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Revisiting the Nature of School Mathematics: Philosophical Foundations and Implications for Contemporary Pedagogy

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Abstract

This paper revisits the nature of school mathematics by examining its philosophical foundations and the implications of these views for contemporary pedagogy. The paper examines how historical and modern conceptions of mathematics, ranging from absolutist philosophies such as Platonism, logicism, and formalism to progressive absolutist views (intuitionism, constructivism), as well as fallibilist perspectives, have shaped mathematics curricula and teaching practices. The analysis highlights the shift from viewing mathematics as a fixed body of infallible knowledge to understanding it as a fallible, evolving, and socially situated discipline. Drawing on both classical insights and recent peer-reviewed studies, we discuss how teachers' beliefs about the nature of mathematics shape instructional models and curriculum implementation. Misalignments between reform-oriented curricula and traditional teacher beliefs are identified as a key challenge in educational change. The paper concludes with recommendations for teacher education and curriculum design, advocating for reflective practice and epistemological plurality to bridge the gap between philosophical ideals and classroom practice.

Keywords

Philosophy of Mathematics, Absolutism, Fallibilism, Constructivism, Mathematics Education

1. Introduction

The question "What is mathematics really all about?" is not merely philosophical—it lies at the heart of debates on how mathematics should be taught (Hersh, 1986: p. 13). As early as the 1970s, scholars like Thom (1973) argued that "whether one wishes it or not, all mathematical pedagogy… rests on a philosophy of mathematics" (p. 204). In other words, every teacher, curriculum designer, or educational reformer operates (consciously or not) with a conception of the nature of mathematics, and this conception profoundly influences their approach to instruction. Hersh (1979) went so far as to claim that controversies in mathematics teaching "cannot be resolved without confronting problems about the nature of Mathematics" (p. 33). Such assertions underscore the critical premise of this paper: the nature of mathematics is not an esoteric concern but a practical determinant of pedagogy and curriculum.

Research in mathematics education supports the idea that teachers' beliefs about the nature of mathematics significantly shape their instructional choices. An absolutist view of mathematics, which presents it as an objective and certain body of knowledge, often corresponds to teacher-centered pedagogies focused on rote learning and the transmission of established procedures (Ernest, 1989). In contrast, a fallibilist or constructivist perspective, which views mathematics as a human construct that is fallible and evolving, tends to align with student-centered, inquiry-based teaching that emphasizes problem-solving and conceptual understanding (Beswick, 2012). These associations have been documented in both historical curriculum developments and recent empirical studies. For example, the rise and decline of the "new math" and "back-to-basics" movements, as well as the emergence of curriculum reforms in the late twentieth century, each reflected different philosophical assumptions. The "new math" reforms were grounded in Platonic absolutism, the "back-to-basics" movement was influenced by an instrumentalist philosophy, and more recent reform initiatives, such as those promoted by the National Council of Teachers of Mathematics (NCTM), have embraced a fallibilist and problem-solving orientation. Analysis of the NCTM Standards supports this interpretation, indicating that its pedagogical principles are fundamentally aligned with a fallibilist conception of mathematics (Toumasis, 1997).

The purpose of this paper is to revisit these philosophical foundations—absolutist and fallibilist, along with their sub-traditions—and examine their implications for contemporary pedagogy and curriculum. We draw on classical philosophical works and current peer-reviewed literature to: 1) outline major philosophies of mathematics (absolutism in its various forms, the progressive absolutism of intuitionism/constructivism, and the fallibilist turn); 2) discuss how each view influences teaching models and curricular trends; and 3) explore challenges in changing entrenched teacher beliefs and implementing reforms. This approach is intended to construct a philosophical-pedagogical framework that informs reflective practice in teacher education and curriculum reform.

To guide this inquiry, the following research questions are posed:

1) In what ways have philosophical conceptions of mathematics influenced curriculum design and pedagogical approaches in school mathematics?

2) How do tensions between absolutist and fallibilist perspectives affect the implementation of curriculum reforms, particularly in relation to teacher beliefs and instructional change?

2. Philosophical Traditions in Mathematics

2.1. Absolutist Foundations: Platonism, Logicism, and Formalism

Throughout much of history, mathematics was viewed through an absolutist lens—as a body of knowledge that is exact, certain, and universally true. Absolutist philosophies assert that mathematical truths are discovered, not invented, and exist independently of human minds or socio-cultural contexts. In this view, often traced back to Plato, mathematical objects (numbers, sets, geometrical forms, etc.) inhabit an abstract realm; mathematicians simply uncover these pre-existing truths. This Platonic realism portrays mathematics as a static field of knowledge built on universally absolute foundations (Steiner, 1987; Bishop, 1996). For example, a Platonist holds that concepts like the number π or the idea of infinity have an objective existence that we gradually reveal. Such a stance implies that mathematical knowledge is infallible: once proven, a theorem is eternally and incorrigibly true, and mathematical results are "value-free" and culture-free facts about reality.

In the late 19th and early 20th centuries, mathematicians and philosophers sought to solidify this absolutist vision by grounding mathematics in rigorous logical foundations. One major initiative was logicism, led by Frege, Russell, and Whitehead. Logicists contended that "all mathematical truths can be proved from the axioms and rules of inference of logic alone" (Ernest, 2004: p. 9). The goal was to show that mathematics is, at root, an extension of formal logic—a quest to ensure that mathematical certainty is as indisputable as logical truth. Frege's program to derive arithmetic from logic, however, hit a famous obstacle: Russell's paradox (Russell, 1902) exposed an inconsistency in naive set theory, undermining Frege's logical foundations. Russell's paradox—the problem of the set of all sets that do not contain themselves—revealed that naive set formation could lead to contradiction, shaking the notion that mathematics was on unassailable logical ground. This contradiction was not merely a technicality; it struck at the absolutist claim of mathematics as a perfectly consistent edifice. As Russell and Whitehead worked to avoid such paradoxes (e.g., via the Theory of Types), it became clear that additional principles beyond pure logic (like the Axiom of Infinity or Axiom of Choice) were required to rebuild mathematics, violating the original logicist ideal. Russell acknowledged that logicism could not account for all of mathematics, as some foundational assumptions, like the infinity of natural numbers, require acceptance beyond purely logical derivation (Russell, 1919).

A parallel absolutist effort was formalism, epitomized by David Hilbert. Hilbert envisioned mathematics as a symbolic system governed by syntactic rules, independent of interpretation, and aimed to establish that such a system would never yield internal contradictions (Hilbert, 2025). Hilbert's program aimed for a finitistic proof of consistency for all of mathematics, hoping to forever dispel the "nightmare scenario" of discovering an inconsistency in arithmetic (Fenn, 2011: p. 1). Two key formalist claims were: 1) mathematics can be reduced to formal axiomatic systems; and 2) the consistency of these systems can be proven within a meta-mathematical framework. However, Hilbert's ambitions, like the logicists', met an insurmountable roadblock. Gödel's Incompleteness Theorems (Gödel, 1931) proved that any sufficiently powerful formal system (capable of encoding basic arithmetic) will inevitably contain true statements that cannot be proved within the system, and that a system cannot demonstrate its own consistency. In essence, Gödel showed that not all true mathematical statements can be proved and that the consistency of a system is itself an unprovable assumption within that system. This revelation was devastating for formalism's second claim: it became clear that one cannot establish the absolute consistency of mathematics from within mathematics itself. The fallout from Gödel's theorems was philosophically profound—it undercut the dream of mathematics as a complete and infallible formal system, reinforcing the notion that some mathematical truths (or consistency itself) might forever rest on postulates accepted on faith (Nagel & Newman, 2001). Despite these setbacks, the absolutist image of mathematics persisted as a last bastion of certainty in the minds of many, upheld by the cultural ideal of mathematics as the model of infallible reasoning (Kline, 1980).

In summary, classical absolutism—embodied by Platonism, logicism, and formalism—portrays mathematics as an objective, unchanging entity: discovered in an abstract realm, reducible to logic or symbol manipulation, and known with certainty through deductive proof. Its legacy in education has been a vision of mathematics teaching that emphasizes theorem-proof rigor, precise definitions, and the conveyance of established truths, largely independent of empirical context or student experience. However, the internal crises of foundations (paradoxes and incompleteness) planted the seeds of a new outlook that would grow more prominent as the 20th century progressed: a view of mathematics as fallible, dynamic, and human-centered.

2.2. Progressive Absolutism: Intuitionism and Constructivism

While Platonism, logicism, and formalism sought to buttress the certainty of mathematics, other thinkers reacted to the foundational crises by modifying what "certainty" meant. Intuitionism, led by the Dutch mathematician L. E. J. Brouwer, has been interpreted as a form of progressive absolutism, a position that retains the pursuit of certainty but grounds it in mental constructability rather than formal logic (Brouwer, 1927; Dummett, 1977). Progressive absolutism, as used here, refers to philosophical views that maintain the goal of objective certainty in mathematics, but relocate its foundation from external logical or Platonic systems to the internal intuition and constructible activity of the mathematician. It differs from classical absolutism, which asserts the existence of a mind-independent mathematical reality, and from pedagogical constructivism, which is concerned with how learners come to understand mathematical ideas through experience rather than with the philosophical justification of mathematical truth (von Glasersfeld, 1995; Sfard, 1991). Intuitionists questioned the universality of classical logic in mathematics, for example, rejecting the blanket use of the Law of Excluded Middle and insisted that mathematical existence means constructability. In Brouwer's intuitionism, mathematical objects are not out there in a Platonic realm; they are constructs of the human mind. A mathematical statement is true only if we can mentally construct a proof of it, and a mathematical object exists only when we can describe how to build it step by step. This philosophy still aims for certainty, but the certainty is grounded in the secure intuition of the mathematician rather than in an external reality. In educational terms, an intuitionist view might value students' personal constructions of mathematical ideas, echoing learning constructivism, yet it remains "foundationist and absolutist" in its pursuit of secure knowledge. However, the foundation for this certainty is no longer a Platonic universe but rather human intuition and constructive proof (Ernest, 2004, p. 29).

A closely related strand is constructivism in the philosophical sense (not to be confused with pedagogical constructivism, though they are historically connected). Philosophical constructivism holds that mathematical knowledge is actively constructed by mathematicians rather than passively discovered. From this perspective, mathematical objects have no meaning until they are constructed and agreed upon through proof. Both intuitionism and constructivism modify the absolutist narrative by acknowledging the role of the human mind and activity in mathematics. In Ernest's analysis, intuitionism and constructivism constitute a form of progressive absolutism: they recognize human mathematical activity as central to the construction of knowledge, yet seek certainty within the confines of what can be rigorously constructed or demonstrated (Ernest, 1998). In other words, an intuitionist can assert that once a statement is proven constructively, it holds true absolutely—but anything that cannot be constructively proven is not admitted as true (thus avoiding non-constructive paradoxes by essentially narrowing the scope of mathematics).

It is worth noting that in the landscape of mathematics education, constructivism as a learning theory (inspired by Piaget and Vygotsky) gained traction in the late 20th century, emphasizing that learners build their own mathematical understanding. There is a philosophical kinship between this pedagogical constructivism and the intuitionist/constructivist philosophy: both value the active construction of knowledge. However, the key distinction lies in their scope: philosophical intuitionism concerns itself with the nature of mathematical truth, asserting that only those concepts which can be mentally constructed are valid, whereas pedagogical constructivism focuses on how individuals learn mathematics, emphasizing that understanding is actively built through cognitive engagement and personal experience. Still, the rise of constructivist learning theories dovetailed with a broader acceptance that mathematics is not simply "out there" to be transmitted, but something that students must actively make sense of, an idea comfortable to progressive absolutists and fallibilists alike.

In summary, progressive absolutism via intuitionism/constructivism kept one foot in the absolutist camp—believing in secure, reliable mathematical knowledge

—but shifted the source of that certainty from an external reality or formal system to the internal consistency of human-constructed proof. This nuanced position paved the way for more radical departures from absolutism by highlighting the mathematician's creative role and the possibility that some classical principles (like proving an existential statement by contradiction alone) might be inadmissible. It softened the ground for the next paradigm: the fallibilist turn, which would cast aside the insistence on certainty altogether.

3. Crises and Critiques of Absolutism: Paradox and Incompleteness

The early 20th-century cracks in the absolutist paradigm—exemplified by Russell's paradox and Gödel's incompleteness—have already been noted, but it is important to underscore their philosophical implications. If mathematics were truly a collection of "incontrovertible" truths built on infallible logic, as absolutists claimed, then consistency was paramount. A single contradiction is catastrophic in an absolutist framework, since from a contradiction, anything follows in logic. The discovery of contradictions in naive set theory (e.g., Russell's paradox) thus created a foundational crisis in mathematics around 1900. The immediate response was the foundationalist movement—logicism, formalism, intuitionism—which can be seen as last-ditch efforts to repair or rebuild the certainty of mathematics on new foundations. Each school had some success but also profound limitations, as discussed. By the 1930s, with Gödel's work, it became clear that no finite system of axioms could be both complete and provably consistent for all of mathematics. This realization does not mean mathematics was crumbling—mathematicians continued doing mathematics successfully—but it forced philosophers to accept that mathematics could not be fully self-justified by any single, mechanical foundation. There would always be an element of fallibility or assumption: an irreducible kernel of unprovable assertions or axioms that we choose to trust (Nagel & Newman, 2001).

Moreover, philosophers of mathematics pointed out that the history of mathematics shows it to be far from infallible. Imre Lakatos, for example, studied how mathematical ideas actually evolve: theorems are proposed, then sometimes refuted by counterexamples, then amended—a process strikingly similar to scientific theory change (Lakatos, 1979). This historical insight was a critique of the timeless certainty narrative. Even Euclid's ancient geometry, long held as the model of absolute truth, was found in the 19th century to have an alternative (non-Euclidean geometry) that is equally consistent. Thus, what was once taken as a necessary truth turned out to be based on an implicit assumption (Euclid's parallel postulate) that could be altered. This and other episodes illustrate what Kitcher (1983) and others emphasized: mathematical knowledge grows and changes over time, and its truths are not immune to revision. Such observations fuel the critique that absolutism presents an idealized, overly sanitized account of mathematics.

In light of these crises and critiques, many 20th-century thinkers began to articulate explicitly non-absolutist philosophies. The stage was set for a fundamental shift—a "turn" in the philosophy of mathematics from a quest for absolute foundations to an embrace of the human and fallible nature of mathematical knowledge. This fallibilist turn is described below, along with its far-reaching consequences for how we think about teaching mathematics.

3.1. The Fallibilist Turn

By the mid-20th century, a new narrative about mathematics gained momentum: one that accepted uncertainty and human agency as intrinsic to mathematics itself. Rather than something discovered in a perfect Platonic heaven or derived infallibly from logic, mathematics came to be seen by many as a human endeavor, empirical in some respects, and embedded in culture. This marked the rise of fallibilism in the philosophy of mathematics—the view that mathematical knowledge is "corrigible, fallible and a changing social product" (Ernest, 1998, para.). Two influential currents in this fallibilist turn were the quasi-empirical approach championed by Imre Lakatos and the social constructivist perspective elaborated by philosophers like Paul Ernest and Reuben Hersh. We also see the educational manifestation of fallibilism in what Ernest called the "problem-solving view" of mathematics. Below, we discuss these developments.

3.2. Quasi-Empiricism and Lakatos

Imre Lakatos, a philosopher of mathematics, played a pivotal role in challenging the absolutist image by carefully examining how mathematics actually develops. In his work Proofs and Refutations (Lakatos, 1976), Lakatos presents a dialogue where a class of students propose a formula (Euler's formula for polyhedra), find a "proof", then discover counterexamples, prompting modifications to definitions and theorems, and so on. Through this case study, Lakatos argued that the growth of mathematical knowledge resembles an empirical process of conjecture and refutation, much like the scientific method. He called this perspective "quasi-empiricism": while mathematics is not an empirical science per se (we do not do lab experiments on numbers), the method of improving conjectures via counterexamples parallels empirical falsification in science.

Lakatos's philosophy implied that mathematics is fallible at any given time—a theorem might be disproven or need revision if a hidden assumption is exposed by a new example. He thus directly contested the notion of mathematics as an incorrigible truth. His claim was shocking to many: mathematics, often held as the paradigm of certain knowledge, was portrayed as provisional and open-ended, subject to revision like any other knowledge (Lakatos, 1976). Indeed, when Lakatos's ideas were first published, even the editors' adding footnotes to his text distanced themselves from the "scandalous" view that mathematics is fallible (Lakatos, 1976). Yet history has vindicated many of Lakatos's insights. The acceptance of multiple geometries, the adjustments to probability theory after paradoxes, and the ongoing refinement of fundamental concepts (like the definition of algorithm or dimension) all attest that mathematics is not a static monument but a living, breathing discipline.

In the context of our discussion, Lakatos represents the empiricist streak within fallibilism. He and others (like George Pólya, who emphasized heuristic problem-solving strategies) reintroduced the idea that experience, albeit thought-experiment experience and trial-and-error, is part of mathematics. Mathematics, from this view, advances by exploring examples, proposing conjectures, and learning from failures. It is "quasi-empirical" in that proofs are scrutinized similarly to how experiments are peer-reviewed in science (Lakatos, 1976). Ernest (1998) builds on this insight, arguing that Lakatos's philosophy provides a foundation for viewing mathematics as fallible and socially embedded, with significant implications for teaching problem-solving and inquiry in the classroom.

3.3. Ernest's Problem-Solving View of Mathematics

In the late 1980s, Paul Ernest—a philosopher of mathematics education—synthesized various philosophical threads into a framework relevant to teaching (Ernest, 1989). He contrasted three dominant views: instrumentalist (mathematics as a toolkit), Platonist (mathematics as a fixed body of truths), and problem-solving (mathematics as a dynamic, social enterprise). The problem-solving view aligns with fallibilism: it portrays mathematics as a "dynamic, continually expanding field of human creation and invention, a cultural product" (Ernest, 1998: p. 23). In the context of our discussion, Lakatos represents the empiricist streak within fallibilism. He and others (like Pólya (1945), who emphasized heuristic problem-solving strategies) reintroduced the idea that experience, albeit thought-experiment experience and trial-and-error, are part of mathematics. Likewise, Videnovic (2021) emphasizes that viewing mathematics as fallible and revisable is essential for fostering creativity and deeper understanding in learners. Polya's work on heuristic strategies and Videnovic's recent empirical analysis add important layers to Ernest's conceptual framework, showing how problem-solving pedagogy has both philosophical and practical grounding.

The problem-solving view aligns closely with the principles of constructivist learning theory and NCTM-style pedagogy. If one believes mathematics is fundamentally about solving problems and creating new knowledge, then teaching should focus on engaging students in authentic problem-solving, encouraging them to conjecture, explore, and even err to learn. In Ernest's model, a teacher with a problem-solving view will likely prioritize understanding, meaning-making, and the process of mathematical inquiry over memorization of formulas or routine drills (Videnovic, 2021). This does not mean basic skills are ignored, but they are seen as tools to be used in broader investigative activities, rather than ends in themselves.

Ernest's articulation provided a conceptual link between the philosophy of mathematics and teachers' beliefs/practices. It became a foundation for many studies examining how a teacher's view of mathematics (e.g., "Mathematics is about discovering patterns and solving puzzles" versus "Mathematics is about getting the right answer by following rules") translates into their classroom behavior. Research confirms that teachers endorsing a problem-solving (fallibilist) conception tend to use more student-centered strategies and rich tasks, whereas those with instrumentalist or Platonist conceptions lean towards teacher explanation and practice of procedures (Kasa, Areaya, & Woldemichael, 2024). Thus, Ernest's problem-solving view is not only a description of mathematics in a philosophical sense; it is also prescriptive for education, suggesting that embracing this view can lead to more effective or at least more contemporary teaching approaches in line with reform recommendations.

3.4. Mathematics as a Socio-Cultural Construct

Perhaps the most radical—and increasingly influential—aspect of the fallibilist turn is the assertion that mathematics is a socially constructed knowledge, deeply intertwined with culture, language, and human activities. This perspective extends fallibilism by arguing not just that mathematical knowledge is fallible and changing, but also that it is created by people in social contexts, rather than having any existence independent of humanity. Ernest (1998) describes mathematics, from a social constructivist perspective, as a cultural product, fallible and shaped by human activity, rather than an infallible body of truths. This perspective rests on two key claims. First, mathematical ideas originate in human needs and activities, such as counting for trade or geometry for land measurement, and are therefore deeply rooted in the experiences and conventions of different societies. Anthropological studies, notably Bishop (1988), provide empirical support for this view by showing how number systems and spatial reasoning practices vary across cultural contexts. Second, the legitimacy of mathematical knowledge is not anchored in objective metaphysics, but in social validation. In this view, a proof gains its authority because it withstands communal scrutiny within the mathematical community, rather than because it corresponds to some external, absolute reality. As part of this view, mathematician-philosophers like Reuben Hersh have argued that mathematics should be seen as part of human culture—what he calls a humanist philosophy of mathematics (Hersh, 1997). Hersh points out, for example, that the way we think about infinity or continuity has changed over centuries through dialogue among mathematicians, and that new definitions or proofs gain status by consensus of experts over time—a fundamentally social phenomenon.

Ludwig Wittgenstein earlier prefigured aspects of social constructivism by suggesting that mathematical certainty is akin to the certainty of language rules: it comes from implicit social agreement on the use of words and statements (mathematics as a special kind of language or "form of life") (Wittgenstein, 2009). According to this view, to call a mathematical statement "true" is to say it follows the accepted rules within a given mathematical language game—rules that humans have created and can modify (Ernest, 1998). For instance, the statement $1 + $1 = $2 is true not because of an abstract Platonic force, but because we have collectively agreed on the meanings of "1", "+", "=", and "2" in standard arithmetic. If we change the context (e.g., in Boolean algebra, 1 + 1 is defined to equal 1), the truth value can differ. Thus, mathematical truths are contextual and contingent on definitions and axioms, which are products of human choice (albeit constrained by the need for consistency and usefulness).

Social constructivist and humanist philosophies do not imply that "anything goes" in mathematics—the physical reality and internal consistency still exert constraints. For example, if a society tried to create a mathematics where 2 + 2 = 5 for everyday objects, it would quickly run into practical contradictions. As Ernest (1998) notes, mathematics remains empirically anchored through its applications and the experiential origins of its concepts. Moreover, once a mathematical theory is developed, it has a certain internal logic that users must adhere to, or they simply are not doing mathematics in that theory. But the broader point is that those theories themselves, and the decision to use one or another, are human-made. Mathematics grows and is validated in a community: proofs are published and vetted through peer review; new conjectures gain interest through communal values of what is important; definitions are adopted because a community finds them fruitful.

This socio-cultural understanding of mathematics has significant implications for pedagogy. It encourages instructional approaches that situate mathematics learning in real contexts, leverage collaboration among students, and treat communication and negotiation of meaning as central to learning mathematics—essentially mirroring the way the mathematical community works (Bishop, 1988). It also encourages acknowledging students' cultural backgrounds, language, and prior experiences as resources in learning mathematics, rather than seeing mathematics as a culture-neutral subject. In recent curriculum reforms around the world, one can see the influence of this philosophy in the emphasis on mathematical discussions, group problem-solving, and real-world problem contexts, which embed mathematics in a human context rather than present it as isolated abstract exercises.

To summarize the fallibilist turn: Over the past century, the philosophy of mathematics shifted from seeking absolute foundations to embracing the human element—mathematics as created, fallible, and evolving. Quasi-empiricists like Lakatos showed how mathematical knowledge is refined through practice and counterexample, Ernest and others promoted a view of mathematics as human inquiry and problem-solving, and social constructivists reframed mathematics as a cultural product sustained by social processes of validation. These perspectives collectively undergird much of the modern thinking in mathematics education, where the goal is not only to teach mathematical results but to induct students into the practices of mathematical thinking—conjecturing, reasoning, communicating, and collaboratively making sense of problems. In the next section, we examine how these differing conceptions of mathematics (absolutist vs fallibilist) translate into distinct models of teaching and learning, and how they have influenced curriculum developments such as the NCTM standards and recent reforms in countries like Ghana.

4. Implications for Pedagogy and Curriculum

4.1. From Philosophy to Classroom: How Conceptions Shape Teaching

The connection between one's view of mathematics and one's approach to teaching it is well documented. A teacher who regards mathematics as a fixed body of truths and procedures (an absolutist stance) will likely teach differently from one who sees mathematics as a creative problem-solving endeavor (a fallibilist stance). In practice, these contrasting philosophical perspectives give rise to distinct models of teaching and learning. For instance, a fallibilist-aligned classroom may include collaborative problem-solving, student-generated conjectures, open-ended investigations, and classroom discussions where students evaluate each other's reasoning. Teachers might use "notice and wonder" routines, encourage multiple solution paths, and explicitly validate errors as a learning tool. Conversely, absolutist-aligned strategies may involve direct instruction, demonstration of formal proofs, and routine skill drills designed to ensure procedural fluency (Boaler, 2016; Sullivan & Lilburn, 2002). The following comparative overview is constructed based on a synthesis of relevant literature, including contributions from Ernest (1989) and Toumasis (1997), who have explored how mathematical worldviews shape instructional practice.

Nature of mathematical knowledge:

Absolutist view: Mathematics is a finished product—a compiled set of facts, formulas, and algorithms that learners must absorb. It is often presented as certain and unambiguous knowledge delivered by the textbook or teacher.

Fallibilist view: Mathematics is an evolving field—an open-ended exploration of problems, patterns, and ideas. Mathematical knowledge is seen as provisional and constructed, open to revision and extension as learners engage with concepts.

Model of learning:

Absolutist: Learning mathematics means receiving and mastering given knowledge. Students are expected to listen, memorize, and practice skills demonstrated by the teacher. Success comes from diligent practice, "hard work and mastery of skills" through drills and exercises (Armah, 2015). Productive learning is often individual and based on repetition to achieve accuracy. Importantly, errors are viewed as missteps to be minimized—signs that the student has not yet correctly absorbed the material. Thus, an absolutist-oriented teacher emphasizes correct performance and might quickly intervene to correct errors, seeing them as impediments to learning.

Fallibilist: Learning mathematics is an active process of inquiry. Students construct understanding by engaging in problem-solving, experimentation, discussion, and reflection. A fallibilist-minded teacher believes knowledge is built, not transferred, so they encourage exploration: e.g., posing questions, using manipulatives, group investigations, and projects. Learning must involve the active construction of understanding, shaped significantly by the learner's interaction with their social environment (Armah, 2015). In this mode, errors are considered natural and even desirable, because they can provoke discussion and lead to deeper understanding. Rather than something to be immediately corrected, errors are opportunities for diagnosing misconceptions and prompting cognitive conflict—the engine of conceptual change. This echoes the fallibilist idea that trial and error are intrinsic to doing mathematics.

Role of the teacher:

Absolutist: The teacher's role is that of an instructor and knowledge authority. The teacher typically demonstrates procedures, presents proofs or problem solutions, and then students practice similarly. The classroom is teacher-centered: the teacher explains, students listen, and emulate. This model can be described as didactic, with the teacher often adopting an authoritarian stance (in the benign sense of being the mathematical authority). The teacher is a "sage on the stage" whose duty is to transmit the established curriculum clearly and efficiently. Classroom interaction in this model often involves the teacher posing closed questions and students responding, with the teacher evaluating for correctness. Collaboration among students is less emphasized; instead, individual effort and discipline are stressed.

Fallibilist: The teacher's role is that of a facilitator or guide. Instead of simply telling students facts, the teacher creates environments where students can discover or construct key ideas for themselves. Phrases like "guide on the side" or "orchestrator of learning" describe this role. A fallibilist teacher designs tasks that allow multiple solution strategies, encourages students to discuss their thinking, and helps them make connections. They often use open-ended questions and rich problems to develop students' ability to think mathematically. The teacher may act as a "conductor of a symphony" of student ideas—coordinating discussions, highlighting insightful approaches, and gently correcting, when necessary, through questioning rather than direct instruction. The teacher-student relationship in this model is more egalitarian and supportive: the teacher often works alongside students as a mentor, and classroom authority can be shared (for instance, students validating each other's solutions). As described by Sullivan and Lilburn (2002), the teacher may take on roles such as counsellor, risk-taker, or devil's advocate to create a classroom environment where students feel free to propose conjectures and challenge ideas.

Goals of instruction and assessment:

Absolutist: The primary goal is skill proficiency and content coverage. Curriculum is often viewed as a sequence of topics to be delivered. Assessment focuses on the right answers and the ability to reproduce standard procedures or proofs. Timed tests, quizzes on formula application, and competition can feature prominently, aligning with the belief that doing mathematics well means doing it accurately and quickly based on learned methods. In this framework, a "good" mathematics lesson is one where students have learned the method and can apply it to similar problems reliably (Skemp, 1976).

Fallibilist: The goals extend to conceptual understanding, problem-solving ability, and mathematical reasoning/communication. A fallibilist teacher wants students to not only get answers but to understand why methods work, to recognize underlying concepts, and to be able to tackle unfamiliar problems. Assessments may include open-ended problems, projects, or oral presentations in which students explain their thinking. The emphasis is on the process as much as the product of solving problems. This approach aligns with modern standards that call for developing mathematical practices such as reasoning, argumentation, and modelling. A successful lesson in this view might be one where students have an insightful discussion or discovery, even if they have not polished every skill yet, because their deeper engagement with mathematical ideas is valued (Boaler, 2016).

It is important to note that these are idealized extremes. In reality, many teachers blend elements of both philosophies in their practice, sometimes out of necessity. As Ernest (2004) observed, teachers may combine absolutist and fallibilist strategies because each has practical merits in different situations. For instance, a teacher might use direct instruction (absolutist style) for a new technique but then engage students in an open-ended project (fallibilist style) to apply it. Nonetheless, the dominant conception a teacher holds can usually be seen in their default approach and how they prioritize various aspects of teaching and learning.

4.2. Historical Curricular Movements and Their Philosophical Roots

The major curriculum movements in mathematics education over the past several decades can be linked to the philosophical conceptions described above. Understanding these links helps explain why certain reforms were promoted and, in some cases, why they encountered resistance or fizzled out. Here we briefly analyze three significant movements—the "New Math" modernist reform of the 1950s-60s, the "Back-to-Basics" reaction of the 1970s, and the problem-solving focused reforms from the 1980s onward (including standards-based movements like NCTM)—in terms of their underlying views of mathematics.

Modern Mathematics ("New Math") movement: In the aftermath of World War II and into the Cold War era, there was a push, especially in the United States and parts of Europe, to reform mathematics education to be more rigorous, abstract, and aligned with contemporary mathematical research (such as set theory and structural algebra). This came to be known as the "New Math". Philosophically, the New Math was inspired by a Platonic/formalist vision of mathematics. It introduced students to set notation, bases other than 10, abstract algebraic structures, and formal geometric axioms—content that reflected the structure-centered approach of Bourbaki-style mathematics. The implicit message was that mathematics is about abstract structures more than about daily computations, and that by exposing students early to the elegant logical structure of mathematics, they would understand the true nature of the discipline. In effect, the New Math assumed an absolutist stance: mathematics was presented as a unified, logically deduced body of knowledge (some New Math textbooks even started with set-theoretic axioms). While intellectually rich, this movement infamously had problems at the implementation level—many teachers and parents did not fully grasp the abstractions, and it seemed disconnected from practical needs. In retrospect, one could say the New Math tried to bring a mathematician's mathematics into the school curriculum without sufficient mediation. It leaned heavily on a formal absolutist philosophy (truth through formal structures), which proved hard to reconcile with how children learn or what society expected from school mathematics at the time. The eventual backlash set the stage for the next movement.

Back-to-Basics movement: By the 1970s, amid dissatisfaction with the New Math, there was a swing back towards basic skills—arithmetic algorithms, computational proficiency, routine problem practice—essentially a return to an instrumentalist-absolutist model. The back-to-basics philosophy was less about lofty visions of mathematics and more about practical outcomes: students should reliably perform arithmetic and basic algebra, presumably because mathematics was viewed as a set of tools for life and other sciences. This corresponds to what Ernest termed the instrumentalist view: mathematics as an instrument or toolbox of rules and facts to be learned for utilitarian purposes (Videnovic, 2021). The underlying notion of mathematics here is still absolutist in that these basic skills are treated as fixed and non-negotiable—one must simply memorize number facts, learn standard algorithms, and practice them to mastery. The pedagogical style associated with back-to-basics was a traditional, teacher-directed one emphasizing memorization and repetitive drills (the educational embodiment of an absolutist outlook focusing on mastery of known truths). Back-to-basics gained political and public support by appealing to the need for "fundamentals" and the perception that the New Math had neglected them. However, critics pointed out that such an approach often ignored the understanding and thinking aspects of mathematics, potentially turning students off from the subject as just rote work. In philosophical terms, back-to-basics might be considered aligned with a narrow instrumental absolutism—it cares less about the grand truth of mathematical structures and more about the utility and correctness of mathematical procedures, but it still treats mathematics as a static body of established procedures to be delivered to students.

Problem-Solving and Standards-Based Reform: In the 1980s and especially with the publication of NCTM's Curriculum and Evaluation Standards for School Mathematics in 1989, a new consensus emerged among leaders in mathematics education: that students need to learn not just mathematical results, but how to think mathematically. The NCTM Standards and subsequent documents (e.g., Principles and Standards for School Mathematics in 2000, Principles to Actions in 2014) placed problem-solving, reasoning, communication, and connections at the forefront of objectives for all grade levels. This reform wave is explicitly rooted in a fallibilist and constructivist philosophy of mathematics. The Standards assume that mathematical knowledge is not static; they encourage exploring multiple solution strategies, understanding underlying concepts, and seeing mathematics as an ongoing human activity. As Toumasis (1997) analyzed, the NCTM Standards implicitly endorse a fallibilist view of mathematics, with many of their pedagogical assumptions and recommended approaches grounded in this philosophical orientation. For example, the emphasis on students conjecturing and justifying solutions mirrors the practice of mathematicians and treats mathematics as something one does, not just something one receives. The focus on communication (students explaining their reasoning to others) resonates with the social constructivist idea that knowledge is established through discourse and shared validation. Likewise, the incorporation of real-world problems and interdisciplinary connections reflects a view of mathematics as embedded in human contexts, not a sealed axiomatic system. In classroom practice, this reform movement translated into strategies like cooperative learning, use of manipulatives and technology, open-ended investigations, and assessment methods beyond timed tests (such as portfolios or projects). Teachers were encouraged to ask higher-order questions and cultivate a classroom culture of inquiry. In short, the reform agenda of the late 20th and early 21st centuries largely carried forward the problem-solving view and the social constructivist perspective from philosophy into mainstream policy.

Globally, many countries have echoed these trends in their curricula. For instance, Ghana's recent educational reforms (the Standards-Based Curriculum introduced in 2019) explicitly adopt an inquiry-driven, student-centered approach to mathematics. The curriculum philosophy in Ghana declares that effective mathematics education "should be inquiry-based" and emphasizes active learning where learners construct knowledge through experience, with teachers acting as facilitators (NaCCA, 2019). The Ghanaian mathematics curriculum documents state: "Mathematics learning is an active contextualized process of constructing knowledge based on learners' experiences rather than acquiring it. Learners are information constructors who operate as researchers. Teachers serve as facilitators by providing the enabling environment that promotes the construction of learners' own knowledge" (NaCCA, 2019: p. v). This language could have been lifted straight from a constructivist manifesto—it reflects the fallibilist, student-centered ideology that learning is done by the student (not delivered by the teacher) and that knowledge is built, not received. The alignment with NCTM-like philosophies is clear: both advocate for inquiry, problem-solving, and real-life connection. By contrast, earlier curricula in Ghana (as in many other places) were more objective-based and content-heavy, reflecting a traditional absolutist approach that prioritized coverage of material and exam preparation over inquiry (Agbofa et al., 2023).

In summary, modern curricular movements have progressively shifted from absolutist to fallibilist orientations. The pendulum swings in the past were instructive: the New Mathematics' formal absolutism proved unsustainable in schools, the back-to-basics absolutism addressed certain needs but was criticized for limiting understanding, and the contemporary standards movement embraced fallibilist ideas to produce a more balanced, robust mathematical education. This historical perspective reinforces the thesis that conceptions of mathematics matter: they drive how curricula are designed and what they emphasize. When misalignments occur—for example, an absolutist-minded teacher tasked with implementing a fallibilist curriculum—problems ensue. This brings us to the critical discussion of challenges faced when philosophical ideals meet classroom reality.

4.3. Discussion

Implementing a new vision of mathematics education is not as simple as writing standards or curriculum guides; it requires changing the deeply held beliefs of teachers and the culture of mathematics teaching. Herein lies one of the greatest challenges of mathematics education reform: shifting teacher beliefs that have been formed over the years (often reflecting the very traditional instruction that reforms seek to move away from). Despite decades of reform efforts worldwide, research consistently finds that many mathematics teachers still hold an absolutist or traditional view of mathematics—seeing it as a fixed body of rules and facts, and believing that teaching means explaining procedures for students to practice (White-Fredette, 2010; Beswick, 2006). As White-Fredette (2010) observed, "Mathematics teachers too often hold true to the traditional view of mathematics as an absolute truth independent of human subjectivity", even while policy discourse encourages more progressive views (p. 21). This disconnect between the philosophical basis of curriculum reforms (usually fallibilist, problem-solving oriented) and the prevailing beliefs of teachers (often absolutist or instrumentalist) is a root cause of many implementation failures in mathematics education.

Teachers' beliefs tend to be ingrained and resistant to change. Psychological research on teacher cognition suggests that beliefs about teaching and learning are often formed early, during one's own school days as a student, and become entrenched through years of experience (a phenomenon sometimes called an "apprenticeship of observation"). Pajares (1992) notes that beliefs are like a "'filtering' mechanism" through which new information must pass, and that beliefs established in early experiences are exceedingly difficult to alter (Shiver, 2010). In the context of mathematics education, a teacher who learned mathematics through lecture and practice, succeeded in that system, and perhaps has taught that way for a long time, may find it challenging to embrace a very different approach like open-ended problem-solving or student-led explorations. Even when teachers express nominal support for reforms, their classroom practice often falls back on familiar patterns, especially under stress or accountability pressures.

Another factor is that teachers may not have been exposed to alternative philosophies of mathematics in their training. If one has never explicitly confronted the question "What is the nature of mathematics?", they might default to the implicit philosophy handed down by their own teachers or textbooks. As some scholars point out, an "explicit discussion of the philosophy of mathematics is missing" in many teacher education programs and professional development efforts (White-Fredette, 2010: p. 21). Without guided reflection on what mathematics is and how it can be learned, teachers might not see why a reform method is designed as it is. For instance, if a teacher sees mathematics as mainly computation, they may regard a collaborative problem-solving task as a waste of time ("they are just talking instead of calculating"). In such a case, the reform practices appear ineffective or odd because they clash with the teacher's frame of reference. This highlights the need for professional development that addresses not just teaching techniques but the underlying epistemology of mathematics. Indeed, researchers argue that as teachers struggle to implement reforms, bringing the philosophy of mathematics into the conversation is crucial (Philipp, 2007; Armah & Robson, 2019). By examining their own beliefs and the rationale behind new approaches, teachers can begin to reconcile or adapt their perspectives.

The philosophical roots of implementation failures can be seen in several historical episodes. The New Math reform failed not only due to flawed materials but because many teachers at the time were not equipped (nor convinced of the need) to teach set theory and abstract structures—their belief was that arithmetic skills were the core of mathematics, so some taught the new content in a rote way or skipped it, thereby undermining the reform's intent. Later, when problem-solving and conceptual understanding became the mantra, some teachers adopted the surface features of reform (like using manipulatives or group work) but without changing their belief that "real math" is the procedural work. This sometimes led to pseudo-implementation, where a classroom has the trappings of reform but not the substance—e.g., students sit in groups but still just practice routines, or they do projects that are cosmetic but not mathematically rich. Such outcomes are often traced back to teachers not fully buying into the fallibilist/constructivist view that justifies the reform practices. If a teacher fundamentally believes that mathematical success is measured by speedy, accurate answers, they will feel anxious in a classroom discussion where students are exploring multiple strategies and making errors, even if that discussion is exactly what a fallibilist curriculum encourages (Armah, 2015). The tension can cause teachers to quietly revert to more controlled, teacher-centric methods, especially when under external pressure to show short-term results (like test scores) that might favor drill.

It is also important to acknowledge systemic and cultural factors. In some educational cultures, the teacher is expected to be the unquestioned authority, and the idea of students constructing knowledge may clash with societal expectations of what teaching looks like. In such contexts, even if an individual teacher is open to change, parents, administrators, or the examination system might reinforce absolutist-traditional approaches. These obstacles are not merely philosophical but are embedded in broader institutional cultures, teacher preparation systems, and assessment regimes that often reinforce traditional models of teaching and learning. For example, high-stakes exams that focus on procedural knowledge will push both teachers and students towards an absolutist, exam-driven mode regardless of the curriculum rhetoric. This was noted in Ghana and other countries implementing standards-based curricula: if assessment practices do not evolve in tandem with curricular philosophy, teachers are incentivized to "teach to the test" in traditional ways, thereby blunting the reform (a case of philosophical misalignment between curriculum and assessment) (Agbofa et al., 2023).

Changing teacher beliefs is not impossible—studies show it can be done, but it often requires significant experiences that challenge existing beliefs and support to develop new ones. For instance, engaging teachers in doing mathematics collaboratively, as learners, can open their eyes to the fallibilist perspective by letting them feel what it is like to explore and conjecture (Li, Cevikbaş, & Kaiser, 2024). Sustained participation in professional learning communities can also help shift teachers' beliefs and identities over time (Rushton, Rawlings Smith, Steadman, & Towers, 2023). Additionally, new generations of teachers who themselves learned under a problem-solving curriculum might bring different beliefs into the profession. In the long run, as more success stories emerge from classrooms using reform-oriented methods, the culture can shift (Kasa, Areaya, & Woldemichael, 2024).

However, a recurring recommendation in the literature is to make the philosophical dimension explicit. When teachers understand why a certain teaching approach is valued (because it aligns with a certain view of what mathematics is and how it is best learned), they are more likely to reflect on their own approach. Recent scholarship stresses that successful curriculum reform in mathematics requires teachers to engage with the underlying philosophy of the subject, not just new methods, so they can critically align their practice with evolving curricular aims (Beswick, 2012). This reflective approach treats teachers as intellectuals and co-designers of reform, rather than as technicians who must implement someone else's vision without context.

In conclusion, the challenges in shifting teacher beliefs and successfully implementing reform are as much philosophical as they are practical. Recognizing the prevailing absolutist image of mathematics among many teachers (and indeed in society at large) is the first step. From there, teacher education and professional development can be designed to gradually introduce and reinforce a more fallibilist, humanized view of mathematics—for example, through exposure to history of mathematics (to see its evolving nature), through solving problems with no immediately evident method (to experience the value of struggle and creativity), and through analyzing student thinking (to appreciate different ways mathematical ideas can manifest). The goal is not to denigrate the old views, but to broaden teachers' perspectives so they can appreciate multiple facets of mathematics. Ideally, a teacher comes to see that while arithmetic skills are essential (absolutist content), the nature of mathematics is bigger—it also thrives on curiosity, pattern-finding, argumentation, and real-world meaning (fallibilist processes). Teachers who reach this blended understanding are likely to be more flexible and effective in their practice, using direct instruction when a skill is needed but also fostering inquiry and discourse to develop deeper understanding.

4.4. Limitations

As a conceptual and philosophical review, this paper does not present original empirical data. While extensive literature has been cited, the synthesis remains interpretive. Future studies may include empirical investigations into how teacher beliefs evolve in response to explicit philosophical engagement or track the classroom impact of philosophy-informed professional development.

5. Conclusion and Recommendations

The nature of school mathematics—whether viewed as a set of eternal truths or as an evolving domain of human inquiry—is far from an abstract debate; it is a driving force behind how mathematics is taught and learned. This paper has traced the philosophical foundations of mathematics from absolutist traditions to fallibilist perspectives and connected these views to pedagogical models and curriculum movements. The evidence is clear that teachers' conceptions of mathematics influence their classroom practice, and that enduring educational change requires engaging with those conceptions. As mathematics education continues to evolve in the 21st century, two broad recommendations emerge:

Foster Reflective, Philosophy-Aware Teacher Education: To bridge the gap between reform ideals and classroom realities, teacher preparation and professional development programs should explicitly address the philosophy of mathematics. Prospective and practicing teachers need opportunities to reflect on questions like "What is mathematics?" and "How is mathematical knowledge generated and validated?" in relation to their teaching. By examining different philosophies—absolutist, instrumentalist, Platonist, problem-solving, social constructivist—teachers can become aware of their own default views and consider alternatives. This kind of reflection can be facilitated through readings (e.g., excerpts from Ernest, Hersh, or Lakatos), group discussions, and analyzing classroom scenarios. The aim is not to impose one "correct" philosophy, but to broaden teachers' perspectives so they realize, for example, that viewing mathematics as a creative problem-solving endeavor has substantial support and can enrich their teaching. Research suggests that without such conscious grappling with underlying beliefs, teachers often interpret new pedagogies through old lenses, diluting their effectiveness (White-Fredette, 2010). On the other hand, when teachers embrace a more humanistic, fallibilist view of mathematics, they become more willing to experiment with student-centered methods and to value student thinking over just answers. In essence, teachers should graduate not only with knowledge of mathematical content and methods, but also with a rich understanding of the nature of mathematics itself—its history, its philosophical debates, and its connection to culture and society. This prepares them to be thoughtful agents of curriculum, rather than passive implementers. As part of this, teacher education can include exploring the history of mathematics to illustrate how mathematical ideas have changed (undermining the notion of absolute, unchanging truth) and how social needs have driven mathematical development (reinforcing the sense of mathematics as a living human endeavor). Moreover, teachers can be engaged in action research in their own classrooms to reflect on how their students learn and what view of mathematics that implies. By cultivating such reflective practitioners, we increase the likelihood that educational reforms take root authentically, because teachers will carry the philosophical rationale for change within themselves.

Embrace Epistemological Plurality in Curriculum Design and Implementation: In implementing curriculum reforms, it is wise to acknowledge that different philosophical perspectives each contribute something valuable to mathematics education, and a balanced approach can be beneficial. An epistemological pluralism means recognizing that absolutist and fallibilist perspectives need not be entirely at odds in practice—elements of each can be aligned to serve educational goals. For instance, there are times when mathematical skills must be mastered to a high degree of accuracy (an absolutist emphasis on correctness and structure), and there are times when open exploration and debate are vital (a fallibilist emphasis on inquiry and conjecture). A robust curriculum can and should incorporate both: ensuring students achieve fluency in essential skills and procedures (acknowledging the structured nature of existing mathematical knowledge), while also engaging them in rich problem-solving, reasoning, and real-world applications (acknowledging the dynamic, applied, and humanistic nature of mathematics). Rather than swinging the pendulum fully to one side, curriculum designers and teachers can strive for a complementary approach. For example, after students learn a standard method (absolutist element), they might be challenged to derive it themselves or apply it in a novel situation (fallibilist element), thus seeing both the reliability of known results and the creativity needed to extend them. Embracing plurality also means being mindful of the transition for teachers and students. Abrupt shifts can cause pushback, but a pluralistic approach might introduce fallibilist practices gradually alongside familiar routines, easing stakeholders into a new philosophy. Additionally, valuing plurality implies a respect for teachers' professional judgment. Rather than mandating one "philosophically pure" method, reform efforts should empower teachers to adapt methods to their context, as long as they maintain the spirit of reform. Many successful teachers intuitively blend expository teaching with investigative learning—such hybridity should be seen as a strength, not a weakness, when aligned with clear learning objectives.

Finally, curriculum reform should include alignment of assessment, resources, and support with the philosophical underpinnings. If a curriculum espouses inquiry and understanding, assessments must go beyond multiple-choice and basic skills to include tasks that allow creative solutions and explanations. Otherwise, the hidden message to teachers and students is that we do not truly value the fallibilist ideals. Similarly, resources (textbooks, technology tools) should reflect the intended balance of procedural and conceptual work, and ongoing support (mentoring, professional learning communities) should be provided as teachers shift practices. Change in education is slow and complex, but it is facilitated when all components of the system are pulling in the same philosophical direction.

In conclusion, "revisiting the nature of school mathematics" is not a one-time scholarly exercise but a continual process that each generation of educators and reformers must engage in. Mathematics is often called the "queen of sciences" for its beauty and power, but it is also a human pursuit, full of struggles, triumphs, and yes, mistakes. When teachers and students come to see it in this richer light—as something alive, evolving, and richly connected to human thought—mathematics education can move beyond the sterile transmission of truths to become a vibrant exploration of ideas. By incorporating philosophical awareness into teacher education and by designing curricula that honor both the certainty and the uncertainty in mathematics, we can cultivate learners who not only can do mathematics but truly understand what mathematics is. This, ultimately, may be the most important lesson we teach in our mathematics classrooms: that mathematics is a human endeavor—one that values rigor and precision, but also creativity, dialogue, and open-mindedness—an endeavor in which they themselves can partake and to which they can contribute.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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