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The First Order Autoregressive Model with Coefficient Contains Non-Negative Random Elements: Simulation and Esimation

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ABSTRACT

This paper considered an autoregressive time series where the slope contains random components with non-negative values. The authors determine the stationary condition of the series to estimate its parameters by the quasi-maximum likelihood method. The authors also simulate and estimate the coefficients of the simulation chain. In this paper, we consider modeling and forecasting gold chain on the free market in Hanoi, Vietnam.

Keywords: Random Coefficient Autoregressive Model; Quasi-Maximum Likelihood; Consistency

1. Introduction

It is well-known that many time series in finance such as stock returns exhibit leptokurtosis, time-varying volatility and volatility clusters. The generalized autoregressive conditional heteroscedasticity (GARCH) and the random coefficient autoregressive (RCA) model have been caturing three characteristics of financial returns.

The RCA models have been studied by several authors [1-3]. Most of their theoreic properties are well-known, including conditions for the existence and the uniqueness of a stationary solution, or for the existence of moments for the stationary distribution. In this paper, we address the stationary conditions for the RCA model, the existence and the uniqueness of a stationary solution and parameter estimation problem for the RCA model with the coefficient have a non-negative random elements.

2. Stationary Conditions of the Series

Consider time series $\{Y_t\}$ satisfying

$$\begin{aligned} &Y_t = \left(\phi + |b_t|\right) Y_{t-1} + e_t \\ &e_t \sim N\left(0, \sigma_e^2\right), b_t \sim N\left(0, \sigma_b^2\right) \end{aligned} \quad (1)$$

where $\left\{(b_t, e_t)\right\}_{t \in z}$ are random vectors with independent identical distribution defined in a certain $(\Omega, \mathcal{F}, P)$ probability space (2)

Firstly, we consider the property of the stochastic variable

$$Y = \sum_{i=0}^{\infty} e_{-i} \prod_{j=0}^{i-1} \left(\phi + |b_{-j}|\right) \quad (3)$$

Let $\log^+ x = \max(\log x, 0)$.

Lemma 1. Suppose that condition (2) satisfied,

$$E \log^+ |e_0| < \infty \text{ and } E \log^+ \left|\phi + |b_0|\right| < \infty \quad (4)$$

If

$$-\infty \le E \log\left|\phi + |b_0|\right| < 0 \quad (5)$$

Y determined by (3) will be absolute convergence with probability 1.

Proof.

Assume $-\infty \le E \log\left|\phi + |b_0|\right| < 0$, according to the law of great numbers, existing stochastic variable i_0 such that:

$$\log\left|\phi + |b_{-1}|\right| + \log\left|\phi + |b_{-2}|\right| + \cdots + \log\left|\phi + |b_{-i}|\right| \le \frac{1}{2} i\gamma \quad (6)$$

with every $i \ge i_0$

where $-\infty < \gamma = E \log\left|\phi + |b_0|\right| < 0$. Then

$$\begin{aligned} |Y| &\le \sum_{i=0}^{i_0} \left(|e_{-i}| \prod_{j=0}^{i-1} \left|\phi + |b_{-j}|\right| \right) + \sum_{i=i_0+1}^{\infty} \left(|e_{-i}| \prod_{k=0}^{i-1} \left|\phi + |b_{-k}|\right| \right) \\ &\le \sum_{i=0}^{i_0} \left(|e_{-i}| \prod_{j=0}^{i-1} \left|\phi + |b_{-j}|\right| \right) + \sum_{i=i_0+1}^{\infty} \left(|e_{-i}| e^{i\gamma/2} \right) \end{aligned} \quad (7)$$

We will prove $P\left\{ \sum_{i=0}^{\infty} |e_{-i}| \cdot e^{i\gamma/2} < \infty \right\} = 1$. Indeed, due to $0 < e^{i\gamma/2} < 1$ and in accordance with lemma Borel-Cantelli, sufficient condition here means proving $\sum_{k=1}^{\infty} P\left\{ |e_{-k}| > \zeta^k \right\} < \infty$ with $\zeta > 1$. We have:

$$\sum_{k=1}^{\infty} P\left\{|e_{-k}| > \zeta^k\right\} = \sum_{k=1}^{\infty} P\left\{\log^+ |e_{-k}| > k\log\zeta\right\}$$
$$= \sum_{k=1}^{\infty} P\left\{\log^+ |e_0| > k\log\zeta\right\}$$
$$\leq \frac{E\log^+|e_0|}{\log\zeta} < \infty.$$

From (7), we have $P\left\{|Y| < \infty\right\} = 1$.

If $E\log\left|\phi + |b_0|\right| = -\infty$, (6) can always correct with some $\gamma < 0$. Therefore, (7) is always true.

Lemma 2. Suppose that (2) and (5) meet $E|b_0|^{\varepsilon} < \infty$ and $E|e_0|^{\varepsilon} < \infty$ with some $\varepsilon > 0$. Then, existing $\delta > 0$ such that $E|Y|^{\delta} < \infty$.

Proof.

Suppose $M(t) = E\left|\phi + |b_0|\right|^t, 0 \leq t \leq \varepsilon$

We have $M(0) = 1$, and owing to $(5): M'(0+) < 0$, $M(t)$ is a decreasing function in the neighborhood of 0. Hence, existing $\delta > 0$ such that $M(\delta) < 1$. Generally less, suppose that $0 < \delta \leq 1$. Due to the convex, we have $(a+b)^{\delta} \leq a^{\delta} + b^{\delta}$ với $a, b \geq 0$.

$$|Y|^{\delta} \leq \left(\sum_{i=0}^{\infty} |e_{-i}| \prod_{j=0}^{i-1} \left|\phi + |b_{-j}|\right|\right)^{\delta}$$
$$\leq \sum_{i=0}^{\infty} |e_{-i}|^{\delta} \prod_{j=0}^{i-1} \left|\phi + |b_{-j}|\right|^{\delta}.$$

Use condition (2) and $M(\delta) < 1$, we obtain:

$$E|Y|^{\delta} \leq E|e_0|^{\delta} \sum_{i=0}^{\infty} E\prod_{j=0}^{i-1} \left|\phi + |b_{-j}|\right|^{\delta}$$
$$= E|e_0|^{\delta} \sum_{i=0}^{\infty} M^i(\delta) < \infty.$$

Lemma 3. Assume (2) and (5) are satisfied with $\alpha \geq 1: E|e_0|^{\alpha} < \infty, E|b_0|^{\alpha} < \infty$ and $E\left|\phi + |b_0|\right|^{\alpha} < 1$. Then $E|Y|^{\alpha} < \infty$.

Proof.

Due to condition (2) and inequality Minkowski $\left(E|Y|^{\alpha}\right)^{1/\alpha} \leq \left(E|e_0|^{\alpha}\right)^{1/\alpha} \cdot \sum_{i=0}^{\infty}\left(E\left|\phi + |b_0|\right|^{\alpha}\right)^{i/\alpha} < \infty$. Hence, $E|Y|^{\alpha} < \infty$.

Theorem 1: Suppose that (1), (4) and (5) satisfied with the almost sure convergence of $Y_k = \sum_{i=0}^{\infty} e_{k-i} \prod_{j=0}^{i-1}\left(\phi + |b_{k-j}|\right)$ and process $\{Y_k; k \in Z\}$ is the stationary solution of (1)

Proof.

Y_k is convergent absolutely, acording to Lemma 1

We have: $Y_k = \sum_{i=0}^{\infty} e_{k-i} \prod_{j=0}^{i-1}\left(\phi + |b_{k-j}|\right)$. Therefore:

$$Y_k = e_k + \sum_{i=1}^{\infty} e_{k-i} \cdot \prod_{j=0}^{i-1}\left(\phi + |b_{k-j}|\right)$$
$$= e_k + e_{k-1}\left(\phi + |b_k|\right) + e_{k-2}\left(\phi + |b_k|\right)\cdot\left(\phi + |b_{k-1}|\right)$$
$$+ \cdots + e_{k-m}\left(\phi + |b_k|\right)\left(\phi + |b_{k-1}|\right)\cdots\left(\phi + |b_{k-m+1}|\right)$$
$$= e_k + \left(\phi + |b_k|\right)\cdot \sum_{i=0}^{\infty} e_{k-1-i} \prod_{j=0}^{i-1}\left(\phi + |b_{k-1-j}|\right)$$
$$= e_k + \left(\phi + |b_k|\right) Y_{k-1}$$

$\Rightarrow Y_k$ is the single solution of (1)

Obviously, $\{Y_k\}$ is a stationary series and $\{Y_k\}$ is independent of $e_t, b_t, t > k$.

3. Estimation of Model Parameters

Suppose that

$$E(b_0, e_0) = (0, 0);$$
$$\operatorname{cov}(b_0, e_0) = \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix},$$
$$\sigma_b^2 > 0, \sigma_e^2 > 0.$$

In this section, we care about estimating vectors of $\theta = \left(\phi, \sigma_b^2, \sigma_e^2\right)$ based on Quasi-Maximum Likelihood method.

With $k \in Z$, we have:

$$E\left(Y_k \mid \mathcal{F}_{k-1}\right) = E\left(\left(\phi + |b_k|\right)Y_{k-1} + e_k \mid \mathcal{F}_{k-1}\right)$$
$$= \left(\phi + E|b_k|\right)Y_{k-1}$$

but $E|b_k| = \sqrt{\frac{2}{\pi}} \cdot \sigma_b$, so

$$E\left(Y_k \mid \mathcal{F}_{k-1}\right) = \left(\phi + \sigma_b\sqrt{\frac{2}{\pi}}\right)Y_{k-1}$$

$$\operatorname{Var}\left(Y_k \mid \mathcal{F}_{k-1}\right) = E\left[\left(Y_k - \left(\phi + \sigma_b\sqrt{\frac{2}{\pi}}\right)Y_{k-1}\right)^2 \middle| \mathcal{F}_{k-1}\right]$$

$$E\left\{\left(|b_k| - \sigma_b\sqrt{\frac{2}{\pi}}\right)^2 Y_{k-1}^2 + e_k^2 \middle| \mathcal{F}_{k-1}\right\}$$

$$= \left[\left(1 + \frac{2}{\pi}\right)\sigma_b^2 - 2\sigma_b\sqrt{\frac{2}{\pi}}E|b_k|\right]Y_{k-1}^2 + \sigma_e^2$$

$$= \left(1 - \frac{2}{\pi}\right)\sigma_b^2 Y_{k-1}^2 + \sigma_e^2.$$

Therefore, we have following likelihood function

$$L_n(u)=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi\left[\left(1-\frac{2}{\pi}\right)xY_{i-1}^2+y\right]}}$$
$$\cdot\exp\left\{-\frac{1}{2}\frac{\left[Y_i-\left(s+\sqrt{\frac{2}{\pi}x}\right)Y_{i-1}\right]^2}{\left(1-\frac{2}{\pi}\right)xY_{i-1}^2+y}\right\}.$$

Maximum likelihood estimators determined by:

$$\sup_{n\in\Gamma}L_n(u)=L_n\left(\hat{\theta}_n\right) \quad (8)$$

where Γ is a certain optional appropriate area of R^3

Let

$$H_n(u)=\frac{1}{n}\sum_{i=1}^{n}g_i(u)$$
$$g_i(u)=\frac{\left[Y_i-\left(s+\sqrt{\frac{2x}{\pi}}\right)Y_{i-1}\right]^2}{\left(1-\frac{2}{\pi}\right)xY_{i-1}^2+y}+\log\left[\left(1-\frac{2}{\pi}\right)xY_{i-1}^2+y\right].$$

Then (8) can be written as $\inf_{n\in\Gamma}H_n(u)=H_n\left(\hat{\theta}_n\right)$

Assume

$$\Gamma=\left\{(s,x,y):-s_0\le s\le s_0,\frac{1}{x_0}\le x\le x_0,\frac{1}{y_0}\le y\le y_0\right\} \quad (9)$$

with $s_0>0,x_0>1,y_0>1$

Now, the consistence of maximum livelihood estimates $\hat{\theta}_n$ is said.

Theorem 2. Suppose (2), (4), (5), (8), (9) satisfied and $P\{(\phi+b_0)e_0=0\}<1$ and $\theta\in\Gamma$. We have $\hat{\theta}_n\to\theta$ a.s $(n\to\infty)$.

Proof.

Let

$$Y_{i-1}(u,\alpha,\beta)=\frac{Y_{i-1}^{\alpha}}{\left(\left(1-\frac{2}{\pi}\right)xY_{i-1}^2+y\right)^{\gamma}},\alpha=0,1,\cdots,2\gamma,\gamma\in N$$

and $\beta(\alpha,\gamma)=E\dfrac{Y_0^{\alpha}}{\left(\left(1-\frac{2}{\pi}\right)xY_0^2+y\right)^{\gamma}},\alpha=0,1,\cdots,2\gamma.$

We will prove $E\inf_{u\in\Gamma}g_1(u)>-\infty$ and $Eg_1(u)$ be continuous on Γ.

Indeed,

$$E\sup_{u\in\Gamma}\left|\log\left[\left(1-\frac{2}{\pi}\right)xY_0^2+y\right]\right|$$
$$\le E\left|\log\left[\frac{\left(1-\frac{2}{\pi}\right)Y_0^2}{x_0}+\frac{1}{y_0}\right]\right|+E\left|\log\left[\left(1-\frac{2}{\pi}\right)x_0Y_0^2+y_0\right]\right|$$
$$+\left|\log(y_0)\right|<\infty$$

$$E\inf_{u\in\Gamma}g_1(u)=E\inf_{u\in\Gamma}\frac{\left[Y_1-\left(s+\sqrt{\frac{2}{\pi}x}\right)Y_0\right]^2}{\left(1-\frac{2}{\pi}\right)xY_0^2+y}$$
$$-E\sup\left[-\log\left(\left(1-\frac{2}{\pi}\right)xY_0^2+y\right)\right]$$
$$\ge E\inf_{u\in\Gamma}\frac{\left[Y_1-\left(s+\sqrt{\frac{2}{\pi}x}\right)Y_0\right]^2}{\left(1-\frac{2}{\pi}\right)xY_0^2+y}$$
$$-E\sup_{u\in\Gamma}\left|\log\left[\left(1-\frac{2}{\pi}\right)xY_0^2+y\right]\right|>-\infty$$

On the other hand,

$$Eg_1(u)=E\frac{\left[Y_1-\left(s+\sqrt{\frac{2}{\pi}x}\right)Y_0\right]^2}{\left(1-\frac{2}{\pi}\right)xY_0^2+y}$$
$$+E\log\left[\left(1-\frac{2}{\pi}\right)xY_0^2+y\right]$$

But

$$\left[Y_1-\left(s+\sqrt{\frac{2}{\pi}x}\right)Y_0\right]^2$$
$$=(\phi-1)^2Y_0^2+\left(|b_1|-\sqrt{\frac{2}{\pi}x}\right)^2Y_0^2$$
$$+2(\phi-s)\cdot\left(|b_1|-\sqrt{\frac{2x}{\pi}}\right)Y_0^2+\sigma_e^2$$

$$E\left(|b_1|-\sqrt{\frac{2x}{\pi}}\right)^2=\left(\sigma_b^2+\frac{2}{\pi}x-\frac{4}{\pi}\sigma_b\sqrt{x}\right)$$
$$E\left(|b_1|-\sqrt{\frac{2x}{\pi}}\right)=\sqrt{\frac{2}{\pi}}\left(\sigma_b-\sqrt{x}\right)$$

Then

$$Eg_1(u) = E\left(\frac{Y_0^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y}\right.$$

$$\cdot\left[(\phi - s)^2 + \left(\delta_b^2 + \frac{2x}{\pi} - \frac{4\delta_b\sqrt{x}}{\pi}\right) + 2\sqrt{\frac{2}{\pi}}\left(\delta_b - \sqrt{x}\right)(\phi - s)\right]$$

$$+E\frac{\delta_e^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y} + E\log\left[\left(1-\frac{2}{\pi}\right)xY_0^2 + y\right] < \infty$$

$Eg_1(u)$ is a continuous function in acordance with $u = (s, x, y)$. Next, we will prove:

$$Eg_1(u) > Eg_1(\theta)\ \ \forall u \neq 0, u \in \Gamma.$$

In fact,

$$Eg_1(u) = E\left(\frac{Y_0^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y}\right.$$

$$\cdot\left[(\phi - s)^2 + \left(\delta_b^2 + \frac{2x}{\pi} - \frac{4\delta_b\sqrt{x}}{\pi}\right) + 2\sqrt{\frac{2}{\pi}}\left(\delta_b - \sqrt{x}\right)(\phi - s)\right]$$

$$+E\frac{\delta_e^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y} + E\log\left[\left(1-\frac{2}{\pi}\right)xY_0^2 + y\right]$$

$$= \left[\phi - s + \sqrt{\frac{2}{\pi}}(\sigma_b - x)\right]^2 \cdot E\frac{Y_0^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y}$$

$$+E\frac{\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y} - E\log\frac{\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2}{\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + y}$$

$$+E\log\left[\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2\right]$$

$$= \left[\phi - s + \sqrt{\frac{2}{\pi}}\left(\sigma_b - \sqrt{x}\right)\right]^2 \cdot E\frac{Y_0^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y}$$

$$+Eh\left(\frac{\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y}\right) + E\log\left[\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2\right]$$

where

$$h(x) = x - \ln x,\ x > 0$$
$$h'(x) = 1 - \frac{1}{x},\ h'(x) = 0 \Leftrightarrow x = 1$$
$$\Rightarrow h(x) > h(1) = 1\ \ \forall x > 0$$
$$\Rightarrow Eg_1(u) \geq Eg_1(\theta)$$

and $Eg_1(u) \geq Eg_1(\theta)$ if and only if

$$\phi - s + \sqrt{\frac{2}{\pi}}(\sigma_b - x) = 0 \text{ and } P\left\{\frac{\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2}{\left(1-\frac{2}{\pi}\right)xY_0^2 + y} = 1\right\} = 1$$

$$\Rightarrow x = \sigma_b^2, y = \sigma_e^2, s = \phi$$

If

$$\left(1-\frac{2}{\pi}\right)\sigma_b^2 Y_0^2 + \sigma_e^2 = \left(1-\frac{2}{\pi}\right)xY_0^2 + y\ \ a.s$$
$$\left(1-\frac{2}{\pi}\right)\left(\sigma_b^2 - x\right)Y_0^2 = y - \sigma_e^2\ \ a.s\ ,$$

If $\sigma_b^2 \neq x$ or $y \neq \sigma_e^2$, $P\{Y_0^2 = c\} = 1$

But $Y_1^2 = (\phi + |b_1|)Y_0^2 + e_1^2 + 2(\phi + |b_1|)e_1 Y_0\ \ (a.s)$

But $\{Y_k, k \in Z\}$ is a stationary series

$$\Rightarrow P\left(Y_1^2 = c\right) = 1$$
$$\Rightarrow c = (\phi + |b_1|)c + e_1^2 + 2(\phi + |b_1|)e_1 Y_0\ \ (a.s)$$

But $EY_0 = 0$, take conditional expectations (e_1, b_1) in both sides, we have:

$$c = (\phi + |b_1|)c + e_1^2\ \ \ (a.s)$$

But $|Y_0| = \sqrt{c}\ a.s \Rightarrow (\phi + |b_1|)e_1 = 0\ \ a.s$

Return to theorem $\inf_{u\in\Gamma} l_n(u) \leq l_n(\theta)$, so

$$\lim_{n\to\infty}\sup \inf_{u\in\Gamma} l_n(u) \leq \lim_{n\to\infty}\sup l_n(\theta)\ .a.s$$

But series $\{g_i(\theta), i \in Z\}$ is stationary and ergodic with $E|g_1(\theta)| < \infty$, according to Ergodic theorem, we have:

$$\lim_{n\to\infty} l_n(\theta) = Eg_1(\theta)\ \ a.s$$
$$\Rightarrow \lim_{n\to\infty}\sup \inf_{u\in\Gamma} l_n(u) \leq Eg_1(\theta)\ \ a.s$$

With each positive integer $n, l_n(u)$ is a continuous function in compact set G, so

$$\hat{\theta}_n \in \Gamma \Rightarrow \lim_{n\to\infty}\sup l_n(\theta_n) \leq Eg_1(\theta)\ \ a.s$$

Let C-compact set in Γ with positive distance to θ. Owing to $g_1(u)$ being continuous in Γ, existing an open sphere $U(u)$ with center u with $r < Eg_1(u), u \in C$

such that:

$$r < E \inf_{t \in U(u)} g_1(t).$$

Sets $U(u), u \in C$ are open covers of C, so C holds such finite open covers, are called $U(u_1), U(u_2), \cdots, U(u_k)$ of C. In accordance with Ergodic thoerem, with every $1 \le j \le k$, we have:

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \inf_{u \in U(u_j)} g_i(u) = E \inf_{u \in U(u_j)} g_1(u) > r \quad a.s$$

See that

$$\inf_{u\in C} l_n(u) \ge \min_{1\le j\le k} \inf_{u\in U(u_j)} l_n(u) \ge \min_{1\le j\le k} \frac{1}{n}\sum_{i=1}^{n} \inf_{u\in U(u_j)} g_i(u)$$

$$\Rightarrow \liminf_{n\to\infty} \inf_{u\in C} l_n(u) \ge \min_{1\le j\le k} \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \inf_{u\in U(u_j)} g_i(u) > r$$

In out of events B_r with $P(B_r) = 0$ with r satisfying: $r < \inf_{u\in C} Eg_1(u)$.

Therefore, $\liminf_{n\to\infty} \inf_{u\in C} l_n(u) \ge \inf_{u\in C} Eg_1(u) \quad a.s$

But $Eg_1(u)$ is continuous and $\theta \notin C$ is singly minimum of $Eg_1(u)$

$$\Rightarrow \liminf_{n\to\infty} \inf_{u\in C} l_n(u) > Eg_1(\theta) \quad a.s$$

Let U is a open sphere with center θ and enough small radius and $U^* = U \cap \Gamma$. If $\hat{\theta}_n \notin U^*$, existing a random subseries n_k such that with $C^* = \Gamma / U^*$, we have:

$$\liminf_{n\to\infty} \inf_{u\in C^*} l_n(u) \le \liminf_{k\to\infty} l_{n_k}\left(\hat{\theta}_{n_k}\right)$$
$$\le \limsup_{n\to\infty} l_n(\theta_n) \quad a.s$$

But $\limsup_{n\to\infty} l_n\left(\hat{\theta}_n\right) \le Eg_1(\theta) < \liminf_{n\to\infty} \inf_{u\in C} l_n(u)$

hence, with each above U^*, existing random variable n_0 such that $\hat{\theta}_n \in U^*, \forall n \ge n_0$.

This completes the proof. □

4. Simulation

In this section, we simulate series (1) with different values of $\theta = (\phi, \sigma_b^2, \sigma_e^2)$. These simulations show stationary and non-stationary series cases.

We simulate series (1) with different values of $\theta = (\phi, \sigma_b^2, \sigma_e^2)$ and in each case we can check the stationary conditions of the series (1) by Lemma 1. In **Figure 1**, we see that the series is not stationary with the negagtive slope $\phi = -1.07$ and in **Figures 2** and **3** we simulate the not stationary series with positive slope $\phi = 0.9$ and $\phi = 0.93$. **Figure 4** presents a stationary but clustering series, **Figures 5-7** present stationary series with parameters are $\phi = -0.7$, $\phi = 0$ and $\phi = 0.7$.

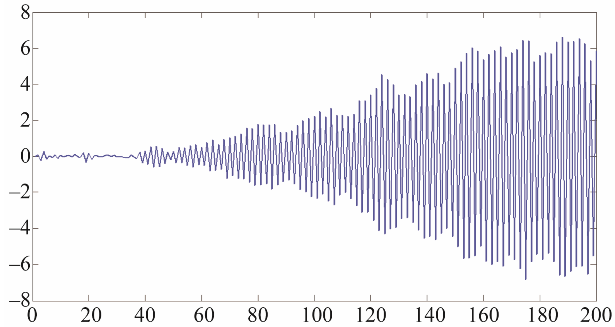

Figure 1. Simulation for series Y_t defined by (1) with $\phi = -1.09; \sigma_b = 0.1; \sigma_e = 0.1$.

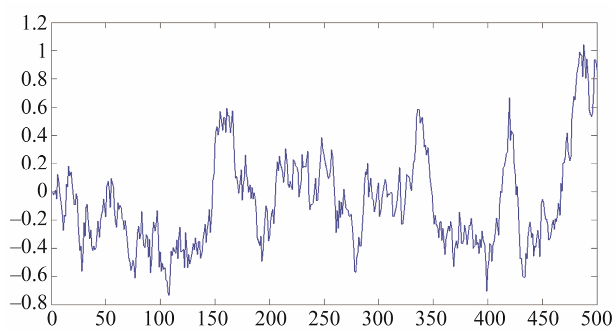

Figure 2. Simulation for series Y_t defined by (1) with $\phi = 0.9; \sigma_b = 0.1; \sigma_e = 0.1$.

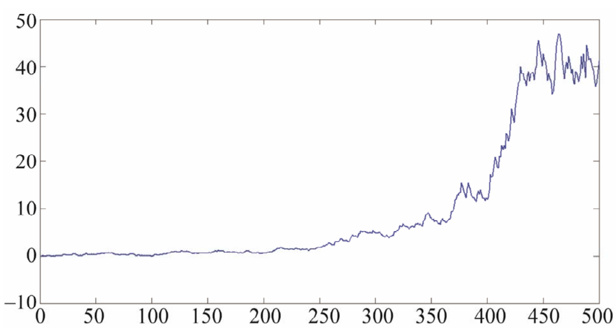

Figure 3. Simulation for series Y_t defined by (1) with $\phi = 0.93; \sigma_b = 0.1; \sigma_e = 0.1$.

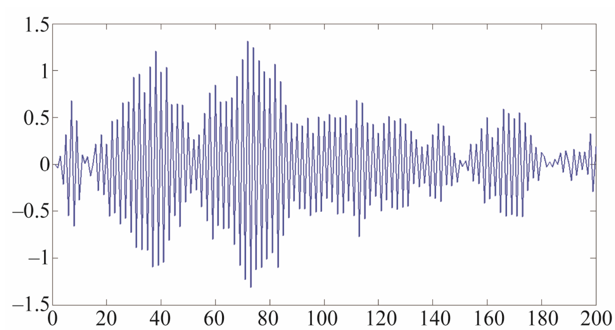

Figure 4. Simulation for series Y_t defined by (1) with $\phi = -1.07; \sigma_b = 0.1; \sigma_e = 0.1$.

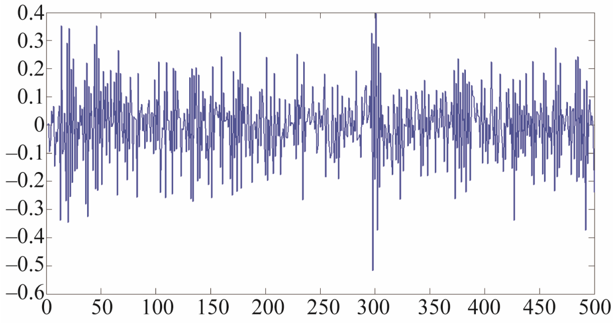

Figure 5. Simulation for series Y_t defined by (1) with $\phi = -0.7; \sigma_b = 0.1; \sigma_e = 0.1$.

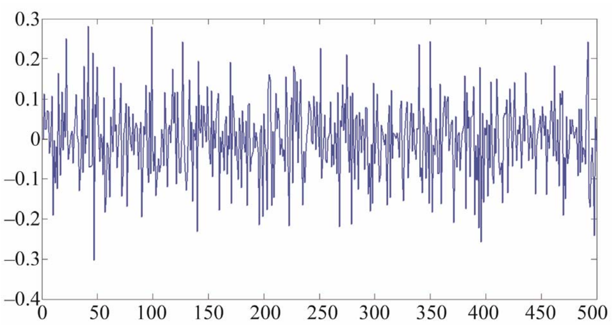

Figure 6. Simulation for series Y_t defined by (1) with $\phi = 0; \sigma_b = 0.1; \sigma_e = 0.1$.

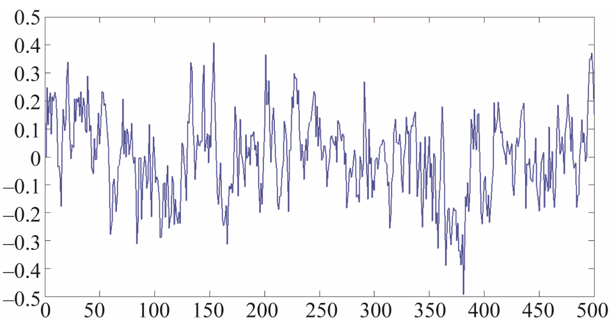

Figure 7. Simulation for series Y_t defined by (1) with $\phi = 0.7; \sigma_b = 0.1; \sigma_e = 0.1$.

5. Application for Real-Time Series

In this section, we use model (1) for the model of return series of the price of gold on the free market in Hanoi, Vietnam. **Figure 8** show the Return series of Gold price r_t.

From the data series we estimate for vector $\theta = \left(\phi, \sigma_b^2, \sigma_e^2\right)$ is $\hat{\theta} = \left(0.0004, 0.0002, 0.0069\right)$. So, we can use the following model to forecast the future value of gold price:

$$r_t = \left(0.0004 + |b_t|\right) r_{t-1} + e_t$$
$$e_t \sim N\left(0, 0.0069\right), b_t \sim N\left(0, 0.0002\right).$$

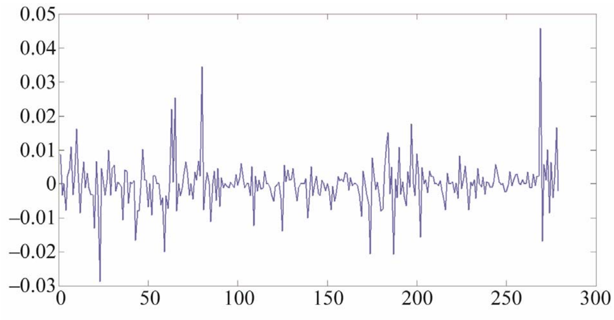

Figure 8. Return series of Gold price r_t.

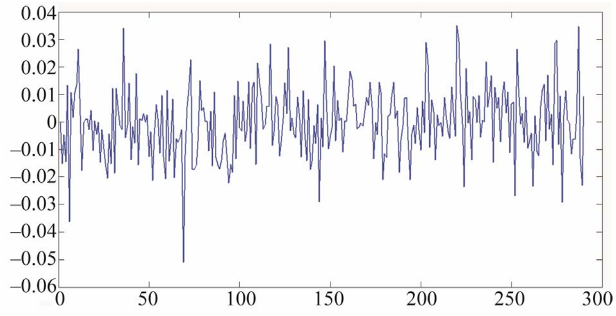

Figure 9. Simulation for series Y_t defined by (1) with $\hat{\theta} = \left(\mathbf{0.0004, 0.0002, 0.0069}\right)$.

Figure 9 below is a simulation of the process (1) with parameters $\hat{\theta} = \left(0.0004, 0.0002, 0.0069\right)$.

6. Conclusion

This paper has solved some problems relating to a kind of first order time series with coefficient regression affected by non-negative random elements. In subsequent studies, the author will consider the asymptotic estimates of the parameters.

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