Social Networking, 2026, 15(2), 11-48
https://www.scirp.org/journal/sn
ISSN Online: 2169-3323
ISSN Print: 2169-3285

Research on a Social Network Group Decision-Making Method for Incomplete Probabilistic Linguistic Information

Miqi Gan, Gaili Xu

School of Mathematics and Statistics, Guilin University of Technology, Guilin, China
Email: gmq18878533942@163.com

How to cite this paper: Gan, M.Q. and Xu, G.L. (2026) Research on a Social Network Group Decision-Making Method for Incomplete Probabilistic Linguistic Information. *Social Networking*, **15**, 11-48.
https://doi.org/10.4236/sn.2026.152002

Received: April 7, 2026
Accepted: April 27, 2026
Published: April 30, 2026

Abstract

In social network group decision-making (SN-GDM), decision-makers usually employ probabilistic linguistic term sets (PLTSs) to express vague and hesitant evaluation information. However, owing to cognitive limitations, incomplete information, and the complexity of social trust relationships, the probabilistic linguistic assessments provided by decision-makers are often incomplete, which reduces the reliability of group evaluation aggregation and increases the difficulty and cost of consensus reaching. To address this issue, this paper proposes a social network group decision-making method for incomplete probabilistic linguistic information. First, a confidence level for each evaluation position is constructed by integrating social trust relationships and the certainty of evaluation information, and incomplete PLTSs are estimated on the basis of the DeGroot model so as to improve the completeness and reliability of individual decision matrices. Second, the comprehensive certainty of evaluation information and the individual-group average similarity are fused, and the Einstein t-norm is used to determine decision-maker weights; on this basis, a group decision matrix and multi-level consensus measures at the evaluation-position, decision-maker, and group levels are established. Furthermore, when group consensus fails to reach a preset threshold, regret theory is introduced to characterize the psychological adjustment pressure generated when decision-makers deviate from group opinion, and, together with confidence levels and information from trusted neighbors, a personalized consensus reaching mechanism containing identification and modification rules is constructed. Finally, the proposed method is validated through a case study on the selection of investment directions for Guangxi's low-altitude economy.

Keywords

Probabilistic Linguistic Term Set, Social Network Group Decision-Making, Incomplete Information, Consensus Reaching

1. Introduction

Against the backdrop of rapid digitalization, networking, and intelligent development, complex decision problems have come to exhibit such characteristics as diversified participants, vague decision information, and networked opinion interaction. As an important tool for integrating the knowledge, experience, and preferences of multiple actors, group decision-making has been widely applied in such fields as alternative selection [1], risk assessment [2], emergency management [3], and other fields. Compared with traditional individual decision making, group decision-making can synthesize the cognitive judgments and value orientations of different decision-makers and obtain more robust and reasonable decisions from multiple alternatives. Meanwhile, the widespread popularization of social media and network communication technologies means that relationships among decision-makers are no longer mutually independent; the formation and revision of individual judgments are often accompanied by trust dependence, opinion diffusion, and social influence [4] [5]. In this context, how to characterize the interactions among decision-makers in social networks and further analyze their effects on the process and outcomes of group decision-making has become an important issue worthy of attention in decision science.

In practical decision-making, decision-makers usually tend to express their preferences in linguistic terms. To represent such information, scholars have proposed various linguistic representation tools. Compared with traditional fuzzy sets [6], linguistic variables are more consistent with natural human expression, but a single linguistic term is insufficient to capture the hesitation involved in evaluations. To address this issue, Rodríguez *et al.* proposed hesitant fuzzy linguistic term sets (HFLTSs) [7], and Chen *et al.* further developed proportional hesitant fuzzy linguistic term sets [8]. However, the former cannot reflect differences in the importance of linguistic terms, while the latter requires the sum of probabilities to equal 1. Considering that information in real decision-making is often uncertain and incomplete, Pang *et al.* proposed probabilistic linguistic term sets (PLTSs) [9]. Since PLTSs allow the sum of probabilities to be less than or equal to 1, they can more flexibly characterize hesitation, uncertainty, and incompleteness. Therefore, this paper adopts PLTSs as the tool for representing decision-makers' linguistic evaluation information.

However, in practical decision-making, the evaluation information provided by decision-makers is often incomplete due to factors such as cognitive limitations and differences in information completeness [10]. This not only weakens the completeness and reliability of individual evaluation matrices, but also further affects the effectiveness of group evaluation aggregation, consensus measurement, and opinion feedback. Existing studies have explored missing-information completion from various perspectives. For example, Zhan *et al.* [11] estimated missing values by integrating trust degree and preference similarity based on K-nearest-neighbor information; Shen *et al.* [12] employed an improved DeGroot model to simulate the opinion formation process of decision-makers so as to obtain complete evaluation information; Zhong *et al.* [13] constructed an optimization model that maximizes reliability to complete missing probabilities. However, most existing methods mainly focus on recovering missing information from the evaluation data itself or with the aid of external numerical information, while paying relatively limited attention to social trust relationships among decision-makers, differences in evaluation stances, and individuals' acceptance of external information in a probabilistic linguistic term set environment. In fact, missing-information completion is not merely a data-repair problem, but should also comprehensively reflect the social relationship structure [11], the certainty of evaluation information [1], and decision-makers' self-maintenance tendencies [14]. Therefore, it is necessary to construct an integrated estimation mechanism for incomplete probabilistic linguistic term sets, so as to provide a more reliable information basis for subsequent group evaluation aggregation and consensus feedback.

When the consensus level falls below a predetermined threshold, individual preferences need to be revised through a feedback mechanism. In recent years, the introduction of social network analysis and bounded rationality theory has promoted the development of consensus reaching mechanisms. In the PLTS environment, some scholars have integrated prospect theory into the consensus reaching process and adjusted feedback on the basis of prospect values [15]. Meanwhile, regret theory, because of its ability to characterize the psychological feelings generated when decision-makers compare different choice outcomes, has gradually been introduced into the field of group decision-making. Zhu *et al.* [16] proposed a three-way consensus model based on regret theory within the PLTS framework, in which an adjustment optimization model was developed to balance consensus attainment and the preservation of individual independence. In social network contexts, Liu *et al.* [17] further combined social networks with regret theory and constructed a personalized consensus feedback strategy by analyzing decision-makers' regret psychology and social influence. In terms of optimizing consensus feedback mechanisms, both the social trust network consensus reaching process considering minimum adjustment cost [18] and the personalized consensus model based on dynamic trust and hesitancy [19] indicate that incorporating social networks can improve consensus efficiency. However, existing methods largely rely on homogeneous adjustment coefficients, failing to reflect differences among decision-makers in confidence levels, social trust, and psychological responses, while also overlooking the psychological motivation triggered by deviation from group opinion. To address this issue, this paper introduces confidence levels and regret theory into the consensus reaching process, internalizes the regret caused by deviation as the driving force for preference revision, and develops a differentiated adjustment model.

Although substantial achievements have been made in existing studies, several shortcomings remain to be further addressed in social network group decision-making with incomplete probabilistic linguistic information.

1) Many existing methods assume that decision information is complete and lack a systematic treatment of incomplete probabilistic linguistic evaluations that commonly arise in social-network environments, making it difficult to complete missing information reasonably and thereby affecting the accuracy of group evaluation aggregation and consensus measurement.

2) Most consensus-feedback mechanisms still regard decision-makers as independent and fully rational individuals and usually adopt uniform or preset adjustment parameters, making it difficult to fully reflect the joint effects of individual differences, social interaction, and behavioral psychological factors. In reality, whether an individual is willing to adjust his or her opinion is affected not only by the social relationship structure, but also by the confidence of the individual's evaluation and by psychological factors such as regret.

3) With respect to decision-maker weighting, some studies mainly rely on entropy indicators or network-structure information and have not simultaneously taken into account evaluation-information quality and the degree of coordination between individual opinions and the group, thereby limiting the rationality and explanatory power of weight allocation.

To overcome the above shortcomings, this paper proposes a social network group decision-making method that considers incomplete PLTS estimation together with confidence level and regret psychology. Using PLTSs as the information-expression tool, the method first constructs the confidence level of each evaluation position by combining social trust relationships with the certainty of evaluation information, and then completes incomplete probabilistic linguistic evaluations on the basis of the DeGroot model. Subsequently, it fuses the comprehensive certainty of evaluation information with the average similarity between the individual and the group and uses the Einstein t-norm to determine decision-maker weights, thereby constructing a group decision matrix and a multi-level consensus measurement system. When the group consensus level fails to reach the preset threshold, regret theory is introduced to characterize the psychological adjustment pressure caused by deviations from the group opinion, and a personalized consensus reaching mechanism is established by combining confidence levels with information from trusted neighbors, so as to improve the rationality and efficiency of consensus feedback as well as the reliability of the final decision results.

The main innovations of this paper are reflected in the following three aspects.

1) For incomplete probabilistic linguistic evaluation information, this paper constructs the confidence level of each evaluation position by integrating social trust relationships and the certainty of evaluation information, and estimates missing information on the basis of the DeGroot model, thereby enhancing the completeness and reliability of individual decision matrices.

2) The comprehensive certainty of evaluation information and the individual-group average similarity are fused, and the Einstein t-norm is employed to determine decision-maker weights. A multi-level consensus measurement system is then established at the evaluation-position, decision-maker, and group levels to characterize group opinion consistency more comprehensively.

3) Regret theory is incorporated into the consensus-feedback process in social networks, and a personalized consensus reaching mechanism that integrates confidence levels, degrees of deviation from group opinion, and information from trusted neighbors is constructed, making opinion adjustment more consistent with the interaction and evolution patterns observed in real social networks.

The remainder of this paper is organized as follows. Section 2 introduces the preliminary knowledge of probabilistic linguistic term sets, social networks, and regret theory. Section 3 proposes an estimation method for incomplete PLTSs based on the DeGroot model. Section 4 develops the group decision-making solution method and the confidence-level-regret-psychology-based consensus reaching mechanism. Section 5 verifies the effectiveness of the proposed method through a case study on the selection of investment directions for Guangxi's low-altitude economy and comparative analyses. Finally, conclusions and future research directions are presented.

2. Preliminaries

This section presents the preliminaries, including probabilistic linguistic term sets, social networks, and regret theory.

2.1. Probabilistic Linguistic Term Sets

Definition 1 [20]. Let $S=\{s_\alpha \mid \alpha=0,1,\cdots,\tau\}$ be a finite and ordered discrete linguistic term set, where s_α denotes a linguistic term used to represent one possible value of a linguistic variable; τ is a positive integer; and the cardinality of S is $\tau+1$.

To improve computational efficiency and reduce information loss, Wang *et al.* [21] proposed the extended linguistic term set $\bar{S}=\{s_\alpha \mid \alpha\in[0,\tau]\}$, and on this basis introduced the following two basic operational rules: 1) $s_\alpha \oplus s_{\alpha'}=s_{\alpha+\alpha'}$; 2) $\xi s_\alpha = s_{\xi\alpha}$, where $s_\alpha, s_{\alpha'}\in\bar{S}$, and $\xi\in[0,1]$.

Definition 2 [22]. Let $S=\{s_\alpha \mid \alpha=0,1,\cdots,\tau\}$ be a linguistic term set. The linguistic scale function $g(\cdot)$ is used to establish an equivalence relation between the linguistic term s_α and the membership degree μ on the interval [0, 1], and is defined as

$$g:[0,\tau]\to[0,1], g(s_\alpha)=\frac{\alpha}{\tau}=\mu \tag{1}$$

Correspondingly, for a given membership degree $\mu\in[0,1]$, the corresponding discrete linguistic term s_α can be obtained through the inverse mapping of the linguistic scale function:

$$g^{-1}:[0,1]\to[0,\tau], g^{-1}(\mu)=s_{\mu\tau}=s_\alpha \tag{2}$$

Definition 3 [9]. Let $S=\{s_\alpha \mid \alpha=0,1,\cdots,\tau\}$ be a linguistic term set. A probabilistic linguistic term set (PLTS) is defined as

$$L(p)=\left\{L^{(k)}\left(p^{(k)}\right) \mid L^{(k)} \in S, p^{(k)} \geq 0, k=1,2,\cdots,\#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1\right\} \tag{3}$$

where $L^{(k)}\left(p^{(k)}\right)$ denotes a probabilistic linguistic element (PLE), representing the k-th linguistic term $L^{(k)}$ and its corresponding probability $p^{(k)}$; and $\#L(p)$ denotes the number of all distinct linguistic terms in $L(p)$.

Definition 4 [9]. Let $L_1(p)=\left\{L_1^{(k)}\left(p_1^{(k)}\right) \mid k=1,2,\cdots,\#L_1(p)\right\}$ and $L_2(p)=\left\{L_2^{(k)}\left(p_2^{(k)}\right) \mid k=1,2,\cdots,\#L_2(p)\right\}$ be two normalized probabilistic linguistic term sets. If $\#L_1(p)>\#L_2(p)$, then $\#L_1(p)-\#L_2(p)$ linguistic elements are added to $L_2(p)$. Each added element takes the smallest linguistic term in $L_2(p)$, and its corresponding probability is 0.

Pang *et al.* [9] further proposed a method for ranking the elements in a probabilistic linguistic term set $L(p)=\left\{L^{(k)}\left(p^{(k)}\right) \mid k=1,2,\cdots,\#L(p)\right\}$. This method sorts the elements in descending order according to the value of $r\left(L^{(k)}\right)p^{(k)}$, where $r\left(L^{(k)}\right)$ denotes the subscript of the linguistic term $L^{(k)}$.

Definition 5 [1]. Let $S=\{s_\alpha \mid \alpha=0,1,\cdots,\tau\}$ be a linguistic term set, and let $L_1(p)=\left\{L_1^{(k)}\left(p_1^{(k)}\right) \mid k=1,2,\cdots,\#L_1(p)\right\}$ and $L_2(p)=\left\{L_2^{(k)}\left(p_2^{(k)}\right) \mid k=1,2,\cdots,\#L_2(p)\right\}$ be any two normalized probabilistic linguistic term sets defined on S. Then, the distance measure between $L_1(p)$ and $L_2(p)$ is

$$\begin{aligned} d_{CH}\left(L_1(p),L_2(p)\right)=\hbar \frac{\sum_{k=1}^{\#L(p)} \sqrt{p_1^{(k)}p_2^{(k)}\left|g\left(L_1^{(k)}\right)-g\left(L_2^{(k)}\right)\right|}}{\#L(p)} \\ +(1-\hbar)\frac{\left|E\left(L_1(p)\right)-E\left(L_2(p)\right)\right|}{\tau} \end{aligned} \tag{4}$$

where $g(\cdot)$ is the scale function given in Definition 2, $\#L(p)$ is the number of linguistic terms (LTs) in the two normalized sets, $E(\cdot)$ is the score function of the probabilistic linguistic term set defined in Equation (5), $r\left(L^{(k)}\right)$ denotes the subscript of $L^{(k)}$, and $\hbar$ is a preference parameter satisfying $0 \leq \hbar \leq 1$.

Definition 6 [23]. Let the linguistic term set be $S=\{s_\alpha \mid \alpha=0,1,\cdots,\tau\}$ and let $L_1(p)=\left\{L_1^{(k)}\left(p_1^{(k)}\right) \mid L_1^{(k)} \in S, k=1,2,\cdots,\#L_1(p)\right\}$ and $L_2(p)=\left\{L_2^{(k)}\left(p_2^{(k)}\right) \mid L_2^{(k)} \in S, k=1,2,\cdots,\#L_2(p)\right\}$ be two normalized probabilistic linguistic term sets. Then, some operational laws of probabilistic linguistic term sets are defined as follows:

1) $L_1(p)\oplus L_2(p)=\bigcup_{L_1^{(k_1)}\in L_1(p),L_2^{(k_2)}\in L_2(p)}\left\{L_3^{(k_3)}\left(p_3^{(k_3)}\right) \mid k_3=1,2,\cdots,\#L_3(p)\right\}$;

2) $\varsigma L_1(p)=\bigcup\left\{\varsigma L_1^{(k_1)}\left(p_1^{(k_1)}\right) \mid k_1=1,2,\cdots,\#L_1(p)\right\}$;

3) $L^{\varsigma}(p)=g^{-1}\left(\bigcup_{\varpi^k\in g(L)}\left\{\left(\varpi^k\right)^{\varsigma}\left(p^k\right)\right\}\right)$.

where $L_3^{(k_3)}=L_1^{(k_1)}\oplus L_2^{(k_2)}$, $p_3^{(k_3)}=p_1^{(k_1)}p_2^{(k_2)}$, $k_i=1,2,\cdots,\#L_i(p)$ $(i=1,2)$; ς is a positive number, and $g(\cdot)$ is the linguistic scale function given in Definition

2. According to the rule in Reference [24], when $L_3^{(k_3)} < s_0$, let $L_3^{(k_3)} = s_0$; when $L_3^{(k_3)} > s_\tau$, let $L_3^{(k_3)} = s_\tau$.

Definition 7 [25]. Suppose the probabilistic linguistic term set is $L^u(p) = \left\{ L^{u(k)}\left(p^{u(k)}\right) \mid k = 1,2,\cdots,\# L_u(p) \right\}$, and its corresponding weight vector is $w = (w_1, w_2, \cdots, w_U)$, satisfying $0 \le w_u \le 1$, $\sum_{u=1}^{U} w_u = 1$. Then the group probabilistic linguistic term set can be expressed as

$$\bar{L}(p) = \left\{ \bar{L}^{(k)}\left(\bar{p}^{(k)}\right) \mid \bar{L}^{(k)} \in S, \bar{p}^{(k)} = \sum_{u=1}^{U} w_u \vartheta^{u(k)}, k = 1,2,\cdots,\# \bar{L}(p) \right\},$$

where $\vartheta^{u(k)} = \begin{cases} p^{u(k)}, & \bar{L}^{(k)}\left(\bar{p}^{(k)}\right) \in L^u(p) \\ 0, & \bar{L}^{(k)}\left(\bar{p}^{(k)}\right) \notin L^u(p) \end{cases}$.

Definition 8 [9]. Let $L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) \mid k = 1,2,\cdots,\# L(p) \right\}$ be a probabilistic linguistic term set (PLTS). Then the score function and deviation degree of $L(p)$ are defined as

$$E(L(p)) = \sum_{k=1}^{\# L(p)} \left(r\left(L^{(k)}\right) \times p^{(k)} \right) \Big/ \sum_{k=1}^{\# L(p)} p^{(k)} \tag{5}$$

$$\partial(L(p)) = \left(\sum_{k=1}^{\# L(p)} \left(p^{(k)} \times \left(r\left(L^{(k)}\right) - \bar{\alpha} \right) \right)^2 \right)^{\frac{1}{2}} \Big/ \sum_{k=1}^{\# L(p)} p^{(k)} \tag{6}$$

where $r\left(L^{(k)}\right)$ denotes the subscript of the linguistic term $L^{(k)}$.

The comparison rules for two probabilistic linguistic term sets $L_1(p)$ and $L_2(p)$ are defined as follows:

1) If $E(L_1(p)) > E(L_2(p))$, then $L_1(p) \succ L_2(p)$;

2) If $E(L_1(p)) = E(L_2(p))$, then;

a) if $\partial(L_1(p)) > \partial(L_2(p))$, then $L_1(p) \prec L_2(p)$;

b) if $\partial(L_1(p)) = \partial(L_2(p))$, then $L_1(p) \sim L_2(p)$;

3) If $E(L_1(p)) < E(L_2(p))$, then $L_1(p) \prec L_2(p)$.

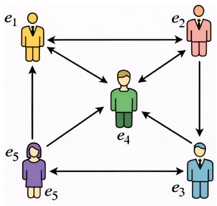
2.2. Social Network

In a social trust network environment, the relationships among individuals in a group decision-making problem do not exist in isolation; rather, they form a network structure with directional and intensity differences through trust ties. To characterize the trust-transmission relationships among decision-makers, graph-theoretic methods can be used to formally describe a social trust network.

Definition 9 [26]. Let there be a directed weighted graph $G = \{E, R, W\}$, where $E = \{e_1, e_2, \cdots, e_U\}$ is the set of nodes representing the decision-makers participating in the decision-making process; R is the set of edges used to describe the trust relationships among decision-makers; and $W = (W_{uv})_{U \times U}$ is the adjacency matrix, whose element W_{uv} represents the degree of trust assigned by decision-maker e_u to decision-maker e_v.

Usually, there are three representation schemes for trust networks, namely graphical representation, algebraic representation, and sociometric representation, as shown in **Table 1**.

Table 1. Examples of different representation methods for social networks.

Graph **representation**	Algebraic representation	Sociometric representation
	e_1Re_2, e_1Re_4; e_2Re_3, e_2Re_4; e_3Re_4, e_3Re_5; e_4Re_1, e_4Re_2; e_5Re_1, e_5Re_4.	$F = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

1) Graphical representation: This can intuitively reflect the directional structure of trust relationships in the network. That is, when there exists $e_u \to e_v$, it indicates that e_u trusts e_v.

2) Algebraic representation: By listing the trust relationships among the subjects, the network connection states are expressed symbolically, which facilitates the logical organization and structural analysis of trust relationships.

3) Sociometric representation: This is usually expressed as an adjacency matrix. Let $F = \left(f_{uv}\right)_{U\times U}$ $f_{uv} \in \{0,1\}$. When $f_{uv} = 1$, it indicates that there is a direct trust relationship from decision-maker e_u to decision-maker e_v; when $f_{uv} = 0$, it indicates that no direct trust relationship exists between them.

However, these representation methods can usually describe only explicit trust relationships, that is, complete trust or complete distrust. In real-world situations, the trust relationships among decision-making agents are often fuzzy in nature, and it is difficult to express them accurately using the above methods. To address this issue, Zhang *et al.* [27] proposed a fuzzy sociometric method.

Definition 10 [27]. Let the fuzzy sociometric relationship defined on the decision-maker set $E = \{e_1, e_2, \cdots, e_U\}$ be $ST = \left(st_{uv}\right)_{U\times U}$. Then it is a relation on the Cartesian product space $E \times E$, and its membership function is $\nu : E \times E \to [0,1]$, with $\nu\left(e_u, e_v\right) = st_{uv}$, where st_{uv} denotes the trust degree (TD) of decision-maker e_u toward decision-maker e_v.

Example 1: The trust relationships among the five decision-makers in Table 1 can be represented by the following fuzzy sociometric matrix:

$$ST = \begin{pmatrix} 0.2 & 0.35 & 0.79 & 0.52 & 0 \\ 0.6 & 0.6 & 0.63 & 0.63 & 0 \\ 0.72 & 0.85 & 0.78 & 0.25 & 1 \\ 0.5 & 0.25 & 0.48 & 0.87 & 0 \\ 0.92 & 0.32 & 0.58 & 0.69 & 0 \end{pmatrix}$$

2.3. Regret Theory

Regret Theory (RT) [28] considers the psychological factors of decision-makers during the decision-making process. Generally speaking, when the option chosen by a decision-maker is better than other alternatives, the decision-maker will feel rejoicing; otherwise, the decision-maker will feel regret. Regret theory characterizes this psychological perception through the rejoicing-regret value.

Definition 11 [28]**.** Let J denote the outcome generated by choosing alternative x. Then the direct utility value of alternative x is usually defined as

$$y(J) = (J)^{\beta} \tag{7}$$

where $0 < \beta < 1$ denotes the risk aversion coefficient of the decision-maker. According to experimental validation results in [29], this parameter is usually recommended to be set to 0.88.

Definition 12 [28]**.** Let J_1 and J_2 denote the outcomes corresponding to choosing alternatives x_1 and x_2, respectively, and let $y(J_1)$ and $y(J_2)$ denote the direct utility values of alternatives x_1 and x_2, respectively. When the decision-maker chooses alternative x_1 instead of alternative x_2, the rejoicing-regret value is expressed as

$$R(J_1, J_2) = 1 - \mathrm{e}^{-\sigma(\Delta y)} = 1 - \mathrm{e}^{-\sigma(y(J_1) - y(J_2))} \tag{8}$$

where $\sigma > 0$ denotes the regret aversion coefficient. Shen *et al.* [30] suggested setting $\sigma = 0.3$. If $R(J_1, J_2) > 0$, it indicates that the decision-maker experiences rejoicing; conversely, if $R(J_1, J_2) < 0$, it indicates that the decision-maker experiences regret.

3. An Estimation Method for Incomplete PLTSs Based on the DeGroot Model

In practical group decision-making situations, decision-makers are often affected by factors such as limited cognition, insufficient information, and differences in evaluation burden, making it difficult for them to provide complete probabilistic linguistic evaluation information for all alternatives and attributes. As a result, missing items may appear in the individual decision-making matrix. If these incomplete evaluations are directly ignored, this will not only damage the informational integrity of the individual decision-making matrix, but also affect the accuracy of subsequent determinations of decision-maker weights, group evaluation results, and consensus level measurement. Therefore, before conducting group aggregation and consensus feedback, it is necessary to reasonably complete the incomplete probabilistic linguistic evaluation information so as to improve the reliability and consistency of the subsequent analysis.

Considering that, in a social network environment, there exist differentiated mutual trust relationships among decision-makers, and that different evaluation positions themselves also carry different degrees of information certainty, this paper starts from two dimensions, namely social trust relationships and the certainty degree of evaluation information, to construct the confidence level of evaluation positions, and thereby realize the estimation of incomplete probabilistic linguistic evaluation information. The basic idea of this mechanism is as follows: for positions with a relatively high confidence level, more of the decision-maker's original evaluation information should be retained; for positions with a relatively low-confidence level, external reference information may be appropriately introduced so as to achieve a reasonable completion of the missing evaluations.

3.1. Acquisition of Confidence Levels Based on the Evaluation Information Dimension and the Trust Relationship Dimension

In the completion process of incomplete PLTSs, a decision-maker's degree of absorption of external reference information is not fixed, but is closely related to their own evaluation state and social network position. Generally speaking, if a decision-maker is confident in their own judgment, they are more inclined to adhere to their original evaluation, and their acceptance of external information will be lower. Therefore, this paper introduces confidence level to characterize the acceptance rate of decision-makers toward external reference opinions, and obtains it objectively from two aspects: the evaluation information dimension and the trust dimension.

Under the trust relationship dimension, the average degree of trust from the other decision-makers toward decision-maker e_u is

$$st_u = \frac{1}{U-1}\sum\nolimits_{v=1,u\neq v}^{U} st_{vu} \tag{9}$$

where the larger st_u is, the more trusted the decision-maker tends to be by others, and the more likely they are to possess stronger social influence and opinion stability. Therefore, in the feedback process, they are more likely to maintain their original judgment.

Under the evaluation information dimension, this paper characterizes the certainty degree of a decision-maker's evaluation based on the probability distribution of the probabilistic linguistic term set, the completeness degree of the probability sum, and the linguistic term set.

First, from the perspective of probability distribution entropy, the more uniform the probability distribution is, the greater the information entropy is, indicating that the evaluation is more uncertain, and the corresponding confidence level is lower. Therefore, we have

$$HD\left(L(p)\right) = -\sum_{k=1}^{\#L(p)}\left(p^{(k)}\log_2 p^{(k)}\right)\Big/\log_2\left(\tau+1\right) \tag{10}$$

Second, from the perspective of the completeness degree of the probability sum, if the given probability mass is closer to 1, then the evaluation is more sufficient, and the corresponding confidence level is higher. Therefore, we have

$$AD\left(L(p)\right) = \sum_{k=1}^{\#L(p)} p^{(k)} \tag{11}$$

Third, from the perspective of the linguistic term set, the more concentrated the terms in a PLTS are, the more concentrated the evaluation information is and the clearer the expression becomes. Therefore, the confidence level is higher [29]. Accordingly, we have

$$CD\left(L(p)\right) = 1 + \sum_{k=1}^{\#L(p)} p^{(k)}\log_2\left(1 - \frac{r\left(L^{(k)}\right) - \bar{\alpha}}{\tau}\right) \tag{12}$$

where $\bar{\alpha}=\sum_{k=1}^{\#L(p)}\left(r\left(L^{(k)}\right)p^{(k)}\right)\Big/\sum_{k=1}^{\#L(p)}p^{(k)}$, and $r\left(L^{(k)}\right)$ is the subscript of $L^{(k)}$. Concentration measures the degree to which each linguistic term clusters around the expected position.

Since the completeness degree of the probability sum, the probability distribution entropy, and the concentration respectively reflect the information characteristics of the probabilistic linguistic term set from different perspectives, relying on only a single indicator is insufficient to comprehensively characterize the certainty degree of the evaluation. The information credibility of $L(p)$ is

$$CCD\left(L(p)\right)=\partial_1\cdot\left(1-HD\left(L(p)\right)\right)+\partial_2\cdot AD\left(L(p)\right)+\partial_3\cdot CD\left(L(p)\right) \tag{13}$$

where $\partial_1,\partial_2,\partial_3\in[0,1]$, and $\partial_1+\partial_2+\partial_3=1$. In this paper, $\partial_1=\partial_2=\partial_3=\frac{1}{3}$. The larger $CCD(\cdot)$ is, the clearer and more stable the evaluation information provided by the decision-maker is, and the lower the uncertainty of the judgment result.

On this basis, by integrating the trust relationship dimension and the evaluation information dimension, the confidence level of decision-maker e_u for alternative x_i under attribute a_j is

$$c_{ij}^u=\psi_1 st_u+\psi_2 CCD\left(q_{ij}^u\right) \tag{14}$$

where $\psi_1,\psi_2\in[0,1]$, and $\psi_1+\psi_2=1$. The larger the confidence level c_{ij}^u, the more the decision-maker tends to retain the original judgment and exhibit stronger self-persistence in subsequent opinion updates; conversely, it indicates that the decision-maker is more easily influenced by external opinions and thus more likely to adjust.

3.2. Estimation of Missing Information

In a social network environment, decision-makers' degrees of absorption of external opinions are not uniform; rather, they are directly affected by differences in trust relationships between themselves and other decision-makers. Generally speaking, decision-makers tend to refer more to the evaluation information provided by neighbors whom they highly trust, while they rely less on opinions from low-trust sources. Based on this understanding, this paper introduces relative trust degree to characterize the differentiated ability of decision-makers to absorb external reference information in the process of completing missing information.

To avoid confusion between probabilistic linguistic term sets and their numerical representations, this paper makes the following distinction in this section: let $q_{ij}^{u,t}$ denote the probabilistic linguistic term set provided by decision-maker e_u for alternative x_i under attribute a_j at time t; let $E\left(q_{ij}^{u,t}\right)$ denote its score function value; and let $\varpi_{ij}^{u,t+1}$ denote the aggregated numerical position constructed during the completion process. In this way, the probabilistic linguistic evaluation object, its scalar representation, and the intermediate mapped variable are kept strictly distinct at the symbolic level.

Let st_{uv} denote the trust degree of decision-maker e_u toward e_v. Then the relative trust degree of e_u toward e_v is defined as

$$\omega_{uv} = \frac{st_{uv}}{\sum_{v=1,v\neq u}^{U} st_{uv}} \tag{15}$$

where ω_{uv} is used to characterize the degree to which decision-maker e_u relies on the external reference evaluation of e_v during the missing-information completion process.

Further, let $q_{ij}^{u,t}$ denote the probabilistic linguistic evaluation of decision-maker e_u at position (i,j), and let $c_{ij}^{u} \in [0,1]$ denote the confidence level at this position determined by Equation (14). To avoid directly performing numerical addition on the PLTS itself, this paper first uses the score function to map the individual's original evaluation and the reference evaluations of trusted neighbors into scalar space. Thus, the external reference score of decision-maker e_u at position (i,j) is defined as

$$\bar{\varpi}_{ij}^{u,t} = \sum_{v=1,u\neq v}^{U} \omega_{uv} \cdot E\left(q_{ij}^{v,t}\right) \tag{16}$$

On this basis, the aggregated numerical position at position (i,j) is constructed as

$$\varpi_{ij}^{u,t+1} = c_{ij}^{u} \cdot E\left(q_{ij}^{u,t}\right) + \left(1 - c_{ij}^{u}\right) \cdot \bar{\varpi}_{ij}^{u,t} \tag{17}$$

It can be seen from Equation (17) that when c_{ij}^{u} is relatively large, the aggregated numerical position is closer to the decision-maker's own original evaluation; conversely, it is more inclined to absorb external reference opinions from trusted neighbors.

Further, according to the inverse mapping of the linguistic scale function, the aggregated numerical position $\varpi_{ij}^{u,t+1}$ can be transformed into a new linguistic term:

$$\tilde{s}_{ij}^{u,t+1} = g^{-1}\left(\varpi_{ij}^{u,t+1}\right) \tag{18}$$

If position (i,j) is partially missing, that is, the original probabilistic linguistic term set $q_{ij}^{u,t}$ already contains some linguistic terms and their corresponding probabilities, but the sum of probabilities satisfies $0 < \sum_{k=1}^{\#q_{ij}^{u,t}} p_{ij}^{u,t(k)} < 1$ then the remaining probability mass is defined as

$$\tilde{p}_{ij}^{u,t+1} = 1 - \sum_{k=1}^{\#q_{ij}^{u,t}} p_{ij}^{u,t(k)} \tag{19}$$

Accordingly, the completed probabilistic linguistic term set can be expressed as

$$\hat{q}_{ij}^{u,t+1} = q_{ij}^{u,t} \cup \left\{\left(\tilde{s}_{ij}^{u,t+1}, \tilde{p}_{ij}^{u,t+1}\right)\right\} \tag{20}$$

If position (i,j) is completely missing, that is, $\sum_{k=1}^{\#q_{ij}^{u,t}} p_{ij}^{u,t(k)} = 0$ then it is directly defined as

$$\hat{q}_{ij}^{u,t+1} = \left\{\left(\tilde{s}_{ij}^{u,t+1}, 1\right)\right\} \tag{21}$$

Accordingly, the completed individual decision matrix can be obtained as

$\hat{Q}_{ij}^{u,t+1} = \left(\hat{q}_{ij}^{u,t+1}\right)_{n \times m}$.

In summary, the incomplete probabilistic linguistic evaluation information completion method proposed in this paper first constructs an aggregated numerical position in scalar space, and then converts it into a newly added linguistic term through the inverse mapping of the linguistic scale function, thereby avoiding improper direct numerical operations on the PLTS itself. Compared with simple mean substitution or fixed-value imputation methods, this method can, on the basis of preserving the decision-maker's original judgment, make full use of the trust relationships in the social network and the certainty characteristics of the evaluation information itself, so as to achieve a more reasonable completion of missing probabilistic linguistic evaluations. For simplicity of subsequent presentation, the completed individual decision matrix $\hat{Q}^{u,t+1}$ is still denoted by Q^u in the following text.

4. A New Method for Solving Group Decision-Making Problems in an Incomplete Probabilistic Linguistic Environment

This section proposes a new consensus measure and designs the corresponding consensus reaching mechanism. First, a consensus measure including both the decision-makers and the collective is constructed to quantify the current consensus state. Then, when the confidence level does not reach the preset threshold, a personalized social-network consensus reaching mechanism based on confidence level and regret theory is further designed.

4.1. Problem Description

Let the set of decision-makers be $E = \{e_1, e_2, \cdots, e_U\}$, the set of alternatives be $X = \{x_1, x_2, \cdots, x_n\}$ $(n \geq 2)$, and the set of attributes be $A = \{a_1, a_2, \cdots, a_m\}$. Under the linguistic term set $S = \{s_\alpha \mid \alpha = 0, 1, \cdots, \tau\}$, the probabilistic linguistic evaluation matrix given by decision-maker e_u is $q_{ij}^u = L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) \mid L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \cdots, \# L(p), \sum_{k=1}^{\# L(p)} p^{(k)} \leq 1 \right\}$, which represents the probabilistic linguistic evaluation of decision-maker e_u for alternative x_i under attribute a_j.

4.2. A Method for Determining Decision-Maker Weights Based on Certainty-Similarity

As the complexity of decision-making problems continues to increase, it has become more convenient for decision-makers to provide decision information in the form of probabilistic linguistic term sets when evaluating alternatives. Since a probabilistic linguistic term set is composed of multiple linguistic terms, and incomplete information may also occur, uncertainty during the decision-making process is unavoidable. Therefore, decision-makers who provide probabilistic linguistic term sets with higher certainty should be assigned greater weights.

According to the information credibility measure (see Equation (13)), the information credibility of the decision matrix $Q^u = \left(q_{ij}^u\right)_{n\times m}$ can be calculated as

$$cs_{uv} = 1 - d_{CH}\left(Q^u, Q^v\right) = 1 - \frac{1}{n\times m}\sum_{i=1}^{n}\sum_{j=1}^{m} d\left(q_{ij}^u, q_{ij}^v\right) \tag{22}$$

where $d(\cdot,\cdot)$ denotes the distance between q_{ij}^u and q_{ij}^v, which is calculated by Equation (4).

The average similarity of decision-maker e_u can be expressed as

$$cs_u = \frac{1}{U-1}\sum_{v=1, v\neq u}^{U} cs_{vu} \tag{23}$$

To simultaneously characterize the certainty of decision information and the coordination of opinions within the group, this paper employs the Einstein T-norm to fuse the comprehensive certainty $CCD\left(Q^u\right)$ and the average similarity cs_u. The Einstein T-norm can preserve the synergistic enhancement effect of the two indicators while avoiding the excessive compression of the fusion result under low-value conditions that may occur with a simple product operator.

Therefore, the initial weight of decision-maker e_u is defined as

$$\tilde{\delta}_u = \frac{CCD\left(Q^u\right)\times cs_u}{1+\left(1-CCD\left(Q^u\right)\right)\times\left(1-cs_u\right)} \tag{24}$$

By further normalizing $\tilde{\delta}_u$, the weight is obtained as

$$\delta_u = \frac{\tilde{\delta}_u}{\sum_{v=1}^{U}\tilde{\delta}_v}, v = 1, 2, \cdots, U \tag{25}$$

Accordingly, the group decision matrix can be obtained and denoted by $O = \left(o_{ij}\right)_{n\times m}$ (calculated using Definition 7), where

$$o_{ij} = \bigoplus_{u=1}^{U} \delta_u \cdot q_{ij}^u \tag{26}$$

where the group evaluation o_{ij} is obtained by aggregating the probabilistic linguistic evaluations of the decision-makers at position (i, j) according to the weight vector δ_u.

4.3. Consensus Level Measurement

The consensus level is defined from three layers, namely the evaluation-position layer, the decision-maker layer, and the group layer, so as to achieve a progressive characterization of the consensus state.

Definition 13 [31]. Let $Q^u = \left(q_{ij}^u\right)_{n\times m}$ be the decision matrix provided by decision-maker e_u, and let $O = \left(o_{ij}\right)_{n\times m}$ be the group decision matrix. Then the consensus level of decision-maker e_u for each alternative x_i under attribute a_j can be defined as

$$CL_{ij}^u = 1 - d\left(q_{ij}^u, o_{ij}\right) \tag{27}$$

where $d\left(q_{ij}^u, o_{ij}\right)$ denotes the distance between q_{ij}^u and o_{ij}, calculated by Equation (4).

Further, the consensus level of decision-maker e_u can be expressed as

$$CL_u = 1 - d_{CH}\left(Q^u, O\right) = 1 - \frac{1}{n \times m}\sum_{i=1}^{n}\sum_{j=1}^{m} d\left(q_{ij}^u, o_{ij}\right) \tag{28}$$

Therefore, the group consensus level can be defined as

$$CCL = \sum_{u=1}^{U} \delta_u \cdot CL_u \tag{29}$$

where δ_u denotes the weight of decision-maker e_u, calculated by Equation (26).

Let the consensus threshold be φ. If $CCL \geq \varphi$, then the group decision-making problem is considered to have reached an acceptable consensus; otherwise, it is regarded as not having reached an acceptable consensus and further adjustment is required. A consensus reaching mechanism will be constructed below.

4.4. A Social-Network Consensus Reaching Mechanism Based on Confidence Level and Regret Psychology

In the consensus reaching process of group decision-making, when the group consensus level is lower than the preset threshold, the decision-makers' original preference information needs to be moderately revised through a feedback mechanism so as to promote the gradual convergence of group opinions. However, existing opinion adjustment methods based on identification rules and modification rules often rely on subjectively preset adjustment coefficients and adopt homogeneous correction methods for different decision-makers. As a result, they have certain limitations in terms of interpretability, fairness, and behavioral realism. Because decision-makers differ in confidence level and trust in other decision-makers, the same deviation does not necessarily lead to the same adjustment behavior. At the same time, such methods usually assume that decision-makers are fully rational and fail to adequately characterize the psychological responses that individuals generate when their opinions deviate from the group judgment.

In view of this, it is necessary to introduce confidence level and regret theory into the social-network consensus feedback process, internalizing the regret psychology triggered when decision-makers' own evaluations deviate from the group opinion into the driving factor for opinion revision, and combining it with the individual's current confidence level to construct a differentiated adjustment mechanism. Based on this idea, this paper further constructs an opinion adjustment model integrating confidence level and regret psychology, so that the adjustment intensity can be determined by both the deviation state and the confidence level. In this way, on the basis of preserving the original decision information as much as possible, the objectivity, rationality, and efficiency of the opinion revision process can be improved. The consensus feedback mechanism introduced below includes two main components: the identification rule and the modification rule.

- Identification Rule

Let the decision matrix of decision-maker e_u be $Q^u = \left(q_{ij}^u\right)_{n \times m}$, and let the group decision matrix be $O = \left(o_{ij}\right)_{n \times m}$. Let $\varphi \in [0,1]$ be the preset consensus threshold. First, identify the set of decision-makers who have not yet met the consensus requirement:

$$NE = \{u \mid CL_u < \varphi\} \tag{30}$$

where CL_u denotes the consensus level between decision-maker e_u and the group decision matrix $O = (o_{ij})_{n\times m}$.

For any $u \in NE$, the set of low-consensus evaluation elements is defined as

$$NSE = \{(i, j) \mid CL_{ij}^u < \varphi\} \tag{31}$$

where CL_{ij}^u denotes the consensus level of decision-maker e_u for alternative x_i under attribute a_j.

- Modification Rule

In a social network environment, the continuous interaction among decision-makers drives their opinions to be continuously adjusted and evolved. Since individuals are more inclined to accept feedback and suggestions from highly trusted neighbors, this paper introduces a social-network-based reference opinion generation mechanism at the consensus feedback stage. Meanwhile, in order to characterize the psychological adjustment pressure generated when a decision-maker's own evaluation deviates from the group opinion, this paper combines regret theory with confidence level to construct a personalized opinion modification rule.

1) Characterization of Adjustment Intensity Based on Regret Theory

Given that the degree to which an evaluation deviates from the group's aggregated opinion directly affects the decision-maker's psychological perception and willingness to revise, this paper further introduces regret psychology to measure the adjustment intensity of evaluations.

For any position to be adjusted, $(u, i, j) \in NSE$, the utility value of the corresponding element q_{ij}^u is expressed as

$$y(q_{ij}^u) = (q_{ij}^u)^\beta \tag{32}$$

where $\beta \in (0,1)$ is the risk aversion coefficient.

At the same time, in the group evaluation matrix $O = (o_{ij})_{n\times m}$ (obtained from Equation (27)), the utility value of element o_{ij} is expressed as

$$y(o_{ij}) = (o_{ij})^\beta \tag{33}$$

The group consensus state can be regarded as an ideal reference in which decision-makers' opinions tend toward consistency. Influenced by psychological factors, decision-makers often compare their original evaluations with this ideal consensus state. When there is a relatively large deviation between their current evaluation and the group reference result, stronger psychological imbalance and revision pressure may arise. Based on this, during the adjustment process, the regret value of the decision-maker's evaluation q_{ij}^u relative to the group consensus evaluation o_{ij} can be characterized as

$$R_{ij}^u = 1 - \mathrm{e}^{-\sigma \cdot \Delta y_u} \tag{34}$$

Considering that subtraction between PLTSs is difficult to define directly, this paper adopts a regret characterization method based on utility distance, namely, $\Delta y_u = -d\left(y\left(q_{ij}^u\right), y\left(o_{ij}\right)\right)$. Thus, the regret value can be further rewritten as

$$R_{ij}^u = 1 - \mathrm{e}^{-\sigma \cdot \Delta y_u} = 1 - \mathrm{e}^{\sigma \cdot d\left(y\left(q_{ij}^u\right), y\left(o_{ij}\right)\right)} = 1 - \mathrm{e}^{\sigma \cdot d\left(\left(q_{ij}^u\right)^\beta, \left(o_{ij}\right)^\beta\right)} \tag{35}$$

where $\sigma \in (0,1)$ denotes the regret aversion coefficient. It can be seen from Equation (36) that when the individual evaluation is closer to the group evaluation, the sense of regret is weaker; conversely, the greater the deviation, the stronger the sense of regret.

Accordingly, the corresponding regret retention coefficient is defined as

$$\tilde{R}_{ij}^u = \mathrm{e}^{R_{ij}^u} \tag{36}$$

Clearly, $\tilde{R}_{ij}^u \in (0,1]$. When the individual evaluation is completely consistent with the group evaluation, $\tilde{R}_{ij}^u = 1$, indicating that there is no revision pressure at this position caused by deviation from the group opinion; as the degree of deviation increases, $\tilde{R}_{ij}^u$ gradually decreases, indicating that the decision-maker's willingness to retain the original evaluation weakens.

2) Modification Function

On the basis of incorporating regret psychology, different decision-makers differ in information certainty, social trust relationships, and degree of self-persistence, so their willingness to retain original opinions also differs significantly. The confidence level constructed in Section 3 reflects the decision-maker's degree of self-persistence regarding the original judgment and the tendency to accept external information. Therefore, this indicator is applicable not only to the process of completing missing information, but also to the feedback adjustment process under low-consensus situations.

Furthermore, by combining the confidence level c_{ij}^u constructed in Section 3 with the regret retention coefficient $\tilde{R}_{ij}^u$, the comprehensive retention coefficient at position (i,j) is defined as

$$\lambda_{ij}^u = c_{ij}^u \cdot \tilde{R}_{ij}^u \tag{37}$$

where $\lambda_{ij}^u \in [0,1]$. The larger λ_{ij}^u is, the more the decision-maker tends to retain the original evaluation; conversely, the more likely the decision-maker is to absorb feedback opinions from the social network and make revisions.

To ensure that the opinion updating process is always carried out within the PLTS space, this paper first constructs the aggregated reference opinion obtained by decision-maker e_u from trusted neighbors at position (i,j):

$$B_{ij}^{u,t} = \overset{U}{\underset{v=1,u\neq v}{\oplus}} \omega_{uv} \cdot q_{ij}^{v,t} \tag{38}$$

where ω_{uv} is given by Equation (15), and $\oplus$ denotes the probabilistic linguistic term set aggregation operation based on Definition 6.

Thus, for any identified position to be revised, $(u,i,j) \in NSE$, the candidate updated result is defined as

$$q_{ij}^{u,t+1} = \bar{\lambda}_{ij}^{u} \cdot q_{ij}^{u,t} \oplus \left(1 - \bar{\lambda}_{ij}^{u}\right) \cdot B_{ij}^{u,t} \tag{39}$$

For positions not belonging to the low-consensus set, $(u,i,j) \notin NSE$, keep

$$q_{ij}^{u,t+1} = q_{ij}^{u,t} \tag{40}$$

The modification rule constructed in this way enables the opinion adjustment process to be jointly influenced by individual confidence level, the degree of deviation from the group, and social trust relationships, thereby more realistically reflecting the dynamic characteristics of group consensus evolution in a social network environment.

Convergence and stopping rule. The proposed feedback rule is designed to move low-consensus evaluations toward a convex combination of the decision-maker's original opinion and the trust-weighted reference opinion from neighbors. Because the update is only applied to the identified low-consensus positions and the adjustment direction is aligned with reducing the deviation from the group opinion, the group consensus level is expected to be non-decreasing in most practical runs.

Nevertheless, since the mechanism involves PLTS aggregation, regret-based retention, and selective position-wise updates, strict analytical monotonicity is not claimed here. Therefore, the iterative process adopts the following stopping rule: the feedback procedure terminates when 1) $CCL \geq \varphi$, or 2) the maximum number of iterations $T_{\max}$ is reached, or 3) the improvement in group consensus between two consecutive rounds is smaller than a preset tolerance ε. In this study, $T_{\max} = 20$, and $\varepsilon = 10^{-4}$.

4.5. Ranking of Alternatives

Once the group consensus level reaches the preset threshold, the ranking process for alternatives can be carried out. Let the final group decision matrix be $GE = \left(ge_{ij}\right)_{n \times m}$, where each element ge_{ij} is a probabilistic linguistic term set. According to the score function in Equation (5) and the deviation degree in Equation (6), the numerical group evaluation is defined as

$$ge_{ij}^{*} = E\left(ge_{ij}\right) \tag{41}$$

where $E\left(ge_{ij}\right)$ denotes the score function value of the probabilistic linguistic evaluation ge_{ij}. It can be seen from this definition that, if an evaluation has a higher score, then its corresponding numerical evaluation is larger, which is more favorable for the subsequent ranking of alternatives. When two alternatives have the same score, they are ranked according to Definition 8.

Therefore, the comprehensive score of each alternative can be calculated as

$$ES\left(x_i\right) = \sum_{j=1}^{m} \omega_j \cdot ge_{ij}^{*}, i = 1, 2, \cdots, n \tag{42}$$

where $\omega = \left(\omega_1, \omega_2, \cdots, \omega_m\right)^{\mathrm{T}}$ is the attribute weight vector. Finally, all alternatives are ranked in descending order according to the magnitude of $ES\left(x_i\right)$, and the alternative with the highest comprehensive score is regarded as the optimal alternative.

4.6. Group Decision-Making Solution Procedure Based on Probabilistic Linguistic Term Sets

Based on the above analysis, the group decision-making process in a probabilistic linguistic term set environment can be summarized as follows:

Input: the original probabilistic linguistic decision matrices of all decision-makers, $Q_u = \left(q_{ij}^u\right)_{n\times m} \left(u = 1, 2, \cdots, U\right)$, and the social network matrix *ST*.

Step 1: Compute the confidence level c_{ij}^u of each decision-maker at each evaluation position according to Equations (9)-(14).

Step 2: Estimate the incomplete probabilistic linguistic term sets according to Equations (15)-(21).

Step 3: Compute the normalized weights δ_u of all decision-makers according to Equations (22)-(26).

Step 4: Aggregate according to Equation (27) to obtain the initial group decision matrix $O = \left(o_{ij}^u\right)_{n\times m}$.

Step 5: Compute the local consensus level CL_{ij}^u, the individual consensus level CL_u, and the group consensus level CCL according to Equations (28)-(30).

Step 6: If $CCL \ge \varphi$, go to Step 9; otherwise, identify the set of decision-makers who have not yet reached the consensus requirement, NE, and the set of their low-consensus evaluation elements, NSE, according to Equations (31)-(32).

Step 7: For any $\left(u, i, j\right) \in NSE$, compute the regret perception, adjustment intensity, and modification coefficient according to Equations (33)-(41), and update the corresponding evaluation value; for evaluation positions that do not belong to the low-consensus set, keep their original values unchanged.

Step 8: Let $t = t + 1$. If $CCL \ge \varphi$, stop; otherwise, if $t \ge T_{\max}$ or $\left|CCL^t - CCL^{t+1}\right| < \varepsilon$, terminate the process and report the best feasible consensus result; else return to Step 5.

Step 9: When the group consensus level reaches the preset threshold, record the final group decision matrix as *GE*, and calculate the comprehensive score of each alternative according to Equations (42)-(43), thereby obtaining the final ranking result.

The above decision-making process is illustrated in **Figure 1**.

5. Case Study on Low-Altitude Economy Investment Selection

This section illustrates the practical application process of the proposed method through a case study. In addition, a comparative analysis is conducted to verify the effectiveness and advantages of the method.

5.1. Problem Description

With the deep integration of the digital economy, intelligent manufacturing, and the new-generation aviation transportation system, the low-altitude economy is becoming an important direction for promoting regional industrial upgrading and fostering new quality productive forces. In recent years, application scenarios such as low-altitude logistics, low-altitude cultural tourism, agricultural and forestry operations, urban patrol, emergency rescue, and supporting infrastructure have continued to expand, indicating that the low-altitude economy has gradually moved from the conceptual cultivation stage to the stage of scenario implementation and industrial development. In this context, how to identify more valuable niche tracks among multiple potential development directions has become an important issue jointly concerned by government departments, industrial capital, and related enterprises.

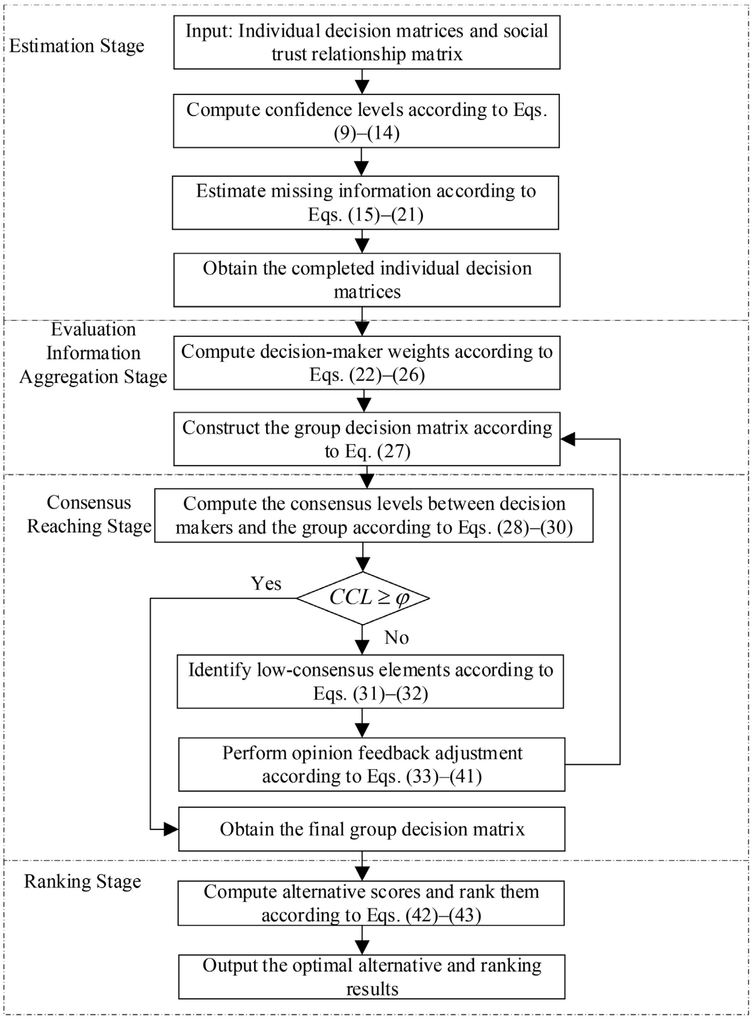


Figure 1. Framework of the decision-making process.

Guangxi has complex terrain and landforms, abundant tourism resources, and diverse agricultural application scenarios. At the same time, it has considerable room for development in regional transportation connectivity, emergency support, and new low-altitude infrastructure construction. Therefore, the low-altitude economy has strong application potential and investment value in Guangxi.

However, different niche tracks differ significantly in terms of technological maturity, scenario adaptability, policy and regulatory requirements, and commercial realization capability. As a result, when making early-stage strategic arrangements, investors often find it difficult to directly judge which direction deserves priority entry. To this end, an investment institution intends to conduct a preliminary study around the Guangxi low-altitude economy sector, and has invited seven decision-makers from the fields of technology, economics, industrial planning, and safety supervision to form an evaluation panel and comprehensively evaluate several candidate niche tracks so as to determine the investment direction with greater potential.

To improve the reproducibility of the case study, the expert panel was formed through purposive selection. The seven decision-makers were chosen from the fields of technology, economics, industrial planning, and safety supervision, with the aim of ensuring that the panel covered the major knowledge domains relevant to low-altitude economy investment assessment. Each participant was required to have practical familiarity with at least one of the candidate development tracks and to be capable of making comparative judgments on technological feasibility, regulatory adaptability, scenario implementation, and market-return potential. The questionnaire survey was conducted independently. Each expert was asked to complete two tasks. First, the expert provided probabilistic linguistic evaluations for the candidate alternatives under the four attributes. Second, the expert reported his or her trust degree toward every other expert in the panel. To reduce mutual interference and anchoring effects, the questionnaires were completed separately before group aggregation. After collection, the responses were checked for completeness and consistency, and ambiguous or missing entries were only confirmed procedurally without altering the substantive judgments of the respondents. The trust relationship data were elicited as continuous real-valued assessments in the interval [0, 1], where 0 denotes complete distrust and 1 denotes complete trust. Each expert could assign any value in [0, 1] to indicate the extent to which he or she would rely on another expert's professional opinion in the current decision context. The resulting trust values were directly used to construct the fuzzy sociometric matrix reported in **Table 2**.

Let the set of candidate niche tracks be $X=\{x_1,x_2,x_3,x_4,x_5\}$, where x_1 denotes low-altitude logistics distribution, x_2 denotes low-altitude cultural tourism sightseeing, x_3 denotes agricultural and forestry planting protection services, x_4 denotes urban patrol and emergency rescue, and x_5 denotes low-altitude infrastructure services. To comprehensively characterize the investment value of different niche tracks, let the evaluation attribute set be $A=\{a_1,a_2,a_3,a_4\}$, where a_1 denotes technological maturity, a_2 denotes adaptability to safety regulation, a_3 denotes feasibility of scenario implementation, and a_4 denotes market return potential.

Technological maturity mainly reflects the development level of the relevant technology route, equipment support capability, and operational stability. Adaptability to safety regulation mainly measures the degree to which the track matches current regulatory requirements in terms of flight safety, low-altitude airspace management, and institutional norms. Feasibility of scenario implementation mainly examines whether the corresponding application scenarios have a clear demand foundation, relatively practical implementation conditions, and room for extension and promotion. Market return potential is used to measure the track's future market expansion capability, prospects for commercial transformation, and investment return level.

Since the problem of selecting investment directions in the low-altitude economy involves strong uncertainty and fuzziness, different decision-makers may hold hesitant and probabilistic judgments regarding the same niche track. Therefore, this paper uses probabilistic linguistic term sets to represent evaluation information. Let the linguistic term set be $S=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6\}$, where s_0, s_1, s_2, s_3, s_4, s_5 and s_6 denote Extremely Low, Very Low, Low, Medium, High, Very High, and Extremely High, respectively.

5.2. Decision-Making Process Based on the Proposed Method

- Parameter settings

Parameter settings. In the case study, the benchmark values were set as follows. The consensus threshold was fixed at $\varphi=0.90$, which is a commonly used high-consensus requirement in consensus reaching studies and reflects the need for sufficiently coordinated collective judgments before final ranking. The risk-aversion coefficient and regret-aversion coefficient were set to $\beta=0.88$ and $\sigma=0.3$, respectively, following the parameter recommendations in [29] [30].

For the confidence-level combination coefficients in Equation (14), the benchmark setting was $\psi_1=0.5$ and $\psi_2=0.5$, with $\psi_1+=\psi_2=1$. This setting was adopted to balance the effects of the trust-relationship dimension and the evaluation-information-certainty dimension. In addition, the three subweights in Equation (13), namely θ_1, θ_2, θ_3 and θ_3, depending on your notation], were set to equal values ([1/3, 1/3, 1/3]) because probability completeness, entropy-based dispersion, and linguistic concentration were regarded as equally important components of evaluation certainty in the absence of prior evidence supporting differential weighting.

Finally, the attribute weights were set as $w=(0.25,0.25,0.25,0.25)^{\mathrm{T}}$. Equal weights were used because the case study aims to illustrate the methodological process rather than to impose exogenous preference bias on any single attribute. This treatment is also appropriate when no validated prior importance ordering among attributes is available.

Input: the trust relationship matrix $ST=(st_{uv})_{7\times7}$ and the decision matrices $Q^u=(q_{ij}^u)_{5\times4}$ $(u=1,2,\cdots,7)$.

According to the questionnaire survey results, the social network among the decision-makers is shown in Table 2, and the decision matrices provided by some decision-makers are shown in Table 3.

Table 2. Social network among decision-makers.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	1	0.87	0.85	0.8	0.74	0.8	0.82
e_2	0.9	1	0.9	0.69	0.69	0.83	0.94
e_3	0.85	0.9	1	0.85	0.82	0.83	0.84
e_4	0.81	0.84	0.8	1	0.84	0.82	0.81
e_5	0.58	0.82	0.5	0.81	1	0.75	0.82
e_6	0.8	0.85	0.8	0.74	0.85	1	0.78
e_7	0.83	0.81	0.4	0.84	0.83	0.41	1

Table 3. Decision matrices of decision-makers.

		a_1	a_2	a_3	a_4
e_1	x_1	$\{s_4(0.5)\}$	$\{(s_3,0.4)\}$	$\{(s_2,1)\}$	$\{(s_2,1)\}$
	x_2	$\{(s_3,0.8),(s_4,0.2)\}$	$\{(s_3,0.5),(s_4,0.5)\}$	$\{(s_3,1)\}$	$\{(s_1,1)\}$
	x_3	$\{(s_3,0.6),(s_4,0.4)\}$	$\{(s_3,1)\}$	$\{(s_6,1)\}$	$\{(s_1,1)\}$
	x_4	$\{(s_2,1)\}$	$\{(s_1,1)\}$	$\{(s_1,0.7),(s_2,0.3)\}$	$\{(s_2,0.3),(s_3,0.4)\}$
	x_5	$\{(s_2,1)\}$	$\{(s_1,0.3),(s_2,0.7)\}$	$\{(s_1,0.5),(s_2,0.5)\}$	$\{(s_2,0.8),(s_3,0.2)\}$
e_2	x_1	$\{(s_3,1)\}$	$\{(s_2,1)\}$	$\{(s_2,1)\}$	$\{(s_2,0.7),(s_3,0.3)\}$
	x_2	$\{(s_2,1)\}$	$\{(s_3,1)\}$	$\{(s_3,0.7),(s_4,0.3)\}$	$\{(s_4,1)\}$
	x_3	$\{(s_4,0.2),(s_5,0.8)\}$	$\{(s_2,1)\}$	$\{(s_4,1)\}$	$\{(s_2,1)\}$
	x_4	$\{(s_2,0.5),(s_3,0.5)\}$	$\{(s_2,1)\}$	$\{(s_2,0.5),(s_3,0.2)\}$	$\{(s_3,1)\}$
	x_5	$\{(s_2,1)\}$	$\{(s_1,0.5),(s_2,0.5)\}$	$\{(s_5,0.5),(s_6,0.2)\}$	$\{(s_4,0.4),(s_5,0.6)\}$
…					
e_7	x_1	$\{(s_1,1.0)\}$	$\{(s_1,0.2),(s_2,0.8)\}$	$\{(s_3,1)\}$	$\{(s_5,1)\}$
	x_2	$\{(s_4,1.0)\}$	$\{(s_1,0.4),(s_2,0.6)\}$	$\{(s_3,1.0)\}$	$\{(s_4,1.0)\}$
	x_3	$\{(s_5,0.2),(s_6,0.8)\}$	$\{(s_2,1)\}$	$\{(s_5,1.0)\}$	$\{(s_1,1.0)\}$
	x_4	$\{(s_4,1)\}$	$\{(s_3,1)\}$	$\{(s_4,1)\}$	$\{(s_3,1)\}$
	x_5	$\{(s_1,1.0)\}$	$\{(s_6,1)\}$	$\{(s_1,1)\}$	$\varnothing$

In the case analysis, the preset consensus threshold is taken as $\varphi = 0.90$, the risk aversion coefficient is set to $\beta = 0.88$, and the regret aversion coefficient is set to $\sigma = 0.3$.

1) Estimation Stage

Step 1: According to Equations (9)-(14), calculate the confidence levels c_{ij}^{u} of the decision-makers at the positions with missing information. The results are shown in Table 4.

Step 2: According to Equations (15)-(21), estimate the incomplete probabilistic linguistic term sets. The completion results for the missing information are shown in Table 4.

Table 4. Completed evaluations of decision-makers.

Decision-maker	Incomplete PLTS	Confidence level c_{ij}^{u}	Completed PLTS
e_1	$q_{11}^{1}=\{s_4(0.5)\}$	$c_{11}^{1}=0.784$	$q_{11}^{1}=\{s_4(0.5),s_{3.574}(0.5)\}$
	$q_{12}^{1}=\{s_3(0.4)\}$	$c_{12}^{1}=0.766$	$q_{12}^{1}=\{s_3(0.4),s_{2.674}(0.6)\}$
	$q_{44}^{1}=\{s_3(0.4),s_2(0.3)\}$	$c_{44}^{1}=0.771$	$q_{44}^{1}=\{s_3(0.4),s_2(0.3),s_{2.742}(0.3)\}$
e_2	$q_{43}^{2}=\{s_2(0.5),s_3(0.2)\}$	$c_{43}^{2}=0.805$	$q_{43}^{2}=\{s_2(0.5),s_3(0.2),s_{2.668}(0.3)\}$
	$q_{53}^{2}=\{s_5(0.5),s_6(0.2)\}$	$c_{53}^{2}=0.805$	$q_{53}^{2}=\{s_5(0.5),s_6(0.2),s_{4.796}(0.3)\}$
e_6	$q_{51}^{6}=\varnothing$	$c_{51}^{6}=0.537$	$q_{51}^{6}=\{s_{2.515}(1)\}$
e_7	$q_{54}^{7}=\varnothing$	$c_{54}^{7}=0.584$	$q_{54}^{7}=\{s_{2.467}(1)\}$

2) Evaluation Information Aggregation Stage

Step 3: According to Equations (22)-(26), calculate the weights of the decision-makers. The results are listed in Table 5.

Table 5. Weights of decision-makers.

Decision-maker	Weight of decision-maker
e_1	0.132
e_2	0.140
e_3	0.143
e_4	0.148
e_5	0.149
e_6	0.144
e_7	0.144

Step 4: According to Equation (27), the group decision matrix can be obtained, namely $O=(o_{ij})_{5\times 4}$. Due to space limitations, the specific results are omitted in the text.

3) Consensus reaching stage

Step 5: Let the consensus threshold be set as $\varphi=0.90$. By Equation (29), the consensus levels of the decision-makers are obtained as $CL_1=0.846$, $CL_2=0.882$, $CL_3=0.876$, $CL_4=0.886$, $CL_5=0.878$, $CL_6=0.867$ and $CL_7=0.855$. According to Equation (30), the group consensus level is $CCL=0.870$.

Step 6: Since $CCL=0.870<\varphi=0.90$, the group has not yet reached consensus. Therefore, according to Equations (31)-(32), the set of evaluation elements that fail to meet the consensus requirement, NSE, is identified, and the results are presented in Table 6.

Furthermore, based on the proposed feedback mechanism, the elements in NSE are adjusted, and the adjustment process is shown below.

Table 6. Consensus reaching process.

Decision-maker	Identified low-consensus positions (i,j)	Corresponding comprehensive retention coefficients $\bar{\lambda}_{ij}^{u}$
e_1	(1, 1), (1, 2), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (5, 3)	0.776, 0.798, 0.845, 0.763, 0.796, 0.788, 0.854, 0.832, 0.864, 0.852, 0.864, 0.744, 0.769
e_2	(1, 1), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 3), (5, 3), (5, 4)	0.888, 0.815, 0.875, 0.876, 0.882, 0.819, 0.770, 0.759, 0.774
e_3	(1, 4), (2, 4), (3, 1), (3, 3), (3, 4), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)	0.826, 0.805, 0.750, 0.822, 0.823, 0.822, 0.763, 0.714, 0.753, 0.734, 0.745
e_4	(1, 1), (1, 2), (1, 3), (3, 1), (3, 3), (3, 4), (4, 2), (4, 3), (5, 1), (5, 2)	0.827, 0.827, 0.828, 0.859, 0.854, 0.861, 0.861, 0.802, 0.784, 0.851
e_5	(1, 1), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 3)	0.854, 0.868, 0.861, 0.846, 0.817, 0.860, 0.857, 0.864, 0.789, 0.863, 0.858
e_6	(1, 1), (1, 3), (1, 4), (2, 1), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (5, 3), (5, 4)	0.827, 0.819, 0.841, 0.828, 0.837, 0.724, 0.803, 0.763, 0.836, 0.837, 0.833, 0.841
e_7	(1, 1), (1, 3), (1, 4), (2, 1), (3, 1), (3, 3), (3, 4), (4, 1), (4, 2), (5, 1), (5, 2), (5, 3)	0.873, 0.863, 0.855, 0.874, 0.810, 0.883, 0.884, 0.879, 0.862, 0.863, 0.773, 0.845

Step 7: According to Equation (38), determine the comprehensive retention coefficient for each low-consensus position in NSE; then further construct the aggregated reference opinions from trusted neighbors according to Equation (39), and combine them with Equation (40) to obtain the adjusted evaluation values.

Step 8: Recalculate the modified group consensus according to Equations (28)-(30). It is obtained that, after the first round of adjustment, the group consensus level is $CCL = 0.908$, which reaches the consensus threshold, and the adjustment process ends.

4) Ranking Stage

Step 9: Rank the alternatives based on their comprehensive scores. Let the attribute weight vector be $\omega = (0.25, 0.25, 0.25, 0.25)^{\mathrm{T}}$. Then, according to Equations (42)-(43), the comprehensive scores of the alternatives are calculated as $E(x_1) = 2.287$, $E(x_2) = 3.028$, $E(x_3) = 3.217$, $E(x_4) = 3.083$, $E(x_5) = 2.478$. Therefore, the ranking result of the alternatives is $x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_1$, and the optimal alternative is x_3.

5.3. Parameter Sensitivity Analysis

To examine the response characteristics of the proposed method to changes in key parameters, this paper further conducts a sensitivity analysis on the basis of completing missing information, assigning decision-maker weights, and performing consensus feedback. Considering that several key parameters in this method respectively correspond to the fusion of trust relationships and evaluation information, the characterization of behavioral psychology, and the control of feedback intensity, the combined confidence-level parameter, the risk aversion coefficient, and the regret aversion coefficient are selected for one-factor perturbation analysis. In each group of experiments, only one parameter is changed, while the remaining parameters are kept at their benchmark values, so as to investigate the influence of different parameter changes on the model output.

To measure the influence of parameter changes on the consensus feedback process, this paper selects the adjustment degree AD, the number of adjusted positions AP, and the number of iterations as evaluation indicators. Among them, AD is used to measure the overall magnitude of opinion adjustment, AP reflects the number of evaluation positions that are modified, and the number of iterations indicates the number of feedback rounds required to reach the preset consensus threshold. Generally speaking, the larger AD is, the higher the adjustment cost paid to achieve consensus; the larger AP is, the wider the scope involved in consensus revision; and the smaller the number of iterations is, the more advantageous the proposed method is in terms of consensus reaching efficiency. Therefore, these three indicators can characterize, in a relatively comprehensive way, the impact of parameter changes on model behavior from the three dimensions of cost, scope, and efficiency.

Definition 14 [16]. Let $Q^u = \left(q_{ij}^u\right)_{n\times m}$ be the initial decision matrix of decision-maker e_u, and let $\tilde{Q}^u = \left(\tilde{q}_{ij}^u\right)_{n\times m}$ be the adjusted decision matrix. Then the adjustment degree AD is defined as

$$AD = \sum_{u=1}^{U}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{d\left(q_{ij}^u,\tilde{q}_{ij}^u\right)}{n\times m} \tag{43}$$

where $d(\cdot,\cdot)$ is the distance measure in Equation (4).

Definition 15 [32]. Let $Q^u = \left(q_{ij}^u\right)_{n\times m}$ be the initial decision matrix of decision-maker e_u, and let $\tilde{Q}^u = \left(\tilde{q}_{ij}^u\right)_{n\times m}$ be the adjusted decision matrix. Then AP is defined as

$$AP = \sum_{u=1}^{U}\sum_{i=1}^{n}\sum_{j=1}^{m}\lambda_{ij}^u,\ \text{where } \lambda_{ij}^u = \begin{cases} 0, & q_{ij}^u = \tilde{q}_{ij}^u \\ 1, & q_{ij}^u \neq \tilde{q}_{ij}^u \end{cases} \tag{44}$$

In the experiments, Figures 2-4 present one-factor sensitivity analyses with respect to the confidence-level combination coefficient ψ_1 (with $\psi_2 = 1-\psi_1$), the regret-theory parameter β, and σ, respectively. Specifically, in each experiment only one parameter is changed, while the remaining parameters are kept at their benchmark values, and AD, AP, and the number of iterations are recorded.

Figure 2 shows the sensitivity results under changes in the comprehensive confidence-level combination coefficient ψ_1. As ψ_1 increases from 0 to 1, the adjustment degree AD increases from about 0.18 to about 0.30, while the number of adjusted positions AP remains at 78 throughout. This result indicates that, in the current case, increasing the weight of the trust-relationship dimension in the comprehensive confidence level will enhance the dependence of the consensus feedback process on revised evaluations, thereby increasing the overall adjustment amplitude. This also shows that the comprehensive confidence-level coefficient mainly affects the strength of opinion updating, while its effect on the identification boundary of low-consensus positions is relatively limited.

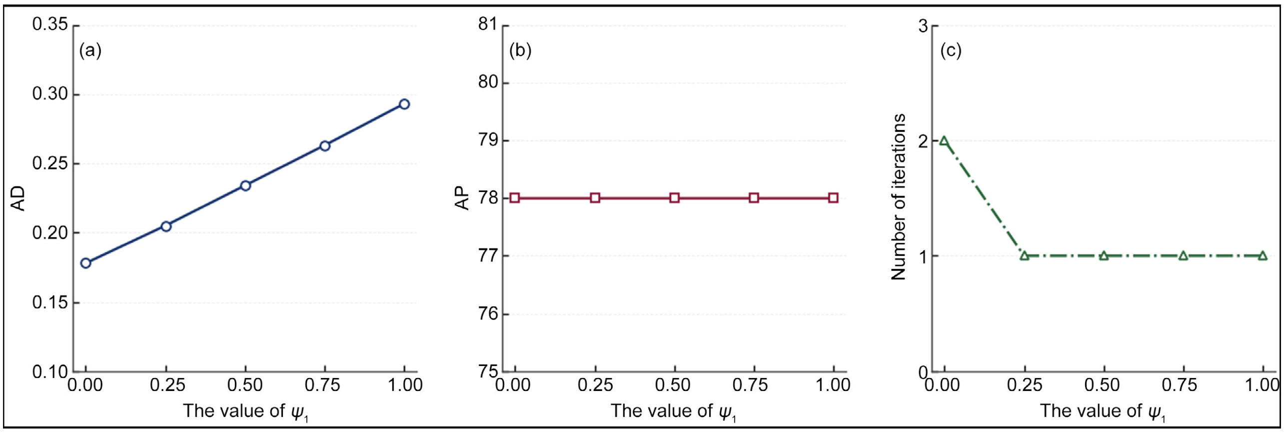


Figure 2. Changes in AD, AP, and the number of iterations under different confidence-level combination coefficients ψ_1.

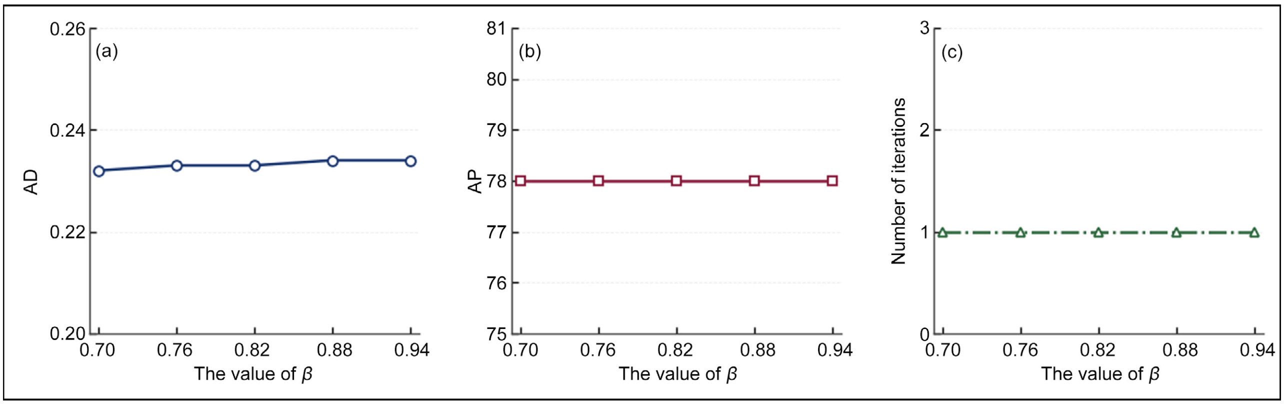


Figure 3. Changes in AD, AP, and the number of iterations under different parameter values of β.

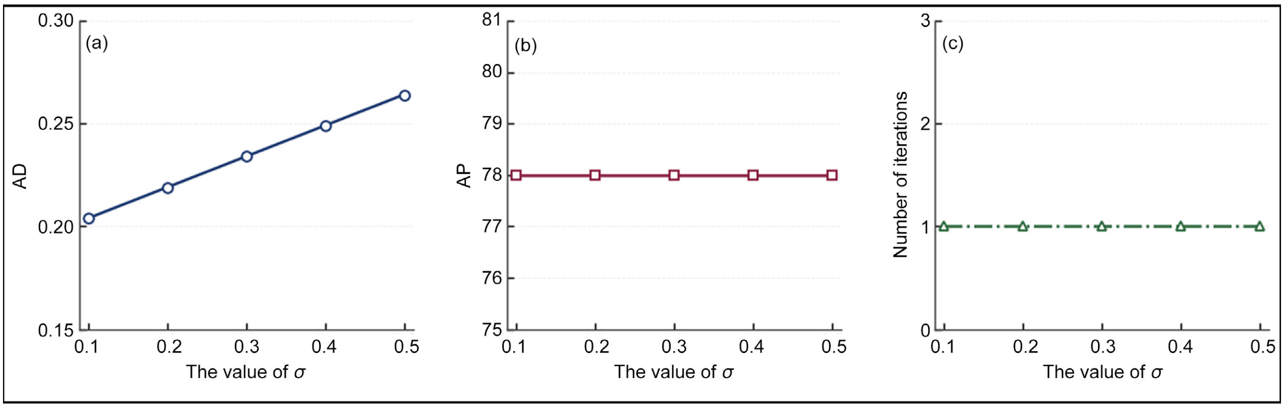


Figure 4. Changes in AD, AP, and the number of iterations under different parameter values of σ.

Figure 3 presents the results under changes in the risk aversion coefficient β. It can be seen that, within the examined interval, β has a relatively small influence on the model output. When β increases from 0.70 to 0.94, the adjustment degree AD fluctuates only within approximately 0.20 to 0.23, AP remains at 78 throughout, and the number of iterations is basically stable. This indicates that, in the current case, the risk aversion coefficient has only a weak perturbation effect on the final consensus feedback result. In other words, the proposed method exhibits good robustness with respect to β: whether in terms of adjustment cost or consensus reaching efficiency, no significant instability is observed.

Figure 4 shows the results under changes in the regret aversion coefficient σ. As σ increases from 0.10 to 0.50, the adjustment degree AD rises from about 0.20 to about 0.26, while the number of adjusted positions AP remains at 78, and the number of iterations changes only slightly. This result indicates that an increase in σ enhances the model's correction intensity for low-consensus positions. That is, the stronger the regret psychology is, the more decision-makers tend to reduce the deviation between their own evaluations and the group opinion, thereby leading to a greater overall adjustment amplitude. At the same time, however, σ has no significant influence on the adjustment scope or the number of feedback rounds.

5.4. Comparative Analysis

To further verify the effectiveness and advantages of the proposed method, this paper analyzes it from four aspects: comparison through ablation experiments, comparison of completion effects under different missing patterns, qualitative comparison with existing methods, and quantitative comparison with existing consensus models. It should be noted that, when evaluating the merits of a consensus method, this paper does not take the final group consensus level alone as the sole criterion; rather, it comprehensively considers the adjustment cost, adjustment scope, ranking stability, and degree of retention of the original evaluation information under the premise of reaching the preset consensus threshold. Based on this evaluation criterion, even if a certain method can achieve a higher final consensus level, if the cost is a significant increase in the magnitude of opinion revision, it does not necessarily mean that it is more suitable as a consensus feedback mechanism in practical decision-making.

- Comparison through Ablation Experiments

To examine the roles of the differentiated social network mechanism, the dynamic confidence-level mechanism, and the missing-information completion mechanism in the proposed method, this paper compares the complete method with three control methods obtained by respectively removing social-network differences, fixing the confidence level, and using simple mean imputation. The comparison results are shown in Table 7.

According to the case data in Section 5.2, under the preset consensus threshold of 0.90, the initial group consensus level of the complete method is 0.870. After one round of feedback adjustment, the final group consensus level increases to 0.908, the adjustment degree is 0.234, and the number of adjusted positions is 78. Compared with the method that removes social network differences, the complete method achieves a slightly higher final consensus level while maintaining a lower adjustment degree, indicating that heterogeneous trust relationships help enhance the targeted use of feedback information, thereby achieving a better balance between consensus improvement and adjustment cost.

Table 7. Quantitative comparison results between the proposed method and different comparison methods.

Method	Initial CCL	Final CCL	AD	AP	$E(x_1)$	$E(x_2)$	$E(x_3)$	$E(x_4)$	$E(x_5)$
Coplete method	0.870	0.908	0.234	78	2.288	3.028	3.217	3.083	2.477
Remoing social network differences	0.870	0.907	0.243	78	2.288	3.028	3.219	3.085	2.473
Fixed confdence level	0.871	0.942	0.486	78	2.266	3.008	3.235	3.049	2.421
Simple median imputtion	0.869	0.907	0.309	77	2.287	3.028	3.217	3.086	2.502

It is worth noting that the fixed-confidence-level method obtains a higher final consensus level in Table 7, but its adjustment degree is significantly higher than that of the proposed method. This indicates that, although ignoring differences among evaluation positions may further raise the final CCL through more intensive opinion revision, it is also more likely to cause excessive intervention in original preferences. In contrast, the proposed method does not seek to increase the final consensus level without limit; instead, it stops feedback once the preset group consensus threshold is reached, thereby preserving the original evaluation information as much as possible while controlling the adjustment cost. Therefore, from the perspective of the "minimum adjustment under an acceptable consensus threshold" principle, the proposed method is more realistic. At the same time, compared with the simple median imputation method, the proposed method achieves a higher initial consensus level and a slightly higher final consensus level, indicating that the proposed completion mechanism can provide a more reliable initial information basis for subsequent group aggregation and feedback adjustment. All four methods ultimately produce the same ranking of alternatives, showing that the proposed method maintains good ranking stability while improving consensus.

- Comparison of Completion Effects Under Different Missing Patterns

Experimental protocol for missing-information tests. To evaluate the performance of the proposed completion method under incomplete information, the complete case data in Section 5.2 were treated as the ground truth, and artificial missingness was introduced under two masking schemes.

1) Random masking: for a given missing rate $r \in \{10\%, 30\%, 50\%\}$, evaluation positions were sampled uniformly at random from the complete entries of the individual decision matrices, and the selected entries were masked as missing.

2) Low-confidence-priority masking: the confidence levels of all complete eval-

uation positions were first computed by Equation (14), and the entries with the lowest confidence values were preferentially masked until the target missing rate was reached. This setting was used to simulate the practical situation in which uncertain or weakly supported evaluations are more likely to be incomplete.

For each missing pattern and each missing rate, the experiment was repeated 30 times. A fixed random seed 2026 was used for reproducibility in the random-masking experiments. The reported results are the averages across repeated runs. To further verify the effectiveness of the proposed incomplete probabilistic linguistic information completion mechanism under different missing intensities and missing patterns, this paper considers two scenarios, namely random missing and low-confidence-position-priority missing, and examines three missing rates, 10%, 30%, and 50%, respectively. The proposed completion method is compared with the simple median imputation method. The comparison indicators include completion error CE, score error SE, and Top1 retention rate. The results are shown in Table 8.

To assess the robustness of the completion results, each masking experiment was repeated N times for every missing pattern and missing rate. For random missingness, a new mask was generated independently in each repetition. For low-confidence missingness, the masking positions were determined according to the ascending order of confidence levels; when ties occurred, tied positions were randomly selected. The reported values of CE, SE, and Top1 retention rate are the mean results over N repetitions, and the corresponding standard deviations are also reported. In the random-masking experiments, a fixed random seed was used to ensure reproducibility.

1) In terms of the two precision indicators, CE and SE, the proposed completion method is overall superior to simple median imputation. Whether under random missingness or low-confidence-position-priority missingness, and across all missing rates from 10% to 50%, the CE and SE of the proposed completion method are always lower than those of simple median imputation. Under random missingness, the CE of the proposed method increases from 0.107 to 0.124, whereas that of simple median imputation remains around 0.145 - 0.146. Under low-confidence missingness, the CE of the proposed method is 0.112 - 0.126, whereas that of simple median imputation is 0.146 - 0.157. This shows that the proposed completion mechanism is more accurate in recovering missing probabilistic linguistic evaluation information.

2) As the missing rate increases, the completion error of all methods generally rises, while ranking stability declines overall. Taking the proposed completion method as an example, under random missingness, CE increases from 0.107 at 10% to 0.124 at 50%, and the Top1 retention rate drops from 93.33% to 66.67%; under low-confidence missingness, the Top1 retention rate also decreases from 76.67% to 56.67%. This indicates that the higher the degree of missingness, the greater the impact on both information recovery and subsequent decision-making stability.

Table 8. Comparison between the proposed completion method and simple median imputation under different missing patterns.

Missing pattern	Missing rate	Method	CE (mean ± SD)	SE (mean ± SD)	Top1 retention (mean ± SD)
Random missingness	10%	Proposed completion	0.107 ± 0.006	0.526 ± 0.021	93.33% ± 4.71%
Random missingness	10%	Simple median imputation	0.146 ± 0.011	0.867 ± 0.034	90.00% ± 3.36%
Random missingness	30%	Proposed completion	0.115 ± 0.012	0.575 ± 0.025	63.33% ± 2.72%
Random missingness	30%	Simple median imputation	0.146 ± 0.09	0.857 ± 0.027	50.00% ± 4.42%
Random missingness	50%	Proposed completion	0.124 ± 0.013	0.648 ± 0.011	66.67% ± 6.35%
Random missingness	50%	Simple median imputation	0.146 ± 0.024	0.863 ± 0.016	50.00% ± 7.73%
Low-confidence missingness	10%	Proposed completion	0.117 ± 0.036	0.602 ± 0.025	86.67% ± 8.43%
Low-confidence missingness	10%	Simple median imputation	0.158 ± 0.025	0.944 ± 0.034	83.33% ± 5.84%
Low-confidence missingness	30%	Proposed completion	0.112 ± 0.021	0.572 ± 0.018	80.00% ± 3.73%
Low-confidence missingness	30%	Simple median imputation	0.146 ± 0.019	0.867 ± 0.027	76.67% ± 6.73%
Low-confidence missingness	50%	Proposed completion	0.126 ± 0.07	0.670 ± 0.025	56.67% ± 5.73%
Low-confidence missingness	50%	Simple median imputation	0.148 ± 0.05	0.879 ± 0.010	40.00% ± 5.43%

Note: $\mathrm{CE} = \frac{1}{|\Omega|}\sum_{(u,i,j)\in\Omega} d\left(\hat{q}_{ij}^{u}, q_{ij}^{u,\mathrm{true}}\right)$ denotes the completion error, where Ω is the set of positions to be completed. $\mathrm{SE} = \frac{1}{|\Omega|}\sum_{(u,i,j)\in\Omega}\left|E\left(\hat{q}_{ij}^{u}\right) - E\left(q_{ij}^{u,\mathrm{true}}\right)\right|$ denotes the score error. The Top1 retention rate indicates the proportion for which the optimal alternative is consistent with that under the complete-data case.

3) The proposed completion method maintains good robustness under high missing rates. Although performance declines as the missing rate rises, the increases in CE and SE of the proposed method remain relatively mild. Moreover, at a 50% missing rate under both random missingness and low-confidence missingness, the Top1 retention rates of the proposed method still reach 66.67% and 56.67%, respectively, which are clearly better than the 50.0% and 40.0% obtained by simple median imputation. This indicates that the proposed method can still maintain relatively stable recovery performance and ranking results under high missingness intensity.

- Qualitative Comparison with Existing Group Decision-Making Methods

To further clarify the characteristics of the proposed method, Table 9 compares the proposed method with existing consensus methods from the aspects of information environment, incomplete-information processing, consideration of psychological behavior, consensus adjustment strategy, and decision-maker weighting strategy.

Table 9. Comparison results among different consensus methods.

Method	Information environment	Handles incomplete information	Considers psychological behavior	Consensus adjustment strategy	Decision-maker weighting strategy
Shen *et al.* [17]	Real numbers	×	√	Personalized feedback based on social networks and regret theory	Based on degree centrality in the social network
Jin *et al.* [33]	PLTSs	×	×	Iterative adjustment based on entropy and similarity	Based on the average fuzzy entropy of PLTSs
Li *et al.* [34]	PLTSs	×	×	Two-stage feedback mechanism based on GRA	Decision-maker weights are not explicitly calculated
Xu *et al.* [35]	PLTSs	×	×	Dynamic-threshold consensus based on SMAA	Decision-maker weights are not explicitly calculated
Shen *et al.* [12]	Real numbers	√	×	DeGroot adjustment based on dynamic trust evolution	Based on external trust and degree centrality in the social network
Method proposed in this paper	PLTSs	√	√	Identification and modification rules; personalized feedback integrating confidence levels and regret psychology	Decision-maker weights computed by fusing comprehensive certainty and average similarity through the Einstein t-norm

The comparative results show that the method proposed in this paper has relatively prominent comprehensive advantages in the following three aspects.

1) In terms of missing-information estimation, this paper incorporates social trust relationships, the certainty of evaluation information, and differences across evaluation positions into the completion process, thereby enhancing both the semantic interpretability and the personalized nature of missing-information recovery. Although [12] considers missing-value estimation, its method still mainly relies on initially incomplete information and is approximately closer to socially influenced numerical imputation, without explaining the generation mechanism of missing evaluations from the perspective of decision-makers' cognitive uncertainty [33]-[35], by contrast, generally assume that PLTS information is complete and thus do not adequately address the pervasive problem of missing evaluations

in real decision-making contexts. In comparison, this paper constructs a completion mechanism from the linguistic semantic characteristics, probability distribution features, and positional differences of PLTSs, so that missing-information estimation achieves both computational rationality and stronger interpretability and specificity.

2) In terms of consensus feedback, this paper introduces regret psychology into the opinion adjustment process, so that the feedback mechanism is influenced not only by the network structure but also jointly by the confidence level of evaluations and the degree to which an individual's opinion deviates from the group opinion. In this way, the behavioral motivations underlying opinion revision are characterized more finely. Although [17] introduces regret theory, it does not sufficiently consider individual differences in information certainty and self-persistence tendency, and therefore its adjustment mechanism still exhibits strong homogeneity. Overall, most existing methods [12] [35] construct feedback rules mainly from the information level or the network-structure level, and their characterization of behavioral mechanisms remains insufficient. By contrast, the present study enhances the model's practical explanatory power regarding decision-makers' revision behavior.

3) In terms of social network modeling, this paper achieves a coordinated integration of trust relationships and individual difference factors. Although [17] constructs trust-based adjustment parameters, its calculation mainly depends on macroscopic indicators such as average trust between subgroups, making it difficult to fully reflect individual heterogeneity. [35] adopts a uniform adjustment rule and lacks a differentiated revision mechanism. [12] [34], although showing some degree of personalization, do not further integrate psychological factors with information certainty. In comparison, this paper simultaneously introduces individual confidence levels into the determination of revision intensity, so that decision-makers' adjustment behavior at different evaluation positions is jointly influenced by multiple factors, including neighbor relationships, self-confidence in evaluation, and the degree of deviation from group opinion. As a result, the proposed mechanism is more consistent with the interaction and evolution characteristics of opinions in real social networks.

4) In terms of decision-maker weighting, this paper employs the Einstein T-norm to fuse the comprehensive certainty of evaluation information with the average individual-group similarity, thereby jointly considering both information quality and group coordination. This makes the weighting mechanism more complete and more interpretable. Existing methods [12] [17] [33] mostly rely on a single entropy indicator or social-network structural information, and thus fail to simultaneously account for the quality of PLTS evaluation information and the degree of individual-group coordination. Consequently, they remain insufficient in terms of the completeness, relevance, and interpretability of their weighting basis.

- Quantitative Comparison with Existing Consensus Models

This section further conducts a quantitative comparative analysis between the proposed method and the existing consensus methods in [33]-[35] in order to verify the above advantages. Based on the dataset in Section 5.2 and the preset consensus threshold $\varphi = 0.90$, the comparison results are shown in **Table 10**.

Fairness setting for comparison. To ensure a fair comparison, all baseline methods and the proposed method were run on the same case dataset from Section 5.2, under the same completed input information, the same consensus threshold $\varphi = 0.90$, and the same attribute-weight vector $w = (0.25, 0.25, 0.25, 0.25)^{\mathrm{T}}$. For methods that do not natively address incomplete information, the same completed decision matrices obtained in the preprocessing stage were used as inputs. No method was given additional prior information unavailable to the others.

Table 10. Quantitative comparison results with existing consensus methods.

Method	Iterations	AD	AP	Alternative ranking
Jin's method [33]	3	0.296	91	$x_3 > x_4 > x_2 > x_5 > x_1$
Li's method [34]	2	0.268	84	$x_3 > x_4 > x_2 > x_5 > x_1$
Xu's method [35]	2	0.251	80	$x_3 > x_4 > x_2 > x_5 > x_1$
The proposed consensus method	1	0.234	78	$x_3 > x_4 > x_2 > x_5 > x_1$

At the quantitative level, this paper further compares the proposed method with the existing consensus methods in [33]-[35], and the results are shown in **Table 10**. As can be seen from **Table 10**, under the same initial decision information and the same preset consensus threshold, the proposed method performs better than the comparative methods in all three indicators: the number of iterations, the adjustment degree AD, and the number of adjusted positions AP. Specifically, the proposed method reaches the preset consensus threshold after only one round of feedback, whereas the comparative methods require two or three rounds of iteration. Meanwhile, the corresponding AD and AP values of the proposed method are 0.234 and 78, respectively, both of which are lower than those of the other methods, indicating that it can achieve consensus revision with a smaller adjustment distance and lower revision cost.

At the same time, all methods obtain the same final ranking of alternatives, namely $x_3 > x_4 > x_2 > x_5 > x_1$, which indicates that the proposed method improves the group consensus level without disrupting the original ranking structure. Instead, it achieves higher feedback efficiency and better ranking stability under a lower adjustment cost. Overall, the proposed method attains a better balance among consensus efficiency, adjustment cost control, and robustness of decision results.

6. Conclusions

This paper proposes a social network group decision-making method for incomplete probabilistic linguistic information by integrating confidence-based completion, certainty-similarity weighting, and regret-based consensus feedback. The method first develops a position-specific confidence structure from social trust relationships and the certainty of evaluation information, and then uses this structure to estimate incomplete PLTSs through a DeGroot-based mechanism. On this basis, decision-maker weights are determined by combining information certainty with individual-group similarity, and a multi-level consensus measurement framework is established. When the preset consensus threshold is not satisfied, regret theory is incorporated into the feedback process to generate a personalized consensus reaching mechanism that accounts for social relationships, confidence levels, and deviations from group opinion.

The empirical results provide three main findings. First, the proposed completion mechanism generally outperforms simple median imputation in terms of completion error and score error, indicating that the joint consideration of trust relationships and information certainty improves the recovery of incomplete probabilistic linguistic evaluations. Second, the weighting scheme enhances the interpretability of group aggregation by jointly considering the quality of evaluation information and the coordination between individual and group opinions. Third, in the case study on investment selection in Guangxi's low-altitude economy, the proposed feedback mechanism increases the group consensus level from 0.870 to 0.908 after one round of adjustment, while keeping the adjustment degree at 0.234 and preserving the final ranking of alternatives. These results suggest that the proposed framework offers a promising way to balance consensus efficiency, adjustment cost, and ranking stability in socially connected decision environments.

Several limitations should also be acknowledged. First, the trust network in the present study is treated as relatively stable, whereas in many real decision settings social trust evolves dynamically over repeated interactions. Second, although regret theory is introduced to capture behavioral heterogeneity, other behavioral factors, such as loss aversion, conformity preference, and belief revision, remain unexplored. Third, the current validation is still mainly based on a single case scenario, which limits the strength of the evidence for generalizability. Future research may therefore extend the framework to dynamic trust networks, incorporate richer behavioral mechanisms, and test the method in additional industry-specific and simulation-based decision settings.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Tian, X., Ma, W., Wu, L., Xie, M. and Kou, G. (2024) Large-Scale Consensus with Dynamic Trust and Optimal Reference in Social Network under Incomplete Probabilistic Linguistic Circumstance. *Information Sciences*, **661**, Article ID: 120123. https://doi.org/10.1016/j.ins.2024.120123

[2] Du, Z., Yu, S. and Chen, Z. (2022) Enhanced Minimum-Cost Conflict Risk Mitigation-Based FMEA for Risk Assessment in a Probabilistic Linguistic Context. *Com-*

puters & Industrial Engineering, **174**, Article ID: 108789. https://doi.org/10.1016/j.cie.2022.108789

[3] Jiang, J., Liu, X., Wang, Z., Ding, W. and Zhang, S. (2024) Large Group Emergency Decision-Making with Bi-Directional Trust in Social Networks: A Probabilistic Hesitant Fuzzy Integrated Cloud Approach. *Information Fusion*, **102**, Article ID: 102062. https://doi.org/10.1016/j.inffus.2023.102062

[4] Liu, P., Li, Y. and Wang, P. (2023) Opinion Dynamics and Minimum Adjustment-Driven Consensus Model for Multi-Criteria Large-Scale Group Decision Making under a Novel Social Trust Propagation Mechanism. *IEEE Transactions on Fuzzy Systems*, **31**, 307-321. https://doi.org/10.1109/tfuzz.2022.3186172

[5] Liu, P., Li, Y. and Wang, P. (2023) Social Trust-Driven Consensus Reaching Model for Multiattribute Group Decision Making: Exploring Social Trust Network Completeness. *IEEE Transactions on Fuzzy Systems*, **31**, 3040-3054. https://doi.org/10.1109/tfuzz.2023.3241145

[6] Liu, Y., Bi, J. and Fan, Z. (2017) Ranking Products through Online Reviews: A Method Based on Sentiment Analysis Technique and Intuitionistic Fuzzy Set Theory. *Information Fusion*, **36**, 149-161. https://doi.org/10.1016/j.inffus.2016.11.012

[7] Rodriguez, R.M., Martinez, L. and Herrera, F. (2012) Hesitant Fuzzy Linguistic Term Sets for Decision Making. *IEEE Transactions on Fuzzy Systems*, **20**, 109-119. https://doi.org/10.1109/tfuzz.2011.2170076

[8] Chen, Z., Chin, K., Li, Y. and Yang, Y. (2016) Proportional Hesitant Fuzzy Linguistic Term Set for Multiple Criteria Group Decision Making. *Information Sciences*, **357**, 61-87. https://doi.org/10.1016/j.ins.2016.04.006

[9] Pang, Q., Wang, H. and Xu, Z. (2016) Probabilistic Linguistic Term Sets in Multi-Attribute Group Decision Making. *Information Sciences*, **369**, 128-143. https://doi.org/10.1016/j.ins.2016.06.021

[10] Zhou, Y., Zheng, C., Wu, P. and Zhou, L. (2024) A Statistical Approach to Large-Scale Group Decision Making with Incomplete Hesitant Fuzzy Linguistic Information by Incorporating Risk Attitude. *Expert Systems with Applications*, **238**, Article ID: 121998. https://doi.org/10.1016/j.eswa.2023.121998

[11] Zhan, J. and Cai, M. (2025) A Cost-Minimized Two-Stage Three-Way Dynamic Consensus Mechanism for Social Network-Large Scale Group Decision-Making: Utilizing k-Nearest Neighbors for Incomplete Fuzzy Preference Relations. *Expert Systems with Applications*, **263**, Article ID: 125705. https://doi.org/10.1016/j.eswa.2024.125705

[12] Shen, Y., Ma, X., Xu, Z., Herrera-Viedma, E., Maresova, P. and Zhan, J. (2024) Opinion Evolution and Dynamic Trust-Driven Consensus Model in Large-Scale Group Decision-Making under Incomplete Information. *Information Sciences*, **657**, Article ID: 119925. https://doi.org/10.1016/j.ins.2023.119925

[13] Zhong, X., Xu, X., Chen, X. and Goh, M. (2022) Reliability-Based Multi-Attribute Large Group Decision Making under Probabilistic Linguistic Environment. *Expert Systems with Applications*, **210**, Article ID: 118342. https://doi.org/10.1016/j.eswa.2022.118342

[14] Ding, Z., Chen, X., Dong, Y. and Herrera, F. (2019) Consensus Reaching in Social Network Degroot Model: The Roles of the Self-Confidence and Node Degree. *Information Sciences*, **486**, 62-72. https://doi.org/10.1016/j.ins.2019.02.028

[15] Wang, Y., Zhan, J., Zhang, C. and Xu, Z. (2024) A Group Consensus Model with Prospect Theory under Probabilistic Linguistic Term Sets. *Information Sciences*, **653**, Article ID: 119800. https://doi.org/10.1016/j.ins.2023.119800

[16] Zhu, J., Ma, X., Kou, G., Herrera-Viedma, E. and Zhan, J. (2023) A Three-Way Consensus Model with Regret Theory under the Framework of Probabilistic Linguistic Term Sets. *Information Fusion*, **95**, 250-274. https://doi.org/10.1016/j.inffus.2023.02.029

[17] Shen, Y., Ma, X., Zhang, H. and Zhan, J. (2024) Fusion Social Network and Regret Theory for a Consensus Model with Minority Opinions in Large-Scale Group Decision Making. *Information Fusion*, **112**, Article ID: 102548. https://doi.org/10.1016/j.inffus.2024.102548

[18] Yang, W., Zhang, L., Shi, J. and Lin, R. (2024) New Consensus Reaching Process with Minimum Adjustment and Feedback Mechanism for Large-Scale Group Decision Making Problems under Social Trust Networks. *Engineering Applications of Artificial Intelligence*, **133**, Article ID: 108230. https://doi.org/10.1016/j.engappai.2024.108230

[19] Guo, J. and Liang, X. (2023) A Dynamic Trust Consensus Model Based on Personalized Adjustment Mechanism under Multi-Granular Probabilistic Linguistic Environment. *Kybernetes*, **52**, 5233-5268. https://doi.org/10.1108/k-03-2022-0448

[20] Herrera, F., Herrera-Viedma, E. and Verdegay, J.L. (1995) A Sequential Selection Process in Group Decision Making with a Linguistic Assessment Approach. *Information Sciences*, **85**, 223-239. https://doi.org/10.1016/0020-0255(95)00025-k

[21] Wang, H., Xu, Z. and Zeng, X. (2018) Hesitant Fuzzy Linguistic Term Sets for Linguistic Decision Making: Current Developments, Issues and Challenges. *Information Fusion*, **43**, 1-12. https://doi.org/10.1016/j.inffus.2017.11.010

[22] Gou, X., Xu, Z. and Liao, H. (2016) Multiple Criteria Decision Making Based on Bonferroni Means with Hesitant Fuzzy Linguistic Information. *Soft Computing*, **21**, 6515-6529. https://doi.org/10.1007/s00500-016-2211-1

[23] Zhang, Y., Xu, Z.S., Wang, H. and Liao, H. (2016) Consistency-Based Risk Assessment with Probabilistic Linguistic Preference Relation. *Applied Soft Computing*, **49**, 817-833. https://doi.org/10.1016/j.asoc.2016.08.045

[24] Wang, H. and Xu, Z. (2016) Interactive Algorithms for Improving Incomplete Linguistic Preference Relations Based on Consistency Measures. *Applied Soft Computing*, **42**, 66-79. https://doi.org/10.1016/j.asoc.2015.09.058

[25] Wu, X. and Liao, H. (2018) An Approach to Quality Function Deployment Based on Probabilistic Linguistic Term Sets and ORESTE Method for Multi-Expert Multi-Criteria Decision Making. *Information Fusion*, **43**, 13-26. https://doi.org/10.1016/j.inffus.2017.11.008

[26] Lin, M., Huang, C., Xu, Z. and Chen, R. (2020) Evaluating IoT Platforms Using Integrated Probabilistic Linguistic MCDM Method. *IEEE Internet of Things Journal*, **7**, 11195-11208. https://doi.org/10.1109/jiot.2020.2997133

[27] Zhang, H., Palomares, I., Dong, Y. and Wang, W. (2018) Managing Non-Cooperative Behaviors in Consensus-Based Multiple Attribute Group Decision Making: An Approach Based on Social Network Analysis. *Knowledge-Based Systems*, **162**, 29-45. https://doi.org/10.1016/j.knosys.2018.06.008

[28] Jin, F., Cai, Y., Zhou, L. and Ding, T. (2023) Regret-Rejoice Two-Stage Multiplicative DEA Models-Driven Cross-Efficiency Evaluation with Probabilistic Linguistic Information. *Omega*, **117**, Article ID: 102839. https://doi.org/10.1016/j.omega.2023.102839

[29] Zhang, S., Zhu, J., Liu, X. and Chen, Y. (2016) Regret Theory-Based Group Decision-Making with Multidimensional Preference and Incomplete Weight Information. *Information Fusion*, **31**, 1-13. https://doi.org/10.1016/j.inffus.2015.12.001

[30] Shen, K., Wang, X., Qiao, D. and Wang, J. (2020) Extended Z-MABAC Method Based on Regret Theory and Directed Distance for Regional Circular Economy Development Program Selection with Z-Information. *IEEE Transactions on Fuzzy Systems*, **28**, 1851-1863. https://doi.org/10.1109/tfuzz.2019.2923948

[31] Xu, Y., Wen, X. and Zhang, W. (2018) A Two-Stage Consensus Method for Large-Scale Multi-Attribute Group Decision Making with an Application to Earthquake Shelter Selection. *Computers & Industrial Engineering*, **116**, 113-129. https://doi.org/10.1016/j.cie.2017.11.025

[32] Zhang, H., Dong, Y., Chiclana, F. and Yu, S. (2019) Consensus Efficiency in Group Decision Making: A Comprehensive Comparative Study and Its Optimal Design. *European Journal of Operational Research*, **275**, 580-598. https://doi.org/10.1016/j.ejor.2018.11.052

[33] Jin, F., Zheng, X., Liu, J., Zhou, L. and Chen, H. (2024) Group Consensus Reaching Process Based on Information Measures with Probabilistic Linguistic Preference Relations. *Expert Systems with Applications*, **249**, Article ID: 123573. https://doi.org/10.1016/j.eswa.2024.123573

[34] Li, P., Xu, Z., Liu, J. and Wei, C. (2023) Social Network Group Decision-Making for Probabilistic Linguistic Information Based on Gra. *Computers & Industrial Engineering*, **175**, Article ID: 108861. https://doi.org/10.1016/j.cie.2022.108861

[35] Xu, Z., Xu, H., Li, P. and Wei, C. (2024) Social Network Group Decision-Making Method Based on Stochastic Multi-Criteria Acceptability Analysis for Probabilistic Linguistic Term Sets. *Information Sciences*, **681**, Article ID: 121269. https://doi.org/10.1016/j.ins.2024.121269