Preface

This volume brings together a series of mathematical papers that explore diverse topics in number theory, abstract algebra, and combinatorial mathematics. The works presented here, originally published in *Advances in Pure Mathematics* between 2016 and 2024, represent significant contributions to their respective fields while maintaining accessibility to mathematically inclined readers. The collection begins with Beatty and Jones' investigation of irrational points in the plane, establishing pedagogically friendly arguments for the irrationality of exponential expressions. This paper sets the tone for the mathematical rigor and creative thinking that characterizes the entire collection. The research develops simple irrationality proofs using Hurwitz polynomials and establishes fundamental results about rational points and their limitations. Following this foundation, we encounter Beatty and Hansen's exploration of Pringsheim convergence and its relationship to the Dirichlet function. This work demonstrates how double sequences exhibit unexpected properties when limit operations commute, providing fascinating insights into convergence phenomena that challenge our classical understanding of limits. The third paper, by Beatty and Legge, delves into the fascinating world of polynomial factorization over finite fields. This comprehensive study characterizes factorization patterns through partition theory, offering probabilistic interpretations of polynomial behavior that have implications for coding theory and cryptographic applications. Perhaps the most intriguing contribution is Beatty's work on small modular solutions to Fermat's Last Theorem. While Wiles famously proved the non-existence of non-trivial integer solutions to $x^n + y^n = z^n$ for $n > 2$, this paper reveals the surprising abundance of solutions in modular arithmetic. The research develops constructive methods for finding these modular solutions and establishes criteria for compatible exponent-modulus pairs. The collection continues with Beatty, Bianco, and Legge's study of generalized Bernoulli-type series, providing recursion formulas for summing important classes

of infinite series. This work extends classical results by Jakob Bernoulli and offers practical computational tools for series evaluation. Two papers by Beatty and collaborators present novel approaches to classical theorems: one offering a geometric proof of Fermat's Little Theorem using polygon colorings and Burnside's Lemma, and another investigating the conditional probabilities of factoring quadratic polynomials over integers. Each paper in this collection not only contributes original mathematics but also demonstrates beautiful connections between different areas of mathematics. The authors consistently find elegant, sometimes unexpected, pathways to solving problems that have challenged mathematicians for centuries. What unifies these works is their commitment to both theoretical depth and practical applicability. The papers provide not just theorems and proofs, but also computational methods, probabilistic interpretations, and algorithmic approaches that make the results accessible and useful to working mathematicians. As you journey through these pages, you will encounter the creativity and precision that characterizes modern mathematical research. The papers build upon classical foundations while exploring new territories, demonstrating that even well-trodden mathematical paths can lead to surprising and beautiful destinations.