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Marketing Mix Optimization in Nigeria's Brewing Industry: A Regression and Geometric Programming Approach (Case Study of Nigerian Breweries PLC)

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Abstract

This study investigates the impact of marketing mix elements—Product, Price, Promotion, and Place (4Ps)—on the revenue and profit of Nigerian Breweries Plc (NBL) from 2013 to 2022, using secondary data. Regression analysis was employed to assess the relationship between the 4Ps and revenue, while geometric programming optimized the marketing mix for profit maximization. Results indicate that Product and Promotion positively influence revenue, whereas Price has a negative effect. Geometric programming revealed the optimal contributions to profit: Product (99.80%), Price (0.09%), Promotion (0.06%), and Place (0.05%). These findings highlight the importance of balancing revenue-driven and profit-oriented strategies, emphasizing that distribution capacity plays a key role in maximizing the effectiveness of other marketing mix elements. Distribution limitations can hinder the impact of production, promotion, and pricing. Therefore, NBL and similar industries should align production with distribution capabilities, tailor promotions to specific distribution channels, and factor distribution costs into pricing strategies. This study contributes to marketing literature by providing empirical evidence on optimizing the marketing mix to enhance revenue and profitability in Nigeria's brewing industry.

Keywords

Marketing Mix Optimization, Regression Analysis, Geometric Programming, Nigerian Breweries Plc, Revenue, Profit

1. Introduction

Nigerian Breweries Plc is Nigeria's pioneering and largest brewing company, incorporated in 1946. The first bottle of its flagship STAR lager beer was produced in 1949, and the company's growth led to the commissioning of a second brewery in Aba in 1957. Following Nigeria's Companies and Allied Matters Act in 1990, the company adopted the name "Nigerian Breweries Plc" to reflect its public limited liability status. Today, Nigerian Breweries operates nine breweries and two malting plants in Aba and Kaduna, with a wide distribution network throughout Nigeria. Its portfolio includes approximately 25 high-quality brands, such as Star lager, Gulder, Maltina, Heineken, Fayrouz, and more recent additions like the Zagg malt-infused energy drink launched in 2022. The company also has an active export business, reaching markets in the UK, Netherlands, U.S., Canada, and regions in Africa, the Middle East, and Asia since 1986.

Beyond brewing, Nigerian Breweries fosters related industries, supporting the production of essential materials like bottles, cans, and packaging and providing opportunities in distribution, marketing, and hospitality sectors. Publicly listed on the Nigerian Exchange Limited (NGX) since 1973, Nigerian Breweries had a market capitalization of ₦337 billion as of December 2022. It has received multiple awards, including recognitions from the NGX for compliance and corporate governance, as well as honors for product quality, marketing excellence, and corporate social responsibility [1]. Despite Nigerian Breweries' impressive history and performance, the relationship between production cost, promotion cost, distribution cost, and price on revenue and profit remains unclear. This ambiguity hinders optimal resource allocation and revenue maximization. The study of the marketing mix at NBL is crucial, given the company's longstanding influence on Nigeria's beverage industry, its economy, and the ancillary sectors it supports. Understanding these dynamics will enable NBL to make data-driven decisions (such as production, promotion, distribution, and pricing strategies) that enhance performance and market share.

The primary objective of this study is to investigate the relationship between production costs, promotion costs, distribution costs, and price with total revenue and/or profit at NBL, employing both multiple regression analysis and geometric programming.

The benefits of studying the marketing mix using Regression analysis and Geometric programming are as follows:

- Multiple regression analysis examines each marketing mix component (product, price, promotion, and place) as an independent variable influencing revenue. This approach provides detailed insights into how each element contributes to overall performance and reveals any interaction effects among variables.
- Geometric programming identifies the most efficient allocation of resources across marketing activities. By setting objectives, such as maximizing revenue or minimizing costs, this method determines the optimal spending levels for

each marketing mix element, guiding strategic decision-making for financial success.

- Marketing effects often exhibit non-linear dynamics, such as diminishing returns on advertising spend. While regression analysis may not fully capture such complexities, geometric programming models non-linear relationships flexibly, offering deeper insights into the varying returns on marketing investments.
- By combining regression analysis to identify key influencers and geometric programming to optimize them, this dual approach provides a nuanced framework for decision-making. It is particularly effective in competitive and dynamic markets, such as Nigeria's beverage industry, where balancing multiple objectives is essential.
- The combined methodology enables scenario testing, allowing NBL to predict how changes in marketing variables impact revenue. This approach equips the company with strategic flexibility to adapt to internal goals or external factors, such as economic shifts or evolving consumer preferences.

This study parallels those cited in the literature review by focusing on the brewing industry [2]-[4] and examining marketing mix elements [5] [6] and other related methodology [7]. However, it diverges from these studies in key ways: it focuses specifically on the Nigerian brewing industry, whereas some of the cited studies explore other African countries such as South Africa, Kenya, and Ethiopia. Additionally, while other studies often examine individual aspects like branding, pricing, or digital marketing, this study provides a comprehensive analysis of the 4Ps. The study also introduces a unique methodological approach by combining multiple regression and geometric programming, which was not employed in the cited studies. This dual approach offers both predictive insights and optimization capabilities, enabling Nigerian Breweries Plc (NBL) to enhance its marketing mix for maximum impact and efficiency. Covering the period from 2013 to 2022, the study also accounts for significant economic shifts and evolving consumer behavior, providing NBL with actionable strategies for long-term adaptability and growth. In sum, the focus on the 4Ps' influence on revenue and profit, combined with advanced analytical techniques, underscores this study's importance for academic research and practical application within NBL and related organizations.

The study is organized into six sections: introduction, literature review, methodology, data analysis, findings and discussion, and conclusion.

2. Literature Review

In this section, we present an overview of the marketing mix, including its components, relevant theories, and key empirical studies. Following this, we delve into various research efforts focusing on the marketing mix within the brewing industry. This discussion highlights areas of convergence and divergence among studies, ultimately identifying a research gap that this study seeks to address.

2.1. Marketing Mix

1) Product

A product encompasses anything offered to a market to satisfy a want or need. This includes physical goods, services, experiences, events, persons, places, organizations, ideas, or properties [8]. The key components of a product are:

- Quality: Ensuring the product meets or exceeds customer expectations [8].
- Design: Combining aesthetics, functionality, and user experience [9].
- Features: Incorporating attributes that differentiate the product [8].
- Branding: Establishing identity through names, logos, and other symbols [10].
- Life Cycle: Recognizing the stages of introduction, growth, maturity, and decline [8].
- Variety: Managing product lines and portfolios [11].

Theories of Product

Three major theories provide insights into product dynamics:

a) Product Life Cycle (PLC) Theory: Suggests that products evolve through stages—introduction, growth, maturity, and decline [8].

b) Feature-Based Segmentation Theory: Proposes that products are designed to meet specific customer needs [5].

c) Brand Equity Theory: Highlights the importance of strong branding in fostering customer loyalty and differentiation [12].

Empirical Evidence

Empirical evidence underscores the role of product-related factors in consumer behavior:

- The impact of product quality on customer loyalty in Nigeria's telecommunications industry was examined by [13]. Using a survey research design and analyzing responses from 300 customers with descriptive and inferential statistics, the study found that product quality significantly influences customer loyalty ($\beta = 0.73$, $p < 0.01$). Customer satisfaction was identified as a mediating factor in this relationship.
- Olaleke investigated the influence of brand identity and image on consumer purchasing decisions [14]. Employing structural equation modeling (SEM) and data from 400 respondents, they revealed that brand identity ($\beta = 0.56$, $p < 0.01$) and brand image ($\beta = 0.43$, $p < 0.05$) are significant determinants of consumer purchase behavior.

2) Place

Place or distribution refers to the processes involved in making a product or service accessible to the target market through various channels, intermediaries, and logistics systems [15]. The key components of distribution include:

- Channel Strategy: Decisions regarding nature of distribution channels [15].
- Logistics: Management of transportation, storage, and inventory [16].
- Market Coverage: Strategies for intensive, selective, or exclusive distribution [8].
- Channel Partners: Collaboration with wholesalers, retailers, or distributors

[15].

- E-Distribution: Leveraging online channels and digital distribution methods [8].

Theories of Distribution

Several theories provide a framework for understanding distribution dynamics:

a) Channel Management Theory: Posits that effective distribution channels enhance product availability [15].

b) Logistics Management Theory: Suggests that efficient logistics lead to improved customer satisfaction and streamlined operations [16].

Marketing Coverage Theory: Highlights that distribution strategies are shaped by the size and complexity of the target market [8].

Empirical Evidence

- Oyedijo in a study titled *"Distribution Channels and Consumer Purchasing Decisions in the Fast-Moving Consumer Goods Sector in Nigeria*" [17], employed a survey research design involving 300 respondents. Using logistic regression analysis, the study revealed that:
  - Distribution channel convenience significantly influences consumer purchasing decisions (OR = 2.53, $p < 0.01$).
  - Distribution channel availability also plays a significant role (OR = 1.83, $p < 0.05$).
- A study on *"Logistics Management and Supply Chain Performance in the Manufacturing Industry in Nigeria"* utilized a case study research design involving five manufacturing firms [18]. Using thematic analysis, the findings showed that effective logistics management reduces lead times and enhances supply chain reliability.

3) Price

Price represents the amount of money customers must pay to acquire a product or service [8]. It plays a critical role in the marketing mix and encompasses several key features:

- Pricing Strategy: Approaches such as penetration pricing, skimming, or competitive pricing [19].
- Price Elasticity: The sensitivity of demand to price changes [8].
- Discounts: Includes promotional pricing, volume discounts, and loyalty discounts [19].
- Value-Based Pricing: Pricing is determined by the perceived value of the product to the customer [20].
- Price Positioning: Pricing relative to competitors to influence market positioning [8].

Theories of Price

Three prominent theories guide the understanding of pricing:

a) Price Elasticity Theory: Explains that changes in price significantly impact demand [19].

b) Value-Based Pricing Theory: Suggests that prices should reflect the product's perceived value to customers [20].

c) Pricing Strategy Theory: Highlights how pricing strategies affect customer behavior and revenue generation [8].

Empirical Evidence

- A study on *"Pricing Strategies and Consumer Purchasing Decisions in the Retail Sector in Nigeria"* was done using a survey research design with 250 respondents and utilized multiple regression analysis [21]. The findings revealed that:
  - Price discounts significantly influenced consumer purchasing decisions ($\beta = 0.61$, $p < 0.01$).
  - Price promotions also had a notable effect ($\beta = 0.45$, $p < 0.05$).
- Again, a study on *"Price Elasticity and Demand for Petroleum Products in Nigeria"* was done using time series data (2009-2018) with Autoregressive Distributed Lag (ARDL) model [22]. The results showed that the price elasticity of demand for petroleum products was -0.83, indicating highly elastic demand.

4) Promotion

Promotion encompasses the activities and strategies used to communicate the value and benefits of a product, service, or idea to potential customers, aiming to influence their purchasing decisions and behaviors [23]. The components of promotion include:

- Advertising: Paid, non-personal communication to inform and persuade target audiences.
- Sales Promotion: Short-term incentives to encourage purchases.
- Public Relations: Activities aimed at maintaining a positive image and fostering goodwill.
- Personal Selling: Direct interactions between sales representatives and customers.
- Digital Marketing: Online platforms for promoting products, such as social media and email marketing.
- Event Marketing: Sponsorships or events to engage customers.
- Experiential Marketing: Creating memorable experiences for consumers.

5) Theories of Promotion

a) Promotion Mix Theory: Combines advertising, sales promotion, public relations, and personal selling as key elements of the promotion strategy [8].

b) Advertising Effectiveness Theory: Highlights how advertising shapes customer attitudes and behaviors [24].

c) Diffusion of Innovations Theory: Explains how innovations spread through promotion and effective communication [25].

6) Empirical Evidence

- A study on the *"Impact of Advertising on Consumer Purchasing Decisions in the Telecommunications Industry in Nigeria"*, was done using a survey research design with 250 respondents and analyzed data using multiple regression, [26]. The findings revealed:

- Advertising awareness significantly influenced consumer purchasing decisions ($\beta = 0.67$, $p < 0.01$).
- Advertising appeal also had a notable impact ($\beta = 0.51$, $p < 0.05$).

- In the same vein [27], a study on *"Effectiveness of Sales Promotion on Consumer Behavior in the Retail Sector in Nigeria"* employed a completely randomized experimental design with 100 participants, divided into four treatment groups (25 respondents each). Using ANOVA for analysis, they found:
  - Sales promotions significantly increased consumer purchasing intentions ($p < 0.01$).
- The *S-Shape of the Sales-Response Curve* as proposed by [28] was investigated [29]. The study utilized secondary data from 18 business organizations over 10 years, and analyzed sales revenue and advertising expenditure through regression and calculus. Findings indicated that: sales-response curve was not S-shaped but rather concave upwards, with sales revenue being partially linearly and partially inversely related to advertising expenditure.

2.2. Research on Breweries

a) Studies in Nigeria

- The impact of branding on consumer loyalty in the Nigerian brewing industry was examined [2]. Using a survey research design, questionnaires were administered to 384 consumers in Lagos, Nigeria. Descriptive statistics and regression analysis revealed that brand identity and brand image significantly influenced consumer loyalty. The independent variables—brand identity and brand image—were measured using a 5-point Likert scale, while the dependent variable, consumer loyalty, was similarly measured. Age, gender, and income served as control variables.
- The effect of pricing strategies on sales volume in the Nigerian beer market was also investigated [3]. An ex-post facto research design was employed, utilizing secondary data from Nigerian Breweries Plc.'s annual reports (2010-2015). Correlation and regression analyses showed that penetration pricing and premium pricing strategies positively impacted sales. Pricing strategies served as the independent variables, while sales revenue (measured in Naira) was the dependent variable. Marketing expenses and advertising costs were used as control variables.
- The role of digital marketing in the Nigerian brewing industry was analyzed [4]. Combining survey and case study approaches, questionnaires were administered to 200 consumers in Enugu, Nigeria, while in-depth interviews were conducted with five digital marketing experts. Descriptive statistics, regression analysis, and thematic analysis revealed that social media marketing significantly enhanced brand awareness and sales. The independent variables—digital marketing activities (social media, email, search engine optimization)—and the dependent variable, sales performance, were measured using a 5-point Likert scale. Firm size and industry experience were control variables.

b) Studies in Other African Countries

- The impact of distribution channels on beer sales in South Africa was done by some researchers [30]. A survey research design was used, with questionnaires administered to 250 beer distributors and retailers. Descriptive statistics and factor analysis revealed that retail stores and taverns were the most effective distribution channels. Distribution channels (retail stores, taverns, supermarkets) were the independent variables, while sales revenue (measured in Rand) was the dependent variable. Marketing support and promotional activities served as control variables.
- The effect of promotional activities on consumer behavior in Kenya's beer market was carried out by some researcher [31]. Using an ex-post facto research design, secondary data from Kenya Breweries Ltd.'s marketing reports (2015-2018) were analyzed through correlation and regression analyses. Findings indicated that sponsorship and events marketing positively influenced consumer behavior. Promotional activities (sponsorship, events, advertising) were the independent variables, while brand preference was the dependent variable. Demographics and product involvement were control variables.
- In a related research, the relationship between the marketing mix and customer satisfaction in Ethiopia's brewing industry was examined [5]. Using a survey research design, questionnaires were administered to 150 consumers in Addis Ababa. Descriptive statistics and regression analysis showed that product quality, price, and promotion significantly affected customer satisfaction. Marketing mix elements were the independent variables, and customer loyalty was the dependent variable. Firm size and industry experience were control variables.

c) Studies Using Geometric Programming

Recent studies have applied geometric programming to optimize the marketing mix:

- In a related research, some researchers utilized geometric programming to optimize marketing strategies, see [12].
- Other researchers applied geometric programming to the marketing mix for a beverage company, see [6].
- In a related development, [32] employed a signomial-geometric programming model to address marketing mix problems.

Common Methodologies Across Studies

The studies cited above share several methodological features, including:

- Survey research designs.
- Use of questionnaires and secondary data collection.
- Analytical tools such as descriptive statistics, regression analysis, factor analysis, and thematic analysis.
- Mixed-method approaches combining quantitative and qualitative methods.
- Application of geometric programming in marketing optimization.

Similarities and Differences

- Similarities:
  - Focus on the brewing industry [2]-[4].
  - Examination of marketing mix elements [5] [7] [32].
- Differences:
  - This study focuses specifically on the Nigerian brewing industry, while others explore different African contexts (e.g., South Africa, Kenya, Ethiopia).
  - Our study delves deeper into the 4Ps of the marketing mix, whereas previous studies often examine specific aspects, such as branding or pricing.
  - Unlike prior research, our study employs a combined approach of multiple regression analysis and geometric programming for a more nuanced exploration of marketing mix relationships and industry performance.

Contribution of This Study

This study offers a unique contribution by integrating multiple regression and geometric programming to provide a comprehensive understanding of the relationships between marketing mix elements and performance in the Nigerian brewing industry.

3. Research Methodology

This section outlines the methodology employed in this research, detailing the data collection, description, and analysis techniques used to investigate the relationships among various financial metrics of Nigerian Breweries (NBL) plc. Following the description of the inferential statistics of Regression Analysis in 3.3, shall be the geometric programming methodology.

3.1. Data

This research uses secondary data collected over a 10-year period from 2013 to 2022, which is considered a long-term period for business analysis [29]. The data was primarily extracted from the annual reports of Nigerian Breweries plc, with the exception of price data, which was directly obtained from the company. In this context, "price" refers to the average price of all drinks produced by NBL per carton/crate. All other variables such as Product, Promotion and Place are represented as Production, Promotion and Distribution Cost respectively and are quantified in billions of naira.

3.2. Description of Data

To effectively describe the data, both graphical methods and correlation analysis will be employed. The following techniques will be utilized:

- Graphical Representation: A scatter plot will be used to visually represent the relationship between the variables. This method allows for an intuitive understanding of how two variables interact. Each point on the scatter plot corresponds to a pair of values from the two variables being analyzed.
- Correlation Analysis: A correlation matrix will be calculated to quantify the strength and direction of relationships between pairs of variables. Correlation

coefficients, such as Pearson's r, will be computed to provide numerical insights into these relationships. A positive correlation indicates that as one variable increases, so does the other, while a negative correlation indicates an inverse relationship.

3.3. Inferential Analysis: Ordinary Least Squares (OLS) Regression

Following the descriptive analysis, as indicated above, inferential statistics will be applied using the Ordinary Least Squares (OLS) regression method. This method estimates the relationships among variables by minimizing the sum of squared differences between observed and predicted values. OLS regression is particularly effective in identifying how changes in independent variables affect a dependent variable.

The Model

$$Y_t = \beta_o + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \varepsilon_t \tag{1}$$

$t = 1, 2, 3, \cdots, 10$.

where:

Y_t = Revenue

X_1 = Product

X_2 = Promotion

X_3 = Place

X_4 = Price

β_o = Intercept

$\beta_i ; i = 1, 2, 3, 4$ = slope coefficients of X_1, X_2, X_3, X_4 respectively.

ε = error associated with Y

t = time (in years).

Assumptions:

i. ε_t follows $N\left(0, \sigma^2\right)$

ii. $E\left(\varepsilon_t \varepsilon_{t'}\right) = 0; t \neq t'$

iii. $E\left(\varepsilon_t \mid X_i\right) = 0$

iv. No multicollinearity.

Normality Assumption

The normality assumption is crucial for mathematical convenience [33] model validity [34] and prediction accuracy [35]. Violating the normality assumption can lead to several consequences, including:

- Inaccurate p-values and confidence intervals [36]
- Reduced model reliability [34]
- Inefficient estimation [33]

To test the normality assumption, [37] will be employed, using the following formula:

$$W = \frac{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2}{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 + \sum_{i=1}^{n} \left(x_i - x_i^*\right)^2} \tag{2}$$

where

x_i = individual data points

$\bar{x}$ = sample mean

x_i^* = expected normal order statistics (from a standard normal distribution).

If the hypothesis of normality is tested at a 5% level of significance, and the p-value is greater than 0.05, we accept the hypothesis that the data is normally distributed; otherwise, we reject it.

Homoscedasticity and Autocorrelation Assumptions

The homoscedasticity assumption implies that all error terms, ε_t have the same variance. The presence of heteroskedasticity does not destroy the unbiasedness and consistency properties of the Ordinary Least Squares (OLS) estimators; however, it results in these estimators being inefficient. Consequently, the t-tests and F-tests based on OLS estimators can yield misleading results, leading to erroneous conclusions [38]. In this research, homoscedasticity will be addressed using White's Robust Standard Errors. In STATA software used for the regression analysis, the model is adjusted to account for heteroskedasticity by use of option, *robust* (r), in the regress command [39]. Autocorrelation refers to the situation where the disturbance term related to any observation is influenced by the disturbance term of another observation. While autocorrelation does not affect the unbiasedness, consistency, and asymptotic normality of OLS estimators, it does render them inefficient. This inefficiency makes the application of t-tests, F-tests, and Chi-square tests inappropriate [38]. In this work, after the regression with r, autocorrelation will be tested using Durbin-Watson (DW) d Statistic given in [38] as:

$$d = \frac{\sum_{t=2}^{n}\left(e_t - e_{t-1}\right)^2}{\sum_{t=1}^{n} e_t^2} \tag{3a}$$

where

d = DW statistic

e = estimated residuals

t = time

$0 < d < 4$

If d = 2, then, there is no autocorrelation; less than 2 indicates positive autocorrelation; while greater than 2 indicates negative autocorrelation. To ensure robust statistical inference in the presence of both heteroskedasticity and autocorrelation, the application of Heteroskedasticity and Autocorrelation Consistent (HAC) or Newey-West Standard Errors is essential [38]. These standard errors provide a reliable adjustment to conventional estimates when dealing with time series data that may exhibit serial correlation and non-constant variance.

The Newey-West formula for estimating HAC standard errors is outlined in [33] as:

$$S_* = S_o + \frac{1}{T}\sum_{j=1}^{L}\sum_{t=j+1}^{T}\left(1 - \frac{j}{L+1}\right)e_t e_{t-j}\left[x_t x_{t-j}' + x_{t-j} x_t'\right] \tag{3b}$$

where: S_* = HAC standard errors

$$S_o = \frac{1}{n}\sum_{i=1}^{n} e_i^2 x_i x_i' \tag{4}$$

T = Number of observations in a time series data

L = maximum lag, which must be determined in advance to be large enough that autocorrelations at lag longer than L are small enough to ignore

$$n = T$$

Multicollinearity Assumption

Multicollinearity arises when there is an exact or near-exact linear relationship among the independent variables. In a business context, it reflects how many economic series tend to move together over time due to shared influences, such as the overall trade cycle or prevailing economic conditions [40]. In this study, multicollinearity is expected to be present, given that data collection spans a period marked by significant political shifts and associated economic policies in Nigeria. Although OLS estimators are BLUE (Best Linear Unbiased Estimators), multicollinearity can increase their variances and covariances, complicating precise estimation. As a result, one or more regressors may have statistically insignificant t-values.

Multicollinearity is less problematic if R^2 is high and the regression coefficients remain individually significant, indicated by high t-values [38]. In marketing research, multicollinearity can reveal equally plausible media-buying options [41]. Various methods are available to assess multicollinearity, including Tolerance (TOL), Variance Inflation Factor (VIF), and Artificial Neural Networks (ANN) [42]. This study will apply the VIF, as detailed by [38]:

$$\operatorname{var}\left(\hat{\beta}_j\right) = \frac{\sigma^2}{\sum_{j=1}^{k}\left(X_j - \overline{X_j}\right)^2} VIF_j \tag{5}$$

where

$\hat{\beta}_j$ = estimated partial regression coefficient of the regressor X_j

$$VIF_j = \frac{1}{1 - R_j^2}$$

R_j^2 = R^2 in the regression of X_j on the remaining $(k-1)$ regressors

k = number of regressors.

The VIF for a variable indicates the degree to which its variance is inflated due to non-orthogonality with other variables in the model [33]. A VIF greater than 10 suggests potential issues with multicollinearity (Torres-Reyna, n.d.). While no definitive solutions exist for eliminating multicollinearity, several approaches are commonly used, including: (i) using prior information, (ii) combining cross-sectional and time-series data, (iii) omitting highly collinear variables, (iv) transforming the data, and (v) obtaining additional data [38].

Omni-Directional Variance Test (OVTEST)

The third assumption, $E\left(\varepsilon_t \mid X_i\right) = 0$, indicates that the error term and each independent variable in the model are uncorrelated. This assumption relates to testing for omitted variable bias, which can be evaluated using the Ramsey RESET test [39]. The test procedure is as follows:

- Ho: The model is correctly specified.
- H_1: The model is misspecified.

Regression Equation:

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon \tag{6}$$

Auxiliary Equation:

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 \hat{Y}^2 + \beta_6 \hat{Y}^3 + \varepsilon \tag{7}$$

OV Statistic = nR^2 (from auxiliary equation).

The test statistic follows an approximate F distribution or an asymptotic Chi-squared distribution, with degrees of freedom equal to the number of auxiliary terms added (e.g., 2 in (7). The null hypothesis (Ho) is accepted if the p-value is greater than the specified level of significance, indicating that the model is likely correctly specified [43].

Link Test

Similar to the OVTEST, the Link Test is a diagnostic tool used to detect model misspeci-fication [44]. The procedure is as follows:

- Ho: The model is correctly specified.
- H_1: The model is misspecified.

Regression Equation:

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon$$

Auxiliary Equation:

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 \hat{Y} + \beta_6 \hat{Y}^2 + \varepsilon \tag{8}$$

Link Statistic = nR^2 (from auxiliary equation). The Link Test statistic follows an approximate F distribution or an asymptotic Chi-squared distribution, with degrees of freedom equal to the number of auxiliary terms added. We accept Ho if the p-value is greater than the specified level of significance, suggesting that the model is correctly specified.

The primary distinctions between the Ramsey RESET test and the Link Test are as follows:

a) The Ramsey RESET test is more general, addressing a broader range of mis-specification types, while the Link Test specifically addresses link function mis-specification.

b) The Ramsey RESET test includes cubed fitted values among the auxiliary terms, whereas the Link Test does not.

These assumptions will be explored and investigated in section 4.0

3.4. Geometric Programming

Optimization model in marketing mix problem takes the form $f(x) = S(x) - C(x)$, see [45], and [46] where $f(x)$ is the profit, $S(t)$ is the sales and $C(t)$ is the cost. The function is called signomial function [47] because it allows the cost coefficient of the function to have negative values, unlike posynomial function that allows only the positive cost coefficient; when the function is optimized, we have signomial programming, which is an extension of geometric (posynomial) programming, see [48] and [49]. In this study, we do not intend to continue with signomial programming because it does not attain global optimal solution, rather, we intend to convert to geometric programming, with the assurance of obtaining a global optimal solution. The attainment of global optimal solution is achieved by the constraint equation being bounded above by unity in the case of the primal solution to the problem. In the case of the dual solution and by the fundamental duality theory, the maximization of the dual problem program is the same as the minimizing of the primal problem [50] and the dual program is a concave function constrained by linear constraints, which is easier to handle. Therefore, instead of minimizing the primal objective function [51], which is a convex function and more difficult to work on, we can maximize the dual objective function subject to orthogonality and normality conditions, which are combined to form a non-homogenous system of linear. Therefore, we maximize the profit function to have:

$$\text{Maximize } f\left(y\right)=\prod_{k=0}^{m}\prod_{j=1}^{n_k}\left(\frac{C_{kj}}{y_{kj}}\sum_{j=1}^{n_k}y_{kj}\right)^{y_{kj}} \tag{9}$$

Subject to:

$$\sum_{j=1}^{n_0}y_{0j}=1 \tag{10}$$

$$\sum_{k=0}^{m}\sum_{j=1}^{n_k}a_{kij}y_{kj}=0\,;\quad i=1,2,\cdots,n \tag{11}$$

where y_{0j} = the dual decision variable from the objective function, y_{kj} = dual decision variables from the constraint equation, m = number of variables, n_k = number of terms in the constraint equation, n_0 = number of terms in the objective function. The necessary condition for optimality is that the constraint equation must be bounded above by unity and the sufficient condition for optimality are the normality and orthogonality conditions. Equations (10) and (11) are combined to form equation (12);

$$Ay=B \tag{12}$$

where A is an ($m\times n$) coefficient matrix, y is a vector of dual decision variables of order ($n\times 1$) and B is a vector of constants of order ($m\times 1$), see [52]. At stationary point the optimal solution becomes:

$$f\left(x^*\right)=f\left(y^*\right)=\prod_{k=0}^{m}\prod_{j=1}^{n_k}\left(\frac{C_{kj}}{y_{kj}^*}\sum_{i=1}^{n_k}y_{kj}^*\right)^{y_{kj}^*} \tag{13}$$

Determination of Optimal Weights of the Dual Decision Variables

We determine the optimal weight of the dual decision variable from equation (12) as follows:

$$y^*=A^{-}B \tag{14}$$

$$y^*=A^{-1}B \tag{15}$$

Equation (14) is a case where the exponent matrix is rectangular while equation (15) is a case where the exponent matrix is a square. Using R-program, we obtain y^* as follows:

$$A <\text{-matrix}\left(c\left(a_{11}, a_{12}, \cdots, a_{1n}, a_{21}, a_{22}, \cdots, a_{2n}, a_{m1}, a_{m2}, \cdots, a_{mn}\right), \text{nrow} = m, \text{ncol} = n\right) \quad (16)$$

$$R <\text{-}t\left(A\right) \quad (17)$$

$$L <\text{-}R\% * \% A \quad (18)$$

$$solve.default\left(L\right) -> P \quad (19)$$

$$T <\text{-}P\% * \% R \quad (20)$$

$$B <\text{-matrix}\left(c\left(0,0,0,0,0,1\right), \text{nrow} = m, \text{ncol} = 1\right) \quad (21)$$

$$y <\text{-}\left(T\right)\% * \% B \quad (22)$$

$$y^* = \begin{bmatrix} y_1^* \\ \vdots \\ y_n^* \end{bmatrix} \quad (23)$$

where y* is the optimal dual decision variables, A^- is the Moore-Penrose generalized inverse, see [52].

Determination of Optimal Objective Function f (y*)

From equation (13), we determine the optimal value of the objective function using R-program as follows:

$$f\left(y^*\right) = \left(\frac{C_1}{y_1^*}\right)^{y_1^*} \times \left(\frac{C_1}{y_2^*}\right)^{y_2^*} \times \cdots \times \left(y_3^* + y_4^*\right)^{y_3^* + y_4^*} \times \cdots \times \left(y_{n-1}^* + y_n^*\right)^{y_{n-1}^* + y_n^*} \quad (24)$$

where $f\left(y^*\right)$ is the optimal value of the objective function which correspond to the optimal profit for the Nigerian brewery PLC.

The optimal primal decision variables are obtained from the relationship in equation (25), see [53];

$$\frac{y^* f\left(x^*\right)}{C_{kj}} = \prod\nolimits_{i=1}^{m} x^{a_{ij}} \quad (25)$$

Determination of Optimal Weights of the Primal Decision Variables

From equation (25), we have;

$$C_j \prod\nolimits_{i=1}^{m} \left(x_i\right)^{a_{ij}} = y_j^* f\left(x^*\right) \quad (26)$$

Taking the Ln of both sides of equation (27), we have

$$\frac{y_j^* f\left(x^*\right)}{C_j} = \left(x_1\right)^{a_{1j}} \cdot \left(x\right)^{a_{2j}} \cdots \left(x\right)^{a_{mj}} \quad (27)$$

Taking the Ln of both sides of equation (27), we have

$$\ln P = a_{1j} \ln x_1 + a_{2j} \ln x_2 + \cdots + a_{mj} \ln x_n ; \quad i = 1, \cdots, m \quad (28)$$

Let

$$w_i = \ln x_i \quad (29)$$

we have

$$\ln P = a_{1j}w_1 + a_{2j}w_2 + \cdots + a_{mj}w_n \tag{30}$$

where $\ln P$ is the vector of constant, B. We obtain the optimal weights of the primal decision variables by taking the exponential of equation (29) to have;

$$x_i^* = \mathrm{e}^{w^*} \tag{31}$$

where the elements of the column vector w* are subset of real number R, [51] [53].

By employing these methodologies, this research aims to provide a comprehensive analysis of financial metrics related to Nigerian Breweries plc, contributing valuable insights into its operational performance over the specified period.

All statistical analyses for this study will be conducted using STATA 15.0 and R software.

4. Analysis of Data

4.1. Data Presentation

Table 1 presents NBL's ten-year financial statement on revenue, product, promotion, place, and price. The data was collected for 10 years (2013-2022). The data has been adequately described in Section 3.1. The graph of the data as well as the correlation matrix will further help describe the data.

Figure 1 presents the graph matrix of revenue and marketing mix variables. All covariates exhibit a strong linear relationship with revenue. Particularly, the graph of price versus place has the same shape with price versus revenue which is a bit concave downward suggesting diminishing return. As price increases, the marginal effect on place (distribution) and revenue decreases. Also, the graph of place versus promotion shows initial concave downwards slope and later straightened out to a straight line. This implies that the concave downward slope at the beginning indicates that, at low levels of promotion, increases in place (distribution) have diminishing returns. As promotion increases, the relationship becomes more linear, suggesting consistent returns. The linearity between place and product suggests consistent returns; increases in product are directly proportional to increases in place (distribution). These phenomena are more explained using the correlation coefficient.

Table 2 presents multiple correlation of data. Below each coefficient is the p-value. The result shows that all the covariates are highly significantly linearly correlated with revenue even at 1% level of significance. The covariates are also significantly linearly correlated among themselves. In particular, the strong positive correlation (86%) between price and place and between price and revenue indicate a significant relationship between them even though there appears some quadratic effect as discussed by the graph matrix above. The same can be said about place versus promotion (r = 96%) and product versus place (r = 98%). This high linear relationship among the covariates may be problematic in the regression analysis that follows.

Table 3 presents multiple regression of revenue and marketing mix variables. With an R^2 value of 99.28%, one expects all the covariates to be significant but promotion did not explain revenue ($p = 0.860$) even when its correlation coefficient with revenue is 93.78%. This calls for further examination of the result, first in terms of multicollinearity, which the correlation matrix indicated above and which result is presented.

Table 4 presents VIF of marketing mix variables. Clearly, the result shows a very high degree of multicollinearity. As was said earlier, a VIF value of more than 10.0 is not acceptable. So, all the variables failed the VIF test. We therefore take some steps, as explained in section 3, to remove the multicollinearity effect from the data.

After applying the various procedures for transforming a multicollinearity set of independent variables, and noting that "place" has the highest VIF value, and noting also the preceding discussions about its relationship with other covariates, the ratio transformation of the covariates coupled with the log transformation of the response gave the most meaningful result and it is presented on Table 5.

Table 5 presents multiple linear regression of Log(revenue) and ratios of covariates, where:

Log revenue = log of revenue i.e. Log (revenue)

prodR2 = product/place

promoR2 = promotion/place

priceR2 = price/place

Normalizing the other marketing mix variables with "place" allows comparison of their relative effects. From the above results, 97% of the total variation in log revenue is explained by the covariates. The covariates are significant at 1% level of significance except "promoR2" which is significant at 10% level of significance. The price coefficient also has the required negative sign suggesting that the higher the price, the lesser the quantity of the products that are purchased. The coefficients of the covariates in the table represent elasticities, measuring percentage changes. The coefficient of the constant term (19.8902) shows the expected log (revenue) when all the covariates are zero. The prodR2 coefficient of 0.3484 means that a 1% increase in production relative to place leads to a 0.3484% increase in log (revenue). The promoR2 coefficient of 0.4407 means that a 1% increase in promotion relative to place leads to a 0.4407% increase in log (revenue). The priceR2 coefficient of −13565.41 means that a 1% increase in price relative to place leads to a 13565.41% decrease in log (revenue). The implication of the findings above are that (1) increasing production or promotion efforts relative to distribution (place) may lead to moderate revenue growth and (2) increasing prices relative to distribution may significantly harm revenue. Checking if there is multicollinearity effect in the result above, the following VIF table is produced in Table 6.

Table 6 presents VIF of covariates ratios. We observe that the ratio transformation has completely eliminated the multicollinearity effect in the covariates. Next, we check if the data satisfies the normality assumption of the OLS regression by using the Shapiro-Wilk test, which is presented in Table 7.

Table 7 presents Shapiro-Wilk Test for Normality. The test, with a p-value of 0.4311 shows that the data follows a normal distribution at 5% level of significance.

We next show that the model is adequate by carrying out omitted variables test (ovtest) and the link test. We first show the result of the ovtest as in Table 8.

Table 8 presents OVTEST for omitted variables. The result shows that there are no omitted variables at 5% level of significance. The link test result is in Table 9.

Table 9 presents Link Test for model adequacy. The link test result shows that the model is adequate since the p-value of "_ hatsq" is 0.565. It is noted that the problem of heterogeneity of variance of the model was *ab initio* taken care of by using robust standard errors. We lastly show if there is autocorrelation in the data as outlined in 3.3b. The result is:

Durbin-Watson d-statistic (4, 10) = 2.087351

Interpretation:

i. (4, 10) represent the number of regressors (4) and the number of observations (10)

ii. 2.087351 is the value of the DW d-statistic.

iii. since the value of DW d-statistic is approximately 2.0, then, there is no autocorrelation. We therefore adopt the model of Table 5 as the best model.

Table 10 presents proportion of contribution of the marketing mix in billions of naira. The Table present marketing mix variables as: Product (A), Promotion (B), Place (C) and Price (D); from where we optimize the profit from the interaction of these marketing mix variables over the years in question.

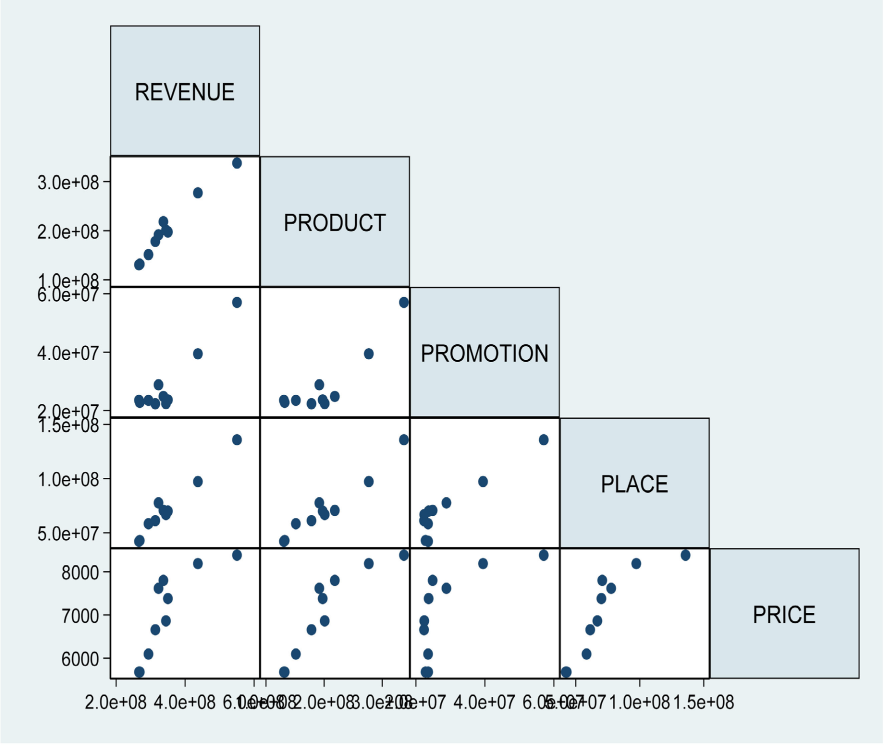

Figure 1. Graph matrix of revenue and marketing mix variables.

Table 1. Ten years financial statement of NBL on revenue, product, promotion, place & price.

S/N	YEAR	REVENUE	PRODUCT	PROMOTION	PLACE	PRICE
1	2013	268,613,518	132,136,476	22,805,957	42,949,612	5,679
2	2014	266,372,475	130,788,296	23,558,010	42,200,086	5,679
3	2015	293,905,792	151,443,890	23,540,657	58,454,978	6,095
4	2016	313,743,147	178,218,528	22,371,313	61,312,319	6,658
5	2017	344,562,517	201,013,357	22,438,092	66,898,905	6,863
6	2018	350,226,472	197,484,694	23,704,811	70,052,363	7,379
7	2019	323,002,120	191,756,513	28,849,136	77,695,289	7,616
8	2020	337,006,267	218,355,350	24,858,334	70,701,538	7,800
9	2021	437,195,534	276,871,996	39,475,853	97,304,194	8,189
10	2022	550,477,627	37,310,437	57,068,804	135,829,790	8,384

Table 2. Multiple correlation of data.

	Revenue	product	promotion	place	price
revenue	1.0000				
product	0.9819	1.0000			
	0.0000				
promotion	0.9378	0.8934	1.0000		
	0.0001	0.0005			
Place	0.9804	0.9753	0.9358	1.0000	
	0.0000	0.0000	0.0001		
Price	0.8246	0.9035	0.6920	0.8698	1.0000
	0.0033	0.0003	0.0266	0.0011	

Table 3. Multiple regression of revenue and marketing mix variables.

Linear regression				Number of obs	=	10
				F (4, 5)	=	4455.28
				Prob > F	=	0.0000
				R-squared	=	0.9928
				Root MSE	=	9.80E+06
		Robust				
revenue	**Coef.**	**Std. Err.**	**t**	**P>t**	**[95% Conf.**	**Interval]**
product	1.189483	0.1747905	6.81	0.001	0.7401696	1.638796
promotion	−0.1490262	0.8045966	−0.19	0.86	2.217308	1.919255
place	1.300058	0.5686086	2.29	0.071	0.1615972	2.761713
price	−28297.69	6916.13	−4.09	0.009	46076.17	−10519.21
_cons	2.18e + 08	3.10E + 07	7.03	0.001	1.38E + 08	2.98E+08

Table 4. VIF of marketing mix variables.

Variable	VIF	1/VIF
Place	44.02	0.022715
product	28.49	0.035102
promotion	16.33	0.06124
Price	10.53	0.094959
Mean VIF	**24.84**	

Table 5. Multiple linear regression of log (revenue) & ratios of covariates.

Linear regression				Number of	obs =	10
				F (3, 6)	=	254.64
				Prob > F	=	0.0000
				R squared	=	0.97
				Root MSE	=	0.04701
		Robust				
logrevenue	Coef.	Std. Err.	t	P > t	[95% Conf. Interval]	
prodR2	0.348365	0.081407	4.28	0.005	0.149169	0.5475598
promoR2	0.440696	0.221593	1.99	0.094	0.101523	0.9829138
priceR2	−13565.4	1014.935	−13.37	0.000	16048.87	−11081.96
_cons	19.89023	0.182398	109.05	0.000	19.44392	20.33654

Table 6. VIF of covariates ratios.

Variable	VIF	1/VIF
priceR2	2.47	0.405244
prodR2	2.05	0.487299
promoR2	1.33	0.75205
Mean VIF	**1.95**	

Table 7. Shapiro-Wilk test for normality.

Variable	Obs	W	V	z	Prob > z
e10	10	0.92827	1.105	0.174	0.4311

Table 8. OVTEST for omitted variable.

Ramsey RESET test using powers of the fitted values of logrevenue
Ho: model has no omitted variables
F (3, 3) = 3.57
Prob > F = 0.1617

Table 9. Link Test for model adequacy.

Source	SS	Df	MS		Number of obs =	10
					F(2, 7) =	119.22
Model	0.4293	2	0.2147		Prob > F =	0.0000
Residual	0.0126	7	0.0018		R-squared =	0.9715
					Adj R-squared =	0.9633
Total	0.4419	9	0.0491		Root MSE =	0.0424
logrevenue	Coef.	Std. Err.	t	P > t	[95% Conf. Interval]	
_hat	−5.7586	11.1942	−0.51	0.623	−32.2287	20.7116
_hatsq	0.1712	0.2835	0.60	0.565	−0.4992	0.8415
_cons	66.7103	110.4976	0.60	0.565	−194.575	327.9956

Table 10. Proportion of contribution of the marketing mix in billion naira.

Proportion	Revenue	Product	Promotion	Place	Price
(Billion, N)	3.485	1.7154	0.2887	0.7234	0.0000703

4.2. Specific Model Development

In developing the marketing mix model for NBL PLC, we define the following marketing mix variables as: Product (A), Promotion (B), Place (C) and Price (D). Our interest is to optimize the profit from the interaction of these marketing mix variables over the years in question, we also define Revenue (R) as the totality of income accrued from the business over the period of time. Each of the variables are randomly assigned values as follows: Let (A) = X_1, (B), = X_2, (C) = X_3, (D) = X_4, and (R) = Totality of sales, which comprise the entire marketing mix variables. Therefore, we proceed as follows:

Let $f(x)$ be the profit, then

$$f(x) = \sum R(x) - \sum C(t) \tag{32}$$

$$f(x) = 3.4851x_1x_2x_3x_4 - 1.7154x_1 - 0.2887x_2 - 0.7234x_3 - 0.00007034x_4 \tag{33}$$

Hence, we can write equation (33) as a signomial programming model

$$\text{Max } f(x) = 3.4851x_1x_2x_3x_4 - 1.7154x_1 - 0.2887x_2 - 0.7234x_3 - 0.00007034x_4 \tag{34}$$

Equation (34) is an unconstrained signomial model, which does not produce a global optimal solution. Therefore, we convert the problem to geometric programming problem by reformulation as follows;

$$\text{Let } u(x) = 3.4851x_1x_2x_3x_4 \tag{35}$$

$$f(t) = -\left[1.7154x_1 + 0.2887x_2 + 0.7234x_3 + 0.00007034x_4\right] \tag{36}$$

Therefore, we minimize the inverse of the objective function subject to the reformulated constraint;

$$\text{Minimize } x_0^{-1} \tag{37}$$

$$\text{Subject to } \frac{x_0}{u(x)} + \frac{f(x)}{u(x)} \le 1 \tag{38}$$

Equations (37) and (38) are equivalent to the geometric programming primal problem, see [32]. The geometric programming model for the problem is now written as:

$$\text{Minimize } f_0(x) = x_0^{-1} \tag{39}$$

Subject to $g_1(x)$:

$$\begin{aligned} &0.2869x_0x_1^{-1}x_2^{-1}x_3^{-1}x_4^{-1} + 0.4922x_2^{-1}x_3^{-1}x_4^{-1} + 0.08283x_1^{-1}x_3^{-1}x_4^{-1} \\ &+ 0.2076x_1^{-1}x_2^{-1}x_4^{-1} + 0.0000202x_1^{-1}x_2^{-1}x_3^{-1} \le 1 \end{aligned} \tag{40}$$

$$\text{Subject to equations (10) and (11)} \tag{41}$$

Equations (39) to (40) are the standard constrained geometric programming model for the marketing mix problem.

4.3. Application of the Developed Model

The degree of difficulty of the problem is:

$K = n - (m+1) = 6 - (5+1) = 0$; where k = degree of difficulty, n= number of terms, m = number of variables. This problem has a zero degree of difficulty and therefore has a unique solution. But since the dual problem is a concave function constrained by linear constraints, see equation (41), and at stationary point, the Min $f(x)$ = Max $f(y)$; then, we apply equation (13). Forming orthogonality and normality, we apply equation (41) and since the problem has a unique solution, we determine the optimal dual decision variables from equation (15).

Forming the orthogonality and normality conditions for the dual decision variables from equation (41), we have

$$-y_1 + y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 = 0$$

$$0y_1 - y_2 + 0y_3 - y_4 - y_5 - y_6 = 0$$

$$0y_1 - y_2 + 0y_3 + 0y_4 - y_5 - y_6 = 0$$

$$0y_1 - y_2 - y_3 - y_4 + 0y_5 - y_6 = 0$$

$$0y_1 - y_2 - y_3 - y_4 - y_5 + 0y_6 = 0$$

$$y_1 + 0y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 = 1$$

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & -1 \\ 0 & -1 & -1 & -1 & 0 & -1 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A < \text{-matrix}\big(c(-1,0,0,0,0,1,1,-1,-1,-1,-1,0,0,0,0,-1,-1,0,0,-1,0,-1,-1,$$
$$0,0,-1,-1,0,-1,0,0,-1,-1,-1,0,0),\text{nrow} = 6,\text{ncol} = 6\big)$$

$$R < \text{-}t(A)$$

$$L < \text{-}R\% * \%A$$

$$solve.default(L)\text{-} > P$$

$$T < \text{-}P\% * \%R$$

$$B < \text{-matrix}\big(c(0,0,0,0,0,1),\text{nrow} = 6,\text{ncol} = 1\big)$$

$$y < \text{-}T\% * \%B$$

$$y^* = \begin{bmatrix} 1 \\ 1 \\ -0.25 \\ 0.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

This is the optimal values of the dual decision variables. But a diagnostic check showed that y_4 does not contribute significantly to the objective function, rather, it leads to degenerate solution; hence, should be removed from the problem, see [54]. The new problem has a rectangular matrix and therefore result into a negative degree of difficulty problem, see [51]. Hence, we apply equation (3.14) and the solution becomes:

$$A < \text{-matrix}\left(c\left(-1,0,0,0,1,1,-1,-1,-1,0,0,0,0,-1,0,0,-1,0,-1,0,0,\right.\right.$$
$$\left.\left.-1,-1,-1,0,0,-1,-1,0,0\right),\text{nrow}=6,\text{ncol}=5\right)$$
$$R < \text{-}t\left(A\right)$$
$$L < \text{-}R\%*\%A$$
$$solve.default\left(L\right)\text{-}>P$$
$$T < \text{-}P\%*\%R$$
$$B < \text{-matrix}\left(c\left(0,0,0,0,0,1\right),\text{nrow}=6,\text{ncol}=1\right)$$
$$y < \text{-}\left(T\right)\%*\%B$$
$$y^* = \begin{bmatrix} 0.6452 \\ -0.4839 \\ 0.6774 \\ -0.2258 \\ 0.3548 \end{bmatrix}$$

The signs on the optimal dual decision variables only shows the nature of interaction while the actual values measure the elasticity between the marketing decision variables, even though our interest is to optimize profit from the marketing mix variables. These signs do not affect the optimal profit [32].

From equation (24), we compute for the optimal objective function as follows:

$$f\left(y^*\right)=\left(\frac{1}{0.6452}\right)^{0.6452}*\left(\frac{0.2869}{0.4839}\right)^{0.4839}*\left(\frac{0.4922}{0.6774}\right)^{0.6774}$$
$$*\left(\frac{0.2076}{0.2258}\right)^{0.2258}*\left(\frac{0.0000202}{0.3548}\right)^{0.3548}*\left(1.7419\right)^{1.7419}$$
$$\therefore f\left(y^*\right)=0.06676854=66768540 \text{ naira}$$

The above is the optimal objective function, which is the optimal profit the business made over the period under study.

From equation (27), we calculate the optimal primal decision variables as follows:

$$17.5786=-\ln x_0+0\ln x_1+0\ln x_2+0\ln x_3+0\ln x_4$$
$$18.5395=\ln x_0-\ln x_1-\ln x_2-\ln x_3-\ln x_4$$
$$18.3361=0\ln x_0+0\ln x_1-\ln x_2-\ln x_3-\ln x_4$$
$$18.1008=0\ln x_0-\ln x_1-\ln x_2+0\ln x_3-\ln x_4$$
$$27.7904=0\ln x_0-\ln x_1-\ln x_2-\ln x_3-0\ln x_4$$

Applying equation (3.29) in conjunction with exact solution to GP problems [53], we have:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & 0 & -1 \end{bmatrix}\begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}=\begin{bmatrix} 17.5786 \\ 18.5395 \\ 18.3361 \\ 18.1008 \end{bmatrix}$$

Applying equation (14), we have

$$A < \text{-matrix}\left(c\left(-1,1,0,0,0,-1,0-1,0,-1,-1,-1,0,-1,-1,-1\right),\text{nrow}=4,\text{ncol}=5\right)$$

$$R < \text{-}t\left(A\right)$$

$$L < \text{-}R\%*\%A$$

$$solve.default\left(L\right)\text{-}>P$$

$$T < \text{-}P\%*\%R$$

$$B < \text{-matrix}\left(c\left(17.5786,18.5395,18.3361,181008\right),\text{nrow}=4,\text{ncol}=1\right)$$

$$y < \text{-}\left(T\right)\%*\%B$$

$$w^* = \begin{bmatrix} -18.5530 \\ 9.0985 \\ -35.1064 \\ -12.4151 \\ 2.1387 \end{bmatrix}$$

Applying equation (31), we have

$$x^* = \begin{bmatrix} 8.761 \\ 8941.843 \\ 5.669 \\ 4.057 \\ 8.489 \end{bmatrix}$$

These are the optimal weights of the primal decision variables.

5. Findings and Discussion

5.1. Findings from Regression Analysis

Analysis of NBL's marketing mix data revealed high multicollinearity, as indicated by the VIF results (Table 4). After testing various transformation methods, using "place" as the normalization variable provided the best results, allowing the coefficients to measure the impact of changes in production, promotion, and price relative to distribution. Key findings include:

(i) Product/Price Ratio: This suggests that increasing production relative to distribution capacity may modestly increase revenue ($p = 0.005$).

(ii) Promotion/Place Ratio: This also suggests that increasing promotional efforts relative to distribution channels may enhance revenue ($p = 0.094$).

(iii) Price/Place Ratio: This implies that raising prices relative to distribution may significantly harm revenue ($p = 0.000$).

5.2. Findings from Geometric Programming

Geometric programming analysis showed that Nigerian Breweries Plc earned a total profit of 66,768,540 naira. Among the marketing mix components, "product" had the highest contribution (8941.84) to profit, followed by "price" (8.45), "promotion" (5.67), and "place" (4.06). In other words, Product contributes 99.80% to profit, Price 0.09%, Promotion 0.06% and Place 0.05%. Relative to Place, Product contributes 220, 242.36%, Price 208.13%, and Promotion 139.66%.

5.3. Discussion

The price/place relationship which shows a significant negative impact on revenue, with a large negative coefficient (−13,656.41, $p = 0.000$) indicates:

- High Positive Correlation ($r = 0.87$): Price strategies vary by location, reflecting pricing adjustments across different markets.
- Negative Revenue Impact: Higher prices generally reduce demand and, consequently, revenue.
- Concave Downwards Trend: Initial price increases lead to steep revenue declines, but further increases have less impact, showing that some markets are highly price-sensitive initially.

The analysis also shows that promotion/place and product/place ratios positively impact revenue, with promotion having a larger coefficient but less statistical significance due to initial concavity in the promotion/place graph. This concave relationship suggests diminishing returns in early promotion stages, making the effect less predictable.

The findings suggest that distribution capacity is crucial in balancing other marketing mix elements, as distribution limitations may restrict the impact of production, promotion, and pricing. Overall, to optimize marketing strategies relative to distribution, NBL and other similar industries should consider:

a) Production Alignment: Match production capacity to distribution capabilities.

b) Promotion Strategy: Tailor promotions to distribution channels, such as in-store promotions.

c) Pricing Strategy: Consider distribution costs when setting prices.

6. Conclusion

Product's impact is consistently high across both regression and geometric programming models. This is in agreement with [13] as explained in Section 2.0. As [55] aptly noted, "bad products" do not sell "for too long." Therefore, companies and industries in Nigeria should prioritize producing high-quality products to maximize both revenue and profit. Price negatively impacts revenue in the regression analysis but positively contributes to profit in geometric programming, suggesting that lower revenue does not necessarily lead to no profit, otherwise, companies or industries will not be in business. The negative impact of price agrees with the price elasticity theory, which is also explained in Section 2.0. Promotion positively affects both revenue and profit, albeit with borderline statistical significance in regression ($p = 0.094$). Nigerian Breweries Plc and other companies facing similar issues should, therefore, consider diversifying their promotional strategies. For instance, advertising expenditures should be strategically "dispersed" rather than heavily "concentrated" [41] to improve revenue outcomes. Place's role is critical, highlighting the need for location-specific strategies and acknowledging its interdependence with product, price, and promotion as outlined above. Finally, the data used in this study is economic in nature and is affected by multicollinearity. To address this, ratio transformation using the variable with the highest variance inflation factor (VIF) was applied and can be applied to similar datasets in other companies or industries. This research has demonstrated the utility of combining regression and geometric programming to optimize marketing mix strategies, offering a replicable framework for other firms in the brewing industry.

Suggestions for Further Research

1. The relationship between the marketing mix variables can be investigated using simultaneous equation models.
2. The impact of Distribution Channel on Marketing Mix Effectiveness could be explored using Structural Equation Model (SEM).

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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