Preface

This volume assembles a series of research papers in number theory published by Ramon Carbó-Dorca and some of his collaborators between 2023 and 2026. The collection focuses on several famous mathematical conjectures—the Collatz Conjecture, the (Strong) Goldbach Conjecture, Fermat's Last Theorem, and the Riemann Hypothesis—and attempts to re-examine these classic problems from a unified perspective of "natural number vector structures."

Natural numbers, the most fundamental concept in mathematics, conceal a structural complexity that lies at the heart of number theory. Traditionally, these conjectures have appeared independent, each with its unique formulation and avenues of proof. However, this collection reveals a novel viewpoint: by reformulating natural numbers as elements within vector spaces and introducing concepts such as "pseudospaces," "vectors of natural powers," "perfect vectors," "Boolean tagged vectors," and the "vector-matrix reversal operator," we can establish a unified framework for these seemingly disparate problems.

In the study of the Collatz Conjecture, we demonstrate how to restrict the problem from the entire set of natural numbers to the set of odd prime numbers, thereby simplifying the path to a proof. For the Strong Goldbach Conjecture, we introduce "prime Boolean vectors" and the vector reversal operation, reformulating the conjecture as a condition of non-orthogonality in a Boolean scalar product. Regarding Fermat's Last Theorem, we extend the problem to arbitrary dimensions and orders through the concepts of "Fermat vectors" and "Fermat polynomials." For the Riemann Hypothesis, we present heuristic graphical evidence, exploring the potential existence of a bijection

between the non-trivial zeros of the Riemann zeta function and prime numbers.

These studies are not only of exploratory theoretical value but are also computationally feasible. We provide several algorithms and computational examples, making the abstract mathematical structures concrete and operable. While this work has not yet completely resolved these famous conjectures, it offers new tools, perspectives, and connections, opening possible avenues for future research.

The organization of this book follows a thematic logic rather than a strict chronological order of publication. We hope readers can grasp the internal coherence of this research program as a whole and draw inspiration from it. The beauty of mathematics often lies in seeing complexity within simplicity and returning from complexity to simplicity. The natural numbers, the earliest mathematical objects we encounter, still hold endless secrets waiting to be discovered.