Date on the Date of the Last Payment (Time *n*)

In this case, the financial equivalence between the value F of the loan and the sequence of periodic payments, with the K^{th} now being denoted by $\hat{P}_k$, implies that we have:

$$F\times\left(1+i\times n\right)=\sum_{k=1}^{n}\hat{P}_k\times\left\{1+i\times\left(n-k\right)\right\} \tag{14}$$

Therefore, in the case of a constant payment equal to $\hat{P}$, it follows that:

$$\hat{P}=\frac{F\times\left(1+i\times n\right)}{n\times\left\{1+\left[i\times\left(n-1\right)/2\right]\right\}} \tag{14'}$$

Therefore, since it is assumed that the constant payment is subdivided into components P^C and P^N, it follows from relations (6), (11) and (14'), that:

$$\frac{F\times(1+i\times n)}{n\times\left[1+i\times(n-1)/2\right]}=\frac{F\times f}{n}+\frac{F}{n}\times\left\{1-f+\frac{f\times i\times(n+1)}{2}\right\} \quad (15)$$

In other words, after some algebras, we have that the weighting factor f is such that:

$$f=\frac{1}{1+i\times(n-1)/2} \quad (16)$$

An expression that is also presented by Forger (2009).

Thus, in the case of our numerical example, observing that $\hat{P}=10616.61$ and $f=0.947867299$, Table 2 presents the evolution of the debt.

Table 2. Evolution of the debt according to Forger, focal date time n.

k	$\hat{P}$	$\hat{A}_k$	$\hat{J}_k$	$\hat{S}_k$
0				120000.00
1	10616.61	9478.67	1137.44	110521.33
2	10616.61	9573.46	1042.65	100947.87
3	10616.61	9668.25	947.87	91279.62
4	10616.61	9763.03	853.08	81516.59
5	10616.61	9857.82	758.29	71658.77
6	10616.61	9952.61	663.51	61706.16
7	10616.61	10047.39	568.72	51658.77
8	10616.61	10142.18	473.93	41516.59
9	10616.61	10236.97	379.15	31279.62
10	10616.61	10331.75	284.36	20947.87
11	10616.61	10426.54	189.57	10521.33
12	10616.61	10521.33	94.79	0.00
$\sum$	127393.36	120000.00	7393.36	

4. Propositions of Mari and Aretusi (2018) and Annibali et al. (2016, 2020)

In its original version, Mari and Aretusi (2018, 2019) focused their attention on the case of a focal date at time *zero*. Moreover, it is explicitly mentioned that this is the only one consistent with the simple interest regime.

As we do not agree with that statement, we will also address the case of focal at time n. As proposed by Annibali et al. (2016, 2020).

4.1. Focal Date in Time *Zero*—Mari and Aretusi (2018)

Denoting, now, to follow the presentation of Mari and Aretusi (2018), by M_k the outstanding balance at time k, and focusing attention on the case of a constant payment equal to P, as given by (13), Mari and Aretusi (2018) postulate that:

$$P=M_{k-1}-M_k+i\times M_{k-1}/\left\{1+i\times(k-1)\right\} \quad (17)$$

where

$$M_k = \left\{ F - P \times \sum_{\ell=1}^{k} (1 + i \times \ell)^{-1} \right\} \times (1 + i \times k) \tag{18}$$

for $k = 1, 2, \cdots, n$.

In other words, it is stipulated that the payment due at time k has the following two components:

$$C_k = M_{k-1} - M_k \tag{19}$$

and

$$I_k = i \times M_{k-1} / \{1 + i \times (k-1)\} \tag{20}$$

for $k = 1, 2, \cdots, n$. Respectively interpreted as the amortization component C_k and interest I_k component of the k^{th} payment.

With this notation, it follows that, in the case of our example, the corresponding evolution of the debt will evolve as shown in Table 3.

Comparing the results respectively presented in Table 1 and Table 3, two aspects must be highlighted.

The first concerns the fact that, effectively, the debt is paid off when the last payment is paid. And that the total of interest is the same in both cases.

The second, which is crucial for the analysis that will be done in Section 5, is that the sequence of differences between the corresponding interest sequences.

$$d_k = J_k - I_k, \ k = 1, 2, \cdots, n \tag{21}$$

presented in the last column of Table 3, has a single signal variation.

Table 3. Evolution of the debt according to Mari & Aretusi, focal date time *zero*.

k	P	C_k	I_k	M_k	d_k
0				120000.00	
1	10638.80	9438.80	1200.00	110561.20	−20.67
2	10638.80	9544.14	1094.67	101071.06	−13.62
3	10638.80	9648.44	990.36	91368.62	−7.59
4	10638.80	9751.73	887.07	81616.90	−2.58
5	10638.80	9854.02	784.78	71762.87	1.44
6	10638.80	9955.35	683.46	61807.53	4.48
7	10638.80	10055.71	583.09	51751.82	6.57
8	10638.80	10155.14	483.66	41592.68	7.72
9	10638.80	10253.65	385.15	31343.03	7.95
10	10638.80	10351.25	287.55	20991.78	7.28
11	10638.80	10447.97	190.83	10543.81	5.72
12	10638.80	10543.81	94.99	0.00	3.29
$\sum$	127665.60	120000.00	7665.60		0.00

4.2. Focal Date in Time *n*

Although also considered in Mari and Aretusi (2018, 2019), we are going to follow the presentation in Annibali et al. (2016, 2020).

In this case, the value of the constant installment $\hat{P}$ is the same as given by (14'). That is:

$$\hat{P} = \frac{F(1+i\times n)}{n\times\left(1+i\times\frac{n-1}{2}\right)} \tag{22}$$

with

$$\hat{M}_k = \hat{P}_k \times (n-k)\times\frac{1+i\times\frac{n-k-1}{2}}{1+i(n-k)} \tag{23}$$

where $\hat{M}_k$ denotes the outstanding balance at time *k*; for $k = 1, 2, \cdots, n$.

With the amortization, $\hat{C}_k$, and interest $\hat{I}_k$ components being, respectively:

$$\hat{C}_k = \hat{M}_{k-1} - \hat{M}_k \tag{24}$$

and

$$\hat{I}_k = \frac{i\times M_{k-1}}{1+i(n-k)} \tag{25}$$

for $k = 1, 2, \cdots, n$.

In Table 4, still relating to our numerical example, we have the corresponding evolution of the debt.

Table 4. Evolution of the debt according to Annibali et al., focal date time *n*.

k	$\hat{P}$	$\hat{C}_k$	$\hat{I}_k$	$\hat{M}_k$	$\hat{d}_k = \hat{J}_k - \hat{I}_k$
0				120.00000	
1	10.61611	9.53503	1.08108	110.46497	56.36
2	10.61611	9.61189	1.00423	100.85308	38.43
3	10.61611	9.69086	92526	91.16222	22.61
4	10.61611	9.77202	84409	81.39021	8.99
5	10.61611	9.85546	76066	71.53475	−2.36
6	10.61611	9.94126	67486	61.59349	−11.35
7	10.61611	10.02951	58660	51.56398	−17.88
8	10.61611	10.12031	49581	41.44367	−21.87
9	10.61611	10.21375	40237	31.22993	−23.22
10	10.61611	10.30994	30618	20.91999	−21.82
11	10.61611	10.40899	20713	10.51100	−17.56
12	10.61611	10.51100	10511	000	−10.32
$\sum$	127.39336	120.00000	7.39336		000

Similarly to the previous case, the debt is effectively redeemed with the payment of the last installment. And, also, that the total interest and installments are the same as those presented in Table 2.

Additionally, considering the corresponding interest sequences, we have the respective differences, given by the relationship:

$$\hat{d}_k = \hat{J}_k - \hat{I}_k,\ \ k = 1, 2, \cdots, n \tag{26}$$

also have a single signal variation.

5. Checking the Financial Consistency of the Models

In de Faro (2014), focusing on the compound interest regime, it was established that an amortization system is financially consistent if the determination of the outstanding balance, or debt status, over the financing term, presents the same results according to each of the three classic procedures. In other words, the results arising from the application of the three classic procedures, the prospective, recurrence, and retrospective methods, must coincide.

Extending the concept of using financial consistency to the case of using the simple interest regime, Lachtermacher and Faro (2023) showed that the three amortization systems proposed by Forger (2009), namely constant installment, constant amortization, and SACRE (increasing amortization system), are also financially consistent.

For the case under study, we will use the financial consistency verification methodology by calculating the outstanding balance for the period $k = 6$ for all amortization systems.

5.1. Forger Method Focal Date Time *Zero*

- Retrospective method:

$$S_k = F - \sum_{\ell=1}^{k} A_\ell$$

$$S_6 = 120000 - \sum_{\ell=1}^{6} A_\ell = 120000 - A_1 - A_2 - \cdots - A_6$$

$$S_6 = 120000 - [9459.48 + 9557.75 + 9656.03 + 9754.31 + 9852.58 + 9950.86]$$

$$= 120000 - 58231.01 = 61768.99$$

- Prospective method:

$$S_k = \sum_{\ell=k+1}^{n} (P_\ell - J_\ell) = \sum_{\ell=k+1}^{n} P_\ell - \sum_{\ell=k+1}^{n} J_\ell = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$

$$S_6 = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$

$$= (12-6) \times 10638.80 - [589.66 + 491.39 + 393.11 + 294.83 + 196.55 + 98.28]$$

$$S_6 = 63832.80 - 2063.82 = 61768.98$$

- Recurrence method:

$$S_k = S_{k-1} + J_k - P_k \Rightarrow S_k = S_{k-2} + J_{k-1} - P_{k-1} + J_k - P_k \text{, etc.}$$
$$S_k = S_0 + \sum_{\ell=1}^{k} J_\ell - \sum_{\ell=1}^{k} P_\ell$$
$$S_6 = S_0 + \sum_{\ell=1}^{6} J_\ell - \sum_{\ell=1}^{6} P_\ell = 120000 + 5601.80 - 63832.81 = 61768.99$$

5.2. Forger Method Focal Date Time *n*

- Retrospective method:

$$S_k = F - \sum_{\ell=1}^{k} A_\ell$$
$$S_6 = 120000 - \sum_{\ell=1}^{6} A_\ell = 120000 - A_1 - A_2 - \cdots - A_6$$
$$S_6 = 120000 - [9478.67 + 9573.46 + 9668.25 + 9763.03 + 9857.82 + 9952.61]$$
$$= 120000 - 58293.84 = 61706.16$$

- Prospective method:

$$S_k = \sum_{\ell=k+1}^{n} (P_\ell - J_\ell) = \sum_{\ell=k+1}^{n} P_\ell - \sum_{\ell=k+1}^{n} J_\ell = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$S_6 = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$= (12-6) \times 10616.114 - [568.72 + 473.93 + 379.15 + 284.36 + 189.57 + 94.79]$$
$$S_6 = 63696.68 - 1990.52 = 61706.16$$

- Recurrence method:

$$S_k = S_{k-1} + J_k - P_k \Rightarrow S_k = S_{k-2} + J_{k-1} - P_{k-1} + J_k - P_k$$
$$\Rightarrow S_k = S_0 + \sum_{\ell=1}^{k} J_\ell - \sum_{\ell=1}^{k} P_\ell$$
$$S_6 = S_0 + \sum_{\ell=1}^{6} J_\ell - \sum_{\ell=1}^{6} P_\ell = 120000 + 5402.84 - 63696.68 = 61706.16$$

5.3. Mari & Aretusi Method Focal Date Time *Zero*

- Retrospective method:

$$S_k = F - \sum_{\ell=1}^{k} A_\ell$$
$$S_6 = 120000 - \sum_{\ell=1}^{6} A_\ell = 120000 - A_1 - A_2 - \cdots - A_6$$
$$S_6 = 120000 - [9438.80 + 9544.14 + 9648.44 + 9751.73 + 9854.02 + 9955.35]$$
$$= 120000 - 58192.48 = 61807.52$$

- Prospective method:

$$S_k = \sum_{\ell=k+1}^{n} (P_\ell - J_\ell) = \sum_{\ell=k+1}^{n} P_\ell - \sum_{\ell=k+1}^{n} J_\ell = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$S_6 = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$= (12-6) \times 10638.80 - [583.09 + 483.66 + 385.15 + 287.55 + 190.93 + 94.99]$$
$$S_6 = 63832.81 - 2025.28 = 61807.53$$

- Recurrence method:

$$S_k = S_{k-1} + J_k - P_k \Rightarrow S_k = S_{k-2} + J_{k-1} - P_{k-1} + J_k - P_k$$
$$\Rightarrow S_k = S_0 + \sum_{\ell=1}^{k} J_\ell - \sum_{\ell=1}^{k} P_\ell$$
$$S_6 = S_0 + \sum_{\ell=1}^{6} J_\ell - \sum_{\ell=1}^{6} P_\ell = 120000 + 5640.34 - 63832.81 = 61807.53$$

5.4. Annibali et al. Method Focal Date Time *n*

- Retrospective method:

$$S_k = C - \sum_{\ell=1}^{k} A_\ell$$
$$S_6 = 120000 - \sum_{\ell=1}^{6} A_\ell = 120000 - A_1 - A_2 - \cdots - A_6$$
$$S_6 = 120000 - [9535.03 + 9611.89 + 9690.86 + 9772.02 + 9855.46 + 9941.26]$$
$$= 120000 - 58406.52 = 61.593.48$$

- Prospective method:

$$S_k = \sum_{\ell=k+1}^{n} (P_\ell - J_\ell) = \sum_{\ell=k+1}^{n} P_\ell - \sum_{\ell=k+1}^{n} J_\ell = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$S_6 = (n-k) \times P - \sum_{\ell=k+1}^{n} J_\ell$$
$$= (12-6) \times 10616.114 - [586.60 + 495.81 + 402.37 + 306.18 + 207.13 + 105.11]$$
$$S_6 = 63696.68 - 2103.20 = 61593.48$$

- Recurrence method:

$$S_k = S_{k-1} + J_k - P_k \Rightarrow S_k = S_{k-2} + J_{k-1} - P_{k-1} + J_k - P_k$$
$$\Rightarrow S_k = S_0 + \sum_{\ell=1}^{k} J_\ell - \sum_{\ell=1}^{k} P_\ell$$
$$S_6 = S_0 + \sum_{\ell=1}^{6} J_\ell - \sum_{\ell=1}^{6} P_\ell = 120000 + 5290.17 - 63696.68 = 61593.49$$

6. Choosing between the Procedures

At first glance, given that the two pair of procedures in question appear to be financially consistent, it could be likely that the choice between them would be a mere matter of taste.

However, considering the perspective of the financing entity, its opportunity cost must be considered.

Denoting ρ the periodic rate that identifies the opportunity cost for the financial institution, we must compare the present values of the corresponding sequences of interest.

6.1. Focal Date Time *Zero*—Forger and Mari & Aretusi

Denoting:

$$V_1(\rho) = \sum_{k=1}^{n} (1+\rho)^{-k} \times J_k \quad \text{Forger Method} \tag{27}$$

and

$$V_2(\rho) = \sum_{k=1}^{n} (1+\rho)^{-k} \times I_k \quad \text{Mari \& Aretusi Method} \tag{28}$$

the respective present values, at the periodic rate, ρ, of the corresponding interest parcels, the financing institution must choose the proposition that presents the lowest present value of the interest sequence.

Now, as can be seen from **Table 3**, considering the sequence $d_k = J_k - I_k$, we see that it presents a single signal variation. In other words, the sequence in question identifies a so-called conventional investment project. Which, according to de Faro (1974), has a single internal rate of return. Which, in this case, as $\sum_{k=1}^{n} J_k - \sum_{k=1}^{n} I_k = 0$, is null.

Consequently, which also happens in the general case of n periods, we have $V_1(\rho) < V_2(\rho)$, for $\rho > 0$. Therefore, the financial institution must choose to implement the procedure suggested by Forger (2009).

To assess the relevance of the differences between the two options, **Tables 5-8** present the behavior of the percentage tax gain:

$$\Delta = \left(\frac{V_1(\rho)}{V_2(\rho)} - 1 \right) \times 100 \tag{29}$$

for monthly rates i ranging from 0.5% to 2%, for terms n ranging from 5 to 30 years, with the interest rate that reflects the lender's opportunity cost, in annual terms and identified as ρ_a, ranging from 5% to 30%.

Table 5. Comparison of Forger and Mari & Aretusi—focal date time 0—i = 0.5% p.m.

Δ	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	−0.1755	−0.3370	−0.4858	−0.6231	−0.7500	−0.8674
10	−0.6062	−1.1413	−1.6124	−2.0271	−2.3924	−2.7150
15	−1.2023	−2.2156	−3.0638	−3.7726	−4.3664	−4.8665
20	−1.9081	−3.4390	−4.6537	−5.6175	−6.3878	−7.0102
25	−2.6867	−4.7341	−6.2733	−7.4363	−8.3285	−9.0259
30	−3.5122	−6.0507	−7.8612	−9.1701	−10.1405	−10.8804

Table 6. Comparison of Forger and Mari & Aretusi—focal date time 0—i = 1.0% p.m.

Δ	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	−0.3109	−0.5967	−0.8598	−1.1025	−1.3268	−1.5343

Continued

10	−0.9957	−1.8731	−2.6447	−3.3235	−3.9213	−4.4492
15	−1.8727	−3.4474	−4.7638	−5.8637	−6.7859	−7.5634
20	−2.8558	−5.1406	−6.9522	−8.3912	−9.5439	−10.4782
25	−3.8964	−6.8571	−9.0837	−10.7709	−12.0705	−13.0915
30	−4.9648	−8.5435	−11.1007	−12.9582	−14.3437	−15.4069

Table 7. Comparison of Forger and Mari & Aretusi—focal date time 0—i = 1.5% p.m.

Δ	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	−0.4199	−0.8058	−1.1609	−1.4883	−1.7907	−2.0706
10	−1.2760	−2.3990	−3.3860	−4.2537	−5.0179	−5.6926
15	−2.3212	−4.2700	−5.8979	−7.2581	−8.3989	−9.3616
20	−3.4575	−6.2191	−8.4078	−10.1477	−11.5435	−12.6768
25	−4.6348	−8.1506	−10.7955	−12.8032	−14.3536	−15.5752
30	−5.8241	−10.0161	−13.0149	−15.1994	−16.8348	−18.0946

Table 8. Comparison of Forger and Mari & Aretusi—focal date time 0—i = 2.0% p.m.

Δ	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	−0.5105	−0.9794	−1.4109	−1.8085	−2.1757	−2.5155
10	−1.4912	−2.8026	−3.9544	−4.9667	−5.8580	−6.6450
15	−2.6500	−4.8724	−6.7279	−8.2782	−9.5789	−10.6772
20	−3.8850	−6.9845	−9.4405	−11.3938	−12.9624	−14.2378
25	−5.1475	−9.0479	−11.9829	−14.2134	−15.9390	−17.3016
30	−6.4102	−11.0198	−14.3201	−16.7290	−18.5370	−19.9335

The values presented are significant and, as they are always negative, they indicate that the financing entity should always opt for the methodology recommended in Forger (2009) adapted by de Faro and Lachtermacher (2023b) since $V_1(\rho) < V_2(\rho)$, for $\rho > 0$ as shown in **Tables 5-8**.

6.2. Focal Date Time *n*—Forger and Annibali et al.

Denoting:

$$V_3(\rho) = \sum_{k=1}^{n} (1+\rho)^{-k} \times \hat{J}_k \quad \text{Forger Method} \tag{30}$$

and

$$V_4(\rho) = \sum_{k=1}^{n} (1+\rho)^{-k} \times \hat{I}_k \quad \text{Annibali et al. Method} \tag{31}$$

the respective present values, at the periodic rate ρ, of the corresponding interest payments, the financial institution must choose the proposition that presents the lowest present value of the interest sequence.

Similarly to the case of focal date at time *zero*, it appears that, as shown in Table 4, the sequence $\hat{d}_k = \hat{J}_k - \hat{I}_k$ also presents a single signal variation, identifying a conventional financing project. For which, as discussed in de Faro (1974), a single internal rate of return is associated. Which, in the case considered, as we also have $\sum_{k=1}^{n} \hat{J}_k = \sum_{k=1}^{n} \hat{I}_k$, the respective internal rate of return is zero.

Therefore, which also happens in the general case of n periods, it follows that $V_3(\rho) > V_4(\rho)$ for $\rho > 0$. Therefore, the financial institution must choose to implement the methodology proposed by Annibali et al. (2016).

Tables 9-12 show the behavior of the respective percentage tax gain:

$$\Delta' = \left(\frac{V_3(\rho)}{V_4(\rho)} - 1 \right) \times 100 \tag{32}$$

also, for monthly rates i ranging from 0.5% to 2%, for annual terms n ranging from 5 to 30 years, with the annual rate ρ_a ranging from 5% to 30%.

Table 9. Comparison of Forger and Annibali et al.—focal date time n—i = 0.5% p.m.

Δ'	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	0.5196	1.0004	1.4456	1.8583	2.2413	2.5971
10	1.7784	3.3772	4.8057	6.0774	7.2072	8.2107
15	3.5104	6.5692	9.1882	11.4080	13.2807	14.8604
20	5.5606	10.2392	14.0513	17.1072	19.5463	21.5002
25	7.8287	14.1611	19.0407	22.7281	25.5154	27.6476
30	10.2442	18.1741	23.9270	28.0242	30.9719	33.1430

Table 10. Comparison of Forger and Annibali et al.—focal date time n—i = 1.0% p.m.

Δ'	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	0.9090	1.7531	2.5373	3.2663	3.9445	4.5759
10	2.8651	5.4638	7.8022	9.8953	11.7626	13.4258
15	5.3427	10.0592	14.1334	17.6042	20.5389	23.0144
20	8.1127	15.0452	20.7385	25.3115	28.9538	31.8585
25	11.0503	20.1346	27.1688	32.4688	36.4466	39.4631
30	14.0768	25.1428	33.1733	38.8453	42.8794	45.8172

Table 11. Comparison of Forger and Annibali et al.—focal date time n—$i = 1.5\%$ p.m.

Δ'	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	1.2144	2.3452	3.3982	4.3792	5.2935	6.1463
10	3.6118	6.9064	9.8845	12.5589	14.9505	17.0839
15	6.4973	12.2756	17.2904	21.5722	25.1944	28.2472
20	9.6229	17.9106	24.7390	30.2222	34.5784	38.0390
25	12.8658	23.5201	31.7768	37.9765	42.6042	46.0925
30	16.1524	28.9273	38.1784	44.6707	49.2537	52.5683

Table 12. Comparison of Forger and Annibali et al.—focal date time n—$i = 2.0\%$ p.m.

Δ'	ρ_a (%)					
n (years)	5%	10%	15%	20%	25%	30%
5	1.4617	2.8258	4.0985	5.2860	6.3944	7.4296
10	4.1619	7.9734	11.4291	14.5393	17.3246	19.8114
15	7.3008	13.8243	19.5012	24.3538	28.4582	31.9137
20	10.6332	19.8331	27.4242	33.5146	38.3418	42.1649
25	14.0448	25.7209	34.7664	41.5380	46.5716	50.3495
30	17.4689	31.3245	41.3350	48.3264	53.2360	56.7706

The results presented, which are also significant, confirm that the financial entity must always opt for the methodology proposed by Annibali et al. (2016, 2020), since $V_3(\rho) > V_4(\rho)$ for $\rho > 0$ as shown in **Tables 9-12**.

7. Conclusion

In this article, we compare methodologies for debt amortization using a simple interest system developed in Brazil and Italy. Two different focal dates were studied due to the type of capitalization proposed in the methodologies. The Constant Installment Amortization Method (French Method) was developed by Forger (2009), and extended by de Faro and Lachtermacher (2023a), in Brazil, and by Mari and Aretusi (2018) and Annibali et al. (2016, 2020) in Italy.

All tested methods presented the same monthly installments, total interest, on both focal dates. However, the corresponding sequences of interest were shown to be different.

Considering the interest sequences on the focal date at time *zero*, the Forger methodology presented lower present values for all opportunity cost rates tested. Therefore, presenting fiscal gains over the Mari & Aretusi methodology should be chosen by the loan financier.

Considering the interest sequences on the focal date at time n, the Annibali et al. methodology presented lower present values for all opportunity cost rates tested.

Therefore, presenting fiscal gains over the Forger methodology, and should be chosen by the loan financier.

Future research should deepen these findings by comparing other types of amortization systems, such as the constant amortization system (SAC in Brazil and Italian-style amortization in Italy).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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