Scientific Research Publishing

Open Journal of Philosophy, 2026, 16(2), 199-222
https://www.scirp.org/journal/ojpp
ISSN Online: 2163-9442
ISSN Print: 2163-9434

The Epistemology of Second-Order Pedagogical Knowledge: A Philosophical Analysis of Teaching Teachers Mathematics

Frank Kwarteng Nkrumah

Department of Mathematics and Computer Studies, Mampong Technical College of Education, Mampong-Ashanti, Ghana
Email: nkrumah.fk@mtce.edu.gh

How to cite this paper: Nkrumah, F. K. (2026). The Epistemology of Second-Order Pedagogical Knowledge: A Philosophical Analysis of Teaching Teachers Mathematics. *Open Journal of Philosophy, 16,* 199-222.
https://doi.org/10.4236/ojpp.2026.162013

Received: February 23, 2026
Accepted: March 24, 2026
Published: March 27, 2026

Abstract

This paper examines the epistemological foundations of Mathematical Knowledge for Teaching Teachers (MKTT) and argues that persistent challenges in mathematics teacher education reflect deeper philosophical tensions rather than mere pedagogical deficiencies. Drawing on established frameworks for Mathematical Knowledge for Teaching, the paper analyzes what is termed the "recursion problem": teaching teachers requires specialized second-order pedagogical knowledge that remains poorly theorized and institutionally under-supported. Five interconnected epistemological tensions are identified: the measurement problem, which privileges quantifiable knowledge while marginalizing practical wisdom and tacit knowing; the individualization of knowledge, which treats knowledge as individual property rather than recognizing its fundamentally relational nature; theory-practice separation in research methodologies; cultural epistemology tensions between universalizing frameworks and epistemological pluralism; and power dynamics that determine what counts as legitimate teacher knowledge. These tensions illuminate why implementation gaps prove resistant to conventional pedagogical solutions. Meaningful progress requires philosophical reconstruction: reconceptualizing teacher knowledge as situated, relational, and culturally embedded; developing research methodologies that honor the integrated nature of knowledge-in-practice; and acknowledging epistemological pluralism while grappling with questions of power and legitimation. This examination contributes to philosophy of mathematics, epistemology, and applied ethics by demonstrating how educational practice raises fundamental questions about the nature and constitution of professional expertise.

Keywords

Epistemology, Mathematical Knowledge for Teaching Teachers, Philosophy of Mathematics, Social Epistemology, Teacher Education

1. Introduction: The Recursion Problem and the Epistemology of Second-Order Knowledge

What kind of knowledge is required to teach someone how to teach mathematics? This apparently simple question conceals a greater philosophical difficulty. Research on Mathematical Knowledge for Teaching has suggested that teaching mathematics requires specialized knowledge that is not reducible to disciplinary expertise (Ball & Phelps, 2008; Ma, 1999). A further, less examined question then arises: if teaching requires specialized pedagogical knowledge, what kind of knowledge is required to teach teachers? While this question has attracted attention in the literature on mathematics teacher educators (e.g., Zaslavsky & Leikin, 2004; Chauvot, 2009; Superfine et al., 2020), its deeper epistemological implications remain comparatively underexplored. This paper proposes that teaching teachers involves second-order pedagogical knowledge, that is, knowledge about pedagogical knowledge itself, and refers to this as the recursion problem in mathematics teacher education.

Before proceeding, it is necessary to clarify several key terms that structure this inquiry. Mathematical Knowledge for Teaching (MKT) refers to the specialized mathematical and pedagogical knowledge required for teaching mathematics to students (Ball et al., 2008). Mathematical Knowledge for Teaching Teachers (MKTT) extends this concept to the knowledge required for teaching pre-service or in-service teachers how to teach mathematics (Zaslavsky & Leikin, 2004; Superfine et al., 2020). Mathematics Teacher Educators (MTEs) are the professionals who engage in this work of teaching teachers. This paper uses the term "second-order knowledge" to refer to knowledge about knowledge itself: knowledge about its justification, use, limits, and conditions of possibility (Kelp & Pedersen, 2015). Applied to pedagogy, this involves a reflective and interpretive capacity to understand, critique, and support others' developing pedagogical reasoning (Biesta, 2015). While MKTT describes the domain of knowledge MTEs require, second-order pedagogical knowledge emphasizes its epistemically distinctive character as knowledge that takes first-order pedagogical knowledge as its object. These distinctions matter because conflating these levels risks obscuring the recursion problem: that teaching teachers may not simply require more sophisticated versions of teaching knowledge, but qualitatively different forms of knowing. MTEs, on this view, work not only with mathematical content and instructional methods, but also with teachers' ways of knowing, their orientations towards mathematics, and their developing professional judgment (Chapman, 2021). Nevertheless, research suggests that MTEs' professional knowledge is rarely the object of systematic preparation and is often developed through informal and idiosyncratic trajectories (Beswick & Goos, 2018; Muir et al., 2017).

This paper's central contribution is threefold. First, it demonstrates that MKTT cannot be adequately theorised through structural extension of MKT categories alone, but requires recognition of its epistemically distinctive character as second-order knowledge involving ethical, cultural, and epistemological judgement along-

side technical pedagogy. Second, it synthesises five epistemological tensions that, whilst individually discussed in the mathematics education literature, have not previously been examined in an integrated framework specifically addressing their manifestation at the level of teaching teachers. Third, it argues that persistent implementation gaps in mathematics teacher education are not primarily technical failures but symptoms of unresolved philosophical tensions embedded in how teacher-educator knowledge is conceptualised, researched, and legitimated.

This paper advances three related claims. First, it suggests that existing frameworks for Mathematical Knowledge for Teaching Teachers (MKTT) tend to view teacher education as a more specialized or advanced form of ordinary teaching, rather than as an epistemically distinctive practice (Chapman, 2021; Superfine et al., 2020). Second, it argues that this tendency reflects deeper assumptions about the nature of knowledge, knowing, and professional judgment, assumptions that are often implicit and rarely examined in mathematics education research (Putnam & Borko, 2000; Vanassche & Berry, 2020). Third, it suggests that several persistent challenges in mathematics teacher education, such as fragmented curricula, theory–practice tensions, difficulties with culturally responsive teaching, and the persistence of negative dispositions toward mathematics, may be better understood, at least in part, as symptoms of unresolved epistemological tensions rather than simply as technical or organizational problems (Scheiner, 2021; Shultz et al., 2023).

From this perspective, difficulties in theorising and institutionalising mathematics teacher education invite closer attention to how educational knowledge itself is conceptualised. A growing body of work in mathematics education and teacher education has questioned views of knowledge as an individual possession and instead emphasised its social, situated, and practice-based character (Ellis, 2007; Putnam & Borko, 2000; Vanassche & Berry, 2020). This raises further questions about the extent to which pedagogical knowledge can be adequately represented through domain-specific frameworks and measurement instruments, how universal such representations can be, and whose ways of knowing they privilege (Harding, 1991; Longino, 2002).

The paper examines these issues through five interconnected epistemological tensions that have been widely discussed, albeit rarely in an integrated way: the privileging of what is measurable over practical and tacit knowing (Longino, 2002; Mosvold et al., 2023); the individualisation of knowledge rather than its relational and social character (Putnam & Borko, 2000; Yandell, 2017); the separation of theory and practice (Essien et al., 2025; Ryle, 2009); tensions between universalising frameworks and epistemological pluralism (Harding, 1991; Scheiner, 2021); and the role of power in shaping what counts as legitimate professional knowledge (Foucault & Gordon, 1980; Fricker, 2007). These tensions become especially visible at the level of second-order pedagogical work and help to explain why many well-intentioned reforms remain difficult to sustain.

This paper proceeds in four stages. First, I trace the conceptual development from Shulman's formulation of pedagogical content knowledge to contemporary

frameworks for MKT and MKTT, attending to their implicit epistemological commitments. Second, I examine empirical studies of mathematics teacher educators' learning and practice to highlight tensions between frameworks and lived work. Third, I analyze persistent implementation challenges through the lens of the five epistemological tensions. Finally, I outline a possible direction for philosophical reconstruction in which teacher knowledge is understood as situated, relational, practice-constituted, and culturally embedded.

The broader suggestion is that mathematics teacher education is not only a technical or empirical enterprise, but also a philosophical one. It is a domain in which questions from epistemology and the philosophy of mathematics—about the nature of knowledge, the relation between knowing and practice, and the exercise of professional judgement—are not abstract but practically and ethically significant.

2. Conceptualizing Knowledge for Teaching: From MKT to the Recursion Problem

2.1. Mathematical Knowledge for Teaching: Shulman's Foundation and Contemporary Elaborations

Shulman's (1986) distinction between subject-matter knowledge and pedagogical content knowledge suggested that pedagogical knowledge has its own intellectual integrity; it is not merely applied mathematics or derived from disciplinary expertise. This framework implied an epistemological claim: legitimate teacher knowledge is not reducible to either subject-matter knowledge or general pedagogical knowledge, but instead emerges at the intersection of these domains, shaped by particular contexts and learner characteristics.

Ball, Thames, and Phelps' (2008) MKT framework represented a methodological advance, moving from theoretical abstraction to empirical operationalization. They distinguished six subdomains organized under two main categories: Subject Matter Knowledge (comprising Common Content Knowledge, Specialized Content Knowledge, and Horizon Content Knowledge) and Pedagogical Content Knowledge (comprising Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Curriculum).

This framework possessed significant analytical power, enabling researchers to identify specific knowledge gaps and examine relationships between teacher knowledge and instructional practice. However, the MKT framework also embodied particular assumptions warranting philosophical scrutiny. It assumes that teacher knowledge can be meaningfully categorized into discrete domains, each measurable and developable. It prioritizes knowledge that is general across contexts rather than deeply situated in particular classrooms or communities. It focuses on individual teacher knowledge rather than collective knowledge systems. It operates largely within an epistemological framework that treats knowledge as something individuals have rather than as something continuously constructed through practice (Pincheira & Alsina, 2021; Scheiner et al., 2023).

Contemporary research has extended and complicated this picture. The onto-semiotic approach integrates mathematical knowledge with teacher competencies through analytical tools addressing not only cognitive dimensions but also affective and interactional domains (Godino et al., 2017). The Knowledge Quartet framework emphasises the interconnectedness of knowledge components and their instantiation in teaching practice (Rowland, 2020). Cross-cultural studies, such as those by Li and Lyublinskaya (2025), reveal how cultural and epistemological traditions shape what counts as legitimate teacher knowledge.

These developments suggest that teacher knowledge is more complex, more culturally situated, and more intertwined with beliefs, dispositions, and contextual factors than early MKT frameworks fully acknowledged. They point to more nuanced, philosophically sophisticated conceptions of what it means to know mathematics for teaching.

2.2. The Recursion Problem: Theorising MKTT

The extension of MKT frameworks to teacher education contexts raises the "recursion problem": the question of what knowledge is required to teach teachers how to teach mathematics. Zaslavsky and Leikin (2004) first conceptualised MKTT as encompassing techniques to guide pre-service teachers' learning, emphasising attunement to pre-service teachers' sensibilities and the evaluation of their mathematical comprehension. Chauvot (2009) extended this by incorporating research knowledge, recognising that MTEs must understand how educational research shapes pedagogical approaches.

More recently, Superfine et al. (2020) developed what may be the most comprehensive framework, conceptualizing MKTT as an extension of MKT domains into two main categories: Subject Matter Knowledge for Teaching Teachers (MTE-SMK) and Pedagogical Content Knowledge for Teaching Teachers (MTE-PCK). This framework distinguishes MKTT from MKT across three dimensions: content (MKT as educational content versus mathematics as a school subject), learners (adult pre-service teachers versus school students), and objectives (equipping teachers versus developing mathematical understanding).

Superfine et al.'s (2020) framework represents a significant theoretical advancement. Nevertheless, it also reveals a philosophical tension: the framework suggests that teaching teachers is sufficiently distinct from teaching students to warrant a distinct theoretical treatment, yet it remains structurally analogous to MKT, merely substituting "for teaching teachers" for "for teaching". This raises legitimate questions about whether MKTT is fundamentally continuous with MKT, differing only in the degree of specialisation, or whether teaching teachers requires qualitatively different forms of knowledge, including epistemological, ethical, and political knowledge alongside technical pedagogical knowledge.

These theoretical developments demonstrate increasing sophistication in the conceptualization of specialized teacher knowledge. They also reveal ongoing philosophical tensions about whether teacher knowledge can be adequately char-

acterized through domain-specific frameworks, whether such frameworks adequately capture the cultural, ethical, and political dimensions of teaching, and how knowledge relates to practice.

3. Empirical Evidence and Theoretical Tensions

Empirical research on MTE development has begun illuminating how MTEs cultivate their specialized knowledge. Masingila et al. (2017) examined MKTT development among novice teacher educators and identified three key developmental processes: determining mathematical learning goals for pre-service teachers, selecting and using problem-solving tasks, and asking facilitative questions. Superfine and Pitvorec (2021) demonstrated that collaborative learning with experienced colleagues significantly supports MTE growth, with benefits extending beyond the novice educator to more experienced colleagues engaged in collective inquiry. Venkat and Askew (2021) found that MTEs developed not only deeper subject-matter and pedagogical content knowledge but also what they describe as "mathematically attuned noticing": the capacity to observe and interpret significant moments in the teaching of mathematics.

These studies collectively suggest that MTE development requires deliberate engagement with mathematical content from teaching perspectives, collaborative inquiry into problems of practice, and sustained engagement with both pre-service teachers and more experienced colleagues. However, research also documents significant gaps. Institutional support varies dramatically (Dick et al., 2022). Professional communities supporting MTE development remain underdeveloped in many contexts, leaving teacher educators isolated (Guberman et al., 2020; MacPhail et al., 2018).

The concept of mathematical relearning emerges as particularly significant in MTE work. Superfine et al. (2020) emphasise that MTEs must help pre-service teachers relearn mathematical content they already know, but now from a teaching perspective. Research using the Knowledge Quartet framework effectively analyses pre-service teachers' knowledge development, revealing challenges in conceptual interpretation, contingency response, and technology integration (De Vetten et al., 2023; Liston, 2015). Nevertheless, significant gaps remain in understanding how MTEs support this relearning process, with some pre-service teachers leaving preparation programmes with knowledge gaps in fundamental mathematical concepts while simultaneously developing negative attitudes that impede their capacity to teach effectively (Beisly et al., 2025).

4. Epistemological Foundations of Implementation Gaps

The empirical evidence reviewed in Section 3 reveals a troubling pattern. Despite increasing sophistication in theoretical frameworks and growing research on MTE development, significant gaps persist between what is known about effective teacher education and what actually occurs in practice. MTEs continue to develop their knowledge through largely informal pathways, institutional support remains

inconsistent (Eryılmaz & Dikilitaş, 2023), and many pre-service teachers leave preparation programmes with both knowledge gaps and negative dispositions toward mathematics (Kumm & Graven, 2024; Sintema & Marbán, 2020).

These persistent implementation gaps are typically framed as practical problems requiring better pedagogical solutions: more effective instructional strategies, improved programme coordination, enhanced technological resources, or refined assessment instruments. However, this framing may fundamentally misdiagnose the issue. What if these implementation gaps are not primarily technical problems but rather symptoms of deeper epistemological tensions embedded in how we conceptualise, research, and legitimate teacher knowledge? What if the very frameworks guiding our work embody philosophical assumptions that inadvertently constrain meaningful progress?

This section examines five interconnected epistemological tensions underlying persistent implementation challenges: the measurement problem (privileging quantifiable knowledge while marginalising practical wisdom); the individualisation problem (treating knowledge as individual property rather than recognising its relational nature); the theory-practice separation problem (studying knowledge apart from its enactment); the cultural epistemology problem (universalising frameworks developed in particular contexts); and the power-legitimation problem (determining whose knowledge counts as legitimate). Understanding these philosophical foundations is essential for moving beyond symptomatic treatments towards a more fundamental reconstruction of mathematics teacher education.

4.1. The Measurement Problem: Epistemological Consequences of Privileging the Quantifiable

Contemporary research on MKT and MKTT increasingly emphasizes empirical measurement and quantification. This methodological orientation has produced genuine benefits: enabling systematic comparison across contexts, identifying specific knowledge gaps, and moving beyond impressionistic claims about teacher quality (Pincheira & Alsina, 2021). However, this focus on measurability creates epistemological consequences that extend far beyond methodology.

The measurement problem operates through a subtle but powerful mechanism. When we privilege what can be reliably measured, we inevitably privilege certain dimensions of knowledge while rendering others invisible or illegitimate. This reflects what might be called epistemological selection, the process by which research methodologies shape what counts as knowledge worth studying and legitimizing.

MKT frameworks emphasize content knowledge and pedagogical content knowledge partly because these can be operationalized through written tasks, interviews, and structured observations. Yet significant dimensions of teacher knowledge resist straightforward measurement: practical wisdom (phronesis in Aristotelian terms), aesthetic sensibility, ethical discernment, tacit knowledge, and relational attunement. These knowledge dimensions are not trivial or periph-

eral to teaching expertise. Experienced teachers and teacher educators consistently identify them as central to effective practice (Stenberg & Maaranen, 2020; Timmerman & Baart, 2022). Unfortunately, these critical knowledge dimensions remain largely absent from dominant frameworks, not because they are unimportant, but because they defy the measurement-focused methodological apparatus.

The philosophical consequence is significant: what we can measure becomes what we recognize as knowledge. As Longino (2002) argues, what gets measured shapes which phenomena are recognized as real and significant. In teacher education research, the consequence is a particular conception of MKTT as decomposable into measurable components, acquirable through systematic instruction, and generalizable across contexts. This conception may fundamentally misrepresent the nature of pedagogical expertise.

This is not an argument against measurement or empirical research. Instead, it is an observation that research methodologies embed particular epistemological commitments that shape what counts as knowledge. Recognizing this opens the possibility of developing complementary research approaches, such as longitudinal case studies, narrative inquiries, and ethnographic studies that treat teacher education as lived practice rather than measured outcomes (Brownell et al., 2019). These methodologies might illuminate knowledge dimensions invisible to measurement-focused research because they embed different epistemological commitments about what counts as knowledge and how it can be understood.

Consider a concrete scenario: an MTE observes a pre-service teacher conducting a rehearsal lesson on fractions. The pre-service teacher correctly executes the planned instructional moves and demonstrates solid content knowledge. However, the MTE senses something amiss in the pre-service teacher's response to student confusion—a subtle dismissiveness, impatience with alternative approaches, a failure to genuinely listen. This "sense" draws on tacit knowledge, relational attunement, and aesthetic judgment about what constitutes mathematically productive interaction. Such knowledge is central to MTE practice, yet it resists decomposition into measurable components. An MTE assessment focused solely on observable pedagogical moves or demonstrable content knowledge would miss precisely what matters most: the pre-service teacher's developing capacity for mathematical responsiveness. The measurement problem thus manifests acutely in second-order work, where the most consequential knowledge often concerns not what pre-service teachers know but how they know, not what they do but how they attend.

4.2. The Individualisation Problem: Knowledge as Property Versus Knowledge as Relation

Putnam and Borko (2000) argue that conventional research on teacher knowledge has been dominated by Western epistemological assumptions that view knowledge as internal mental content located within individual cognition and measurable

through individual performance, rather than as socially situated and distributed across contexts. Vanassche and Berry (2020) contrast two views of teacher educator knowledge: one that treats knowledge as a stable quality that individual teacher educators acquire, possess, and perform, and another that understands knowledge as continually developing through practice. Ellis (2007) similarly argues that problematic epistemological assumptions, including dualism, objectivism, and individualism, characterise traditional approaches to teacher education. Scheiner (2021) explicitly critiques the notion of individual teachers as sole "transformers" and "gatekeepers" of knowledge, contending instead that knowledge transformation is a socially and institutionally embedded process. Yandell (2017) advances this position most directly by arguing that knowledge is "born from social engagement" and exists in social activity, rather than as the possession of an individual.

Much teacher knowledge is fundamentally relational and socially constructed (Hopper et al., 2017). When a teacher skillfully responds to a student's unexpected mathematical question, the knowledge displayed emerges not solely from the teacher's individual mind but from the relationship between teacher and student, the particular context of their interaction, the mathematical traditions and practices available in their community, and the shared language and conceptual resources developed through prior interactions. The knowledge that emerges in productive professional development communities often transcends what any individual participant could generate on their own (Superfine & Pitvorec, 2021).

This tension becomes especially visible when MTEs facilitate mathematical "relearning" with pre-service teachers. An MTE working individually with a pre-service teacher might help that teacher develop a deeper understanding of, for instance, why invert-and-multiply works for fraction division. However, when the same MTE facilitates collective inquiry among a group of pre-service teachers exploring multiple solution strategies, justifying their reasoning to one another, and encountering alternative representations, qualitatively different knowledge emerges. The group develops shared mathematical language, negotiates what counts as a valid explanation, and constructs collective understanding that exceeds any individual contribution. The MTE's knowledge-in-practice here is fundamentally relational: knowing how to structure interactions, recognize emerging collective understanding, identify whose voices are marginalized, and facilitate knowledge co-construction. Individual-focused frameworks cannot adequately capture this relational dimension of MTE knowledge because they treat knowledge as located within individual minds rather than as constituted through social practice.

This connects to philosophical debates in social epistemology about the social constitution of knowledge. Goldman (2002) notes significant disagreements about what constitutes knowledge (ranging from mere belief to true belief) and whether social epistemology should replace or extend classical epistemology. Shevchenko (2022) emphasizes the importance of both external socio-political contexts and internal knowledge production mechanisms in the creation of collective knowledge.

Levy and Alfano (2020) suggest that epistemic dispositions considered vices at the individual level might actually be virtues in social contexts.

This matters both practically and philosophically. Practically, research examining individual MTE knowledge may miss the most significant learning that occurs in collaborative contexts. An MTE working in isolation might develop certain competencies; the same educator embedded in a community engaged in collective inquiry might develop qualitatively different kinds of knowledge that emerge precisely through collaborative engagement. Philosophically, treating teacher knowledge as an individual possession fundamentally misrepresents the complex, socially embedded nature of educational understanding. Biesta (2015) argues that education is fundamentally about relation rather than production.

This tension helps explain persistent implementation gaps. If teacher knowledge is fundamentally relational, then teacher education programmes that isolate pre-service teachers in individual coursework, assess them through individual examinations, and treat learning as individual achievement will inevitably struggle to develop the collaborative, relational knowledge essential to effective teaching.

4.3. The Theory-Practice Separation Problem: Methodological Irony in Mathematics Education Research

Current research methods in mathematics teacher education struggle to capture both theoretical knowledge and practical teaching dynamics simultaneously. Mosvold et al. (2023) explicitly address this dilemma, noting that measurement and cognitive interviews are "removed from the dynamics of teaching," while observation methods are "closer to practice but mostly involve an outsider perspective." Critically, they emphasize that "few methods tap into the tacit and often invisible demands that teachers encounter in teaching." The complexity emerges because understanding teaching practice requires "deconstructing teachers' knowledge and practice using existing typologies and frameworks." However, these frameworks may not capture the full spectrum of in-the-moment decision-making.

MKTT, by definition, exists in the integration of theory and practice. An MTE's knowledge is not something possessed independently of practice; it is enacted, demonstrated, and refined through the actual work of supporting pre-service teachers' learning (Pansell, 2023). The knowledge involved in responding to a pre-service teacher's mathematical misconception, designing a task that will challenge learners' thinking, or facilitating productive mathematical discussion is inseparable from its instantiation in practice. It must be understood as what Pansell (2023) terms "didactic praxeology", in which theoretical understanding and practical enactment are co-constitutive.

This position resonates with broader philosophical debates about the nature of knowledge. Ryle's (2009) distinction between "knowing-that" (propositional knowledge) and "knowing-how" (practical knowledge) underscores that practical knowledge cannot be reduced to propositional forms; it is exhibited through skillful performance. Similarly, Polanyi and Sen (2009) contend that much practical

expertise is tacit: embodied, situated, and resistant to explicit articulation. If pedagogical knowledge is fundamentally practical, then research methodologies that isolate knowledge from practice risk being epistemologically and methodologically inadequate.

More practice-oriented approaches may therefore offer richer insight into MKTT. These include video analyses of teaching, longitudinal case studies of MTEs in authentic contexts, collaborative interpretation of practice artefacts, and structured rehearsals that render pedagogical reasoning visible (Gibbons et al., 2023; Liston, 2015). Such methods treat practice not as a site for applying theory, but as the primary locus for constituting, negotiating, and developing knowledge.

This epistemological tension helps explain the persistence of theory–practice divides in teacher education. When research methodologies enact a separation between knowledge and practice, they tacitly reinforce the very dualism they seek to overcome. As a result, pre-service teachers may internalize the view that theory and practice are distinct domains, struggling to bring theoretical insights into classrooms, not from a deficit of skill, but because the knowledge they acquire remains abstracted from the contexts in which it must live and breathe.

Consider an MTE reviewing a video of a pre-service teacher's lesson and making real-time pedagogical decisions: when to pause for analysis, which mathematical moment warrants attention, how to frame questions that make pre-service teachers' reasoning visible, and whether to foreground the mathematical error or the pedagogical move. This decision-making exemplifies knowledge-in-practice that cannot be meaningfully separated into discrete components assessed independently. The MTE's knowledge exists in the integration of mathematical understanding, pedagogical insight, and interpretive judgment enacted in the moment. Research methodologies that study MTE content knowledge separately from practice (through written assessments) or study practice separately from knowledge (through observation without access to reasoning) inevitably fragment what is integrated. The theory-practice separation thus produces a methodological irony: we study teacher educators' knowledge through approaches that cannot, by their very nature, capture the integrated, practice-constituted character of the very knowledge we seek to understand.

4.4. Cultural Responsiveness and Epistemological Pluralism

Prevailing frameworks for conceptualizing teacher knowledge, such as MKT, MKTT, and the Knowledge Quartet, have emerged from specific cultural and epistemological contexts that are predominantly Western, Anglo-American, and empiricist in methodological orientation, privileging individual cognition and measurable outcomes. Nevertheless, such frameworks have traveled globally and have been adopted and adapted across educational systems embedded in diverse epistemological traditions (Scheiner, 2021).

This global mobility raises a deeper philosophical question: can a framework forged within one epistemic paradigm claim universality across diverse knowledge traditions? Comparative and validation studies suggest otherwise, revealing not

only contextual variation but also conceptual dissonance, in that categories that are intelligible in one culture may be reductive in another (Delaney, 2012; Kwon et al., 2012). Scheiner, (2021) demonstrate this tension through contrasts between English-speaking and German-speaking pedagogical epistemologies, the former being pragmatic and practice-oriented, the latter normative and theoretically grounded.

The problem becomes sharper when viewed through the lens of culturally responsive teaching and ethnomathematics. Indigenous and community-based mathematical traditions often embody epistemologies that are relational, embodied, and communal rather than individualist or propositional (Ezeife, 2002). Western-derived models may thus enact what postcolonial scholars term epistemic violence: the erasure of alternative ways of knowing through the universalization of particular ways of knowing.

Following Harding's (1991) standpoint epistemology, knowledge must be understood as situated and perspectival. From this view, MKT frameworks express not a neutral epistemology of teaching but a culturally contingent one, grounded in Western empiricism. Recognizing this calls for epistemological humility: the acknowledgment that Western frameworks represent one possible grammar of educational knowledge among many. Philosophical pluralism, then, does not entail relativism, but rather an openness to multiple traditions of reasoning about mathematics, pedagogy, and human understanding.

This tension manifests concretely when MTEs work with pre-service teachers from diverse epistemological traditions. For instance, an MTE facilitating a discussion of mathematical proof might encounter a pre-service teacher whose cultural background emphasises collective verification through community consensus rather than individual logical deduction, or who values algorithmic efficiency and memorisation as foundations for understanding rather than as obstacles to it. If the MTE's framework privileges Western norms of individual sense-making and discovery-oriented learning, the pre-service teacher's alternative pedagogical sensibilities may be misrecognised as deficits rather than as reflecting legitimate but different epistemological commitments about mathematical knowing. The MTE thus faces a philosophical dilemma that universalising frameworks cannot resolve: how to honour epistemological pluralism whilst maintaining mathematical integrity, how to acknowledge that effective teaching is culturally situated whilst preparing teachers for assessment systems that encode particular cultural norms. This is not a technical problem requiring better instructional strategies but an epistemological problem requiring engagement with questions about what counts as mathematics, whose ways of knowing are legitimate, and how power operates in determining these answers.

4.5. Power, Standards, and the Politics of Legitimation

Beneath every framework for understanding teacher knowledge lies an assumption often left implicit: that we can establish standards for good teacher knowledge,

that expertise can be identified and measured, and that some forms of knowledge are more legitimate than others. These assumptions appear commonsensical yet warrant philosophical scrutiny.

The question of who decides what counts as mathematical knowledge for teaching is deceptively simple. It is not merely an academic inquiry but a question of power. When researchers develop frameworks such as MKT, when teacher education programmes establish standards for adequate teacher knowledge, and when policies mandate particular forms of teacher preparation, these acts simultaneously legitimise certain forms of knowledge while excluding or marginalising others.

Whose mathematics is valued? MKT frameworks typically prioritise formal, academic mathematics. However, mathematical practices are embedded in diverse cultural, occupational, and community contexts. When assessments privilege formal knowledge, they implicitly devalue the mathematical reasoning present in everyday practices and local traditions (Ezeife, 2002; Siswono et al., 2018). Which pedagogical traditions are recognised? Contemporary frameworks tend to reflect pedagogical norms prevalent in Western, middle-class educational settings, emphasising individual sense-making and discovery-oriented learning. However, many traditions value collective reasoning, memorisation as a basis for conceptual understanding, or reverence for teacher authority. When frameworks implicitly privilege one pedagogical ethos, they risk delegitimising others and marginalising teachers whose practices reflect alternative cultural logics (Li & Lyublinskaya, 2025).

This dynamic aligns with Foucault's & Gordon (1980) conception of power/knowledge: the view that power and knowledge are mutually constitutive, and that what is recognized as knowledge is inseparable from the structures of authority that produce and sustain it. From this perspective, MKT frameworks do not merely describe teacher knowledge; they participate in the construction of what counts as valuable professional expertise.

Empirical patterns illustrate this interplay of knowledge and power. Teachers from marginalised or under-resourced communities often score lower on conventional MKT measures. One interpretation, the deficit view, attributes this to an actual lack of knowledge. A second, more critical interpretation suggests that these frameworks encode epistemological and pedagogical norms reflective of dominant groups, failing to register alternative but equally valid forms of mathematical and pedagogical knowledge rooted in lived experience. These competing interpretations carry distinct ethical and political consequences. The first naturalises inequity by treating the framework as neutral; the second exposes the framework as a cultural artefact that privileges certain epistemic standpoints while marginalising others.

Fricker's (2007) concept of epistemic injustice illuminates this dynamic. Epistemic injustice arises when individuals or groups are wronged as knowers, when their contributions are dismissed, their epistemic credibility is denied, or their

ways of knowing are rendered illegitimate. In this light, when frameworks fail to acknowledge the pedagogical expertise of teachers from non-dominant backgrounds, the problem is not simply a methodological bias but a form of epistemic injustice embedded in the framework's epistemological architecture.

This critique is not an argument against standards or the possibility of evaluating expertise. Rather, it calls for epistemic reflexivity: an awareness of how frameworks are constituted, whose perspectives they embody, what forms of knowledge they render visible or invisible, and how they might inadvertently perpetuate inequities even when developed with egalitarian intent (Lunn Brownlee et al., 2017; LeBlanc & Kinsella, 2016).

Consider how this operates in MTE practice: an MTE evaluates a pre-service teacher's performance using an MKTT-based assessment instrument. The pre-service teacher, who brings extensive informal mathematics knowledge from managing a family business and a deep understanding of mathematical reasoning in their indigenous community, scores poorly on formal academic mathematics items. The assessment framework, designed to ensure rigorous standards, registers this as a knowledge deficit. However, a critical interpretation recognizes that the framework privileges particular forms of mathematical knowledge—abstract, decontextualized, aligned with academic mathematics—while rendering invisible the situated, practical, community-embedded mathematical reasoning the pre-service teacher possesses. The MTE thus participates, however unintentionally, in epistemic injustice: the pre-service teacher is wronged as a knower, their legitimate mathematical expertise dismissed because it does not conform to the framework's implicit norms. This illustrates how power operates through epistemological frameworks themselves: not through overt exclusion but through the very categories that define what counts as mathematical knowledge worthy of recognition.

5. Synthesis: Understanding Implementation Gaps

The five epistemological tensions illuminate specific implementation gaps in mathematically plausible ways, though the causal relationships warrant appropriate epistemic modesty. The measurement problem helps explain curricular fragmentation: when research privileges what is measurable, teacher education programmes emphasise discrete, assessable knowledge components whilst neglecting integration and practical wisdom. The individualisation problem contributes to difficulties with collaborative learning and professional community development. When knowledge is treated as individual property, institutional structures isolate rather than connect MTEs and pre-service teachers. The theory-practice separation perpetuates the persistent divide between coursework and field experiences: when research methodologies enact this separation, programmes structurally reproduce it. The cultural epistemology tension illuminates challenges with culturally responsive teaching: when frameworks universalise particular epistemological norms, they structurally marginalise alternative pedagogical traditions. The power-legitimation problem helps explain dispositional difficulties and mathematics

anxiety. When frameworks privilege forms of knowledge aligned with dominant groups, pre-service teachers from marginalised backgrounds may internalise deficit narratives about their mathematical capacity. These connections are plausible rather than deterministic, suggestive rather than conclusive. However, they reframe implementation gaps as symptoms of deeper philosophical tensions rather than merely technical problems amenable to better pedagogy.

The five epistemological tensions help explain why persistent implementation challenges prove so resistant to conventional reforms. The fragmentation of content and pedagogy reflects epistemological choices in how we study teacher education, as well as Western epistemology's tendency to separate disciplinary knowledge from applied knowledge (Hanke et al., 2021; Larsson et al., 2018). The prevalence of negative attitudes and mathematics anxiety reflects deeper epistemological assumptions about mathematics itself, often hindering academic performance and requiring attention to both individual and contextual factors (Barroso et al., 2020; Cipora et al., 2022; Putwain & Wood, 2022). The difficulty of implementing culturally responsive teaching stems from the tension between universalist frameworks and the recognition that teacher knowledge is plural, contextual, and culturally embedded (Bennouna et al., 2021; Kieran & Anderson, 2018). Even technological integration struggles reveal that technology is often conceived as an additive tool rather than as a catalyst for reconceptualising what mathematical knowing and teaching might mean.

Understanding these interlocking tensions implies that genuine progress requires more than improved techniques. It demands a philosophical reconstruction: a re-examination of what counts as legitimate teacher knowledge, of how research can honour the situated and relational character of knowing, of whose epistemological traditions are centred, of how effectiveness is defined within inequitable systems, and of how power structures determine what is valued as knowledge.

A defender of measurement-based and universalising frameworks might reasonably object that this critique risks abandoning necessary standards and rigour. They would argue that: 1) measurement enables accountability and quality assurance in teacher preparation; 2) common frameworks allow cross-context comparison and accumulated knowledge; 3) universal standards protect against relativism and ensure all students receive competent instruction; 4) epistemic pluralism, taken too far, provides no basis for distinguishing effective from ineffective practice. These concerns warrant serious engagement rather than dismissal. This paper's critique does not imply that measurement is impossible or undesirable, that standards are inherently problematic, that all knowledge claims are equally valid, or that cross-cultural comparison is futile. Rather, it establishes several boundary conditions: First, measurement and assessment remain valuable tools but cannot be the sole or primary arbiters of what counts as legitimate teacher knowledge. Second, frameworks provide necessary conceptual organisation but must be recognised as provisional, culturally situated, and partial rather than comprehensive

or neutral. Third, maintaining standards requires ongoing philosophical negotiation about what those standards should honour, whose perspectives they embody, and how they might inadvertently perpetuate inequities. Fourth, epistemological pluralism is not relativism; it requires grappling seriously with how to adjudicate amongst competing knowledge claims whilst acknowledging that adjudication criteria themselves are culturally embedded. The path forward thus requires neither abandoning rigour nor uncritically defending existing frameworks, but rather maintaining both conviction about what matters in mathematics teaching and humility about whose perspectives current frameworks privilege.

6. Towards a Philosophical Reconstruction

From the analysis of the current literature, five epistemological commitments emerge to guide a more inclusive and reflexive reimagining of mathematics teacher education. First, knowledge is situated and relational. Teacher knowledge is not a measurable possession but a relational capacity that arises within communities of practice (Wake et al., 2015). Pedagogical expertise develops through sustained engagement, collaboration, and collective inquiry (Adler, 2024). Teacher education must therefore be restructured around community-based learning, long-term relationships with practice, and the recognition that knowledge is distributed across social and material networks rather than confined to individual minds (Beswick & Goos, 2018).

Second, knowledge as integrated practice, not applied theory. Pedagogical knowledge is knowledge-in-action (Kaiser et al., 2018); it arises from the integration of theory and practice. Skilled teaching is not the application of theory but the enactment of understanding through practice (Herbst et al., 2017). Research and teacher education must therefore privilege practice-based pedagogies, including approximations, video analysis, case inquiry, and rehearsals, that make reasoning visible and treat teaching itself as a site of epistemic activity.

Third, epistemological pluralism is foundational. Frameworks of teacher knowledge must recognize that what counts as legitimate expertise differs across cultures and epistemological traditions (Trinick & Allen, 2024). This requires serious engagement with non-Western pedagogies and indigenous mathematical systems (Meyer & Aikenhead, 2021; Xu & Ball, 2024), the development of frameworks that honor multiple knowledge traditions, and humility in recognizing that good teaching is a culturally inflected notion rather than a universal standard (Mellone et al., 2018).

Fourth, multiple ways of knowing are legitimate. Pedagogical expertise includes not only propositional and measurable knowledge but also phronesis, aisthesis, tacit and relational knowing, and ethical discernment (Nielsen & Lund, 2020; Salloum, 2016). These dimensions are central, not peripheral, to good teaching (Abtahi, 2021). Methodologically, this calls for plural research approaches, including narrative, ethnographic, and longitudinal studies, that explore teaching as lived, embodied practice rather than as abstract competence.

Fifth, critical reflexivity about power and legitimation. Frameworks are not neutral instruments; they are artefacts of power that define whose knowledge counts (Fúnez-Flores et al., 2024; Tanswell & Rittberg, 2020). Ongoing critical examination must address whose interests frameworks serve, the assumptions they embed, and how they may perpetuate inequities under the guise of egalitarian rhetoric (Padilla et al., 2024). Bringing feminist epistemology, postcolonial theory, and critical social thought into dialogue with mainstream research can illuminate these dynamics and open new possibilities for justice-oriented teacher education (Swanson, 2017).

Implications

For mathematics teacher educators, these commitments suggest the importance of engaging in ongoing professional learning grounded in the collaborative examination of practice; maintaining critical reflexivity about one's own epistemological assumptions; cultivating multiple forms of knowledge that encompass content expertise alongside practical wisdom, and pedagogical skill alongside ethical discernment; modelling epistemological humility; and making pedagogical reasoning visible to pre-service teachers.

For teacher education programmes, these commitments call for designing curricula around integrative principles rather than disciplinary silos; structuring learning experiences that emphasise collaboration and knowledge co-construction; implementing practice-based pedagogies from the outset; creating sustained field experiences; addressing dispositional development explicitly; integrating culturally responsive pedagogy throughout; and developing assessment approaches that honour multiple forms of knowledge.

For institutional structures, the implications include creating systematic, valued pathways for MTE development; establishing collaborative professional communities that support ongoing MTE learning; allocating resources that recognize the specialized and demanding nature of teacher education; and restructuring institutions to enable boundary-crossing collaboration between mathematics and education faculty.

For research on teacher knowledge, these commitments invite developing methodologies that study knowledge-in-practice; employing research approaches that honor the relational dimensions of knowledge; engaging multiple epistemological traditions; maintaining reflexivity about how research methodologies shape what counts as knowledge; and conducting cross-cultural research examining how teacher knowledge frameworks function differently across contexts.

7. Conclusion: Teacher Education as Philosophical Practice

This examination of Mathematical Knowledge for Teaching Teachers reveals a field requiring philosophical reconstruction. The persistent fragmentation of content, pedagogy, and epistemology reflects deeper institutional contradictions and epistemological disagreements about what counts as valid knowledge. The preva-

lence of mathematics anxiety and negative dispositions suggests inadequately addressed affective dimensions. Culturally responsive and inclusive pedagogies reveal tensions between ideals and institutional constraints, between recognising epistemological pluralism and maintaining mathematical rigour. Emerging technologies offer promising possibilities, but their integration depends fundamentally on how we conceptualise teaching and learning.

Philosophical honesty requires acknowledging that some tensions lack tidy resolution. Epistemological reconstruction does not eliminate dilemmas but rather helps us understand and navigate them more thoughtfully. How do we maintain intellectual rigour and professional standards whilst honouring epistemological pluralism? How do we conduct research when much pedagogical knowledge resists measurement? How do we develop theoretical frameworks that help us understand teacher knowledge whilst recognising that knowledge exists in practice? How do we prepare teachers for schools as they currently exist, whilst maintaining a commitment to transforming education towards greater justice? These questions require ongoing dialogue across epistemological traditions, a willingness to examine our assumptions, and the capacity to distinguish between epistemological relativism and epistemological pluralism.

This analysis suggests that mathematics teacher education is not primarily a technical enterprise requiring better pedagogical techniques, though technique matters, nor primarily an empirical enterprise requiring more research, though research matters. Rather, it is fundamentally a philosophical practice: work requiring ongoing engagement with questions about epistemology (what constitutes legitimate knowledge? how does knowledge develop?), ethics (what are our obligations to pre-service teachers, to their future students, to communities?), politics (whose knowledge is valued? how does power operate in educational systems?), and aesthetics (what makes teaching beautiful, elegant, graceful?).

Treating teacher education as philosophical practice means maintaining reflexivity about our own assumptions, engaging seriously with alternative perspectives, recognizing that many important questions lack definitive answers but require ongoing dialogue and deliberation, cultivating both conviction and humility, and understanding that effectiveness cannot be defined purely technically but always involves value judgments about what matters in teaching and learning.

The recursion problem with which this paper began ultimately reveals that pedagogical knowledge is more complex, more relational, more culturally situated, and more philosophically profound than frameworks that treat it as a measurable individual competency acknowledge. Teaching teachers requires not only mathematical and pedagogical content knowledge but also practical wisdom, ethical discernment, political awareness, aesthetic sensibility, relational capacity, epistemological sophistication, and critical consciousness. These forms of knowledge cannot be acquired primarily through coursework or measured primarily through instruments. They develop through sustained engagement in practice, collaborative inquiry with colleagues, ongoing reflection, and accumulated experience, sup-

ported by communities that value the complexity of this work and provide conditions for its flourishing.

The field of mathematics teacher education has made substantial progress over the past few decades. Increasingly sophisticated frameworks for understanding teacher knowledge have been developed, evidence on effective pedagogical approaches has accumulated, and innovations to address persistent challenges have been proposed. However, this progress remains incomplete without a philosophical reconstruction that attends to epistemological foundations, acknowledges limitations, grapples with tensions, and maintains reflexivity about whose interests are served by particular conceptualizations.

The path forward requires neither abandoning progress made nor uncritically accepting existing frameworks. Rather, it demands ongoing philosophical engagement: treating frameworks as provisional tools rather than definitive truths, maintaining openness to alternative perspectives, recognizing that what counts as good teaching is partly culturally and politically determined, and acknowledging that tensions among competing values often lack simple resolution.

This ongoing philosophical work is not peripheral to improving mathematics teacher education but central to it; without philosophical clarity about what we are trying to accomplish, whose perspectives matter, what counts as legitimate knowledge, and how power operates in educational systems, pedagogical innovations and empirical research risk reproducing problematic patterns even while intending transformation. The work of mathematics teacher educators is difficult, complex, and consequential. It deserves recognition, support, and ongoing collective inquiry into how we might do it more thoughtfully, more justly, and more effectively.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Abtahi, Y. (2021). What If I Was Harmful? Reflecting on the Ethical Tensions Associated with Teaching the Dominant Mathematics. *Educational Studies in Mathematics, 110,* 149-165. https://doi.org/10.1007/s10649-021-10117-1

Adler, J. (2024). Social Practice Theory and Mathematics Teacher Education: A Conversation between Theory and Practice. *NOMAD Nordic Studies in Mathematics Education, 8,* 31-53. https://doi.org/10.7146/nomad.v8i3.146710

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education, 59,* 389-407. https://doi.org/10.1177/0022487108324554

Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2020). A Meta-Analysis of the Relation between Math Anxiety and Math Achievement. *Psychological Bulletin, 147,* 134-168. https://doi.org/10.1037/bul0000307

Beisly, A., Evans, S., & Latta, L. (2025). Process over Product: Associations among Math Anxiety, Feelings about Math, and Instructional Beliefs in Early Childhood Preservice Teachers. *Journal of Mathematics Teacher Education, 28,* 33-49.

https://doi.org/10.1007/s10857-023-09613-3

Bennouna, C., Brumbaum, H., McLay, M. M., Allaf, C., Wessells, M., & Stark, L. (2021). The Role of Culturally Responsive Social and Emotional Learning in Supporting Refugee Inclusion and Belonging: A Thematic Analysis of Service Provider Perspectives. *PLOS ONE, 16,* e0256743. https://doi.org/10.1371/journal.pone.0256743

Beswick, K., & Goos, M. (2018). Mathematics Teacher Educator Knowledge: What Do We Know and Where to from Here? *Journal of Mathematics Teacher Education, 21,* 417-427. https://doi.org/10.1007/s10857-018-9416-4

Biesta, G. (2015). How Does a Competent Teacher Become a Good Teacher? In R. Heilbronn, & L. Foreman-Peck (Eds.), *Philosophical Perspectives on Teacher Education* (p. 1). Wiley. https://doi.org/10.1002/9781118977859.ch1

Brownell, M. T., Benedict, A. E., Leko, M. M., Peyton, D., Pua, D., & Richards-Tutor, C. (2019). A Continuum of Pedagogies for Preparing Teachers to Use High-Leverage Practices. *Remedial and Special Education, 40,* 338-355. https://doi.org/10.1177/0741932518824990

Castro Superfine, A., Prasad, P. V., Welder, R. M., Olanoff, D., & Eubanks-Turner, C. (2020). Exploring Mathematical Knowledge for Teaching Teachers: Supporting Prospective Elementary Teachers' Relearning of Mathematics. *The Mathematics Enthusiast, 17,* 367-402. https://doi.org/10.54870/1551-3440.1492

Chapman, O. (2021). Mathematics Teacher Educator Knowledge for Teaching Teachers. In M. Goos, & K. Beswick (Eds.), *The Learning and Development of Mathematics Teacher Educators* (pp. 403-416). Springer International Publishing. https://doi.org/10.1007/978-3-030-62408-8_21

Chauvot, J. B. (2009). Grounding Practice in Scholarship, Grounding Scholarship in Practice: Knowledge of a Mathematics Teacher Educator-Researcher. *Teaching and Teacher Education, 25,* 357-370. https://doi.org/10.1016/j.tate.2008.09.006

Cipora, K., Santos, F. H., Kucian, K., & Dowker, A. (2022). Mathematics Anxiety—Where Are We and Where Shall We Go? *Annals of the New York Academy of Sciences, 1513,* 10-20. https://doi.org/10.1111/nyas.14770

De Vetten, A., Keijzer, R., Schoonenboom, J., & Van Oers, B. (2023). Pre-Service Primary School Teachers' Knowledge during Teaching Informal Statistical Inference. *Statistics Education Research Journal, 22,* Article 12. https://doi.org/10.52041/serj.v22i2.424

Delaney, S. (2012). A Validation Study of the Use of Mathematical Knowledge for Teaching Measures in Ireland. *ZDM, 44,* 427-441. https://doi.org/10.1007/s11858-012-0382-5

Dick, L. K., Appelgate, M. H., Gupta, D., & Soto, M. M. (2022). Continuous Improvement Lesson Study: A Model of MTE Professional Development. *Mathematics Teacher Educator, 10,* 111-128. https://doi.org/10.5951/mte.2020.0077

Ellis, V. (2007). Taking Subject Knowledge Seriously: From Professional Knowledge Recipes to Complex Conceptualizations of Teacher Development. *The Curriculum Journal, 18,* 447-462. https://doi.org/10.1080/09585170701687902

Eryılmaz, R., & Dikilitaş, K. (2023). Identity Tensions of In-Service Teacher Educators: A Narrative Inquiry. *Language Teaching Research, 29,* 3448-3466. https://doi.org/10.1177/13621688231216816

Essien, A. A., Evans, T., Inprasitha, M., Salanzar, S. M., & Pitta-Pantazi, D. (2025). *Mathematics Education Research Must Be Useful for the Classroom.* arXiv: 2508.14433.

Ezeife, A. N. (2002). Mathematics and Culture Nexus: The Interactions of Culture and Mathematics in an Aboriginal Classroom. *International Education Journal, 3,* 176-187.

Foucault, M., & Gordon, C. (1980). *Power/Knowledge: Selected Interviews and Other*

Writings 1972-1977. Vintage.

Fricker, M. (2007). *Epistemic Injustice: Power and the Ethics of Knowing.* University of London. https://doi.org/10.1093/acprof:oso/9780198237907.001.0001

Fúnez-Flores, J. I., Ataide Pinheiro, W., Ávila Mendoza, A., Phelps, R., & Cherry Shive, E. (2024). The Sociopolitical Turn in Mathematics Education and Decolonial Theory. *London Review of Education, 22,* Article 13. https://doi.org/10.14324/lre.22.1.13

Gibbons, L., Chapin, S., Feldman, Z., & Starks, R. (2023). Pre-Paring for Ambitious Mathematics Teaching: Examining the Role of Video in Supporting Mathematics Teacher Educators. *Educational Studies in Mathematics, 113,* 331-351. https://doi.org/10.1007/s10649-023-10229-w

Godino, J. D., Giacomone, B., Batanero, C., & Font, V. (2017). Enfoque Ontosemiótico de los Conocimientos y Competencias del Profesor de Matemáticas. *Bolema: Boletim de Educação Matemática, 31,* 90-113. https://doi.org/10.1590/1980-4415v31n57a05

Goldman, A. I. (2002). *Pathways to Knowledge: Private and Public.* Oxford University Press.

Guberman, A., Ulvik, M., MacPhail, A., & Oolbekkink-Marchand, H. (2020). Teacher Educators' Professional Trajectories: Evidence from Ireland, Israel, Norway and the Netherlands. *European Journal of Teacher Education, 44,* 468-485. https://doi.org/10.1080/02619768.2020.1793948

Hanke, E., Hehner, S., & Bikner-Ahsbahs, A. (2021). Reducing Fragmentation in University Pre-Service Teacher Education. *EDeR. Educational Design Research, 5,* Article 39. https://doi.org/10.15460/eder.5.2.1613

Harding, S. (1991). *Whose Science? Whose Knowledge? Thinking from Women's Lives.* Cornell University Press.

Herbst, P., Boileau, N., Clark, L., Milewski, A., Chieu, V. M., Gürsel, U., & Chazan, D. (2017). *Directing Focus and Enabling Inquiry with Representations of Practice: Written Cases, Storyboards, and Teacher Education.* North American Chapter of the International Group for the Psychology of Mathematics Education.

Hopper, T. F., Sanford, K., & Fu, H. (2017). Finding the Connective Tissue in Teacher Education: Creating New Spaces for Professional Learning to Teach. *McGill Journal of Education, 51,* 1013-1035. https://doi.org/10.7202/1039626ar

Kaiser, G., Forgasz, H., Graven, M., Kuzniak, A., Simmt, E., & Xu, B. (2018). *Invited Lectures from the 13th International Congress on Mathematical Education.* Springer. https://doi.org/10.1007/978-3-319-72170-5

Kelp, C., & Pedersen, N. J. L. L. (2015). *Second-Order Knowledge.* Routledge.

Kieran, L., & Anderson, C. (2018). Connecting Universal Design for Learning with Culturally Responsive Teaching. *Education and Urban Society, 51,* 1202-1216. https://doi.org/10.1177/0013124518785012

Kumm, M., & Graven, M. H. (2024). Exploring Pre-Service Teachers' Knowledge of Efficient Calculation Strategies. *South African Journal of Childhood Education, 14,* a1494. https://doi.org/10.4102/sajce.v14i1.1494

Kwon, M., Thames, M. H., & Pang, J. (2012). To Change or Not to Change: Adapting Mathematical Knowledge for Teaching (MKT) Measures for Use in Korea. *ZDM, 44,* 371-385. https://doi.org/10.1007/s11858-012-0397-y

Larsson, J., Airey, J., Danielsson, A. T., & Lundqvist, E. (2018). A Fragmented Training Environment: Discourse Models in the Talk of Physics Teacher Educators. *Research in Science Education, 50,* 2559-2585. https://doi.org/10.1007/s11165-018-9793-9

LeBlanc, S., & Kinsella, E. A. (2016). Toward Epistemic Justice: A Critically Reflexive Ex-

amination of 'Sanism' and Implications for Knowledge Generation. *Studies in Social Justice, 10,* 59-78. https://doi.org/10.26522/ssj.v10i1.1324

Levy, N., & Alfano, M. (2020). Knowledge from Vice: Deeply Social Epistemology. *Mind, 129,* 887-915. https://doi.org/10.1093/mind/fzz017

Li, Y., & Lyublinskaya, I. (2025). Effect of American-Based Professional Development Program on Acculturation Strategies of Kazakhstan Mathematics Faculty. *Trends in Higher Education, 4,* Article 4. https://doi.org/10.3390/higheredu4010004

Liston, M. (2015). The Use of Video Analysis and the Knowledge Quartet in Mathematics Teacher Education Programmes. *International Journal of Mathematical Education in Science and Technology, 46,* 1-12. https://doi.org/10.1080/0020739x.2014.941423

Longino, H. E. (2002). Science and the Common Good: Thoughts on Philip Kitcher's *science, Truth, and Democracy*. *Philosophy of Science, 69,* 560-568. https://doi.org/10.1086/344618

Lunn Brownlee, J., Ferguson, L. E., & Ryan, M. (2017). Changing Teachers' Epistemic Cognition: A New Conceptual Framework for Epistemic Reflexivity. *Educational Psychologist, 52,* 242-252. https://doi.org/10.1080/00461520.2017.1333430

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* Lawrence Erlbaum Associates.

MacPhail, A., Ulvik, M., Guberman, A., Czerniawski, G., Oolbekkink-Marchand, H., & Bain, Y. (2018). The Professional Development of Higher Education-Based Teacher Educators: Needs and Realities. *Professional Development in Education, 45,* 848-861. https://doi.org/10.1080/19415257.2018.1529610

Masingila, J. O., Olanoff, D., & Kimani, P. M. (2017). Mathematical Knowledge for Teaching Teachers: Knowledge Used and Developed by Mathematics Teacher Educators in Learning to Teach via Problem Solving. *Journal of Mathematics Teacher Education, 21,* 429-450. https://doi.org/10.1007/s10857-017-9389-8

Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2018). Cultural Transposition: Italian Didactic Experiences Inspired by Chinese and Russian Perspectives on Whole Number Arithmetic. *ZDM, 51,* 199-212. https://doi.org/10.1007/s11858-018-0992-7

Meyer, S., & Aikenhead, G. (2021). Indigenous Culture-Based School Mathematics in Action: Part I: Professional Development for Creating Teaching Materials. *The Mathematics Enthusiast, 18,* 100-118. https://doi.org/10.54870/1551-3440.1516

Mosvold, R., Wæge, K., & Fauskanger, J. (2023). Teacher Time Out as a Site for Studying Mathematical Knowledge for Teaching. *The Journal of Mathematical Behavior, 70,* Article ID: 101037. https://doi.org/10.1016/j.jmathb.2023.101037

Muir, T., Wells, J., & Chick, H. (2017). Developing an Understanding of What Constitutes Mathematics Teacher Educator PCK: A Case Study of a Collaboration between Two Teacher Educators. *Australian Journal of Teacher Education, 42,* 60-79. https://doi.org/10.14221/ajte.2017v42n12.4

Nielsen, B. L., & Lund, J. H. (2020). Different Dimensions of Knowledge in Teacher Education - A General Typification. *Nordic Journal of Comparative and International Education (NJCIE), 4,* 9-25. https://doi.org/10.7577/njcie.3722

Padilla, A., Lambert, R., Tan, P., & White-Smith, K. (2024). Conceptualizing Political Knowledges Needed to Teach Inclusive Mathematics: Theorizing through Counterstories. *ZDM—Mathematics Education, 56,* 461-472. https://doi.org/10.1007/s11858-024-01598-4

Pansell, A. (2023). Mathematical Knowledge for Teaching as a Didactic Praxeology. *Fron-*

tiers in Education, 8, Article 1165977. https://doi.org/10.3389/feduc.2023.1165977

Pincheira, N., & Alsina, Á. (2021). Teachers' Mathematics Knowledge for Teaching Early Algebra: A Systematic Review from the MKT Perspective. *Mathematics, 9,* Article 2590. https://doi.org/10.3390/math9202590

Polanyi, M., & Sen, A. (2009). *The Tacit Dimension.* University of Chicago Press.

Putnam, R. T., & Borko, H. (2000). What Do New Views of Knowledge and Thinking Have to Say about Research on Teacher Learning? *Educational Researcher, 29,* 4-15. https://doi.org/10.3102/0013189x029001004

Putwain, D. W., & Wood, P. (2022). Anxiety in the Mathematics Classroom: Reciprocal Relations with Control and Value, and Relations with Subsequent Achievement. *ZDM—Mathematics Education, 55,* 285-298. https://doi.org/10.1007/s11858-022-01390-2

Rowland, T. (2020). Frameworks for Conceptualizing Mathematics Teacher Knowledge. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 299-302). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_63

Ryle, G. (2009). *The Concept of Mind.* Routledge. https://doi.org/10.4324/9780203875858

Salloum, S. (2016). The Place of Practical Wisdom in Science Education: What Can Be Learned from Aristotelian Ethics and a Virtue-Based Theory of Knowledge. *Cultural Studies of Science Education, 12,* 355-367. https://doi.org/10.1007/s11422-015-9710-8

Scheiner, T. (2021). Examining Assumptions about the Need for Teachers to Transform Subject Matter into Pedagogical Forms Accessible to Students. *Teachers and Teaching, 28,* 1-11. https://doi.org/10.1080/13540602.2021.2016688

Scheiner, T., Buchholtz, N., & Kaiser, G. (2023). Mathematical Knowledge for Teaching and Mathematics Didactic Knowledge: A Comparative Study. *Journal of Mathematics Teacher Education, 27,* 1083-1104.

Shevchenko, A. A. (2022). "Social" in Social Epistemology. *Siberian Journal of Philosophy, 20,* 10-18. https://doi.org/10.25205/2541-7517-2022-20-2-10-18

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, 15,* 4-14. https://doi.org/10.3102/0013189x015002004

Shultz, M., Close, E., Nissen, J., & Van Dusen, B. (2023). Enacting Culturally Relevant Pedagogy When "Mathematics Has No Color": Epistemological Contradictions. *International Journal of Research in Undergraduate Mathematics Education, 10,* 486-515. https://doi.org/10.1007/s40753-023-00219-x

Sintema, E. J., & Marbán, J. M. (2020). Pre-Service Secondary Teachers' Mathematical Pedagogical Content Knowledge Self-Concept Related to Their Content Knowledge of Functions and Students. *International Electronic Journal of Mathematics Education, 15,* em0598. https://doi.org/10.29333/iejme/8327

Siswono, T. Y. E. et al. (2018). Exploring the Effect of Lesson Study with Ethnomathematics Exploration Approach on Pre-Service Teachers' Understanding of Culturally Responsive Teaching. *Journal on Mathematics Education, 9,* 231-244.

Stenberg, K., & Maaranen, K. (2020). Promoting Practical Wisdom in Teacher Education: A Qualitative Descriptive Study. *European Journal of Teacher Education, 45,* 617-633. https://doi.org/10.1080/02619768.2020.1860012

Superfine, A. C., & Pitvorec, K. (2021). Using Community Artifacts to Support Novice Math Teacher Educators in Teaching Prospective Teachers. *International Journal of Science and Mathematics Education, 19,* 59-75. https://doi.org/10.1007/s10763-021-10152-7

Swanson, D. M. (2017). Mathematics Education and the Problem of Political Forgetting: In Search of Research Methodologies for Global Crisis. *Journal of Urban Mathematics Ed-*

ucation, 10, 7-15. https://doi.org/10.21423/jume-v10i1a337

Tanswell, F. S., & Rittberg, C. J. (2020). Epistemic Injustice in Mathematics Education. *ZDM, 52,* 1199-1210. https://doi.org/10.1007/s11858-020-01174-6

Timmerman, G., & Baart, A. (2022). The Continuing Formation of Relational Caring Professionals. *Medicine, Health Care and Philosophy, 25,* 587-602. https://doi.org/10.1007/s11019-022-10104-0

Trinick, T., & Allen, P. (2024). Wayfinding in an Indigenous Initial Teacher Education Mathematics Programme. *ZDM—Mathematics Education, 56,* 485-495. https://doi.org/10.1007/s11858-024-01589-5

Vanassche, E., & Berry, A. (2020). Teacher Educator Knowledge, Practice, and S-STTEP Research. In J. Kitchen, et al. (Eds.), *International Handbook of Self-Study of Teaching and Teacher Education Practices* (pp. 177-213). Springer. https://doi.org/10.1007/978-981-13-6880-6_6

Venkat, H., & Askew, M. (2021). Development in South African Primary Mathematics Teacher Educators' Work with In-Service Teachers. *International Journal of Science and Mathematics Education, 19,* 39-58. https://doi.org/10.1007/s10763-021-10161-6

Wake, G., Swan, M., & Foster, C. (2015). Professional Learning through the Collaborative Design of Problem-Solving Lessons. *Journal of Mathematics Teacher Education, 19,* 243-260. https://doi.org/10.1007/s10857-015-9332-9

Xu, H., & Ball, R. (2024). *Multiple Forms of Knowing in Mathematics: A Scoping Literature Study.* arXiv: 2406.16921.

Yandell, J. (2017). Knowledge, English and the Formation of Teachers. *Pedagogy, Culture & Society, 25,* 583-599. https://doi.org/10.1080/14681366.2017.1312494

Zaslavsky, O., & Leikin, R. (2004). Professional Development of Mathematics Teacher Educators: Growth through Practice. *Journal of Mathematics Teacher Education, 7,* 5-32. https://doi.org/10.1023/b:jmte.0000009971.13834.e1