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A Personalized Consensus-Reaching Method for Large-Scale Group Decision-Making Based on Similarity-Corrected Trust Network

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Abstract

Social network large-scale group decision-making (SN-LSGDM) has become an important research topic in the field of decision science. However, the current methods have some limitations: the trust network ignores the influence of opinions, the weights of decision-makers are subjective, and the consensus adjustment efficiency is low while ignoring individual differences. Therefore, this paper proposes a personalized consensus-reaching method for large-scale group decision-making based on similarity-corrected trust network. Firstly, a trust network correction method based on opinion similarity is constructed to adaptively optimize the initial trust relationship. Secondly, the entropy weight method is applied to realize the objective determination of decision maker weights based on opinion similarity and trust degree, avoiding the subjective bias caused by traditional preset coefficients. Thirdly, a reference matrix is determined based on the trust relationship, and differentiated personalized adjustment coefficients are designed, thereby forming an efficient personalized consensus reaching mechanism. Finally, the practicality of the proposed model is verified through an illustrative example of live-streaming e-commerce decision-making. In addition, simulation experiments and comparative analyses are conducted to highlight the superiority of the proposed model.

Keywords

Large-Scale Group Decision Making (LSGDM), Social Network, Probabilistic Linguistic Term Set (PLTS), Decision-Maker Weights, Consensus Reaching Process (CRP)

1. Introduction

Group decision making (GDM) achieves a unified solution by integrating opinions from multiple decision makers (DMs), and has been extensively studied in the fields of management science and operations research [1]-[3]. Driven by the increasing complexity of decision-making scenarios and the diversified participation of stakeholders, large-scale group decision making (LSGDM) has emerged, focusing on decision problems involving no fewer than 20 decision makers with complex interactive relationships [4]-[6]. Boosted by social media and digital platforms, social network large-scale group decision making (SN-LSGDM) has become a research hotspot in modern decision science [7]-[9]. By incorporating mechanisms of trust, influence and opinion propagation, SN-LSGDM makes the decision-making scenario more realistic, and has been widely applied in healthcare management [10], urban construction [11] [12], agricultural development [13], microgrid planning [14] and other domains.

To accurately characterize subjective and vague evaluation information, linguistic expression models have been continuously developed and improved. Zadeh [15] first introduced linguistic variables, which use qualitative language to express preferences instead of precise numerical values. Herrera and Martínez [16] proposed the 2-tuple linguistic model to avoid information loss during computation. Rodríguez *et al.* [17] constructed the hesitant fuzzy linguistic term set (HFLTS), which allows decision makers to hesitate among multiple linguistic terms. However, HFLTS assumes equal importance for each linguistic term and cannot reflect differences in preference intensity. To overcome this limitation, Pang *et al.* [18] proposed the probabilistic linguistic term set (PLTS), which assigns probability information to each linguistic term and enables a more accurate and comprehensive representation of complex decision-maker preferences. PLTS has now become one of the most widely applied expression tools in uncertain large-scale group decision-making [19]-[21].

In SN-LSGDM, the reasonable determination of decision-maker weights directly affects the credibility of decision results. Determining decision-maker weights based on trust networks is a common method in social network decision-making problems [22]-[24]. However, relying solely on trust networks may be somewhat one-sided, and some scholars have incorporated other factors into the measurement of decision-maker weights in their research. Chen *et al.* [25] measured authority based on decision-makers' identity, experience, and other attributes, and determined decision-maker weights by combining authority with the hybrid centrality derived from the trust network. Wang *et al.* [26] defined the matching deviation degree to measure the fitness between experts and decision problems, and then combined it with trust degree to obtain hybrid decision-maker weights.

The Consensus Reaching Process (CRP) is the core step in social network large-scale group decision-making to resolve conflicts and improve group consensus. Xu *et al.* [27] aggregated the decision matrices of experts within subgroups and then adjusted them with reference to the group opinion. Wang *et al.* [25] identified all decision makers whose consensus levels were below the threshold as the adjustment targets, and promoted consensus by modifying the elements with the maximum distance between their preferences and the group aggregation matrix. Tu *et al.* [22] not only adjusted opinions but also adopted a weight penalty mechanism to identify and manage uncooperative subgroups. Liu *et al.* [23] constructed an optimization model aiming at maximizing the group identification level to determine the classification threshold, and then proposed a consensus feedback mechanism with minimal cross-classification adjustments, thereby improving consensus efficiency. Tian *et al.* [28] calculated personalized adjustment parameters using trust loss and determined the optimal reference point to achieve dual consensus requirements both within and between subgroups. Wang *et al.* [29] employed a group pressure mechanism to determine adjustment coefficients, and then adjusted all decision makers in the subgroup with the lowest consensus level based on these coefficients.

Although the above research achievements on SN-LSGDM have solved many decision-making problems, there still exist some shortcomings that need to be further addressed.

1) In the research on SN-LSGDM, many studies [30] [31] conduct decision-making directly on the initial social network, ignoring the influence of other factors on the trust relationships among decision makers. Ahlim *et al.* [32] and Yang *et al.* [33] obtained a comprehensive network by combining similarity, trust relationship and other indicators, yet failed to address the subjectivity of preset weights and the compensatory issue of multiple factors. Xing *et al.* [34] proposed a trust incentive mechanism based on similarity, but overlooked that similarity does not always positively promote trust relationships. Thereby, improving the research on how factors such as similarity affect the initial trust relationship constitutes an important research direction.

2) In terms of determining decision-maker weights, relying solely on trust relationships to derive expert weights [22]-[24] is overly one-sided. It tends to ignore differences in expert preferences, leading to distorted decision results. Chen *et al.* [25] and Wang *et al.* [26] considered more factors, but their weight determination relies on preset coefficients with strong subjectivity, which may result in unreasonable weight assignment to decision makers. Therefore, it is necessary to develop reasonable methods for objectively determining decision-maker weights.

3) Most existing consensus methods [19] [23] [24] [26] [27] adopt a uniform adjustment scheme when modifying all decision matrices to be adjusted, which results in low adjustment efficiency and may cause excessive adjustment. Among them, Tu *et al.* [23] and Liu *et al.* [24] designed different adjustment strategies to refine the adjustment process to a certain extent, yet still ignored the individual differences among decision makers. Tian *et al.* [28] and Wang *et al.* [29] proposed various methods to determine personalized adjustment coefficients for decision makers, but were restricted to a single adjustment strategy. Therefore, developing a complete and efficient consensus reaching method under social networks is a worthwhile research topic.

To address the above issues, this paper investigates the SN-LSGDM problem with PLTS and proposes a framework based on similarity-corrected trust networks and a personalized consensus reaching mechanism to solve SN-LSGDM problems in the probabilistic linguistic environment. To reflect the evolution of the initial trust network among decision makers, a method for correcting the trust network based on similarity is presented. On the basis of the corrected trust network, the Louvain algorithm is employed to cluster decision makers into different subgroups. Subsequently, consensus level and trust degree are identified as influencing factors of decision-maker weights, which are then objectively determined following the idea of the entropy weight method. Next, the consensus levels at different levels are measured, and a personalized consensus reaching mechanism is proposed to determine the reference matrix based on trust relationships. Finally, the alternatives are ranked according to their expected scores. Accordingly, a novel method is developed to solve the SN-LSGDM problem with PLTS.

The main contributions of this paper are as follows.

1) A method for correcting the trust network based on similarity is proposed. The initial trust relationships are adaptively corrected using the similarity of evaluation opinions among decision makers, which can more realistically reflect the dynamic changes of trust relationships in the decision-making process and improve the rationality of the trust network.

2) The objective determination of decision-maker weights is realized based on the entropy weight method. Opinion similarity and trust degree are taken as indicators for weight calculation. With the entropy weight method, the weights of the two indicators are objectively assigned and integrated to obtain the comprehensive weights of decision makers, which effectively avoids the subjective bias caused by artificially preset coefficients in traditional methods.

3) A personalized consensus reaching mechanism based on trust reference is constructed. A trust reference matrix is established through trust relationships, and differentiated personalized adjustment coefficients are designed, which significantly improves the convergence efficiency of consensus.

The remainder of this paper is organized as follows. Section 2 reviews some concepts of PLTSs and social networks. Section 3 proposes a novel method for solving SN-LSGDM problems with PLTSs. Section 4 provides a case study to illustrate the application of the proposed method. Finally, the stability and superiority of the method are verified through comparative analysis and numerical experiments in Section 5. The conclusions are drawn in Section 6.

2. Preliminaries

2.1. Probabilistic Linguistic Term Set

This section reviews some basic preliminary knowledge related to PLTSs, including expectation, variance, and other related concepts.

Definition 1. [35] Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set (LTS), where τ is a positive integer. S is a set composed of a series of ordered linguistic terms used to represent linguistic evaluation information. Among them, the middle term s_0 indicates a linguistic value of "indifference" or "medium"; positive and negative values are symmetrically distributed on both sides of s_0. $s_{-\tau}$ and s_τ represent the lower and upper bounds of the linguistic variable values, respectively, and $2\tau+1$ is the granularity of S.

Definition 2. [36] Let s_α be a linguistic term in the linguistic term set S, and ξ_α be the characteristic value of s_α, which represents the membership degree of s_α ranging from 0 to 1. For any $s_\alpha \in \{s_{-\tau},\cdots,s_\tau\}$, the linguistic scale function can be expressed as

$$\xi_\alpha = f(s_\alpha) = \frac{\alpha+\tau}{2\tau} \tag{1}$$

Definition 3. [18] Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set. Then the PLTS is defined as follows:

$$L(p)=\left\{s_\alpha^{(l)}\left(p^{(l)}\right) \mid s_\alpha^{(l)} \in S, p^{(l)} \geq 0, l=1,2,\cdots \#L, \sum_{l=1}^{\#L} p^{(l)} \leq 1\right\} \tag{2}$$

where $s^{(l)}p^{(l)}$ is a probabilistic linguistic term composed of the linguistic term $s^{(l)}$ and its corresponding probability $p^{(l)}$. $\#L$ denotes the number of linguistic terms.

Definition 4. [18] For a PLTS $L(p)$, it is said to be non-standard when $\sum_{l=1}^{\#L} p^{(l)} < 1$. In this case, the ratio assignment method is adopted to normalize it.

$$\hat{p}^{(l)} = p^{(l)} \Big/ \sum_{l=1}^{\#L} p^{(l)} \tag{3}$$

Definition 5. [36] Given a normalized PLTS $L(p)$, the expectation of $L(p)$ is defined as

$$E(L(p)) = \sum_{l=1}^{\#L} f\left(s_\alpha^{(l)}\right) \cdot p^{(l)} \tag{4}$$

where $f(\cdot)$ is a linguistic scale function, as shown in Equation (1).

Definition 6. [36] Given a normalized probabilistic linguistic term set $L(p)$, the variance of $L(p)$ is defined as

$$\sigma^2(L(p)) = \sum_{l=1}^{\#L}\left[\left(f\left(s_\alpha^{(l)}\right)-E(L(p))\right)^2 \cdot p^{(l)}\right] \tag{5}$$

Definition 7. [18] Given k standardized PLTS $L(p)$, the PLWA operator can aggregate multiple decision evaluation information. The calculation formula of the PLWA operator is as follows:

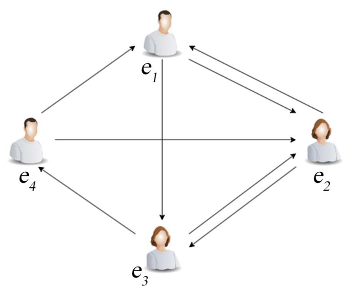
$$\begin{aligned} &\mathrm{PLWA}(L_1(p),L_2(p),\cdots,L_k(p)) = \varphi_1 L_1(p) \oplus \varphi_2 L_2(p) \oplus \cdots \oplus \varphi_k L_k(p) \\ &= \bigcup_{s_\alpha^{1(l)} \in L_1(p)} \left\{\varphi_1 s_\alpha^{1(l)} p^{1(l)}\right\} \oplus \bigcup_{s_\alpha^{2(l)} \in L_2(p)} \left\{\varphi_1 s_\alpha^{2(l)} p^{2(l)}\right\} \oplus \cdots \oplus \bigcup_{s_\alpha^{k(l)} \in L_k(p)} \left\{\varphi_1 s_\alpha^{k(l)} p^{k(l)}\right\} \end{aligned} \tag{6}$$

2.2. Social Network Analysis

Social network analysis can effectively characterize the correlation features among social actors. In the field of decision-making research, social trust networks serve as an important tool for analyzing the relationships between decision makers. Generally speaking, a social trust network mainly consists of three components: first, decision makers; second, the connections among decision makers; and third, the attribute characteristics of decision makers themselves. The above elements can fully reflect the interactive relationships among social entities. Therefore, this paper adopts a social trust network to describe and characterize the relationships between decision makers.

A social trust network can be abstracted as a network structure composed of several nodes and an edge set connecting these nodes. In this study, decision makers are regarded as network nodes. The node set consisting of l decision makers can be denoted as $E=\{e_1,e_2,\cdots,e_l\}$, and the corresponding social trust network structure is shown in **Table 1**.

Table 1. Specific representation of the social network.

<table>
<tr><th>Graph</th><th>Algebra</th><th>Social matrix</th></tr>
<tr><td>
</td><td>$e_1\Re e_2$, $e_1\Re e_3$
$e_2\Re e_1$, $e_2\Re e_3$
$e_3\Re e_2$, $e_3\Re e_4$
$e_4\Re e_1$, $e_4\Re e_2$</td><td>$OT=\begin{pmatrix}0&1&1&0\\1&0&1&0\\0&1&0&1\\1&1&0&0\end{pmatrix}$</td></tr>
</table>

1) Graph: The social trust network can be represented graphically, where nodes correspond to individual DMs, and the edges between nodes depict the mutual relationships among decision makers. If $e_1\to e_2$ exists, it indicates that DM e_1 has a direct trust relationship with DM e_2.

2) Algebra: The social trust network can be expressed in algebraic form to describe various relationships. $e_1\Re e_2$ indicates that hat DM e_1 has a direct trust relationship with DM e_2.

3) Social matrix: A social trust network can use elements in the adjacency matrix to indicate whether a direct trust relationship exists between DMs. For example, when $ot_{1,4}=0$, it means DM e_1 has no direct trust in DM e_4. When $ot_{1,2}\neq 0$, it means DM e_1 has direct trust in DM e_2.

Definition 8. [8] The fuzzy sociometric approach $OT=(ot_{kh})_{l\times l}$ can be used to represent the social trust relationships among l decision makers, where $ot_{kh}\in[0,1]$ is used to quantify the degree of trust of decision maker e_k in decision maker e_h. To ensure the universality of the research, this paper adopts the fuzzy sociometric method to describe the trust relationships between decision makers.

An example is given below for illustration. The social trust relationship matrix $OT=(ot_{kh})_{4\times 4}$ of four decision makers can be expressed as:

$$OT = \begin{pmatrix} - & 0.60 & - & 0.80 \\ 0.75 & - & 0.88 & - \\ 0.80 & - & - & 0.65 \\ - & 0.60 & 0.90 & - \end{pmatrix}$$

3. Large Group Decision Making Based on Similarity-Corrected Social Network

3.1. Problem Description

In the social network large group decision-making problem, let the set of decision makers be denoted as $E = \{e_1, \cdots, e_k, \cdots, c_l\}$, the set of decision alternatives be denoted as $X = \{x_1, \cdots, x_i, \cdots, x_n\}$, and the set of attributes be denoted as $C = \{c_1, \cdots, c_j, \cdots, c_m\}$ with its weight vector $W = \{w_1, \cdots, w_j, \cdots, w_m\}^{\mathrm{T}}$, satisfying $\sum_{j=1}^{m} w_j = 1$. The decision evaluation matrix of decision maker e_k is expressed as $\boldsymbol{B}_k = \left(L_{ij}^k(p)\right)_{n \times m}$, where $L_{ij}^k(p)$ represents the probabilistic linguistic term evaluation value given by decision maker e_k with respect to attribute c_j of alternative x_i.

3.2. Correction of Trust Relationships

In the actual decision-making environment, the similarity of opinions among decision makers will affect the original trust relationships. Although Xing *et al.* [34] realized the promoting effect of opinion similarity on trust relationships by constructing a trust incentive mechanism, they ignored that decision makers may reduce their trust in other decision makers with low opinion similarity. Therefore, this paper corrects the trust relationships in the social network based on opinion similarity.

Definition 9. Let the decision matrices of decision makers e_k and e_h be $\boldsymbol{B}_k = \left(L_{ij}^k(p)\right)_{n \times m}$ and $\boldsymbol{B}_h = \left(L_{ij}^h(p)\right)_{n \times m}$ respectively. The distance between them is defined as

$$d(e_k, e_h) = d(\boldsymbol{B}_k, \boldsymbol{B}_h) = \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} w_j \left[E\left(L_{ij}^k(p)\right) - E\left(L_{ij}^h(p)\right)\right]^2}{n}} \tag{7}$$

where w_j is the weight of attribute c_j, and $\sum_{j=1}^{m} w_j = 1$.

Definition 10. To measure the opinion consistency of different decision makers in the current decision-making process, the opinion similarity between decision makers e_k and e_h is defined as

$$s_{kh} = 1 - d(e_k, e_h) \tag{8}$$

Then the group average similarity $\bar{s}$ can be calculated by Equation (8), which reflects the average level of opinion similarity in the group.

$$\bar{s} = \frac{2}{l(l-1)} \sum_{k=1}^{l-1} \sum_{h=k+1}^{l} s_{kh} \tag{9}$$

Combined with Equations (8) and (9), the similarity standard deviation σ_s is calculated, which is used to measure the dispersion degree of similarity among all decision makers and reflect the fluctuation of opinion differences between decision makers, so as to facilitate the subsequent determination of parameters for correcting trust relationships.

$$\sigma_s = \sqrt{\frac{2}{l(l-1)}\sum_{k=1}^{l-1}\sum_{h=k+1}^{l}\left(s_{kh}-\overline{s}\right)^2} \tag{10}$$

The interval formed by "group average similarity ± similarity standard deviation" is used as the correction threshold. Only when the opinion similarity between decision makers is significantly higher or lower than the overall group level can the opinion difference be regarded as large enough to strengthen or weaken the initial trust relationship. If the similarity falls within this interval, it is regarded as normal deviation and the original trust degree remains unchanged, so as to avoid excessive correction and ensure the stability and rationality of the trust network.

Definition 11. The corrected trust relationship can be determined by the following equation.

$$ct_{kh} = \begin{cases} \min\left\{ot_{kh}\cdot\left(1+\dfrac{s_{kh}-\left(\overline{s}+\sigma_s\right)}{1-\left(\overline{s}+\sigma_s\right)}\right),1\right\}, & s_{kh} > \overline{s}+\sigma_s \\ ot_{kh}, & \overline{s}-\sigma_s \le s_{kh} \le \overline{s}+\sigma_s \\ \max\left\{ot_{kh}\cdot\left(1-\dfrac{\left(\overline{s}-\sigma_s\right)-s_{kh}}{\overline{s}-\sigma_s}\right),0\right\}, & s_{kh} < \overline{s}-\sigma_s \end{cases} \tag{11}$$

The trust network correction based on opinion similarity is conducted only once before the consensus reaching process starts. The corrected trust network remains fixed and unchanged throughout the subsequent consensus iteration process. On the one hand, this assumption simplifies the evolutionary complexity of dynamic social networks and improves decision-making efficiency. On the other hand, considering that decision makers mainly make appropriate compromises to achieve group consensus during subsequent opinion adjustments, their core trust relationships will not change frequently due to short-term opinion adjustments. Therefore, keeping the trust network stable in the consensus iteration stage is more consistent with the logic of actual decision-making behavior.

3.3. Clustering Process

To reduce the complexity of large group decision-making problems, clustering methods can be used to divide the large-scale decision maker group into several subgroups, thereby effectively improving decision-making efficiency. Louvain clustering [37] is one of the most mainstream and efficient non-overlapping community detection algorithms at present. Its core is to automatically identify compact subgroups in the network by maximizing modularity, without pre-specifying the number of clusters, and it can efficiently handle ultra-large-scale networks.

Before applying the Louvain algorithm for clustering, the corrected directed trust network is first transformed into an undirected trust network. Specifically, the weight of the undirected edge between any two decision makers i and j is taken as the average of their bidirectional corrected trust degrees, so as to meet the requirement of the Louvain algorithm for undirected graphs.

Definition 12. [37] Modularity is a core indicator for measuring the quality of community division, with a value range of [−1, 1]. A larger value indicates a denser interior and sparser exterior of the community. Its definition is given as follows:

$$Q = \frac{1}{2\eta}\sum_{k,h}\left(ue_{kh} - \frac{\gamma_k \gamma_h}{2\eta}\right)\cdot \delta_{kh} \tag{12}$$

where ue_{kh} denotes t the undirected edge between node *k* and node *h*. In this paper, ue_{kh} is the average of the bidirectional corrected trust relationships between decision-maker e_k and e_h, that is $ue_{kh} = \left(ct_{kh} + ct_{hk}\right)/2$. γ_k and γ_h are the sum of all edge weights of node *k* and node *h* respectively, $\eta = \frac{1}{2}\sum_k \gamma_k$ is the total weight of edges in the social network, and δ_{kh} is the indicator function which equals 1 if node *k* and node *h* belong to the same community and 0 otherwise.

Definition 13. [37] To evaluate the effectiveness of algorithm iteration, the modularity gain index ΔQ is introduced, whose definition is given as follows:

$$\Delta Q = \left[\frac{\Sigma_{in}}{2\eta} - \frac{\left(\Sigma_{tot}\right)^2}{4\eta^2}\right] - \left[\frac{\Sigma'_{in}}{2\eta} - \frac{\left(\Sigma'_{tot}\right)^2}{4\eta^2}\right] \tag{13}$$

where Σ_{in} is the sum of internal edge weights of the original community, Σ_{tot} is the sum of all edge weights of the original community, and Σ'_{in} and Σ'_{tot} are the corresponding values of the new community after the movement.

The basic steps of the Louvain community detection algorithm are as follows.

1) Each node forms a single community, that is, the number of communities is equal to the number of nodes;

2) Try to move each node into the community where its adjacent nodes are located in turn, calculate the modularity gain ΔQ after movement, and record the node with the maximum ΔQ. If $\max \Delta Q > 0$, move the node into the corresponding community; otherwise, keep it unchanged;

3) Repeat (2) until no node movement can improve the modularity;

4) After division, merge each community into a new node. The sum of all internal edge weights of the original community is reconstructed as the weight of the new node, and the sum of all edge weights between every two original communities is reconstructed as the edge weight between new nodes.

5) Repeat (1) - (4) until the modularity remains unchanged. The final clustering result is obtained at this time, and the clustering set is denoted as $G = \left\{G_1, \cdots, G_r, \cdots, G_R\right\}$.

3.4. An Objective Method for Determining the Weights of Decision Makers

In large group decision-making problems, people usually tend to trust individuals with outstanding professional competence and high decision-making credibility. Individuals with stronger decision-making ability generally have a greater say in the group. In view of this, this paper quantifies the decision-making ability of decision makers by using average similarity and average trusted degree. The higher the decision-making level of a decision maker, the greater the reference value of the decision-making information provided, and the higher the corresponding decision weight assigned.

Definition 14. The average opinion similarity between decision maker e_k and other decision makers can be calculated by

$$AS_k = \frac{\sum_{h=1,h\neq k}^{l} s_{kh}}{l-1} \tag{14}$$

where s_{kh} is the opinion similarity between decision maker e_k and e_h, which can be obtained by Equation (8).

Definition 15. The average trusted degree of decision maker e_k can be calculated by

$$TS_k = \frac{\sum_{h=1,h\neq k}^{l} ct_{kh}}{l-1} \tag{15}$$

where ct_{kh} is the corrected trust relationship between decision maker e_k and e_h, which can be obtained by Equation (11).

In this paper, the entropy weight method [38] is employed to quantify the information uncertainty of the two dimensions (similarity and trust degree), so as to objectively determine the weights of these two dimensions and further determine the weights of decision makers. First, the data of the two dimensions are normalized to eliminate differences in data characteristics.

$$NAS_k = \frac{AS_k - \min(AS_k)}{\max(AS_k) - \min(AS_k)} \tag{16}$$

$$NTS_k = \frac{TS_k - \min(TS_k)}{\max(TS_k) - \min(TS_k)} \tag{17}$$

Convert the data of each dimension into relative probabilities, reflecting the contribution proportion of each decision maker in that dimension, using the following formula.

$$p_{k1} = \frac{NAS_k}{\sum_{k=1}^{l} NAS_k} \tag{18}$$

$$p_{k2} = \frac{NTS_k}{\sum_{k=1}^{l} NTS_k} \tag{19}$$

Based on Shannon entropy in information theory, the information uncertainty of each dimension is quantified. The entropy H_{as} for the consensus level dimension and the entropy H_{ts} for the trust score dimension are respectively given by

$$H_s = -\frac{1}{\log l}\sum_{k=1}^{l}\left(p_{k1}\cdot\log p_{k1}\right) \tag{20}$$

$$H_t = -\frac{1}{\log l}\sum_{k=1}^{l}\left(p_{k2}\cdot\log p_{k2}\right) \tag{21}$$

The smaller the entropy value, the more effective information contained. Based on this principle, the weight of the similarity dimension λ_s and the weight of the trust dimension λ_t are obtained by

$$\lambda_s = \frac{1-H_s}{2-H_s-H_t} \tag{22}$$

$$\lambda_t = \frac{1-H_t}{2-H_s-H_t} \tag{23}$$

Furthermore, the weight φ_k of decision maker e_k is calculated by

$$\varphi_k = \frac{\lambda_s\cdot AS_k + \lambda_t\cdot TS_k}{\sum_{k=1}^{l}\left(\lambda_s\cdot AS_k + \lambda_t\cdot TS_k\right)} \tag{24}$$

The community weight is obtained by summing the weights of decision makers within the community, that is

$$\phi_r = \sum_{e_k\in G_r}\varphi_k \tag{25}$$

3.5. Construction of the Consensus Model

3.5.1. Measurement of Consensus Level

Affected by differences in decision makers' knowledge reserves and cognitive levels, it is often difficult to achieve fully consistent decision results in actual large-scale group decision-making processes. Generally, a consensus threshold can be preset according to practical needs, the consensus level is measured, and decision makers' evaluation information is iteratively revised through a feedback mechanism until the decision consensus reaches a convergent state.

Let the decision matrix of decision maker e_k be $\boldsymbol{B}_k = \left(L_{ij}^k(p)\right)_{n\times m}$, and the group decision matrix be $\hat{\boldsymbol{B}} = \left(\hat{L}_{ij}(p)\right)_{n\times m}$. Based on the PLWA aggregation operator, the decision matrices of all decision makers are integrated into the group decision matrix, as shown in the following formula:

$$\hat{L}_{ij}(p) = \varphi_1 L_{ij}^1(p)\oplus\varphi_2 L_{ij}^2(p)\oplus\cdots\oplus\varphi_l L_{ij}^l(p) \tag{26}$$

Definition 16. The individual consensus level of decision maker e_k is defined as

$$CL_k = 1 - d\left(\boldsymbol{B}_k, \hat{\boldsymbol{B}}\right) \tag{27}$$

where $\boldsymbol{B}_k$ is the decision matrix of decision maker e_k, $\hat{\boldsymbol{B}}$ is the group decision matrix, and the calculation formula $d\left(\boldsymbol{B}_k,\hat{\boldsymbol{B}}\right)$ is shown in Equation (7).

Definition 17. The consensus level of community G_r is defined as

$$SCL_r = \sum_{e_k \in G_r} \varphi_k CL_k \tag{28}$$

Definition 18. The group consensus level is defined as

$$GCL = \phi_r SCL_r \tag{29}$$

The larger the GCL, the higher the consensus degree of group decision-making. When the group consensus level reaches the preset threshold, the group decision matrix meets the requirements; otherwise, the consensus reaching process is activated to adjust the decision information of some decision makers, so as to achieve consensus convergence.

3.5.2. Consensus Reaching Process

Consensus reaching is a key step for large group decision-making to evolve from scattered opinions to unified judgments, and also a core foundation to ensure scientific, credible and implementable decision results. Through the combined effects of group interaction, opinion game and information revision, individual preferences gradually converge and conflicts are continuously resolved, eventually forming a consistent conclusion accepted by most participants. A sound trust relationship among decision makers can reduce the cost of information communication, minimize opinion conflicts and doubts, and encourage individuals to be more willing to accept others' viewpoints and adjust their own preferences. Therefore, this paper incorporates consensus level and trust relationship into the decision adjustment process, takes individual differences of decision makers to be adjusted into account, promotes consensus improvement, and rationally utilizes trust relationships to facilitate consensus reaching.

Let the consensus threshold be Φ. All decision makers with the lowest individual consensus level in communities where $SCL < \Phi$ are marked as the decision makers to be adjusted. In the consensus reaching process, the reference set is composed of decision makers whose individual consensus level is higher than the group consensus level ($CL > GCL$). The reason for selecting such decision makers as references is that their evaluation opinions are more consistent with the overall group opinion, with higher credibility and representativeness, which can provide a stable and reasonable reference direction for the decision makers to be adjusted.

If multiple decision makers satisfy $CL > GCL$ simultaneously, the one with the highest trusted degree is selected as the reference. If the trusted degrees are equal, the one with the highest consensus level is chosen. If these are also equal, the decision maker with the smallest index number is selected. The specific adjustments can be found in **Strategy 1**.

If the trust relationship with the referenced decision maker is lower than the trust threshold, the group opinion matrix is used as the reference. The trust threshold can be set according to actual decision-making situations, and in this paper, it is set as $\varpi = 0.6$. The specific adjustments can be found in **Strategy 2**.

Strategy 1: Suppose that in the t-th iteration, decision maker e_k is identified as the one to be adjusted, with decision maker e_h as its reference. The adjustment formula is given as

$$\boldsymbol{B}_k^{(t+1)} = \mu_k^{(t)} \boldsymbol{B}_k^{(t)} + \left(1 - \mu_k^{(t)}\right) \boldsymbol{B}_h^{(t)} \tag{30}$$

where $\mu_k^{(t)}$ is the personalized adjustment coefficient of decision maker e_k, and $\mu_k^{(t)} = \exp\left[-\vartheta \cdot \left(\Phi - CL_k\right) \cdot \left(1 - \varpi + ct_{kh}\right)\right]$, Φ is the consensus threshold, CL_k is the individual consensus level of decision maker e_k, ct_{kh} is the revised trust relationship from decision maker e_k to decision maker e_h, and ϑ is the amplification coefficient used to magnify the effect of the personalized adjustment coefficient, which is set as $\vartheta = 10$ in this paper.

Strategy 2: Suppose that in the t-th iteration, decision maker e_k is identified as the one to be adjusted, and its trust relationship with the reference decision maker e_h is lower than the trust threshold ϖ. Then the group decision matrix is selected as the reference, and the adjustment formula is given as

$$\boldsymbol{B}_k^{(t+1)} = \nu_k^{(t)} \boldsymbol{B}_k^{(t)} + \left(1 - \nu_k^{(t)}\right) \hat{\boldsymbol{B}}^{(t)} \tag{31}$$

where $\nu_k^{(t)}$ is the personalized adjustment coefficient of decision maker e_k, and $\nu_k^{(t)} = \exp\left[-\vartheta \cdot \left(\Phi - CL_k\right)\right]$.

3.6. Alternative Selection Process

On the basis of forming a basically unified opinion through the consensus reaching process, the subsequent work enters the alternative selection stage. The comprehensive score of each alternative is calculated and ranked, and the alternative with the highest score is selected as the optimal solution.

Suppose in the t-th iteration, $GCL^{(t)} \geq \Phi$ and the group decision matrix is $\hat{\boldsymbol{B}}^{(t)} = \left(\hat{L}_{ij}^{(t)}(p)\right)_{n \times m}$, then the comprehensive score of alternative x_i can be calculated by

$$score(x_i) = \sum_{j=1}^{m} \left[w_j E\left(\hat{L}_{ij}^{(\tau)}(p)\right) \right] \tag{32}$$

where w_j denotes the weight of attribute c_j, and $E\left(\hat{L}_{ij}^{(\tau)}(p)\right)$ represents the expectation of the PLTS for alternative x_i with respect to attribute c_j in the group decision matrix $\hat{\boldsymbol{B}}^{(t)}$, which can be obtained by Equation (4).

3.7. SN-LSGDM Framework Based on Similarity-Corrected Trust Network

Aiming at the multi-attribute large group decision making problem in social networks, this paper revises the initial trust network based on similarity, uses the Louvain clustering algorithm to conduct community detection on the network, and determines the weights of opinion similarity and trust relationship based on information entropy to obtain the decision makers' weights. A personalized consensus reaching model with consensus level and trust relationship as the core is constructed. In summary, the specific procedure of the proposed decision making method is as follows.

Step 1. Revise the trust relationship. Based on the opinion similarity between decision makers, the initial social trust network is revised by Equations (7) - (11) to obtain the corrected trust network $CT = (ct_{kh})_{l\times l}$.

Step 2. Divide the community structure. Based on the adjusted trust network among decision makers, the Louvain community detection algorithm is employed to conduct structural division of the large group network, and the clustering set is obtained as $G = \{G_1, \cdots, G_r, \cdots, G_R\}$.

Step 3. Calculate the weights of decision makers and communities. Based on the idea of information entropy, the weights of the two parameters, opinion similarity and trust relationship, are determined by Equations (14) - (23). Furthermore, the weights of decision makers are calculated by Equation (24), and the community weights are obtained by Equation (25).

Step 4. Calculate the group consensus level GCL. The community and group decision matrices are aggregated by Equation (26), and the consensus levels at the individual, community and group levels are calculated by Equations (27) - (29). If the group consensus level $GCL \geq \Phi$, go to **Step 6**; otherwise, set $t = t + 1$ (initially $t = 0$) and proceed to **Step 5**.

Step 5. Implement the consensus reaching process. Identify the decision maker with the lowest individual consensus level among all communities where $SCL < \Phi$ as the decision maker to be adjusted. The decision matrix of this decision maker is adjusted by Equation (30) or Equation (31) according to the available reference information, and then return to **Step 4**.

Step 6. Rank the alternatives. Based on the group decision matrix after consensus reaching, the comprehensive score of each alternative is calculated by Equation (32), and the optimal alternative is selected after ranking.

4. Case Analysis

4.1. Problem Background

With the deep integration of the digital economy and new retail, live-streaming e-commerce has become a mainstream transaction form in China's consumer market with a scale exceeding one trillion yuan and more than 500 million users. Many enterprises are eager to broaden sales channels and enhance brand influence by leveraging the live-streaming e-commerce economy. However, the selection of live-streaming e-commerce schemes requires multi-party participation in decision-making, which is essentially a typical large-scale group decision-making problem. Research on large group decision-making mechanisms, opinion evolution, and behavior aggregation for live-streaming e-commerce scenarios can not only improve the theoretical system of group decision-making in complex network environments, but also provide scientific support for platform governance, consumption guidance, and marketing optimization.

A daily necessities company plans to launch live-streaming e-commerce to promote its products. After preliminary evaluation, four alternatives are available for selection: daily in-store live streaming (x_1), influencer marketing live streaming

(x_2), public welfare project co-hosted live streaming (x_3), and factory traceability live streaming (x_4). To evaluate the four alternatives, the company invited 20 decision makers from relevant industries to assess each alternative against the following attributes:

1) Operational economic benefit (c_1): refers to the comprehensive performance of the live-streaming e-commerce scheme in terms of input cost, sales revenue, profit margin and input-output ratio, used to measure the profitability and commercial monetization capability of the scheme.

2) Brand influence improvement (c_2): reflects the enhancement effect of live streaming activities on brand awareness, reputation and word-of-mouth communication, and embodies the growth of brand value, market influence and the effect of shaping long-term user cognition.

3) Risk controllability (c_3): mainly measures the risks of the scheme in commodity quality, public opinion, supply chain, compliant operation and other aspects, as well as the capabilities of risk early warning, response and control, so as to ensure the stable operation of live streaming activities.

4) Scenario sustainability (c_4): focuses on whether the live streaming mode can operate stably in the long run, including operational replicability, user retention, and mode adaptability, which determines whether the live streaming scheme can be continuously implemented and promoted on a large scale.

Considering both decision quality and efficiency, this paper sets the consensus threshold $\Phi = 0.9$ and the attribute weights as $W = \{0.35, 0.20, 0.25, 0.20\}$. The decision matrices of 20 decision-makers are presented in **Appendix A**. The social trust network among decision makers is shown in **Appendix B**.

4.2. Decision-Making Process

The initial decision matrices of some decision makers are presented as follows.

$$\boldsymbol{B}_{4\times4}^{1} = \begin{pmatrix} \{s_{-1}(0.5), s_0(0.5)\} & \{s_0(0.7), s_1(0.3)\} & \{s_2(0.65), s_0(0.35)\} & \{s_{-2}(0.8), s_{-1}(0.2)\} \\ \{s_1(0.5), s_3(0.5)\} & \{s_1(0.4), s_2(0.6)\} & \{s_1(0.9), s_2(0.1)\} & \{s_{-1}(0.4), s_2(0.6)\} \\ \{s_0(0.55), s_2(0.45)\} & \{s_2(0.4), s_3(0.6)\} & \{s_2(0.79), s_3(0.21)\} & \{s_0(0.6), s_2(0.4)\} \\ \{s_{-1}(0.8), s_{-3}(0.2)\} & \{s_0(0.25), s_1(0.75)\} & \{s_0(0.33), s_2(0.67)\} & \{s_{-3}(0.5), s_{-1}(0.5)\} \end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{2} = \begin{pmatrix} \{s_{-2}(0.6), s_1(0.4)\} & \{s_1(0.8), s_2(2)\} & \{s_1(0.4), s_2(0.6)\} & \{s_{-3}(0.7), s_{-2}(0.3)\} \\ \{s_0(0.6), s_2(0.4)\} & \{s_0(0.3), s_1(0.7)\} & \{s_0(0.5), s_1(0.5)\} & \{s_1(0.2), s_2(0.2), s_3(0.4)\} \\ \{s_{-1}(0.65), s_2(0.35)\} & \{s_1(0.5), s_2(0.5)\} & \{s_1(0.7), s_2(0.3)\} & \{s_1(0.6), s_3(0.4)\} \\ \{s_1(0.8), s_2(0.2)\} & \{s_2(0.4), s_3(0.6)\} & \{s_0(0.3), s_2(0.7)\} & \{s_0(0.4), s_1(0.6)\} \end{pmatrix}$$

Step 1. Revise the initial social trust network by Equations (7) - (11) to obtain the corrected trust network $CT = (ct_{kh})_{20\times20}$, as shown in **Appendix C**.

Step 2. Employ the Louvain community detection algorithm to conduct structural division of the large group network. The clustering results are shown in Table 2 and Figure 1, where $\#G_r$ denotes the number of decision makers in community G_r.

Table 2. Clustering results.

G_r	$\#G_r$	$\{e_k \mid e_k \in G_r\}$
G_1	8	$\{e_1, e_8, e_9, e_{13}, e_{14}, e_{15}, e_{16}, e_{19}\}$
G_2	7	$\{e_2, e_3, e_4, e_{17}, e_{18}, e_{20}\}$
G_3	5	$\{e_5, e_6, e_7, e_{10}, e_{11}, e_{12}\}$

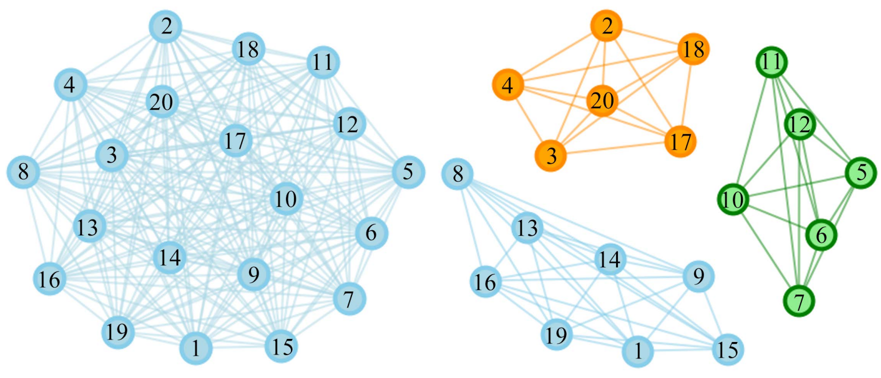


Figure 1. (a) Initial social trust network, (b) Clustering results of the Louvain algorithm.

Step 3. The weights of opinion similarity and trust relationship are determined as $\lambda_s = 0.4156$ and $\lambda_t = 0.5844$ by Equations (14) - (23). Furthermore, the weights of decision makers are calculated by Equation (24), as shown in Table 3, and the community weights $\phi_1 = 0.3912$, $\phi_2 = 2952$ and $\phi_3 = 0.3136$ are obtained by Equation (25).

Table 3. The detailed community information.

G_1	$\phi_1 = 0.3912$		G_2	$\phi_2 = 2952$		G_3	$\phi_3 = 0.3136$	
	$SCL_1^{(0)} = 0.8068$			$SCL_2^{(0)} = 0.8042$			$SCL_3^{(0)} = 0.8093$	
DM	CL_k	φ_k	DM	CL_k	φ_k	DM	CL_k	φ_k
e_1	0.8117	0.0509	e_2	0.8020	0.0515	e_5	0.7989	0.0572
e_8	0.7829	0.0419	e_3	0.7852	0.0507	e_6	0.7746	0.0515
e_9	0.8580	0.0567	e_4	0.8132	0.0484	e_7	0.8387	0.0499
e_{13}	0.7195	0.0468	e_{17}	0.8148	0.0444	e_{10}	0.8207	0.0525
e_{14}	0.8322	0.0511	e_{18}	0.8077	0.0507	e_{11}	0.8085	0.0485
e_{15}	0.8594	0.0511	e_{20}	0.8042	0.0496	e_{12}	0.158	0.0540
e_{16}	0.8212	0.0493						
e_{19}	0.7432	0.0435						

Step 4. Aggregate the group decision matrix by Equation (26) and then calculate the consensus levels at the individual, community and group levels by Equations (27) - (29). The detailed community information is shown in **Table 3**. The initial group consensus level is calculated as $GCL^{(0)} = 0.8068 < 0.9$, thus the consensus reaching process is initiated.

Step 5. Implement the consensus reaching process. At iteration $t = 1$, all communities with $SCL^{(0)} < 0.9$ are identified as G_1, G_2 and G_3. The decision makers with the lowest individual consensus level in each low-consensus community are e_{13}, e_3 and e_6 respectively, which are determined as the decision makers to be adjusted. Since the consensus levels of all initial decision makers are below 0.9, the decision matrices of these three decision makers are adjusted by Equation (31) based on the group decision matrix. After adjustment, the group consensus level is recalculated as $GCL^{(1)} = 0.8337 < 0.9$, and the iterative adjustment is continued.

At iteration $t = 3$, the decision makers to be adjusted are identified as e_8, e_{18} and e_{12}. Among them, e_{18} and e_{12} are adjusted by Equation (30) with reference to the decision matrix of the most trusted decision maker e_6 and e_5. For e_8, the trust strength with its most trusted decision maker e_6 is insufficient ($ct_{18,6} = 0.41$), so the group decision matrix is adopted as the reference for adjustment via Equation (31). After adjustment, the group consensus level is recalculated as $GCL^{(3)} = 0.8741 < 0.9$, and the iterative adjustment is continued.

The iteration process is repeated until $t = 5$. After adjustment, the group consensus level is recalculated as $GCL^{(5)} = 0.9015 > 0.9$, at which point consensus is achieved.

Step 6. Based on the group decision matrix after consensus achievement, the comprehensive scores of the alternatives are calculated by Equation (32).

$$score(x_1) = 0.4528, score(x_2) = 0.5604, score(x_3) = 0.6785, score(x_4) = 0.5190$$

The ranking of the schemes is $x_3 \succ x_2 \succ x_4 \succ x_1$, and the optimal scheme selected is x_3.

5. Simulation Experiments and Comparative Analysis

5.1. Simulation Experiments

Simulation experiments can intuitively observe the influence of parameter variations on decision results, quantify the stability of the model, and provide a reliable basis for parameter selection. Therefore, this section conducts simulation experiments on the following three aspects: 1) consensus threshold; 2) number of decision makers and matrix scale; 3) parameters ϖ and ϑ.

To ensure strict reproducibility of the experimental results, a fixed random seed of 42 is used for all simulation experiments. The data generation process is specified as follows:

1) The trust matrix of decision makers is randomly generated from a uniform distribution over the interval [0, 1].

2) The probabilistic linguistic decision matrices are randomly generated based on the standard PLTS with a granularity of 7, where the probabilities satisfy the normalization constraint.

3) Each experiment is independently repeated 100 times, and the average value is taken as the final result to reduce the impact of random fluctuations.

5.1.1. Influence of Consensus Threshold

In LSGDM problems, the consensus threshold serves as the critical criterion for measuring opinion consistency, and is a core parameter that triggers the consensus adjustment process and defines decision boundaries, which directly determines the efficiency and fairness of decision-making. A reasonable threshold can balance the demands of different groups, gain wide recognition, reduce resistance and risk of disputes in implementation, and avoid situations of minority domination or "majority tyranny". Therefore, this section conducts an in-depth analysis of the consensus threshold and investigates its sensitivity to the number of decision-making iterations. The analysis range of the threshold is set between 0.86 and 0.94 with a step size of 0.02. Figure 2 shows the average number of iterations required to reach consensus in 100 trials under different thresholds.

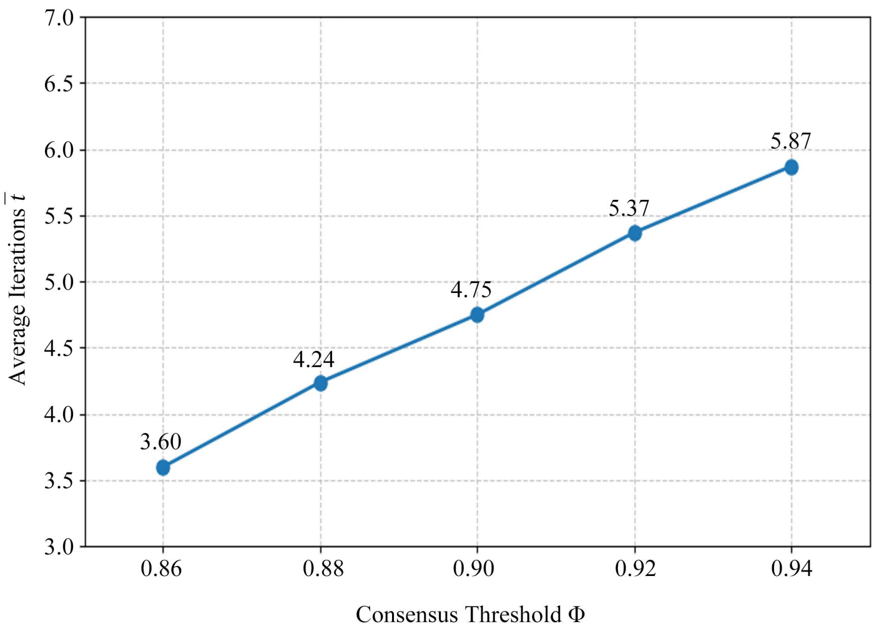


Figure 2. Influence of the consensus threshold.

The results in Figure 2 show that the average number of iterations $\bar{t}$ exhibits moderate sensitivity to the consensus threshold Φ. As shown in the figure, $\bar{t} = 3.60$ (at $\Phi = 0.86$) increases gently to $\bar{t} = 5.87$ (at $\Phi = 0.94$). It can also be observed that the average number of iterations $\bar{t}$ remains at a relatively low level within the analyzed range of the consensus threshold, indicating that the feedback adjustment mechanism of the proposed consensus reaching model is efficient and stable.

5.1.2. Influence of the Number of Decision Makers and Matrix Scale

The number of decision makers directly determines the representativeness and diversity of opinions, and the matrix scale reflects the complexity of decision information dimensions. In this section, simulation experiments are carried out on the pairwise combinations of the number of decision makers being 20, 40, 60, 80, 100 and the decision matrix scale being 4 × 4, 5 × 5, 6 × 6 respectively, and the average number of iterations required to reach consensus in 100 trials is counted, with the results shown in Figure 3.

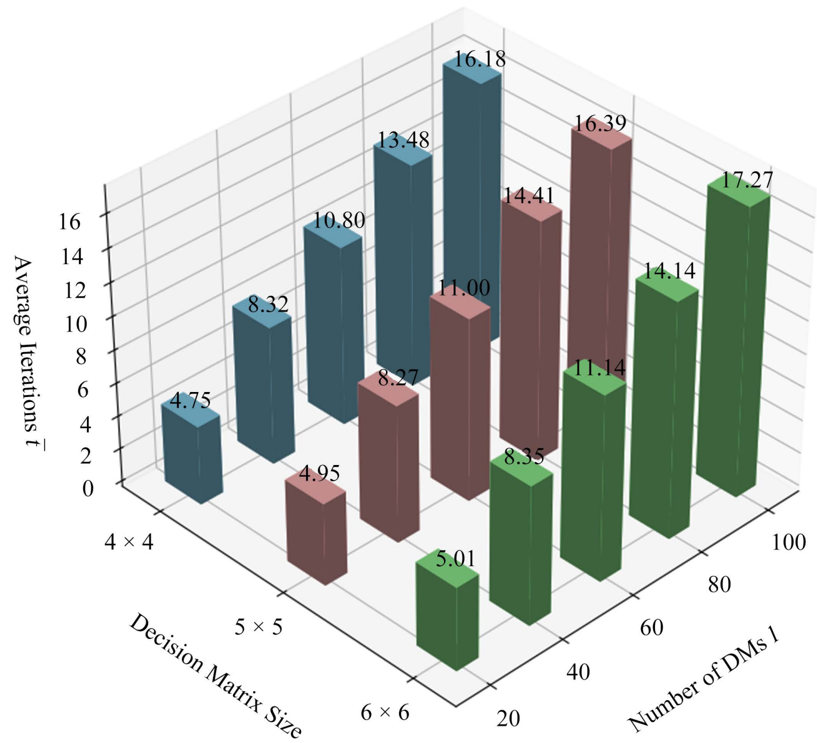


Figure 3. Influence of the number of decision makers and matrix scale.

The results in Figure 3 show that the average number of iterations $\bar{t}$ is significantly sensitive to the number of decision makers. For example, when the decision matrix scale is 4 × 4, $\bar{t} = 4.75$ ($l = 20$) increases to $\bar{t} = 16.18$ ($l = 100$) at a relatively fast rate. Similar trends are observed under other matrix scales. This may be due to the fact that the sharp increase in the number of decision makers leads to more complex differences of opinions, and the model requires multiple rounds of iterations to converge and reach consensus, thus the number of iterations rises rapidly.

On the contrary, the average number of iterations $\bar{t}$ is not sensitive to the decision matrix scale. For example, when the number of decision makers $l = 40$, $\bar{t} = 8.32$ (4×4) changes to $\bar{t} = 8.27$ (5×5) and $\bar{t} = 8.35$ (6×6). Overall, the average number of iterations $\bar{t}$ varies slightly and sometimes fluctuates under different decision matrix scales, indicating that the model can easily handle complex multi-dimensional decision information.

5.1.3. Influence of Parameters ϖ and ϑ

Analyzing the parameters in the model can quantitatively identify the influence degree of parameter changes on decision results, so as to determine excellent and robust parameters and ensure the efficiency and quality of the decision model. Therefore, in this section, when the amplification coefficient ϑ in the model ranges from 6 to 14 with a step size of 2, and the trust threshold ϖ ranges from 0.4 to 0.8 with a step size of 0.1, the average number of iterations required to reach consensus in each 100 trials is analyzed. The results are shown in Figure 4.

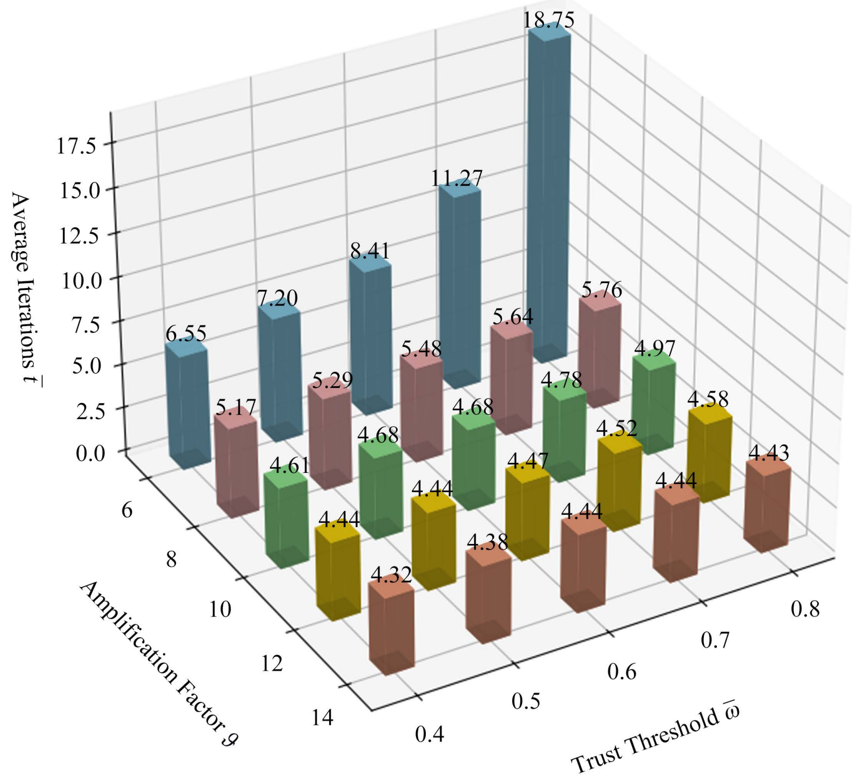


Figure 4. Influence of parameters ϖ and ϑ.

It can be seen from Figure 4 that the average number of iterations $\bar{t}$ increases with the increase of the trust threshold ϖ. This is because the increase of the trust threshold ϖ means stricter trust requirements for the reference decision makers. When the trust threshold is too high, most decision makers can only adopt strategy 2 to adjust with reference to the group decision matrix. Since the efficiency of strategy 2 is lower than that of strategy 1, the average number of iterations increases. Considering practical situations, setting an excessively low trust threshold will ignore the adjustment decision makers' willingness to trust the reference, which is inconsistent with the actual decision-making context. Meanwhile, it is observed from Figure 4 that the influence of the trust threshold ϖ on the average number of iterations $\bar{\tau}$ gradually decreases when $\varpi < 0.6$ (especially when $\vartheta = 6$). Therefore, $\varpi = 0.6$ is set in the model of this paper.

The average number of iterations $\overline{t}$ decreases with the increase of the amplification coefficient ϑ. This is because the increase of ϑ enlarges the difference between the adjusted decision maker and the reference matrix, improves the adjustment range, and accelerates the speed of consensus reaching. It is also noted that when ϑ increases to more than 10, its influence on $\overline{t}$ becomes very weak. For example, when $\varpi = 0.6$, $\overline{t}$ decreases from $\overline{t} = 5.48$ ($\vartheta = 8$) to $\overline{t} = 4.68$ ($\vartheta = 10$), with a decrease of 14.60%. While $\overline{t}$ decreases from $\overline{t} = 4.68$ ($\vartheta = 10$) to $\overline{t} = 4.47$ ($\vartheta = 12$), with a decrease of only 4.49%. Figure 5 shows the decrease of $\overline{t}$ caused by each increase of ϑ in the experiment when $\varpi = 0.6$. Therefore, $\vartheta = 10$ is set in the model of this paper.

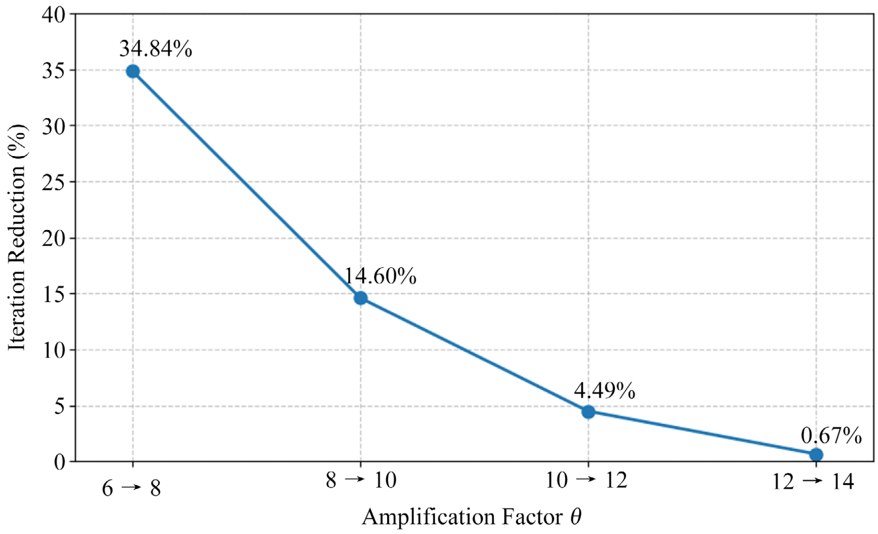


Figure 5. Percentage decrease in $\overline{t}$ caused by the increase in ϑ ($\varpi = 0.6$).

5.2. Comparative Analysis

To further demonstrate the advantages of the proposed method, this section conducts comparative analyses from two aspects: the consensus reaching method and the model theory.

5.2.1. Comparative Analysis with Other Literature on CRP

To further verify the advantages of the proposed consensus reaching method, comparisons are conducted based on the following dimensions in comparison with the existing methods [26] [28] [29].

1) NI (Number of Iterations): The number of iterations required to reach the preset consensus threshold.

2) NAD (Number of Adjusted DMs): The total number of decision makers adjusted during consensus process.

3) NAT (Number of Adjustment Times): The total adjustment times of all decision makers.

4) TD (Total Deviation): The total deviation between the initial decision matrix and the final adjusted matrix.

To ensure fairness and verifiability of the comparison, all benchmark methods involved in this paper adopt exactly the same input data, consensus threshold, and iteration stopping rule. The comparison results are shown in **Table 4**.

Table 4. Comparative analysis with CRP methods.

CRP methods	NI	NAD	NAT	TD	final GCL
Wang *et al.* [26]	11	20	175	2.3097	0.9023
Tian *et al.* [28]	5	20	76	2.3907	0.9062
Wang *et al.* [29]	5	20	34	2.3289	0.9202
The proposed method	5	14	14	2.1540	0.9015

As shown in **Table 4**, the proposed CRP method in this paper has the following advantages.

1) Under the same consensus threshold (0.9), the proposed method has the least number of iterations and the highest efficiency. It can be seen from **Table 4** that the proposed method, method [28] and method [29] all reach consensus with only 5 iterations, while method [26] requires 11 iterations. Meanwhile, the NAD and NAT of the proposed method are the lowest among the compared methods. The other three methods adjust every decision maker, some even repeatedly. This is because Wang *et al.* [26] only adjusts the element with the maximum distance of all decision makers below the threshold in each iteration, and the low efficiency of consensus reaching leads to repeated adjustments. Tian *et al.* [28] includes all experts below the consensus threshold in the adjustment scope, and the excessively large scope results in a cumbersome consensus reaching process. Wang *et al.* [29] selects all experts in the subgroup with the lowest subgroup consensus level for adjustment, with NAD and NAT being 20 and 34 respectively, indicating a similar problem but less serious. By comparison, the high efficiency of the proposed method is further verified.

2) In the consensus reaching process, the proposed method has higher accuracy and can retain more original decision information. As can be seen from **Table 4**, this method only adjusts 14 decision makers without repeated adjustments, and the total deviation (TD) between the initial matrices and the adjusted matrices is 2.5140, which is the lowest among all methods in the table. In addition, it can be observed that the final GCL of the proposed method is the smallest, which shows that the proposed method balances consensus accuracy while emphasizing efficiency, and can terminate iterations in time to avoid excessive adjustment, thus preserving more original information.

5.2.2. Theoretical Comparative Analysis with Other Literature

Based on the overall consideration of the decision-making method, the following is the theoretical comparative analysis between the proposed method and existing methods through **Table 5**, so as to highlight the advantages of the method proposed in this paper.

Table 5. Theoretical comparative analysis.

Reference	linguistic context	improved trust network	DM weights		CRP		
			consider trust	consider other factors	objective	multi-strategy	personalization
Liu *et al.* [22]	IFS	×	√	×	-	√	×
Tu *et al.* [23]	reciprocal preference	×	√	×	-	√	×
Liu *et al.* [24]	PLTS	×	√	×	-	√	×
Sun *et al.* [9]	FPR	×	√	×	-	√	√
Chen and Wang [25]	FPR	×	√	√	×	√	×
Wang *et al.* [26]	PLTS	×	√	√	×	×	×
Tian *et al.* [28]	PLTS	×	√	×	-	×	√
Wang *et al.* [29]	PLTS	×	√	×	-	×	√
Sun and Zhu [30]	LD	×	√	×	-	×	×
Han *et al.* [31]	PLTS	×	×	√	-	√	√
Ahlim *et al.* [32]	reciprocal preference	√	√	×	-	×	×
Yang and Wang [33]	IFS	√	√	×	-	√	√
Xing *et al.* [34]	2-tuple linguistic	√	√	×	-	√	√
The proposed method	PLTS	√	√	√	√	√	√

1) In real decision-making, differences in the similarity of evaluation information among decision makers will affect their trust relationships. Many scholars [32]-[34] have considered this point and used a linear combination of similarity and trust relationship to expand the initial trust network. However, these improved methods ignore the compensatory effect of linear combination, which may lead to deviation of decision-maker relationships from reality and affect clustering and decision results. The method proposed in this paper uses the similarity of decision makers to revise the initial trust network: high similarity enhances trust while low similarity weakens trust, making the relationships among decision makers more consistent with actual decision scenarios, and solving the compensation problem caused by expanding the initial trust network via linear combination.

2) The trust relationship among decision makers is an important feature in social networks. However, determining decision maker weights solely based on trust relationships may lead to collusion. Some methods [25] [26] [31] take other factors into account, such as group size and similarity, effectively remedying the one-sidedness of relying only on single trust information. Methods for objectively determining decision maker weights under multiple factors within the probabilistic linguistic term set environment have not received extensive attention. Moreover, determining decision maker weights under multiple factors by relying only on preset weight parameters [25] [26] entails strong subjectivity, which may result in unreasonable weight allocation. On the basis of integrating similarity and trust

relationships into decision maker weight calculation, this paper objectively determines the weight coefficients of the two influencing factors using the uncertainty idea of the entropy weight method, solving the shortcomings of the above studies.

3) The consensus reaching process is a crucial part in large-scale group decision-making problems. Multi-strategy adjustment [22]-[25] [31] [33] [34] means distinguishing different decision scenarios and providing refined and accurate adjustment strategies. Personalized adjustment [28] [29] [31] [33] [34] indicates that the adjustment mechanism fully considers individual differences among decision makers. Both can effectively improve the efficiency of consensus reaching and avoid insufficient or excessive adjustment caused by a single adjustment strategy. The personalized consensus reaching mechanism based on trust reference proposed in this paper realizes multi-strategy and personalized adjustment, which helps to improve the efficiency of consensus reaching.

6. Conclusions

This paper proposes a large-scale group decision-making method based on similarity-corrected trust networks, and investigates its application in alternative selection under the background of live-streaming e-commerce economy. The main contributions are threefold:

1) A method for correcting trust networks via similarity is presented. The opinion similarity between decision makers is adopted to revise the initial trust relationship, which effectively reflects the dynamic changes of trust among decision makers.

2) The entropy weight method is used to objectively determine decision makers' weights. Opinion similarity and trust degree are taken as two factors to measure decision makers' weights. The weights of these two factors are determined based on the idea of the entropy weight method, and the comprehensive weights of decision makers are obtained through weighted summation, which effectively avoids the subjectivity of setting coefficients artificially.

3) A personalized consensus reaching process based on trust reference is proposed. The reference matrix is determined by trust relationships, and personalized adjustment coefficients are designed, which takes individual differences of decision makers into account and effectively promotes consensus reaching. Finally, taking live-streaming e-commerce model selection as an example, the effectiveness and robustness of the model are verified through simulation experiments and comparative analysis.

This paper still has research limitations regarding large-scale group decision-making problems under the probabilistic linguistic term set environment, and future work mainly includes the following aspects:

1) The clustering method adopted in this paper still has certain limitations. Existing clustering is mainly based on experts' trust relationships or preference similarity, and cannot thoroughly describe dynamic interaction, information transmission paths and community structure evolution in social networks. Innovative

clustering methods based on the characteristics of social networks will be explored in future research.

2) This paper mainly conducts research and analysis on decision-making problems in the context of live-streaming e-commerce. In fact, large-scale group decision-making problems exist in various fields of daily life, such as catering and transportation. Therefore, in future research, the proposed decision-making method can be applied to different practical fields.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A: The Decision Matrices of 20 Decision-Makers

$$\boldsymbol{B}_{4\times4}^{1}=\begin{pmatrix}\{s_{-1}(0.5),s_{0}(0.5)\} & \{s_{0}(0.7),s_{1}(0.3)\} & \{s_{0}(0.35),s_{2}(0.65)\} & \{s_{-2}(0.8),s_{-1}(0.2)\}\\ \{s_{1}(0.5),s_{3}(0.5)\} & \{s_{1}(0.4),s_{2}(0.6)\} & \{s_{1}(0.9),s_{2}(0.1)\} & \{s_{-1}(0.4),s_{2}(0.6)\}\\ \{s_{0}(0.55),s_{2}(0.45)\} & \{s_{2}(0.4),s_{3}(0.6)\} & \{s_{2}(0.79),s_{3}(0.21)\} & \{s_{0}(0.6),s_{2}(0.4)\}\\ \{s_{-3}(0.2),s_{-1}(0.8)\} & \{s_{0}(0.25),s_{1}(0.75)\} & \{s_{0}(0.33),s_{2}(0.67)\} & \{s_{-3}(0.5),s_{-1}(0.5)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{2}=\begin{pmatrix}\{s_{-2}(0.6),s_{1}(0.4)\} & \{s_{1}(0.8),s_{2}(0.2)\} & \{s_{1}(0.4),s_{2}(0.6)\} & \{s_{-3}(0.7),s_{-2}(0.3)\}\\ \{s_{0}(0.6),s_{2}(0.4)\} & \{s_{0}(0.3),s_{1}(0.7)\} & \{s_{0}(0.5),s_{1}(0.5)\} & \{s_{1}(0.2),s_{2}(0.4),s_{3}(0.4)\}\\ \{s_{-1}(0.65),s_{2}(0.35)\} & \{s_{1}(0.5),s_{2}(0.5)\} & \{s_{1}(0.7),s_{2}(0.3)\} & \{s_{1}(0.6),s_{3}(0.4)\}\\ \{s_{1}(0.8),s_{2}(0.2)\} & \{s_{2}(0.4),s_{3}(0.6)\} & \{s_{0}(0.3),s_{2}(0.7)\} & \{s_{0}(0.4),s_{1}(0.6)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{3}=\begin{pmatrix}\{s_{2}(0.4),s_{3}(0.6)\} & \{s_{1}(1.0)\} & \{s_{0}(0.15),s_{1}(0.85)\} & \{s_{1}(0.6),s_{2}(0.4)\}\\ \{s_{1}(0.3),s_{2}(0.7)\} & \{s_{2}(1)\} & \{s_{0}(0.14),s_{1}(0.86)\} & \{s_{0}(0.8),s_{2}(0.2)\}\\ \{s_{-1}(1)\} & \{s_{1}(0.59),s_{2}(0.41)\} & \{s_{-1}(0.7),s_{1}(0.3)\} & \{s_{1}(0.4),s_{2}(0.6)\}\\ \{s_{0}(1)\} & \{s_{-1}(1)\} & \{s_{-1}(0.3),s_{1}(0.7)\} & \{s_{0}(0.1),s_{1}(0.9)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{4}=\begin{pmatrix}\{s_{1}(0.5),s_{2}(0.5)\} & \{s_{0}(0.23),s_{2}(0.77)\} & \{s_{-2}(0.5),s_{-1}(0.5)\} & \{s_{-3}(0.6),s_{-1}(0.4)\}\\ \{s_{0}(0.6),s_{2}(0.4)\} & \{s_{1}(0.38),s_{2}(0.62)\} & \{s_{-1}(0.37),s_{2}(0.63)\} & \{s_{0}(0.85),s_{1}(0.15)\}\\ \{s_{2}(0.34),s_{3}(0.66)\} & \{s_{1}(0.3),s_{2}(0.7)\} & \{s_{1}(0.5),s_{2}(0.5)\} & \{s_{0}(0.3),s_{2}(0.7)\}\\ \{s_{0}(0.65),s_{1}(0.35)\} & \{s_{0}(0.8),s_{2}(0.2)\} & \{s_{-2}(0.5),s_{0}(0.5)\} & \{s_{-2}(0.6),s_{-1}(0.4)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{5}=\begin{pmatrix}\{s_{0}(0.3),s_{2}(0.7)\} & \{s_{0}(0.32),s_{1}(0.68)\} & \{s_{1}(0.6),s_{2}(0.4)\} & \{s_{1}(0.67),s_{3}(0.33)\}\\ \{s_{-1}(0.3),s_{0}(0.7)\} & \{s_{0}(0.8),s_{1}(0.2)\} & \{s_{-1}(0.67),s_{1}(0.33)\} & \{s_{-2}(0.5),s_{-1}(0.5)\}\\ \{s_{0}(0.7),s_{1}(0.3)\} & \{s_{1}(0.75),s_{2}(0.25)\} & \{s_{0}(0.67),s_{2}(0.33)\} & \{s_{1}(0.5),s_{3}(0.5)\}\\ \{s_{-2}(0.8),s_{-1}(0.2)\} & \{s_{-1}(0.4),s_{0}(0.6)\} & \{s_{0}(0.34),s_{1}(0.66)\} & \{s_{-3}(0.7),s_{-1}(0.3)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{6}=\begin{pmatrix}\{s_{-1}(0.35),s_{0}(0.65)\} & \{s_{-2}(0.7),s_{-1}(0.3)\} & \{s_{-2}(0.5),s_{-1}(0.5)\} & \{s_{-1}(0.7),s_{1}(0.3)\}\\ \{s_{-2}(0.3),s_{0}(0.7)\} & \{s_{-2}(0.65),s_{0}(0.35)\} & \{s_{-2}(0.3),s_{-1}(0.4),s_{0}(0.3)\} & \{s_{-2}(0.34),s_{0}(0.66)\}\\ \{s_{1}(0.4),s_{2}(0.6)\} & \{s_{0}(0.7),s_{2}(0.3)\} & \{s_{1}(0.2),s_{2}(0.8)\} & \{s_{1}(0.3),s_{3}(0.7)\}\\ \{s_{2}(0.7),s_{3}(0.3)\} & \{s_{2}(0.7),s_{3}(0.3)\} & \{s_{1}(0.65),s_{2}(0.35)\} & \{s_{1}(0.7),s_{3}(0.3)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{7}=\begin{pmatrix}\{s_{-2}(0.9),s_{-1}(0.1)\} & \{s_{0}(0.7),s_{1}(0.3)\} & \{s_{-2}(0.7),s_{-1}(0.3)\} & \{s_{-2}(0.53),s_{1}(0.47)\}\\ \{s_{1}(0.67),s_{2}(0.33)\} & \{s_{0}(0.2),s_{1}(0.8)\} & \{s_{-1}(0.7),s_{1}(0.3)\} & \{s_{0}(0.8),s_{2}(0.2)\}\\ \{s_{2}(0.74),s_{3}(0.26)\} & \{s_{1}(0.7),s_{2}(0.3)\} & \{s_{1}(0.5),s_{2}(0.5)\} & \{s_{0}(0.3),s_{2}(0.7)\}\\ \{s_{0}(0.2),s_{1}(0.8)\} & \{s_{0}(0.5),s_{2}(0.5)\} & \{s_{0}(0.5),s_{1}(0.5)\} & \{s_{1}(0.7),s_{2}(0.3)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{8}=\begin{pmatrix}\{s_{-2}(0.4),s_{-1}(0.3)\} & \{s_{0}(0.7),s_{1}(0.3),s_{0}(0.3)\} & \{s_{-3}(0.3),s_{-2}(0.7)\} & \{s_{-1}(0.75),s_{0}(0.25)\}\\ \{s_{-1}(0.4),s_{0}(0.6)\} & \{s_{-2}(0.8),s_{0}(0.2)\} & \{s_{-1}(0.3),s_{0}(0.7)\} & \{s_{-2}(0.4),s_{-1}(0.6)\}\\ \{s_{0}(0.4),s_{1}(0.6)\} & \{s_{0}(0.2),s_{2}(0.8)\} & \{s_{1}(0.67),s_{2}(0.33)\} & \{s_{2}(0.5),s_{3}(0.5)\}\\ \{s_{1}(0.77),s_{2}(0.23)\} & \{s_{0}(0.29),s_{2}(0.71)\} & \{s_{1}(0.35),s_{2}(0.65)\} & \{s_{2}(0.5),s_{3}(0.5)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{9}=\begin{pmatrix}\{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_0(1.0)\} & \{s_{-1}(1)\} & \{s_{-1}(0.7),s_0(0.3)\}\\ \{s_0(0.3),s_1(0.7)\} & \{s_1(1.0)\} & \{s_{-1}(0.2),s_0(0.8)\} & \{s_0(0.2),s_1(0.8)\}\\ \{s_1(0.12),s_2(0.88)\} & \{s_0(1)\} & \{s_1(1)\} & \{s_2(0.6),s_3(0.4)\}\\ \{s_0(0.25),s_1(0.75)\} & \{s_1(1.0)\} & \{s_{-3}(0.3),s_{-1}(0.7)\} & \{s_{-1}(0.9),s_0(0.1)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{10}=\begin{pmatrix}\{s_{-1}(0.6),s_0(0.4)\} & \{s_{-3}(0.8),s_{-1}(0.2)\} & \{s_{-3}(0.5),s_{-2}(0.5)\} & \{s_0(0.36),s_2(0.64)\}\\ \{s_0(0.25),s_1(0.75)\} & \{s_0(0.8),s_2(0.2)\} & \{s_0(0.8),s_1(0.2)\} & \{s_0(0.7),s_1(0.3)\}\\ \{s_0(0.4),s_1(0.6)\} & \{s_2(0.2),s_3(0.8)\} & \{s_0(0.6),s_1(0.4)\} & \{s_1(0.8),s_2(0.2)\}\\ \{s_{-1}(0.1),s_0(0.9)\} & \{s_{-1}(0.67),s_0(0.34)\} & \{s_0(0.3),s_2(0.7)\} & \{s_{-1}(0.53),s_0(0.47)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{11}=\begin{pmatrix}\{s_{-1}(1)\} & \{s_{-1}(0.8),s_0(0.2)\} & \{s_{-3}(1.0)\} & \{s_{-2}(0.7),s_0(0.3)\}\\ \{s_2(1.0)\} & \{s_1(0.8),s_2(0.2)\} & \{s_1(0.3),s_2(0.7)\} & \{s_1(0.6),s_2(0.4)\}\\ \{s_0(1)\} & \{s_2(1)\} & \{s_0(0.3),s_2(0.7)\} & \{s_0(0.15),s_1(0.85)\}\\ \{s_{-1}(1.0)\} & \{s_0(0.75),s_1(0.25)\} & \{s_0(0.3),s_1(0.7)\} & \{s_{-1}(0.9),s_0(0.1)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{12}=\begin{pmatrix}\{s_1(0.2),s_2(0.8)\} & \{s_0(0.7),s_1(0.3)\} & \{s_0(0.73),s_1(0.27)\} & \{s_1(0.5),s_2(0.5)\}\\ \{s_1(0.72),s_2(0.28)\} & \{s_{-1}(0.7),s_0(0.3)\} & \{s_{-1}(0.34),s_0(0.66)\} & \{s_0(0.3),s_1(0.7)\}\\ \{s_0(0.63),s_1(0.37)\} & \{s_1(0.3),s_2(0.7)\} & \{s_0(0.8),s_1(0.2)\} & \{s_0(0.4),s_1(0.6)\}\\ \{s_{-2}(0.67),s_{-1}(0.33)\} & \{s_{-2}(0.33),s_0(0.67)\} & \{s_{-2}(0.8),s_{-1}(0.2)\} & \{s_{-2}(0.5),s_{-1}(0.5)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{13}=\begin{pmatrix}\{s_1(0.7),s_2(0.3)\} & \{s_1(0.7),s_2(0.3)\} & \{s_0(0.4),s_1(0.6)\} & \{s_1(0.2),s_2(0.8)\}\\ \{s_0(0.3),s_2(0.7)\} & \{s_2(1.0)\} & \{s_0(0.8),s_1(0.2)\} & \{s_0(0.8),s_1(0.2)\}\\ \{s_{-1}(1)\} & \{s_{-2}(1)\} & \{s_{-3}(0.8),s_{-1}(0.2)\} & \{s_{-3}(0.25),s_{-2}(0.75)\}\\ \{s_{-1}(0.25),s_0(0.75)\} & \{s_1(1.0)\} & \{s_0(0.25),s_1(0.75)\} & \{s_{-2}(0.2),s_0(0.8)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{14}=\begin{pmatrix}\{s_{-2}(0.7),s_{-1}(0.3)\} & \{s_0(1.0)\} & \{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_{-2}(0.8),s_0(0.2)\}\\ \{s_0(0.7),s_2(0.3)\} & \{s_1(1.0)\} & \{s_{-2}(0.8),s_{-1}(0.2)\} & \{s_0(0.7),s_2(0.3)\}\\ \{s_1(0.75),s_2(0.25)\} & \{s_1(1)\} & \{s_2(0.6),s_3(0.4)\} & \{s_2(0.8),s_3(0.2)\}\\ \{s_{-2}(0.2),s_{-1}(0.8)\} & \{s_1(1.0)\} & \{s_{-1}(0.75),s_0(0.25)\} & \{s_0(0.3),s_1(0.7)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{15}=\begin{pmatrix}\{s_1(1)\} & \{s_{-1}(0.3),s_0(0.7)\} & \{s_0(0.6),s_1(0.4)\} & \{s_{-3}(0.6),s_{-1}(0.4)\}\\ \{s_0(0.66),s_2(0.34)\} & \{s_0(0.21),s_1(0.79)\} & \{s_{-1}(0.7),s_0(0.3)\} & \{s_2(0.25),s_3(0.75)\}\\ \{s_0(0.2),s_1(0.8)\} & \{s_2(1)\} & \{s_0(0.35),s_1(0.65)\} & \{s_1(0.3),s_2(0.7)\}\\ \{s_{-1}(0.8),s_0(0.2)\} & \{s_0(0.9),s_2(0.1)\} & \{s_{-1}(0.3),s_0(0.7)\} & \{s_{-2}(0.1),s_{-1}(0.9)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{16}=\begin{pmatrix}\{s_{-2}(0.8),s_0(0.2)\} & \{s_{-1}(1)\} & \{s_{-2}(0.8),s_0(0.2)\} & \{s_{-2}(0.6),s_{-1}(0.4)\}\\ \{s_2(1.0)\} & \{s_{-1}(0.8),s_1(0.2)\} & \{s_0(0.7),s_1(0.3)\} & \{s_0(0.1),s_1(0.9)\}\\ \{s_1(1.0)\} & \{s_3(1)\} & \{s_{-1}(0.7),s_0(0.3)\} & \{s_1(0.85),s_2(0.15)\}\\ \{s_{-1}(0.7),s_0(0.3)\} & \{s_1(1.0)\} & \{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_{-1}(0.25),s_0(0.75)\}\end{pmatrix}$$

$$\boldsymbol{B}_{4\times4}^{17}=\begin{pmatrix}\{s_{-1}(0.4),s_0(0.6)\} & \{s_{-1}(0.7),s_0(0.3)\} & \{s_{-2}(0.66),s_{-1}(0.34)\} & \{s_{-1}(0.4),s_1(0.6)\}\\ \{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_{-2}(0.4),s_{-1}(0.6)\} & \{s_{-1}(0.5),s_0(0.5)\} & \{s_{-2}(0.5),s_0(0.5)\}\\ \{s_0(0.2),s_2(0.8)\} & \{s_0(0.7),s_1(0.3)\} & \{s_0(0.6),s_2(0.4)\} & \{s_1(0.7),s_2(0.3)\}\\ \{s_0(0.66),s_2(0.34)\} & \{s_0(0.2),s_1(0.8)\} & \{s_0(0.4),s_1(0.6)\} & \{s_2(1)\}\end{pmatrix}$$

$$B_{4\times4}^{18}=\begin{pmatrix}\{s_1(0.2),s_2(0.8)\} & \{s_0(0.7),s_1(0.3)\} & \{s_0(0.6),s_1(0.4)\} & \{s_2(0.6),s_3(0.4)\}\\ \{s_{-1}(0.2),s_0(0.8)\} & \{s_{-1}(0.1),s_0(0.9)\} & \{s_{-2}(0.4),s_{-1}(0.6)\} & \{s_{-1}(0.7),s_0(0.3)\}\\ \{s_0(0.7),s_1(0.3)\} & \{s_0(0.7),s_1(0.3)\} & \{s_1(0.8),s_2(0.2)\} & \{s_2(0.6),s_3(0.4)\}\\ \{s_{-2}(0.4),s_{-1}(0.6)\} & \{s_{-1}(0.3),s_0(0.7)\} & \{s_{-1}(0.28),s_0(0.72)\} & \{s_{-2}(0.4),s_{-1}(0.6)\}\end{pmatrix}$$

$$B_{4\times4}^{19}=\begin{pmatrix}\{s_1(0.8),s_2(0.2)\} & \{s_1(1.0)\} & \{s_3(1)\} & \{s_{-1}(0.2),s_0(0.8)\}\\ \{s_0(0.7),s_1(0.3)\} & \{s_1(1)\} & \{s_{-2}(0.2),s_0(0.8)\} & \{s_1(0.2),s_2(0.8)\}\\ \{s_0(0.2),s_1(0.8)\} & \{s_{-1}(1.0)\} & \{s_{-3}(0.23),s_{-2}(0.77)\} & \{s_{-1}(0.8),s_0(0.2)\}\\ \{s_0(1.0)\} & \{s_{-2}(1.0)\} & \{s_0(0.7),s_1(0.3)\} & \{s_1(0.8),s_2(0.2)\}\end{pmatrix}$$

$$B_{4\times4}^{20}=\begin{pmatrix}\{s_{-2}(0.6),s_0(0.4)\} & \{s_0(0.8),s_1(0.2)\} & \{s_1(0.4),s_3(0.6)\} & \{s_{-2}(0.6),s_{-1}(0.4)\}\\ \{s_0(0.7),s_1(0.3)\} & \{s_0(0.2),s_1(0.8)\} & \{s_{-2}(0.3),s_{-1}(0.7)\} & \{s_0(0.2),s_1(0.6),s_2(0.2)\}\\ \{s_0(0.6),s_1(0.4)\} & \{s_1(0.8),s_2(0.2)\} & \{s_1(0.5),s_2(0.5)\} & \{s_0(0.4),s_2(0.6)\}\\ \{s_1(0.85),s_2(0.15)\} & \{s_0(0.7),s_1(0.3)\} & \{s_{-3}(0.7),s_{-2}(0.3)\} & \{s_0(0.8),s_1(0.2)\}\end{pmatrix}$$

Appendix B: The Initial Social Trust Network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0.1	0.7	0.1	0.5	0.7	0.8	0.5	0.5	0.1	0.4	0.4	0.9	0.5	0.8	0.4	0.2	0.9	0.8	0.4
2	0.1	0	0.5	0.5	0.8	0.6	0.5	0.6	0.4	0.1	0.1	0.9	0.8	0.3	0.9	0.5	0.1	0.9	0.6	0.9
3	0.7	0.1	0	0.6	0.8	0.9	0.3	0.5	0.7	0.2	0.8	0.8	0.1	0.6	0.4	0.2	0.6	1	0.6	0.1
4	0.5	0.8	1	0	0.7	0.1	1	0.6	0.1	0.1	0.1	0.7	0.2	0.8	0.1	0.5	0.4	0.1	0.1	0.9
5	1	0.1	0.7	1	0	0.9	0.9	0.3	0.7	0.5	0.9	0.7	0.8	0.9	0.4	0.7	0.1	0.2	0.9	0.4
6	0.7	0.6	0.3	0.6	0.9	0	0.3	0.3	0.2	0.2	0.8	0.7	1	0.7	1	0.2	0.2	0.2	0.4	1
7	0.2	0.5	0.1	0.1	0.5	0.6	0	0.5	0.9	0.9	1	0.6	0.3	0.1	0.8	0.2	0.1	0.7	0.1	0.6
8	0.2	0.4	0.1	0.4	0.3	0.3	0.1	0	0.6	0.7	0.2	0.1	0.3	0.7	0.3	0.6	0.1	0.7	0.1	0.2
9	0.7	0.5	0.6	0.3	0.7	0.3	0.5	0.4	0	0.9	0.8	0.5	0.8	0.6	0.7	0.5	0.6	0.2	0.9	0.6
10	0.8	0.6	0.1	0.1	0.9	0.7	0.9	0.9	0.2	0	1	0.6	0.7	0.5	0.3	0.5	0.5	0.3	0.3	0.4
11	0.3	0.1	0.8	0.4	0.4	0.8	0.7	0.4	0.3	0.8	0	0.2	0.5	0.4	1	0.2	0.6	0.5	0.1	0.2
12	0.9	0.8	0.2	0.2	0.9	0.6	0.7	0.9	0.3	0.5	0.9	0	0.2	0.3	0.9	0.8	0.5	0.1	0.8	0.3
13	0.8	0.3	0.2	0.7	0.4	0.4	0.4	0.5	0.5	0.6	0.1	0.7	0	0.5	0.8	0.3	0.7	0.2	0.7	0.7
14	0.5	0.5	1	0.4	0.1	0.3	0.1	0.9	0.8	0.2	0.6	0.2	0.9	0	0.4	0.1	0.6	0.1	0.9	0.9
15	0.6	0.2	0.8	0.1	0.1	0.3	0.1	0.9	0.7	0.8	0.7	0.2	0.3	0.6	0	0.8	0.6	0.3	0.8	0.4
16	0.6	0.8	0.7	0.5	0.6	0.2	0.4	0.8	0.5	0.1	0.8	0.2	0.1	0.6	0.8	0	0.1	0.4	0.5	0.1
17	0.6	0.3	0.7	0.5	0.5	0.2	0.8	0.1	0.3	0.1	0.1	0.1	0.5	0.1	0.1	0.4	0	0.7	0.2	0.7
18	0.6	0.9	0.8	1	0.4	0.9	0.3	0.3	0.4	0.1	0.7	0.1	0.6	0.1	0.4	0.3	0.9	0	0.3	0.4
19	0.3	0.1	0.6	0.3	0.3	0.3	0.4	0.8	0.2	0.1	0.3	0.6	0.3	0.1	0.4	1	0.9	0.4	0	0.4
20	0.6	0.3	0.2	0.6	0.5	1	1	0.3	0.1	0.2	0.4	0.4	0.4	0.1	0.3	0.4	0.8	0.1	0.6	0

Appendix C: The Corrected Social Trust Network

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0.1	0.7	0.1	0.5	0.68	0.8	0.5	0.5	0.1	0.42	0.4	0.87	0.5	0.9	0.4	0.2	0.9	0.8	0.4
2	0.1	0	0.5	0.5	0.8	0.6	0.5	0.6	0.4	0.1	0.1	0.9	0.78	0.3	0.92	0.5	0.1	0.89	0.6	0.9
3	0.7	0.1	0	0.6	0.8	0.86	0.3	0.49	0.7	0.2	0.8	0.82	0.1	0.59	0.4	0.2	0.6	1	0.6	0.1
4	0.5	0.8	1	0	0.7	0.1	1	0.6	0.11	0.1	0.1	0.7	0.19	0.8	0.11	0.5	0.4	0.1	0.1	0.9
5	1	0.1	0.7	1	0	0.89	0.9	0.3	0.7	0.5	0.9	0.83	0.8	0.9	0.4	0.7	0.1	0.3	0.9	0.4
6	0.68	0.6	0.29	0.6	0.89	0	0.32	0.41	0.2	0.2	0.8	0.67	0.88	0.7	1	0.2	0.28	0.2	0.37	1
7	0.2	0.5	0.1	0.1	0.5	0.64	0	0.57	1	0.91	1	0.6	0.28	0.13	0.8	0.22	0.11	0.7	0.1	0.6
8	0.2	0.4	0.1	0.4	0.3	0.41	0.11	0	0.6	0.7	0.2	0.1	0.27	0.7	0.3	0.6	0.14	0.7	0.09	0.2
9	0.7	0.5	0.6	0.33	0.7	0.3	0.66	0.4	0	0.9	0.8	0.5	0.78	0.76	0.73	0.55	0.6	0.2	0.9	0.68
10	0.8	0.6	0.1	0.1	0.9	0.7	0.91	0.9	0.2	0	1	0.6	0.68	0.5	0.3	0.52	0.5	0.3	0.29	0.4
11	0.32	0.1	0.8	0.4	0.4	0.8	0.71	0.4	0.3	0.93	0	0.2	0.48	0.4	1	0.23	0.6	0.5	0.09	0.2
12	0.9	0.8	0.2	0.2	1	0.57	0.7	0.87	0.3	0.5	0.9	0	0.2	0.3	0.97	0.8	0.5	0.13	0.8	0.3
13	0.78	0.29	0.2	0.66	0.4	0.35	0.38	0.45	0.49	0.59	0.1	0.7	0	0.45	0.8	0.29	0.67	0.2	0.7	0.67
14	0.5	0.5	0.99	0.4	0.1	0.3	0.13	0.9	1	0.2	0.6	0.2	0.82	0	0.41	0.1	0.6	0.1	0.86	0.9
15	0.68	0.2	0.8	0.11	0.1	0.3	0.1	0.9	0.73	0.8	0.7	0.22	0.3	0.61	0	0.81	0.6	0.3	0.8	0.4
16	0.6	0.8	0.7	0.5	0.6	0.2	0.44	0.8	0.55	0.1	0.94	0.2	0.1	0.63	0.81	0	0.1	0.4	0.49	0.1
17	0.6	0.3	0.7	0.5	0.5	0.28	0.89	0.14	0.3	0.1	0.1	0.1	0.48	0.1	0.1	0.4	0	0.7	0.2	0.7
18	0.6	0.89	0.8	1	0.61	0.9	0.3	0.3	0.4	0.1	0.7	0.13	0.6	0.1	0.4	0.3	0.9	0	0.3	0.4
19	0.3	0.1	0.6	0.3	0.3	0.27	0.39	0.73	0.2	0.1	0.28	0.6	0.3	0.1	0.4	0.97	0.9	0.4	0	0.4
20	0.2	0.7	0.7	0.3	0.6	0.1	0.3	0.3	0.91	0.6	0.5	0.4	0.57	0.5	0.3	0.8	0.8	0.4	0.4	0