

Chapter 1

Nonlinear-Parametric Effects and a Dynamic Chaos

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1.1. Opening Remarks

In the classical theory of nonlinear oscillations the examination of the nonlinear force resonance and the parametric one are artificially separated [171-173]. From the procedural point of view, this seems quite correct and natural since it allows the provision of the model's mathematical description by a simple manner and to achieve their closed analytical consideration. At that, the key distinctive features of the nonlinear systems can be revealed, as well as their principal differences from the linear systems. This is quite enough to form the world outlook by the persons beginning to study the fundamentals of the oscillation theory.

However, at mentioned approach some problems will raise. The first one is noted quite exactly in [172]: "... examination of the case of direct force impact is not completely correct without the simultaneous influence on the parameters of the system. ...If to consider that the forced periodic process obligated in its origin to the direct impact will cause, in turn, periodic variations of parameters of nonlinear system, then it becomes clear that the resulting resonance phenomena may have a rather complicated character... This circumstance does not allow the complete separation of two mentioned types of resonance phenomena for nonlinear systems." We would like to add that if a motion in the oscillating system is far from the periodic one, then the difficulties will repeatedly increase.

Not less important problem consists in the fact that a classical theory of nonlinear oscillations examines, as a rule, motions in dynamic systems with rather small dimensions of a phase space. Motions in these systems usually represent the motions closed to periodic ones. Nevertheless, for the modern theory of oscillations, the increased interest to stochastic motions (Hamiltonian systems) and to chaotic motions (dissipative systems) is now typical. To date, it is absolutely clear that such types of dynamic system motions are so much natural as, for instance, the quiescent state or a limit cycle. It should be mentioned that study of the determined chaos attracts the researchers not only from the point of view of familiarization of new knowledge, but it is generally recognized due to its applied orientation [31, 32, 62, 65, 75, 87, 89, 100-103, 105-119, 174 etc.].

One more interesting problem relates to identification of an attractor of the dynamic systems with non-traditional behavior. For the first time, the concept of a "strange attractor" was introduced in the paper of Ruelle and Tackens [24] and since that time it was permanently specified [16]. The special interest is caused by the strange non-chaotic attractor, and some researchers believe that at boundary between regular and chaotic motions (so-called, "edge of chaos") one can observe the processes similar to a process of evolution and the information processing [175]. As a rule, a presence of the strange non-chaotic attractor can be identified to biharmonic impact on the nonlinear oscillator with the incommensurable frequencies [179, 184-193].

Therefore, a search of autonomous dynamic systems or systems with the one-frequency impact, which have the phase space, where one can see the "rough" strange non-chaotic attractor (SNCA) (in the sense, which was laid in this term by A.A. Andronov and L.S. Pontriagin [194]), is a issue of the day for the modern physics and the nonlinear dynamics.

In connection with above-mentioned, the following tasks are stated and solved in the present chapter.

1) Mathematical model construction for the non-autonomous oscillating system with continuous time containing the nonlinear capacitance and having the small number of freedom degree, in which one could observe all above-mentioned phenomena and processes at variation of parameters.

2) This model should be physically realizable and, moreover, represent the reasonable combination of components traditionally used in electronics.

3) Investigation of the model offered in detail by means of numerical methods.

4) Implementation of the laboratory dummy and fulfillment of a series of a full-scale experiment.

5) Comparison of numerical and full-scale experiments.

1.2. Mathematical Model Construction of the General Kind

Let us consider the non-autonomous circuit, which electrically can be represented by **Figure 1.1** and contains the external source of the harmonic impact $A \cos pt$, linear dissipative elements R_1, R_2, R_S , the nonlinear capacitance C_N , two linear induction elements L_1, L_2 , and the source of a bias voltage E . The elements' destination is the following. Dissipative elements R_1, R_2, R_S , reflect the energy losses in the reactive elements L_1, L_2 and C , respectively. The induction coil L_1 serves together with the nonlinear capacitance for the tuning in resonance with the external impact frequency p , the induction coil L_2 forms the bias circuit for the nonlinear capacitance.

Using the Kirchhoff laws we can write the equation system for currents and voltages in the circuit under consideration as:

$$\begin{cases} L_1 \frac{di_1}{dt} = A \cos pt - R_1 i_1 - R_S (i_1 - i_2) - u_C, \\ L_2 \frac{di_2}{dt} = u_C + R_S (i_1 - i_2) - R_2 i_2, \\ \frac{d}{dt}(C_N u_C) = i_1 - i_2. \end{cases} \quad (1.1)$$

This system of equations has the third order but the presence of the external force increases the order by one. Hence, the net phase space dimension of the flow under consideration is equal to four (two freedom degrees), which, as the following calculations will show, is enough for provision of the rich and various model's dynamics.

The last equation in (1.1) can be rewritten in the form:

$$C_D \frac{du_C}{dt} = \left(C_N + u_C \frac{dC_N}{du_C} \right) \frac{du_C}{dt} = i_1 - i_2 \quad (1.2)$$

where C_D is a dynamical capacitance of the nonlinear element.

Let us define concretely the volt-farad characteristic of the nonlinear capacitance. We can chose it maximally closed to the respective characteristic or the real varicaps with the sharp $p-n$ junction [176]:

$$C_N = C_0 \sqrt{\frac{g}{g - u_C}}, \quad g = E + \varphi_0 \quad (1.3)$$

In the last equation the following designations are used: E is the bias voltage of the varicap, φ_0 is contact potential difference, C_0 is the junction capacitance in the operating point at E bias.

Equation (1.2) for the chosen type of nonlinearity (1.3) can be rewritten after necessary transformations in the form:

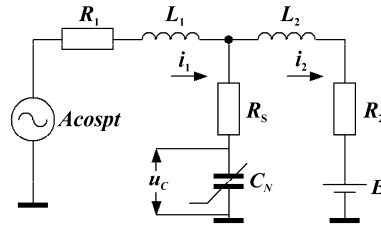


Figure 1.1. Schematic diagram of the non-autonomous electrical circuit with nonlinear capacitance.

$$C_0 m(u_C) \frac{du_C}{dt} = i_1 - i_2 \quad (1.4)$$

in which the multiplier

$$m(u_C) = \sqrt{\frac{g}{g-u_C} \frac{g-0.5u_C}{g-u_C}} \quad (1.5)$$

is the coefficient of modulation of nonlinear capacitance in the operating point.

The offered model (1.1), (1.4), (1.5) suits for the calculations at the arbitrarily meaning of parameters of the dissipative and reactive elements at fulfillment the only restriction $u_C < g$. The sense of this restriction is simple enough: there are prohibited the conditions for the varicap at which the direct currents flow through $p-n$ junction, otherwise, the approximation (1.3) will loose the sense.

The bifurcation phenomena and processes observed at parameter variations of the model (1.1) are so much plural and various, that it is impossible to examine all of them in the frames of the present chapter. That is why, we shall be limited here by the consideration of the resonance case only, which has in addition the most practical interest.

1.3. Mathematical Model in the Resonance Case

Let the dynamic system under consideration has a resonance frequency ω_0 in small-signal case, i.e. for $m(u_C) \approx 1$. It allows the introduction of non-dimensional time $\tau = \omega_0 t$ and since $d/dt = \omega_0 d/d\tau$, we can transform the equation system (1.1) as:

$$\begin{cases} X_1 \frac{di_1}{d\tau} = A \cos \Omega \tau - R_1 i_1 - R_s (i_1 - i_2) - u_C \\ X_2 \frac{di_2}{d\tau} = u_C + R_s (i_1 - i_2) - R_2 i_2 \\ Y_C m(u_C) \frac{du_C}{d\tau} = i_1 - i_2 \end{cases} \quad (1.6)$$

Here we used the following designations: $X_1 = \omega_0 L_1$, $X_2 = \omega_0 L_2$ are the reactive impedance of induction coils; $Y_C = \omega_0 C_0$ is a varicap static conductance at the frequency of small-signal resonance; $\Omega = p/\omega_0$ is the normalized frequency of the external impact. Since Y_C as well as R_s is the name-plate value for each specific type of the varicap, it is expedient to find out the suitable expressions for description the other reactivities having expressed them through exactly this value.

Let us write the expression for the circuit impedance, which is seen from the excitation source at ω_0 frequency:

$$Z(j\omega_0) = R_1 + jX_1 + \frac{R_2 R_s + X_2 X_C + j(R_s X_2 - R_2 X_C)}{R_2 + R_s + j(X_2 - X_C)} \quad (1.7)$$

$$X_C = \frac{1}{Y_C}, j = \sqrt{-1}$$

Usually, the losses in the reactive elements are insignificant and one can neglect them in the first approximation. Such approximation is fully righteous due to another reason that the anharmonic and anisochronous properties of the nonlinear system will certainly become apparent with increasing of amplitude of the external impact and due to this the resonance frequency will inevitably change. In our case the nonlinearity C_N is the “soft” one and we can expect the resonance curve inclination to the left and the displacement downward of the resonance frequency [172]. This allows the simplification of

equation (1.7) to the following kind:

$$Z(j\omega_0) = j \frac{X_1 X_2 - X_C (X_1 + X_2)}{X_2 - X_C} \quad (1.8)$$

It is absolutely evident that for effective usage of the excitation voltage with frequency $p \approx \omega_0$ the absence of damping pole at this frequency is necessary. Therefore, we shall demand that $X_2 \neq X_C$ and assume that $X_2 = \alpha X_C$, $\alpha \neq 1$. Furthermore, from the condition of equality to zero of the imagine part of $Z(j\omega_0)$ and from (1.8) it follows that $X_1 = \alpha X_C / (\alpha - 1)$ $\alpha > 1$. From this we get right away the required relations between parameters of the equation system (1.6):

$$X_1 = \alpha \frac{1}{Y_C}, \quad X_2 = \frac{\alpha}{\alpha - 1} \frac{1}{Y_C} \quad (1.9)$$

The parameter α has the simple physical sense. Let us explain it. The analysis of the impedance $Z(j\omega)$ behavior in the frequency range (we miss it as not related to the essence of the problem under investigation and available to a graduate student) conducted at approximation of low losses permit to obtain that besides the resonance frequency $\omega_0 = \sqrt{(L_1 + L_2) / (C_0 L_1 L_2)}$ there is a damping pole at frequency $\omega_\infty = 1 / \sqrt{C_0 L_2}$. The parameter α defines the following connection between these frequencies: $\alpha = (\omega_0 / \omega_\infty)^2$.

Now the system (1.6) added by equations (1.9) allows the final formulation of the mathematical model as:

$$\begin{cases} \frac{di_1}{d\tau} = \frac{\alpha - 1}{\alpha} Y_C [A \cos \Omega \tau - R_1 i_1 - R_S (i_1 - i_2) - u_C] \\ \frac{di_2}{d\tau} = \frac{Y_C}{\alpha} [u_C + R_S (i_1 - i_2) - R_2 i_2] \\ \frac{du_C}{d\tau} = \frac{1}{m(u_C) Y_C} (i_1 - i_2) \end{cases} \quad (1.10)$$

The numerical examination results of this model are given in the next section, and now we shall prove for flow (1.10) the existence of an attractor – the limit attracting variety in a phase space. The attractor exists if and only if the divergence of D flow is negative. Actually, in this case any initial volume V_0 of the phase space behaves itself in time τ as follows:

$$V(\tau) = V_0 \exp(D\tau) \quad (1.11)$$

It means that if $D < 0$ all phase trajectories starting from V_0 will “settle” in time on the closed attracting variety of zero volume, i.e. on the attractor [17,171].

For the flow (1.10) the expression for the phase flow divergence calculation (the Lee derivative) gets the following form:

$$D = \frac{\partial}{\partial i_1} \left(\frac{di_1}{d\tau} \right) + \frac{\partial}{\partial i_2} \left(\frac{di_2}{d\tau} \right) + \frac{\partial}{\partial u_C} \left(\frac{du_C}{d\tau} \right) = D_1 + D_2 (i_1 - i_2) \quad (1.12)$$

where

$$D_1 = -\frac{Y_C}{\alpha} [(\alpha - 1)(R_1 + R_2) + R_2 + R_S]$$