Open Journal of Optimization, 2021, 10, 101-125
https://www.scirp.org/journal/ojop
ISSN Online: 2325-7091
ISSN Print: 2325-7105

# Predictive Mathematical and Statistical Modeling of the Dynamic Poverty Problem in Burundi: Case of an Innovative Economic Optimization System

**Fulgence Nahayo, Ancille Bagorizamba, Marc Bigirimana, Irene Irakoze**

ISTA/University of Burundi, Bujumbura, Burundi
Email: fulgence.nahayo@ub.edu.bi, ancille.bagorizamba@ub.edu.bi, mackinanix1994@gmail.com, irene.irakoze@ub.edu.bi

**How to cite this paper:** Nahayo, F., Bagorizamba, A., Bigirimana, M. and Irakoze, I. (2021) Predictive Mathematical and Statistical Modeling of the Dynamic Poverty Problem in Burundi: Case of an Innovative Economic Optimization System. *Open Journal of Optimization*, **10**, 101-125.
https://doi.org/10.4236/ojop.2021.104008

**Received:** December 10, 2021
**Accepted:** December 27, 2021
**Published:** December 30, 2021



## Abstract

The mathematical and statistical modeling of the problem of poverty is a major challenge given Burundi's economic development. Innovative economic optimization systems are widely needed to face the problem of the dynamic of the poverty in Burundi. The Burundian economy shows an inflation rate of −1.5% in 2018 for the Gross Domestic Product growth real rate of 2.8% in 2016. In this research, the aim is to find a model that contributes to solving the problem of poverty in Burundi. The results of this research fill the knowledge gap in the modeling and optimization of the Burundian economic system. The aim of this model is to solve an optimization problem combining the variables of production, consumption, budget, human resources and available raw materials. Scientific modeling and optimal solving of the poverty problem show the tools for measuring poverty rate and determining various countries' poverty levels when considering advanced knowledge. In addition, investigating the aspects of poverty will properly orient development aid to developing countries and thus, achieve their objectives of growth and the fight against poverty. This paper provides a new and innovative framework for global scientific research regarding the multiple facets of this problem. An estimate of the poverty rate allows good progress with the theory and optimization methods in measuring the poverty rate and achieving sustainable development goals. By comparing the annual food production and the required annual consumption, there is an imbalance between different types of food. Proteins, minerals and vitamins produced in Burundi are sufficient when considering their consumption as required by the entire Burundian population. This positive contribution for the latter comes from the fact that some cows, goats, fishes, ⋯, slaughtered in Burundi come from neighboring countries. Real production remains in deficit. The lipids, acids, calcium, fibers and carbohydrates produced in Burundi are insufficient for consumption. This negative contribution proves a Burundian food deficit. It is a decision-making indicator for the design and updating of agricultural policy and implementation programs as well as projects. Investment and economic growth are only possible when food security is mastered. The capital allocated to food investment must be revised upwards. Demographic control is also a relevant indicator to push forward Burundi among the emerging countries in 2040. Meanwhile, better understanding of the determinants of poverty by taking cultural and organizational aspects into account guides managers for poverty reduction projects and programs.

## Keywords

Poverty Problem, Mathematical Modeling, Applied Statistics, Operational Research, Symplectic Partitioned Runge Kutta Algorithm, Dynamic Programming, Matlab and Simulink, AMPL, KNITRO, Gurobi, Economic Optimization, Technology Transfer, Incubation of Results, Sustainable Development Goals

## 1. Introduction

Burundi and the African continent exist four billion years ago. Burundi remains poor in view of the world economic balance. Burundi has more resources and more potential than many countries in the world. Africa remains poor despite its human resources, raw materials and natural wealth. Several systems of governance have followed one another and yet poverty remains an exponential function in some regions. The question is to know the reason of this almost eternal poverty.

The government of Burundi Reta Nkozi, Reta Mvyeyi has made its priority: the eradication of poverty. All Burundi population must have in a dignified and eternal manner Inoti y'ibihumbi cumi (10,000 BIF) mu mufuko. ISTA wants to make its contribution to the resolution and scientific mastery of the problem of poverty in Burundi.

In this paper, it is about developing a predictive mathematical and statistical modeling of the poverty problem in Burundi. Concretely, it is a question of identifying the variables of poverty and their mathematical interactions by modeling the dynamics of the problem of poverty in Burundi, the National efficiency objective function in view of all its variables and national potential and finally the production and consumption constraints of all layers of the Burundian population.

**National positioning:** The mathematical and statistical modeling of the poverty problem in Burundi is a major challenge in view of the economic place of Burundi in the concert of nations. Innovative economic optimization systems are widely needed in order to address the problem of poverty in Burundi. Indeed, the national GDP is 310 US Dollars per capita per year for a population of 12,044,164 in 2019. The national area is 27,834 km$^2$ while the total revenue (excluding donations) in 2018 is estimated at 5562.4 billion BIF. The Burundian economy shows an inflation rate of −1.5% in 2018 for a real GDP growth rate of 2.8% in 2016. In this paper, it is a question of finding models contributing to the resolution of the problem of poverty in Burundi. The results of this research come to fill the knowledge gap in the modeling and optimization of the Burundian economic system. It is a question of finding an optimization model combining the variables of production, consumption, budget, human resources and available raw materials.

**International positioning:** The scientific modeling of the problem of poverty and its optimal resolution allows measuring the poverty rate well and allowing comparing the levels of poverty from various countries on the basis of a new comparison approach using advanced knowledge internationally. In addition, knowledge of the other aspects that eliminate poverty will make it possible to properly orient development aid to developing countries and thus achieve their objectives of growth and the fight against poverty. This theme provides a new and innovative framework for scientific research on a global scale in view of the multiple facets of this problem [1] [2].

**Scientific positioning:** Better estimating the poverty rate will allow good progress with the theories and optimization methods in the measurement of the rate of poverty in general. In particular, the completion of this research work will make it possible to correctly assess the level of achievement of the sustainable development goals. In addition, getting to know better the killers of poverty, taking into account cultural and organizational aspects, will help to orient those in charge of projects and programs to fight against poverty and take effective and efficient actions.

**Problem:** The main problem of this research is to develop a scientific model of the problem of poverty in view of the available resources and ‘eradicate and plan well the economic future of Burundi.

**Research hypotheses:** Hypothesis 1: The current statistical and economic models are far from adequate to face poverty and the exponential population. Hypothesis 2: The development of a scientific model of the problem of poverty in view of the available resources is essential for a good mastery of the indicators of aid to decision at the micro and macroeconomic level.

**Research objective:** The objectives of the projects are:

- To develop a more complete scientific model of the problem of poverty in view of the resources available in Burundi for mastery of indicators of aid to poverty decision at the micro and macroeconomic level.
- Eradicate and plan well the economic future of Burundi.
- Establish a normative component providing a series of indicators for those responsible for the fight against poverty to enable them to better understand the economic, administrative and organizational stakes in the fight against

poverty.

- To support the choice of economic and political decision-makers within the framework of interventions in favor of Burundi.
- Model and plan the process of transfer and incubation of research results.

For the mathematical modeling of the problem of poverty in Burundi, it is concretely to work on: Identification of Variables, Modeling of the dynamics of poverty, Constraints Modeling, Objective Function Modeling, Modeling of the Optimal Poverty Control Problem, Resolution digital by an Algorithmic implementation under a Matlab programming language with global optimization tool-boxes Simulink, the Demonstration of algorithmic convergence and commutation, Extraction of the solutions which are the indicators of the decision on the quantity of production, consumption, monetary and non-monetary yield in relation to the variables of the problem identified including the technical variables of control and mastery poverty in Burundi. Finally, it is about the Popularization and Validation of decision indicators, the Process of setting up a reference center on the incubation of the results of this research, the Knowledge transfer process to the community, the technology sustainability process, the Burundi economic future prediction process, Innovation at the service of the community. A responsible and enlightened elite is the future of the nation. Wealth begins with "the thinking design", Twese turi hamwe vyose birashoboka \og together we can \fg.

Figure 1 shows that this research touches on several sustainable development objectives. This figure clarifies the interaction between these SDGs and all the variables of the model according to their relevance degree.

## 2. Mathematical Modeling of the Poverty Problem in Burundi

The mathematical modeling of the poverty problem in Burundi mainly concerns the identification of variables, the modeling of the dynamics of poverty, the modeling constraints, the modeling of the objective function and finally the combination of all these functions for a complete modeling of the optimal poverty control problem.

### 2.1. Identification of State Variables and Technical Control Variables

We have identified variables according to the different categories of raw resources available across all areas of agricultural, industrial and artisanal production. These are mainly the following resources:

- **Raw materials:** Gold, coltan, nickel, limestone, aluminum, silver, zinc, clay sand, rocks, peat, quartz, monazite, uranium, diamond, etc.
- **Agricultural products:** Tea, coffee, mushrooms, beans, corn, wheat, sorghum, eulesine, palm, bananas, potato, sweet potato, cassava, colocases, soybeans, cabbage, carrots, cotton, tobacco, pineapples, mangoes, oranges, lemons, papayas,

| SDGs involved in the project | Description | The project variables in relation to the SDG |
|---|---|---|
| SDG 1: No poverty | End poverty in all its forms and everywhere | Population<br>National Budget<br>Production companies<br>Consummer products |
| SDG 2: Zero hunger | End hunger, achieve food security and improved nutrition and promote sustainable agriculture | Population<br>National Budget<br>Production companies<br>Consummer products |
| SDG 3: Good Health and well being | Ensure healthy lives and promote well-being for all at all ages | Population<br>National Budget<br>Production companies<br>Consummer products |
| SDG 8: Decent work and economic growth | Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all | Economic growth<br>Population<br>The National Budget<br>Production companies<br>Consummer products |
| SDG 9: Industry, Innovation and Infrastructure | Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation | Raw materials<br>Population<br>The National Budget<br>Production companies<br>Consummer products |
| SDG 12: Responsible consumption and production | Ensure sustainable consumption and production patterns | Raw materials<br>Population<br>The National Budget<br>Production companies<br>Consummer products |

**Figure 1.** The sustainable development goals involved in the project.

strawberries, avocados, pasta, sunflower, sugar cane, rice, peas, peanuts, etc.

- **Livestock products:** Cow meat, goat meat, pork meat sheep meat, rabbit meat, duck meat, chicken meat, fish, eggs, butters, etc.
- **Industrial products:** Yoghurt, cow cheese, goat cheese, organo-mineral fertilizers, agricultural lime, primus, small primus, primus nyongera, amstel blonde 65 cl, amstel royal, amstel bock, amstel blonde 50 cl, lemonades, wine, kinju, aquavie, sangwe, fungus, akezamutima juice, maracuja juice, pineapple juice, cement, mattress, soap, wheat flour, corn flour, corn flour cassava, sorghum flour, alien flour, sesame flour, cotebu loincloths, pona pharmaceutical products, siphar products, bread, nails, baking iron, metal bars, tôles, etc.
- **Consumption variables:** Men, women and children.

This model considers animals as production variables. Mathematically, we have the state variables $y_1,\cdots,y_n$ which give a vector function $y=\left(y_1,\cdots,y_n\right)^{\mathrm{T}}$, Technical variables for poverty control $w=\left(w_1,\cdots,w_p\right)^{\mathrm{T}}$ as well as their interaction for the complete modeling of the mathematical model.

## 2.2. Modeling the Dynamics of Poverty [1]-[7]

The dynamics of poverty is a system of first-order, non-linear and non-convex differential equations, all the more so as the problem of poverty is complex and multi-variable with implications for consumption, depletion and regeneration of resources.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f\left(t, y_1(t), \cdots, y_n(t), w(t)\right) \tag{1}$$

$$= FGT_\alpha$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{z-R_i}{z}\right)^{\alpha} I\left(R_i \le z\right) \tag{2}$$

with:

$$I\left(R_i \le z\right) = \begin{cases} 1 & \text{si } R_i \le z \\ 0 & \text{sinon} \end{cases}$$

when $\dot{y} = FGT_\alpha$, et $R_i = PC_r/N$, the income per individual. The component of the dynamics of poverty is modeled as follows:

$$\dot{y}_i = f\left(T_i, K_i, E_i, D_i\right) = m \times r_i \times T_i^a K_i^\beta E_i^\gamma D_i^S \tag{3}$$

While the Production function $P$ is given by the following model:

$$P = \sum_{i=1}^{n} m \times r_i \times T_i^a K_i^\beta E_i^\gamma D_i^S \tag{4}$$

when $m$ is a parameter, $z$ is the poverty line, $\alpha$ is the aversion coefficient which will determine the significance of the index. If $\alpha$ is high, a large weight of the poor will be taken into account. $r = \left(r_1, \cdots, r_n\right)$ is the productivity, $T_i = \frac{y_i}{r_i}$ the work component, $K_i = \frac{y_i}{a'}$ is the component of capital where $a'$ is the yield, $E_i = c'R_i + E_0$ with $c' = \frac{\Delta E_i}{\Delta R_i}$ the consumption component. The term $C_r$ is the function of the reduced costs [3]. Overall we have that $T\left(y_1, \cdots, y_n\right) = \left(T_1, \cdots, T_n\right)$ is the work with $T_i$ which depends of $y_i, i = 1, \cdots, n$, $K\left(y_1, \cdots, y_n\right) = \left(K_1, \cdots, K_n\right)$ is the Capital, $E\left(y_1, \cdots, y_n\right) = \left(E_1, \cdots, E_n\right)$ is the consumption, $D\left(y_1, \cdots, y_n, w\right) = \left(D_1, \cdots, D_n\right)$ represents the volume of financial items. The differential equation $f\left(t, y_1(t), \cdots, y_n(t), w(t)\right)$ is the function defining the dynamics of poverty in Burundi. By mathematical transformation, we have:

$$\begin{aligned} P &= \sum_{i=1}^{n} m \times r_i \times \left(\frac{y_i}{r_i}\right)^a \left(\frac{y_i}{a'}\right)^\beta \left(c'R_i + E_0\right)^\gamma D_i^S \\ &= \sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^\beta}\left(c'R_i + E_0\right)^\gamma D_i^S. \end{aligned} \tag{5}$$

Considering the fluctuations of financial elements, their volume $D_i$ can be expressed by the following stochastic differential equation:

$$\begin{aligned} \mathrm{d}D_i(t) &= N_i(t)\mathrm{d}c_i(t) + \mu_i(t)c_i(t)\mathrm{d}t - E_i(t)\mathrm{d}t \\ &= \{\tau(t)N_0(t)c_0(t) + [\lambda_i(t)+\mu_i(t)]N_i(t)c_i(t) - E_i(t)\}\mathrm{d}t \\ &\quad + N_i(t)c_i(t)\sigma_i(t)\mathrm{d}W(t) \\ &= \{\tau(t)D_i(t) + [\lambda_i(t)+\mu_i(t)-\tau(t)]u_i(t) - E_i(t)\}\mathrm{d}t + \sigma_i(t)u_i(t)\mathrm{d}W(t), \end{aligned} \tag{6}$$

$$\dot{D}_i = \tau(t)D_i(t) + [\lambda_i(t)+\mu_i(t)-\tau(t)]u_i(t) - E_i(t) + \sigma_i(t)u_i(t)\frac{\mathrm{d}W(t)}{\mathrm{d}t} \tag{7}$$

$$= q(t, \tau(t), \lambda(t), \mu_i(t), \sigma_i(t), u_i(t), D_i(t), E_i(t), W(t)) \tag{8}$$

where $\tau(t)$, $\lambda(t)$, $\mu_i(t)$, $\sigma_i(t)$ et $u_i(t) = N_i(t)c_i(t), i = 0, \cdots, n$ are respectively the interest rate, the appreciation rate, the dividend rate, the volatility and the wealth of the investment on the $i^{th}$ variable.

The Derivation of the individual income function is modeled by replacing $P$ by its value in the individual income $R_i$:

$$R_i = \frac{\left[\sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^{\beta}} (c'R_i + E_0)^{\gamma} D_i^S\right] C_r}{N}. \tag{9}$$

The technical control variables $w(t)$ are given by:

$$\begin{aligned} w(t) &= (w_1(t), w_2(t), w_3(t), w_4(t), w_5(t), w_6(t)) \\ &= (P(t), T(t), K(t), E(t), D(t), C(t)) \end{aligned} \tag{10}$$

By injecting all these equations into (2), the real dynamic therefore takes the following form:

$$\dot{D}_i = q(t, \tau(t), \lambda(t), \mu_i(t), \sigma_i(t), u_i(t), D_i(t), E_i(t), W(t))$$

$$= \tau(t)D_i(t) + [\lambda_i(t)+\mu_i(t)-\tau(t)]u_i(t) - E_i(t) + \sigma_i(t)u_i(t)\frac{\mathrm{d}W(t)}{\mathrm{d}t} \tag{11}$$

$$\dot{y} = f(t, y_1(t), \cdots, y_n(t), w(t)) \tag{12}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{z - \dfrac{\left[\sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^{\beta}} (c'R_i + E_0)^{\gamma} D_i^S\right] C_r}{N}}{z}\right)^{\alpha} I(R_i \le z) \tag{13}$$

$$= \frac{1}{N^2}\sum_{i=1}^{N}\left(\frac{Nz - \left[\sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^{\beta}} (c'R_i + E_0)^{\gamma} D_i^S\right] C_r}{z}\right)^{\alpha} I(R_i \le z) \tag{14}$$

The next section shows the relationship between state variables and control variables with the required raw resources.

### 2.3. Constraints of the Poverty Problem in Burundi [1]-[6] [8]-[23]

Constraints are parameters and equations defining the problem. Considering the mathematical and economic complexity of the poverty problem, the constraints have ramifications for consumption, resource availability, and production. The combination of the required primary resources, the available primary resources and the variables constitute the mixed constraints given by the following system:

$$\begin{cases} a_{11}y_1 + a_{12}y_2 + a_{13}y_3 + a_{14}y_4 + \cdots + a_{1n}y_n \le b_1 \\ a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + a_{24}y_4 + \cdots + a_{2n}y_n \le b_2 \\ a_{31}y_1 + a_{32}y_2 + a_{33}y_3 + a_{34}y_4 + \cdots + a_{3n}y_n \le b_3 \\ \vdots \\ a_{m1}y_1 + a_{m2}y_2 + a_{m3}y_3 + a_{m4}y_4 + \cdots + a_{mn}y_n \le b_m \\ y_i \ge 0 \end{cases} \tag{15}$$

where $f(t, y_1, \cdots, y_n, w)$ is the function defining the dynamics of poverty in Burundi in time and space, the $a_{ij}$, $i = 1, \cdots, m$, $j = 1, \cdots, n$, are the required primary resources and the $b_i$ are the available primary resources. In practice, the constraint matrix will be modeled on the basis of data collected in the field in the pilot provinces and will follow a generalization of the problem throughout Burundi country.

Figure 2 shows the constraints matrix A where Tech = technology, En = energy, Eau = investment or production water, HR = human resources, Arg = investment money, Fum org = organic manure, Fum. non org = non-organic manure, Insect = insecticide, Lev = yeast, Sorg = sorghum, four = animal fodder, far = flour, Suc = sugar, salt, oil, Sav = soap, milk, dye.

### 2.4. Objective Function

The cost function of the poverty problem is $J(y_1, \cdots, y_n, w)$. It means the national yield function in view of the primary resources sold and consumed. This function is mathematically modeled as follows:

$$\begin{aligned} J(y_1(t), \cdots, y_n(t), w(t)) &= c_{m1}(t)\, y_1(t) + \cdots + c_{mn}(t)\, y_n(t) \\ c_{mi}(t) &= \frac{(\lambda t)^m \mathrm{e}^{-\lambda t}}{m!}, \; m = 1, 2, 3, 4; \; i = 1, \cdots, n \end{aligned} \tag{16}$$

where $m$ is the cropping season index. This function will be submitted to the maximization operator by ruling on the national maximum yield. In other words, it is a question of modeling the budgetary function if we consider all the possible economic variants. The $C(t) = (c_1(t), \cdots, c_n(t))$ is the function of the costs of the available resources sold. This function is vectorial with n variables defined above, namely $r = (r_1, \cdots, r_n)$ productivity $T = (T_1, \cdots, T_n)$ work, $K = (K_1, \cdots, K_n)$ capital, $E = (E_1, \cdots, E_n)$ consumption and $D = (D_1, \cdots, D_n)$ volume of financial items, which include, in addition to the profits, all the items of the operating accounts more or less rebellious to the decomposition in volume and price index (interest, indirect taxes, etc.). In the absence of objective bases,

| Variables | $y_i$ | Tech | En | Eau | RH | Arg. | Fum org. | Fum non org. | Insect | Lev | Sorg | four | far | suc | sel | huile | Sav | Lait | Dye |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | $y_1$ | $a_{11}$ | $a_{21}$ | $a_{31}$ | $a_{41}$ | $a_{51}$ | $a_{61}$ | $a_{71}$ | $a_{81}$ | $a_{91}$ | $a_{101}$ | $a_{111}$ | $a_{121}$ | $a_{131}$ | $a_{141}$ | $a_{151}$ | $a_{161}$ | $a_{171}$ | $a_{181}$ |
| Coltan | $y_2$ | $a_{12}$ | $a_{22}$ | $a_{32}$ | $a_{42}$ | $a_{52}$ | $a_{62}$ | $a_{72}$ | $a_{82}$ | $a_{92}$ | $a_{102}$ | $a_{112}$ | $a_{122}$ | $a_{132}$ | $a_{142}$ | $a_{152}$ | $a_{162}$ | $a_{172}$ | $a_{182}$ |
| Nickel | $y_3$ | $a_{13}$ | $a_{23}$ | $a_{33}$ | $a_{43}$ | $a_{53}$ | $a_{63}$ | $a_{73}$ | $a_{83}$ | $a_{93}$ | $a_{103}$ | $a_{113}$ | $a_{123}$ | $a_{133}$ | $a_{143}$ | $a_{153}$ | $a_{163}$ | $a_{173}$ | $a_{183}$ |
| Limestone | $y_4$ | $a_{14}$ | $a_{24}$ | $a_{34}$ | $a_{44}$ | $a_{54}$ | $a_{64}$ | $a_{74}$ | $a_{84}$ | $a_{94}$ | $a_{104}$ | $a_{114}$ | $a_{124}$ | $a_{134}$ | $a_{144}$ | $a_{154}$ | $a_{164}$ | $a_{174}$ | $a_{184}$ |
| Aluminium | $y_5$ | $a_{15}$ | $a_{25}$ | $a_{35}$ | $a_{45}$ | $a_{55}$ | $a_{65}$ | $a_{75}$ | $a_{85}$ | $a_{95}$ | $a_{105}$ | $a_{115}$ | $a_{125}$ | $a_{135}$ | $a_{145}$ | $a_{155}$ | $a_{165}$ | $a_{175}$ | $a_{185}$ |
| The silver metal. | $y_6$ | $a_{16}$ | $a_{26}$ | $a_{36}$ | $a_{46}$ | $a_{56}$ | $a_{66}$ | $a_{76}$ | $a_{86}$ | $a_{96}$ | $a_{106}$ | $a_{116}$ | $a_{126}$ | $a_{136}$ | $a_{146}$ | $a_{156}$ | $a_{166}$ | $a_{176}$ | $a_{186}$ |
| Zinc | $y_7$ | $a_{17}$ | $a_{27}$ | $a_{37}$ | $a_{47}$ | $a_{57}$ | $a_{67}$ | $a_{77}$ | $a_{87}$ | $a_{97}$ | $a_{107}$ | $a_{117}$ | $a_{127}$ | $a_{137}$ | $a_{147}$ | $a_{157}$ | $a_{167}$ | $a_{177}$ | $a_{187}$ |
| Clay | $y_8$ | $a_{18}$ | $a_{28}$ | $a_{38}$ | $a_{48}$ | $a_{58}$ | $a_{68}$ | $a_{78}$ | $a_{88}$ | $a_{98}$ | $a_{108}$ | $a_{118}$ | $a_{128}$ | $a_{138}$ | $a_{148}$ | $a_{158}$ | $a_{168}$ | $a_{178}$ | $a_{188}$ |
| Sand | $y_9$ | $a_{19}$ | $a_{29}$ | $a_{39}$ | $a_{49}$ | $a_{59}$ | $a_{69}$ | $a_{79}$ | $a_{89}$ | $a_{99}$ | $a_{109}$ | $a_{119}$ | $a_{129}$ | $a_{139}$ | $a_{149}$ | $a_{159}$ | $a_{169}$ | $a_{179}$ | $a_{189}$ |
| Rock | $y_{10}$ | $a_{110}$ | $a_{210}$ | $a_{310}$ | $a_{410}$ | $a_{510}$ | $a_{610}$ | $a_{710}$ | $a_{810}$ | $a_{910}$ | $a_{1010}$ | $a_{1110}$ | $a_{1210}$ | $a_{1310}$ | $a_{1410}$ | $a_{1510}$ | $a_{1610}$ | $a_{1710}$ | $a_{1810}$ |
| Peat | $y_{11}$ | $a_{111}$ | $a_{211}$ | $a_{311}$ | $a_{411}$ | $a_{511}$ | $a_{611}$ | $a_{711}$ | $a_{811}$ | $a_{911}$ | $a_{1011}$ | $a_{1111}$ | $a_{1211}$ | $a_{1311}$ | $a_{1411}$ | $a_{1511}$ | $a_{1611}$ | $a_{1711}$ | $a_{1811}$ |
| quartz | $y_{12}$ | $a_{112}$ | $a_{212}$ | $a_{312}$ | $a_{412}$ | $a_{512}$ | $a_{612}$ | $a_{712}$ | $a_{812}$ | $a_{912}$ | $a_{1012}$ | $a_{1112}$ | $a_{1212}$ | $a_{1312}$ | $a_{1412}$ | $a_{1512}$ | $a_{1612}$ | $a_{1712}$ | $a_{1812}$ |
| Monazite | $y_{13}$ | $a_{113}$ | $a_{213}$ | $a_{313}$ | $a_{413}$ | $a_{513}$ | $a_{613}$ | $a_{713}$ | $a_{813}$ | $a_{913}$ | $a_{1013}$ | $a_{1113}$ | $a_{1213}$ | $a_{1313}$ | $a_{1413}$ | $a_{1513}$ | $a_{1613}$ | $a_{1713}$ | $a_{1813}$ |
| Uranium | $y_{14}$ | $a_{114}$ | $a_{214}$ | $a_{314}$ | $a_{414}$ | $a_{514}$ | $a_{614}$ | $a_{714}$ | $a_{814}$ | $a_{914}$ | $a_{1014}$ | $a_{1114}$ | $a_{1214}$ | $a_{1314}$ | $a_{1414}$ | $a_{1514}$ | $a_{1614}$ | $a_{1714}$ | $a_{1814}$ |
| Diamond | $y_{15}$ | $a_{115}$ | $a_{215}$ | $a_{315}$ | $a_{415}$ | $a_{515}$ | $a_{615}$ | $a_{715}$ | $a_{815}$ | $a_{915}$ | $a_{1015}$ | $a_{1115}$ | $a_{1215}$ | $a_{1315}$ | $a_{1415}$ | $a_{1515}$ | $a_{1615}$ | $a_{1715}$ | $a_{1815}$ |
| Tea | $y_{16}$ | $a_{116}$ | $a_{216}$ | $a_{316}$ | $a_{416}$ | $a_{516}$ | $a_{616}$ | $a_{716}$ | $a_{816}$ | $a_{916}$ | $a_{1016}$ | $a_{1116}$ | $a_{1216}$ | $a_{1316}$ | $a_{1416}$ | $a_{1516}$ | $a_{1616}$ | $a_{1716}$ | $a_{1816}$ |
| Coffee | $y_{17}$ | $a_{117}$ | $a_{217}$ | $a_{317}$ | $a_{417}$ | $a_{517}$ | $a_{617}$ | $a_{717}$ | $a_{817}$ | $a_{917}$ | $a_{1017}$ | $a_{1117}$ | $a_{1217}$ | $a_{1317}$ | $a_{1417}$ | $a_{1517}$ | $a_{1617}$ | $a_{1717}$ | $a_{1817}$ |
| mushrooms | $y_{18}$ | $a_{118}$ | $a_{218}$ | $a_{318}$ | $a_{418}$ | $a_{518}$ | $a_{618}$ | $a_{718}$ | $a_{818}$ | $a_{918}$ | $a_{1018}$ | $a_{1118}$ | $a_{1218}$ | $a_{1318}$ | $a_{1418}$ | $a_{1518}$ | $a_{1618}$ | $a_{1718}$ | $a_{1818}$ |
| Beans | $y_{19}$ | $a_{119}$ | $a_{219}$ | $a_{319}$ | $a_{419}$ | $a_{519}$ | $a_{619}$ | $a_{719}$ | $a_{819}$ | $a_{919}$ | $a_{1019}$ | $a_{1119}$ | $a_{1219}$ | $a_{1319}$ | $a_{1419}$ | $a_{1519}$ | $a_{1619}$ | $a_{1719}$ | $a_{1819}$ |
| Maize | $y_{20}$ | $a_{120}$ | $a_{220}$ | $a_{320}$ | $a_{420}$ | $a_{520}$ | $a_{620}$ | $a_{720}$ | $a_{820}$ | $a_{920}$ | $a_{1020}$ | $a_{1120}$ | $a_{1220}$ | $a_{1320}$ | $a_{1420}$ | $a_{1520}$ | $a_{1620}$ | $a_{1720}$ | $a_{1820}$ |
| Corn | $y_{21}$ | $a_{121}$ | $a_{221}$ | $a_{321}$ | $a_{421}$ | $a_{521}$ | $a_{621}$ | $a_{721}$ | $a_{821}$ | $a_{921}$ | $a_{1021}$ | $a_{1121}$ | $a_{1221}$ | $a_{1321}$ | $a_{1421}$ | $a_{1521}$ | $a_{1621}$ | $a_{1721}$ | $a_{1821}$ |
| Sorghum | $y_{22}$ | $a_{122}$ | $a_{222}$ | $a_{322}$ | $a_{422}$ | $a_{522}$ | $a_{622}$ | $a_{722}$ | $a_{822}$ | $a_{922}$ | $a_{1022}$ | $a_{1122}$ | $a_{1222}$ | $a_{1322}$ | $a_{1422}$ | $a_{1522}$ | $a_{1622}$ | $a_{1722}$ | $a_{1822}$ |
| Eulesin | $y_{23}$ | $a_{123}$ | $a_{223}$ | $a_{323}$ | $a_{423}$ | $a_{523}$ | $a_{623}$ | $a_{723}$ | $a_{823}$ | $a_{923}$ | $a_{1023}$ | $a_{1123}$ | $a_{1223}$ | $a_{1323}$ | $a_{1423}$ | $a_{1523}$ | $a_{1623}$ | $a_{1723}$ | $a_{1823}$ |
| Palm | $y_{24}$ | $a_{124}$ | $a_{224}$ | $a_{324}$ | $a_{424}$ | $a_{524}$ | $a_{624}$ | $a_{724}$ | $a_{824}$ | $a_{924}$ | $a_{1024}$ | $a_{1124}$ | $a_{1224}$ | $a_{1324}$ | $a_{1424}$ | $a_{1524}$ | $a_{1624}$ | $a_{1724}$ | $a_{1824}$ |
| Bananas | $y_{25}$ | $a_{125}$ | $a_{225}$ | $a_{325}$ | $a_{425}$ | $a_{525}$ | $a_{625}$ | $a_{725}$ | $a_{825}$ | $a_{925}$ | $a_{1025}$ | $a_{1125}$ | $a_{1225}$ | $a_{1325}$ | $a_{1425}$ | $a_{1525}$ | $a_{1625}$ | $a_{1725}$ | $a_{1825}$ |
| Potato | $y_{26}$ | $a_{126}$ | $a_{226}$ | $a_{326}$ | $a_{426}$ | $a_{526}$ | $a_{626}$ | $a_{726}$ | $a_{826}$ | $a_{926}$ | $a_{1026}$ | $a_{1126}$ | $a_{1226}$ | $a_{1326}$ | $a_{1426}$ | $a_{1526}$ | $a_{1626}$ | $a_{1726}$ | $a_{1826}$ |
| Yam | $y_{27}$ | $a_{127}$ | $a_{227}$ | $a_{327}$ | $a_{427}$ | $a_{527}$ | $a_{627}$ | $a_{727}$ | $a_{827}$ | $a_{927}$ | $a_{1027}$ | $a_{1127}$ | $a_{1227}$ | $a_{1327}$ | $a_{1427}$ | $a_{1527}$ | $a_{1627}$ | $a_{1727}$ | $a_{1827}$ |
| Cassava | $y_{28}$ | $a_{128}$ | $a_{228}$ | $a_{328}$ | $a_{428}$ | $a_{528}$ | $a_{628}$ | $a_{728}$ | $a_{828}$ | $a_{928}$ | $a_{1028}$ | $a_{1128}$ | $a_{1228}$ | $a_{1328}$ | $a_{1428}$ | $a_{1528}$ | $a_{1628}$ | $a_{1728}$ | $a_{1828}$ |
| Colocases | $y_{29}$ | $a_{129}$ | $a_{229}$ | $a_{329}$ | $a_{429}$ | $a_{529}$ | $a_{629}$ | $a_{729}$ | $a_{829}$ | $a_{929}$ | $a_{1029}$ | $a_{1129}$ | $a_{1229}$ | $a_{1329}$ | $a_{1429}$ | $a_{1529}$ | $a_{1629}$ | $a_{1729}$ | $a_{1829}$ |
| Soy | $y_{30}$ | $a_{130}$ | $a_{230}$ | $a_{330}$ | $a_{430}$ | $a_{530}$ | $a_{630}$ | $a_{730}$ | $a_{830}$ | $a_{930}$ | $a_{1030}$ | $a_{1130}$ | $a_{1230}$ | $a_{1330}$ | $a_{1430}$ | $a_{1530}$ | $a_{1630}$ | $a_{1730}$ | $a_{1830}$ |
| Cabbage | $y_{31}$ | $a_{131}$ | $a_{231}$ | $a_{331}$ | $a_{431}$ | $a_{531}$ | $a_{631}$ | $a_{731}$ | $a_{831}$ | $a_{931}$ | $a_{1031}$ | $a_{1131}$ | $a_{1231}$ | $a_{1331}$ | $a_{1431}$ | $a_{1531}$ | $a_{1631}$ | $a_{1731}$ | $a_{1831}$ |
| Carrots | $y_{32}$ | $a_{132}$ | $a_{232}$ | $a_{332}$ | $a_{432}$ | $a_{532}$ | $a_{632}$ | $a_{732}$ | $a_{832}$ | $a_{932}$ | $a_{1032}$ | $a_{1132}$ | $a_{1232}$ | $a_{1332}$ | $a_{1432}$ | $a_{1532}$ | $a_{1632}$ | $a_{1732}$ | $a_{1832}$ |
| Coton | $y_{33}$ | $a_{133}$ | $a_{233}$ | $a_{333}$ | $a_{433}$ | $a_{533}$ | $a_{633}$ | $a_{733}$ | $a_{833}$ | $a_{933}$ | $a_{1033}$ | $a_{1133}$ | $a_{1233}$ | $a_{1333}$ | $a_{1433}$ | $a_{1533}$ | $a_{1633}$ | $a_{1733}$ | $a_{1833}$ |
| Tabac | $y_{34}$ | $a_{134}$ | $a_{234}$ | $a_{334}$ | $a_{434}$ | $a_{534}$ | $a_{634}$ | $a_{734}$ | $a_{834}$ | $a_{934}$ | $a_{1034}$ | $a_{1134}$ | $a_{1234}$ | $a_{1334}$ | $a_{1434}$ | $a_{1534}$ | $a_{1634}$ | $a_{1734}$ | $a_{1834}$ |
| Ananas | $y_{35}$ | $a_{135}$ | $a_{235}$ | $a_{335}$ | $a_{435}$ | $a_{535}$ | $a_{635}$ | $a_{735}$ | $a_{835}$ | $a_{935}$ | $a_{1035}$ | $a_{1135}$ | $a_{1235}$ | $a_{1335}$ | $a_{1435}$ | $a_{1535}$ | $a_{1635}$ | $a_{1735}$ | $a_{1835}$ |
| Mangoes | $y_{36}$ | $a_{136}$ | $a_{236}$ | $a_{336}$ | $a_{436}$ | $a_{536}$ | $a_{636}$ | $a_{736}$ | $a_{836}$ | $a_{936}$ | $a_{1036}$ | $a_{1136}$ | $a_{1236}$ | $a_{1336}$ | $a_{1436}$ | $a_{1536}$ | $a_{1636}$ | $a_{1736}$ | $a_{1836}$ |
| Oranges | $y_{37}$ | $g_{137}$ | $a_{237}$ | $a_{337}$ | $a_{437}$ | $a_{537}$ | $a_{637}$ | $a_{737}$ | $a_{837}$ | $a_{937}$ | $a_{1037}$ | $a_{1137}$ | $a_{1237}$ | $a_{1337}$ | $a_{1437}$ | $a_{1537}$ | $a_{1637}$ | $a_{1737}$ | $a_{1837}$ |
| Citrons | $y_{38}$ | $a_{138}$ | $a_{238}$ | $a_{338}$ | $a_{438}$ | $a_{538}$ | $a_{638}$ | $a_{738}$ | $a_{838}$ | $a_{938}$ | $a_{1038}$ | $a_{1138}$ | $a_{1238}$ | $a_{1338}$ | $a_{1438}$ | $a_{1538}$ | $a_{1638}$ | $a_{1738}$ | $a_{1838}$ |
| Papayas | $y_{39}$ | $a_{139}$ | $a_{239}$ | $a_{339}$ | $a_{439}$ | $a_{539}$ | $a_{639}$ | $a_{739}$ | $a_{839}$ | $a_{939}$ | $a_{1039}$ | $a_{1139}$ | $a_{1239}$ | $a_{1339}$ | $a_{1439}$ | $a_{1539}$ | $a_{1639}$ | $a_{1739}$ | $a_{1839}$ |
| Strawberries | $y_{40}$ | $a_{140}$ | $a_{240}$ | $a_{340}$ | $a_{440}$ | $a_{540}$ | $a_{640}$ | $a_{740}$ | $a_{840}$ | $a_{940}$ | $a_{1040}$ | $a_{1140}$ | $a_{1240}$ | $a_{1340}$ | $a_{1440}$ | $a_{1540}$ | $a_{1640}$ | $a_{1740}$ | $a_{1840}$ |
| Avocado | $y_{41}$ | $a_{141}$ | $a_{241}$ | $a_{341}$ | $a_{441}$ | $a_{541}$ | $a_{641}$ | $a_{741}$ | $a_{841}$ | $a_{941}$ | $a_{1041}$ | $a_{1141}$ | $a_{1241}$ | $a_{1341}$ | $a_{1441}$ | $a_{1541}$ | $a_{1641}$ | $a_{1741}$ | $a_{1841}$ |
| Watermelons | $y_{42}$ | $a_{142}$ | $a_{242}$ | $a_{342}$ | $a_{442}$ | $a_{542}$ | $a_{642}$ | $a_{742}$ | $a_{842}$ | $a_{942}$ | $a_{1042}$ | $a_{1142}$ | $a_{1242}$ | $a_{1342}$ | $a_{1442}$ | $a_{1542}$ | $a_{1642}$ | $a_{1742}$ | $a_{1842}$ |
| Tournesol | $y_{43}$ | $a_{143}$ | $a_{243}$ | $a_{343}$ | $a_{443}$ | $a_{543}$ | $a_{643}$ | $a_{743}$ | $a_{843}$ | $a_{943}$ | $a_{1043}$ | $a_{1143}$ | $a_{1243}$ | $a_{1343}$ | $a_{1443}$ | $a_{1543}$ | $a_{1643}$ | $a_{1743}$ | $a_{1843}$ |
| peanuts | $y_{44}$ | $a_{144}$ | $a_{244}$ | $a_{344}$ | $a_{444}$ | $a_{544}$ | $a_{644}$ | $a_{744}$ | $a_{844}$ | $a_{944}$ | $a_{1044}$ | $a_{1144}$ | $a_{1244}$ | $a_{1344}$ | $a_{1444}$ | $a_{1544}$ | $a_{1644}$ | $a_{1744}$ | $a_{1844}$ |
| avocado oil | $y_{45}$ | $a_{145}$ | $a_{245}$ | $a_{345}$ | $a_{445}$ | $a_{545}$ | $a_{645}$ | $a_{745}$ | $a_{845}$ | $a_{945}$ | $a_{1045}$ | $a_{1145}$ | $a_{1245}$ | $a_{1345}$ | $a_{1445}$ | $a_{1545}$ | $a_{1645}$ | $a_{1745}$ | $a_{1845}$ |
| Palm | $y_{46}$ | $a_{146}$ | $a_{246}$ | $a_{346}$ | $a_{446}$ | $a_{546}$ | $a_{646}$ | $a_{746}$ | $a_{846}$ | $a_{946}$ | $a_{1046}$ | $a_{1146}$ | $a_{1246}$ | $a_{1346}$ | $a_{1446}$ | $a_{1546}$ | $a_{1646}$ | $a_{1746}$ | $a_{1846}$ |
| Sugar cane | $y_{47}$ | $a_{147}$ | $a_{247}$ | $a_{347}$ | $a_{447}$ | $a_{547}$ | $a_{647}$ | $a_{747}$ | $a_{847}$ | $a_{947}$ | $a_{1047}$ | $a_{1147}$ | $a_{1247}$ | $a_{1347}$ | $a_{1447}$ | $a_{1547}$ | $a_{1647}$ | $a_{1747}$ | $a_{1847}$ |
| Rice | $y_{48}$ | $a_{148}$ | $a_{248}$ | $a_{348}$ | $a_{448}$ | $a_{548}$ | $a_{648}$ | $a_{748}$ | $a_{848}$ | $a_{948}$ | $a_{1048}$ | $a_{1148}$ | $a_{1248}$ | $a_{1348}$ | $a_{1448}$ | $a_{1548}$ | $a_{1648}$ | $a_{1748}$ | $a_{1848}$ |
| Peas | $y_{49}$ | $a_{149}$ | $a_{249}$ | $a_{349}$ | $a_{449}$ | $a_{549}$ | $a_{649}$ | $a_{749}$ | $a_{849}$ | $a_{949}$ | $a_{1049}$ | $a_{1149}$ | $a_{1249}$ | $a_{1349}$ | $a_{1449}$ | $a_{1549}$ | $a_{1649}$ | $a_{1749}$ | $a_{1849}$ |
| Peanut oil | $y_{50}$ | $a_{150}$ | $a_{250}$ | $a_{350}$ | $a_{450}$ | $a_{550}$ | $a_{650}$ | $a_{750}$ | $a_{850}$ | $a_{950}$ | $a_{1050}$ | $a_{1150}$ | $a_{1250}$ | $a_{1350}$ | $a_{1450}$ | $a_{1550}$ | $a_{1650}$ | $a_{1750}$ | $a_{1850}$ |
| Cow Meat | $y_{51}$ | $a_{151}$ | $a_{251}$ | $a_{351}$ | $a_{451}$ | $a_{551}$ | $a_{651}$ | $a_{751}$ | $a_{851}$ | $a_{951}$ | $a_{1051}$ | $a_{1151}$ | $a_{1251}$ | $a_{1351}$ | $a_{1451}$ | $a_{1551}$ | $a_{1651}$ | $a_{1751}$ | $a_{1851}$ |
| Goat Meat | $y_{52}$ | $a_{152}$ | $a_{252}$ | $a_{352}$ | $a_{452}$ | $a_{552}$ | $a_{652}$ | $a_{752}$ | $a_{852}$ | $a_{952}$ | $a_{1052}$ | $a_{1152}$ | $a_{1252}$ | $a_{1352}$ | $a_{1452}$ | $a_{1552}$ | $a_{1652}$ | $a_{1752}$ | $a_{1852}$ |
| Pork meat | $y_{53}$ | $a_{153}$ | $a_{253}$ | $a_{353}$ | $a_{453}$ | $a_{553}$ | $a_{653}$ | $a_{753}$ | $a_{853}$ | $a_{953}$ | $a_{1053}$ | $a_{1153}$ | $a_{1253}$ | $a_{1353}$ | $a_{1453}$ | $a_{1553}$ | $a_{1653}$ | $a_{1753}$ | $a_{1853}$ |
| Meat of sheep | $y_{54}$ | $a_{154}$ | $a_{254}$ | $a_{354}$ | $a_{454}$ | $a_{554}$ | $a_{654}$ | $a_{754}$ | $a_{854}$ | $a_{954}$ | $a_{1054}$ | $a_{1154}$ | $a_{1254}$ | $a_{1354}$ | $a_{1454}$ | $a_{1554}$ | $a_{1654}$ | $a_{1754}$ | $a_{1854}$ |
| Rabbit meat | $y_{55}$ | $a_{155}$ | $a_{255}$ | $a_{355}$ | $a_{455}$ | $a_{555}$ | $a_{655}$ | $a_{755}$ | $a_{855}$ | $a_{955}$ | $a_{1055}$ | $a_{1155}$ | $a_{1255}$ | $a_{1355}$ | $a_{1455}$ | $a_{1555}$ | $a_{1655}$ | $a_{1755}$ | $a_{1855}$ |
| Duck meat | $y_{56}$ | $a_{156}$ | $a_{256}$ | $a_{356}$ | $a_{456}$ | $a_{556}$ | $a_{656}$ | $a_{756}$ | $a_{856}$ | $a_{956}$ | $a_{1056}$ | $a_{1156}$ | $a_{1256}$ | $a_{1356}$ | $a_{1456}$ | $a_{1556}$ | $a_{1656}$ | $a_{1756}$ | $a_{1856}$ |
| Chicken meat | $y_{57}$ | $a_{157}$ | $a_{257}$ | $a_{357}$ | $a_{457}$ | $a_{557}$ | $a_{657}$ | $a_{757}$ | $a_{857}$ | $a_{957}$ | $a_{1057}$ | $a_{1157}$ | $a_{1257}$ | $a_{1357}$ | $a_{1457}$ | $a_{1557}$ | $a_{1657}$ | $a_{1757}$ | $a_{1857}$ |
| fish | $y_{58}$ | $a_{158}$ | $a_{258}$ | $a_{358}$ | $a_{458}$ | $a_{558}$ | $a_{658}$ | $a_{758}$ | $a_{858}$ | $a_{958}$ | $a_{1058}$ | $a_{1158}$ | $a_{1258}$ | $a_{1358}$ | $a_{1458}$ | $a_{1558}$ | $a_{1658}$ | $a_{1758}$ | $a_{1858}$ |
| eggs | $y_{59}$ | $a_{159}$ | $a_{259}$ | $a_{359}$ | $a_{459}$ | $a_{559}$ | $a_{659}$ | $a_{759}$ | $a_{859}$ | $a_{959}$ | $a_{1059}$ | $a_{1159}$ | $a_{1259}$ | $a_{1359}$ | $a_{1459}$ | $a_{1559}$ | $a_{1659}$ | $a_{1759}$ | $a_{1859}$ |
| Beurres | $y_{60}$ | $a_{160}$ | $a_{260}$ | $a_{360}$ | $a_{460}$ | $a_{560}$ | $a_{660}$ | $a_{760}$ | $a_{860}$ | $a_{960}$ | $a_{1060}$ | $a_{1160}$ | $a_{1260}$ | $a_{1360}$ | $a_{1460}$ | $a_{1560}$ | $a_{1660}$ | $a_{1760}$ | $a_{1860}$ |
| Yaourt | $y_{61}$ | $a_{161}$ | $a_{261}$ | $a_{361}$ | $a_{461}$ | $a_{561}$ | $a_{661}$ | $a_{761}$ | $a_{861}$ | $a_{961}$ | $a_{1061}$ | $a_{1161}$ | $a_{1261}$ | $a_{1361}$ | $a_{1461}$ | $a_{1561}$ | $a_{1661}$ | $a_{1761}$ | $a_{1861}$ |
| Cows Fromage | $y_{62}$ | $a_{162}$ | $a_{262}$ | $a_{362}$ | $a_{462}$ | $a_{562}$ | $a_{662}$ | $a_{762}$ | $a_{862}$ | $a_{962}$ | $a_{1062}$ | $a_{1162}$ | $a_{1262}$ | $a_{1362}$ | $a_{1462}$ | $a_{1562}$ | $a_{1662}$ | $a_{1762}$ | $a_{1862}$ |
| Goat Fromage | $y_{63}$ | $a_{163}$ | $a_{263}$ | $a_{363}$ | $a_{463}$ | $a_{563}$ | $a_{663}$ | $a_{763}$ | $a_{863}$ | $a_{963}$ | $a_{1063}$ | $a_{1163}$ | $a_{1263}$ | $a_{1363}$ | $a_{1463}$ | $a_{1563}$ | $a_{1663}$ | $a_{1763}$ | $a_{1863}$ |
| organo-mineral fertilizers | $y_{64}$ | $a_{164}$ | $a_{264}$ | $a_{364}$ | $a_{464}$ | $a_{564}$ | $a_{664}$ | $a_{764}$ | $a_{864}$ | $a_{964}$ | $a_{1064}$ | $a_{1164}$ | $a_{1264}$ | $a_{1364}$ | $a_{1464}$ | $a_{1564}$ | $a_{1664}$ | $a_{1764}$ | $a_{1864}$ |
| agricultural lime | $y_{65}$ | $a_{165}$ | $a_{265}$ | $a_{365}$ | $a_{465}$ | $a_{565}$ | $a_{665}$ | $a_{765}$ | $a_{865}$ | $a_{965}$ | $a_{1065}$ | $a_{1165}$ | $a_{1265}$ | $a_{1365}$ | $a_{1465}$ | $a_{1565}$ | $a_{1665}$ | $a_{1765}$ | $a_{1865}$ |
| Primus | $y_{66}$ | $a_{166}$ | $a_{266}$ | $a_{366}$ | $a_{466}$ | $a_{566}$ | $a_{666}$ | $a_{766}$ | $a_{866}$ | $a_{966}$ | $a_{1066}$ | $a_{1166}$ | $a_{1266}$ | $a_{1366}$ | $a_{1466}$ | $a_{1566}$ | $a_{1666}$ | $a_{1766}$ | $a_{1866}$ |
| Small Primus | $y_{67}$ | $a_{167}$ | $a_{267}$ | $a_{367}$ | $a_{467}$ | $a_{567}$ | $a_{667}$ | $a_{767}$ | $a_{867}$ | $a_{967}$ | $a_{1067}$ | $a_{1167}$ | $a_{1267}$ | $a_{1367}$ | $a_{1467}$ | $a_{1567}$ | $a_{1667}$ | $a_{1767}$ | $a_{1867}$ |
| Primus Nyongera | $y_{68}$ | $a_{168}$ | $a_{268}$ | $a_{368}$ | $a_{468}$ | $a_{568}$ | $a_{668}$ | $a_{768}$ | $a_{868}$ | $a_{968}$ | $a_{1068}$ | $a_{1168}$ | $a_{1268}$ | $a_{1368}$ | $a_{1468}$ | $a_{1568}$ | $a_{1668}$ | $a_{1768}$ | $a_{1868}$ |
| Amstel | $y_{69}$ | $a_{169}$ | $a_{269}$ | $a_{369}$ | $a_{469}$ | $a_{569}$ | $a_{669}$ | $a_{769}$ | $a_{869}$ | $a_{969}$ | $a_{1069}$ | $a_{1169}$ | $a_{1269}$ | $a_{1369}$ | $a_{1469}$ | $a_{1569}$ | $a_{1669}$ | $a_{1769}$ | $a_{1869}$ |
| Amstel Blonde 65 cl | $y_{70}$ | $a_{170}$ | $a_{270}$ | $a_{370}$ | $a_{470}$ | $a_{570}$ | $a_{670}$ | $a_{770}$ | $a_{870}$ | $a_{970}$ | $a_{1070}$ | $a_{1170}$ | $a_{1270}$ | $a_{1370}$ | $a_{1470}$ | $a_{1570}$ | $a_{1670}$ | $a_{1770}$ | $a_{1870}$ |
| Amstel Royal | $y_{71}$ | $a_{171}$ | $a_{271}$ | $a_{371}$ | $a_{471}$ | $a_{571}$ | $a_{671}$ | $a_{771}$ | $a_{871}$ | $a_{971}$ | $a_{1071}$ | $a_{1171}$ | $a_{1271}$ | $a_{1371}$ | $a_{1471}$ | $a_{1571}$ | $a_{1671}$ | $a_{1771}$ | $a_{1871}$ |
| Amstel Bock | $y_{72}$ | $a_{172}$ | $a_{272}$ | $a_{372}$ | $a_{472}$ | $a_{572}$ | $a_{672}$ | $a_{772}$ | $a_{872}$ | $a_{972}$ | $a_{1072}$ | $a_{1172}$ | $a_{1272}$ | $a_{1372}$ | $a_{1472}$ | $a_{1572}$ | $a_{1672}$ | $a_{1772}$ | $a_{1872}$ |
| Amstel Blonde 50 cl | $y_{73}$ | $a_{173}$ | $a_{273}$ | $a_{373}$ | $a_{473}$ | $a_{573}$ | $a_{673}$ | $a_{773}$ | $a_{873}$ | $a_{973}$ | $a_{1073}$ | $a_{1173}$ | $a_{1273}$ | $a_{1373}$ | $a_{1473}$ | $a_{1573}$ | $a_{1673}$ | $a_{1773}$ | $a_{1873}$ |
| Limonades | $y_{74}$ | $a_{174}$ | $a_{274}$ | $a_{374}$ | $a_{474}$ | $a_{574}$ | $a_{674}$ | $a_{774}$ | $a_{874}$ | $a_{974}$ | $a_{1074}$ | $a_{1174}$ | $a_{1274}$ | $a_{1374}$ | $a_{1474}$ | $a_{1574}$ | $a_{1674}$ | $a_{1774}$ | $a_{1874}$ |
| Wine | $y_{75}$ | $a_{175}$ | $a_{275}$ | $a_{375}$ | $a_{475}$ | $a_{575}$ | $a_{675}$ | $a_{775}$ | $a_{875}$ | $a_{975}$ | $a_{1075}$ | $a_{1175}$ | $a_{1275}$ | $a_{1375}$ | $a_{1475}$ | $a_{1575}$ | $a_{1675}$ | $a_{1775}$ | $a_{1875}$ |
| Kinju | $y_{76}$ | $a_{176}$ | $a_{276}$ | $a_{376}$ | $a_{476}$ | $a_{576}$ | $a_{676}$ | $a_{776}$ | $a_{876}$ | $a_{976}$ | $a_{1076}$ | $a_{1176}$ | $a_{1276}$ | $a_{1376}$ | $a_{1476}$ | $a_{1576}$ | $a_{1676}$ | $a_{1776}$ | $a_{1876}$ |
| Aquavie | $y_{77}$ | $a_{177}$ | $a_{277}$ | $a_{377}$ | $a_{477}$ | $a_{577}$ | $a_{677}$ | $a_{777}$ | $a_{877}$ | $a_{977}$ | $a_{1077}$ | $a_{1177}$ | $a_{1277}$ | $a_{1377}$ | $a_{1477}$ | $a_{1577}$ | $a_{1677}$ | $a_{1777}$ | $a_{1877}$ |
| Sangwe | $y_{78}$ | $a_{178}$ | $a_{278}$ | $a_{378}$ | $a_{478}$ | $a_{578}$ | $a_{678}$ | $a_{778}$ | $a_{878}$ | $a_{978}$ | $a_{1078}$ | $a_{1178}$ | $a_{1278}$ | $a_{1378}$ | $a_{1478}$ | $a_{1578}$ | $a_{1678}$ | $a_{1778}$ | $a_{1878}$ |
| Fungus | $y_{79}$ | $a_{179}$ | $a_{279}$ | $a_{379}$ | $a_{479}$ | $a_{579}$ | $a_{679}$ | $a_{779}$ | $a_{879}$ | $a_{979}$ | $a_{1079}$ | $a_{1179}$ | $a_{1279}$ | $a_{1379}$ | $a_{1479}$ | $a_{1579}$ | $a_{1679}$ | $a_{1779}$ | $a_{1879}$ |
| Akezamutima juice | $y_{80}$ | $a_{180}$ | $a_{280}$ | $a_{380}$ | $a_{480}$ | $a_{580}$ | $a_{680}$ | $a_{780}$ | $a_{880}$ | $a_{980}$ | $a_{1080}$ | $a_{1180}$ | $a_{1280}$ | $a_{1380}$ | $a_{1480}$ | $a_{1580}$ | $a_{1680}$ | $a_{1780}$ | $a_{1880}$ |
| Maracuja juice | $y_{81}$ | $a_{181}$ | $a_{281}$ | $a_{381}$ | $a_{481}$ | $a_{581}$ | $a_{681}$ | $a_{781}$ | $a_{881}$ | $a_{981}$ | $a_{1081}$ | $a_{1181}$ | $a_{1281}$ | $a_{1381}$ | $a_{1481}$ | $a_{1581}$ | $a_{1681}$ | $a_{1781}$ | $a_{1881}$ |
| Ananas juice | $y_{82}$ | $a_{182}$ | $a_{282}$ | $a_{382}$ | $a_{482}$ | $a_{582}$ | $a_{682}$ | $a_{782}$ | $a_{882}$ | $a_{982}$ | $a_{1082}$ | $a_{1182}$ | $a_{1282}$ | $a_{1382}$ | $a_{1482}$ | $a_{1582}$ | $a_{1682}$ | $a_{1782}$ | $a_{1882}$ |
| ciment | $y_{83}$ | $a_{183}$ | $a_{283}$ | $a_{383}$ | $a_{483}$ | $a_{583}$ | $a_{683}$ | $a_{783}$ | $a_{883}$ | $a_{983}$ | $a_{1083}$ | $a_{1183}$ | $a_{1283}$ | $a_{1383}$ | $a_{1483}$ | $a_{1583}$ | $a_{1683}$ | $a_{1783}$ | $a_{1883}$ |
| Matelas | $y_{84}$ | $a_{184}$ | $a_{284}$ | $a_{384}$ | $a_{484}$ | $a_{584}$ | $a_{684}$ | $a_{784}$ | $a_{884}$ | $a_{984}$ | $a_{1084}$ | $a_{1184}$ | $a_{1284}$ | $a_{1384}$ | $a_{1484}$ | $a_{1584}$ | $a_{1684}$ | $a_{1784}$ | $a_{1884}$ |
| Soap | $y_{85}$ | $a_{185}$ | $a_{285}$ | $a_{385}$ | $a_{485}$ | $a_{585}$ | $a_{685}$ | $a_{785}$ | $a_{885}$ | $a_{985}$ | $a_{1085}$ | $a_{1185}$ | $a_{1285}$ | $a_{1385}$ | $a_{1485}$ | $a_{1585}$ | $a_{1685}$ | $a_{1785}$ | $a_{1885}$ |
| Wheat flour | $y_{86}$ | $a_{186}$ | $a_{286}$ | $a_{386}$ | $a_{486}$ | $a_{586}$ | $a_{686}$ | $a_{786}$ | $a_{886}$ | $a_{986}$ | $a_{1086}$ | $a_{1186}$ | $a_{1286}$ | $a_{1386}$ | $a_{1486}$ | $a_{1586}$ | $a_{1686}$ | $a_{1786}$ | $a_{1886}$ |
| Maize flour | $y_{87}$ | $a_{187}$ | $a_{287}$ | $a_{387}$ | $a_{487}$ | $a_{587}$ | $a_{687}$ | $a_{787}$ | $a_{887}$ | $a_{987}$ | $a_{1087}$ | $a_{1187}$ | $a_{1287}$ | $a_{1387}$ | $a_{1487}$ | $a_{1587}$ | $a_{1687}$ | $a_{1787}$ | $a_{1887}$ |
| Cassava flour | $y_{88}$ | $a_{188}$ | $a_{288}$ | $a_{388}$ | $a_{488}$ | $a_{588}$ | $a_{688}$ | $a_{788}$ | $a_{888}$ | $a_{988}$ | $a_{1088}$ | $a_{1188}$ | $a_{1288}$ | $a_{1388}$ | $a_{1488}$ | $a_{1588}$ | $a_{1688}$ | $a_{1788}$ | $a_{1888}$ |
| Sorghum flour | $y_{89}$ | $a_{189}$ | $a_{289}$ | $a_{389}$ | $a_{489}$ | $a_{589}$ | $a_{689}$ | $a_{789}$ | $a_{889}$ | $a_{989}$ | $a_{1089}$ | $a_{1189}$ | $a_{1289}$ | $a_{1389}$ | $a_{1489}$ | $a_{1589}$ | $a_{1689}$ | $a_{1789}$ | $a_{1889}$ |
| Eulsin flour | $y_{90}$ | $a_{190}$ | $a_{290}$ | $a_{390}$ | $a_{490}$ | $a_{590}$ | $a_{690}$ | $a_{790}$ | $a_{890}$ | $a_{990}$ | $a_{1090}$ | $a_{1190}$ | $a_{1290}$ | $a_{1390}$ | $a_{1490}$ | $a_{1590}$ | $a_{1690}$ | $a_{1790}$ | $a_{1890}$ |
| Sesame flour | $y_{91}$ | $a_{191}$ | $a_{291}$ | $a_{391}$ | $a_{491}$ | $a_{591}$ | $a_{691}$ | $a_{791}$ | $a_{891}$ | $a_{991}$ | $a_{1091}$ | $a_{1191}$ | $a_{1291}$ | $a_{1391}$ | $a_{1491}$ | $a_{1591}$ | $a_{1691}$ | $a_{1791}$ | $a_{1891}$ |
| Loincloths from Cotebu | $y_{92}$ | $a_{192}$ | $a_{292}$ | $a_{392}$ | $a_{492}$ | $a_{592}$ | $a_{692}$ | $a_{792}$ | $a_{892}$ | $a_{992}$ | $a_{1092}$ | $a_{1192}$ | $a_{1292}$ | $a_{1392}$ | $a_{1492}$ | $a_{1592}$ | $a_{1692}$ | $a_{1792}$ | $a_{1892}$ |
| Pona Pharm. products | $y_{93}$ | $a_{193}$ | $a_{293}$ | $a_{393}$ | $a_{493}$ | $a_{593}$ | $a_{693}$ | $a_{793}$ | $a_{893}$ | $a_{993}$ | $a_{1093}$ | $a_{1193}$ | $a_{1293}$ | $a_{1393}$ | $a_{1493}$ | $a_{1593}$ | $a_{1693}$ | $a_{1793}$ | $a_{1893}$ |
| SIPHAR Products | $y_{94}$ | $a_{194}$ | $a_{294}$ | $a_{394}$ | $a_{494}$ | $a_{594}$ | $a_{694}$ | $a_{794}$ | $a_{894}$ | $a_{994}$ | $a_{1094}$ | $a_{1194}$ | $a_{1294}$ | $a_{1394}$ | $a_{1494}$ | $a_{1594}$ | $a_{1694}$ | $a_{1794}$ | $a_{1894}$ |
| Bread | $y_{95}$ | $a_{195}$ | $a_{295}$ | $a_{395}$ | $a_{495}$ | $a_{595}$ | $a_{695}$ | $a_{795}$ | $a_{895}$ | $a_{995}$ | $a_{1095}$ | $a_{1195}$ | $a_{1295}$ | $a_{1395}$ | $a_{1495}$ | $a_{1595}$ | $a_{1695}$ | $a_{1795}$ | $a_{1895}$ |
| Clous | $y_{96}$ | $a_{196}$ | $a_{296}$ | $a_{396}$ | $a_{496}$ | $a_{596}$ | $a_{696}$ | $a_{796}$ | $a_{896}$ | $a_{996}$ | $a_{1096}$ | $a_{1196}$ | $a_{1296}$ | $a_{1396}$ | $a_{1496}$ | $a_{1596}$ | $a_{1696}$ | $a_{1796}$ | $a_{1896}$ |
| Concrete Iron | $y_{97}$ | $a_{197}$ | $a_{297}$ | $a_{397}$ | $a_{497}$ | $a_{597}$ | $a_{697}$ | $a_{797}$ | $a_{897}$ | $a_{997}$ | $a_{1097}$ | $a_{1197}$ | $a_{1297}$ | $a_{1397}$ | $a_{1497}$ | $a_{1597}$ | $a_{1697}$ | $a_{1797}$ | $a_{1897}$ |
| Metal bar | $y_{98}$ | $a_{198}$ | $a_{298}$ | $a_{398}$ | $a_{498}$ | $a_{598}$ | $a_{698}$ | $a_{798}$ | $a_{898}$ | $a_{998}$ | $a_{1098}$ | $a_{1198}$ | $a_{1298}$ | $a_{1398}$ | $a_{1498}$ | $a_{1598}$ | $a_{1698}$ | $a_{1798}$ | $a_{1898}$ |
| Tles | $y_{99}$ | $a_{199}$ | $a_{299}$ | $a_{399}$ | $a_{499}$ | $a_{599}$ | $a_{699}$ | $a_{799}$ | $a_{899}$ | $a_{999}$ | $a_{1099}$ | $a_{1199}$ | $a_{1299}$ | $a_{1399}$ | $a_{1499}$ | $a_{1599}$ | $a_{1699}$ | $a_{1799}$ | $a_{1899}$ |
| Plastic | $y_{100}$ | $a_{1100}$ | $a_{2100}$ | $a_{3100}$ | $a_{4100}$ | $a_{5100}$ | $a_{6100}$ | $a_{7100}$ | $a_{8100}$ | $a_{9100}$ | $a_{10100}$ | $a_{11100}$ | $a_{12100}$ | $a_{13100}$ | $a_{14100}$ | $a_{15100}$ | $a_{16100}$ | $a_{17100}$ | $a_{18100}$ |
| Wood | $y_{101}$ | $a_{1101}$ | $a_{2101}$ | $a_{3101}$ | $a_{4101}$ | $a_{5101}$ | $a_{6101}$ | $a_{7101}$ | $a_{8101}$ | $a_{9101}$ | $a_{10101}$ | $a_{11101}$ | $a_{12101}$ | $a_{13101}$ | $a_{14101}$ | $a_{15101}$ | $a_{16101}$ | $a_{17101}$ | $a_{18101}$ |
| Charbon | $y_{102}$ | $a_{1102}$ | $a_{2102}$ | $a_{3102}$ | $a_{4102}$ | $a_{5102}$ | $a_{6102}$ | $a_{7102}$ | $a_{8102}$ | $a_{9102}$ | $a_{10102}$ | $a_{11102}$ | $a_{12102}$ | $a_{13102}$ | $a_{14102}$ | $a_{15102}$ | $a_{16102}$ | $a_{17102}$ | $a_{18102}$ |
| Suggar | $y_{103}$ | $a_{1103}$ | $a_{2103}$ | $a_{3103}$ | $a_{4103}$ | $a_{5103}$ | $a_{6103}$ | $a_{7103}$ | $a_{8103}$ | $a_{9103}$ | $a_{10103}$ | $a_{11103}$ | $a_{12103}$ | $a_{13103}$ | $a_{14103}$ | $a_{15103}$ | $a_{16103}$ | $a_{17103}$ | $a_{18103}$ |
| Levure | $y_{104}$ | $a_{1104}$ | $a_{2104}$ | $a_{3104}$ | $a_{4104}$ | $a_{5104}$ | $a_{6104}$ | $a_{7104}$ | $a_{8104}$ | $a_{9104}$ | $a_{10104}$ | $a_{11104}$ | $a_{12104}$ | $a_{13104}$ | $a_{14104}$ | $a_{15104}$ | $a_{16104}$ | $a_{17104}$ | $a_{18104}$ |
| hubro | $y_{105}$ | $a_{1105}$ | $a_{2105}$ | $a_{3105}$ | $a_{4105}$ | $a_{5105}$ | $a_{6105}$ | $a_{7105}$ | $a_{8105}$ | $a_{9105}$ | $a_{10105}$ | $a_{11105}$ | $a_{12105}$ | $a_{13105}$ | $a_{14105}$ | $a_{15105}$ | $a_{16105}$ | $a_{17105}$ | $a_{18105}$ |
| Water | $y_{106}$ | $a_{1106}$ | $a_{2106}$ | $a_{3106}$ | $a_{4106}$ | $a_{5106}$ | $a_{6106}$ | $a_{7106}$ | $a_{8106}$ | $a_{9106}$ | $a_{10106}$ | $a_{11106}$ | $a_{12106}$ | $a_{13106}$ | $a_{14106}$ | $a_{15106}$ | $a_{16106}$ | $a_{17106}$ | $a_{18106}$ |

**Figure 2.** The constraints matrix A ([8], [10]-[23]).

$D$ can be calculated by admitting that it varies, for example, like the volume of all the factors of production.

### 2.5. Mathematical Modeling of Optimal Control of the Poverty Problem

The combination of all the above equations demonstrated and their mathematical transformation allow us to pose the problem. The mathematical modeling of the optimal control of the poverty problem is therefore an optimization problem written as follows:

$$\begin{cases} \max\limits_{\mathbf{y}\in\mathbf{Y}} J\left(y_1(t),\cdots,y_n(t),w(t)\right) \\ \dot{y} = f\left(t,y_1(t),\cdots,y_n(t),w(t)\right) \\ A\left(t,y_1(t),\cdots,y_n(t),w(t)\right) \le b \\ y_i \ge 0 \end{cases} \tag{17}$$

In this system, the first equation is the objective function of the national yield applied to a maximal optimization operator, the second differential equation is the dynamics of poverty and finally the last equations represent the system of constraints involved in the model. In order to simplify the writings for algorithmic rewriting questions, the problem takes the following form:

$$\begin{cases} \max\limits_{\mathbf{y}\in\mathbf{Y}} J\left(y(t),w(t)\right) \\ \dot{y} = f\left(t,y(t),w(t)\right) \\ A\left(t,y(t),w(t)\right) \le b \\ y_i \ge 0 \end{cases} \tag{18}$$

## 3. Algorithmic Resolution Approach and Commutation

### 3.1. 4th Order Runge Kutta Algorithm

1) Given a time step $h$, a maximum number of iterations $N$ and an initial condition $\left(t_0, y_{1,0}, y_{2,0}, \cdots, y_{m,0}\right)$;

2) a) For $0 \le n \le N$

For $i = 1,2,3,\cdots,m$: $k_{i,1} = hf_i\left(t_n, y_{1,n}, y_{2,n}, \cdots, y_{m,n}\right)$

b) For $i = 1,2,3,\cdots,m$: $k_{i,2} = hf_i\left(t_n + \frac{h}{2}, y_{1,n} + \frac{k_{1,1}}{2}, y_{2,n} + \frac{k_{2,1}}{2}, \cdots, y_{m,n} + \frac{k_{m,1}}{2}\right)$

c) For $i = 1,2,3,\cdots,m$: $k_{i,3} = hf_i\left(t_n + \frac{h}{2}, y_{1,n} + \frac{k_{1,2}}{2}, y_{2,n} + \frac{k_{2,2}}{2}, \cdots, y_{m,n} + \frac{k_{m,3}}{2}\right)$

d) For $i = 1,2,3,\cdots,m$: $k_{i,4} = hf_i\left(t_n + h, y_{1,n} + k_{1,3}, y_{2,n} + k_{2,3}, \cdots, y_{m,n} + k_{m,3}\right)$

e) For $i = 1,2,3,\cdots,m$: $y_{i,n+1} = y_{i,n} + \frac{1}{6}\left(k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4}\right)$

f) $t_{n+1} = t_n + h$

g) For $i = 1,2,3,\cdots,m$: $A_a\left(y_{i,n}, w_{i,n}\right) \le 0$

h) $\max\limits_{\mathbf{y}\in\mathbf{Y}} J\left(y_1,\cdots,y_n,w_n\right) = c_1y_1 + \cdots + c_ny_n + rT^aK^\beta E^\gamma D^S$

i) Display $t_{n+1}$ et $\vec{y}_{n+1}$, $J\left(\vec{y},\vec{w}\right)$;

3) Stop.

### 3.2. Symplectic Partitioned Runge Kutta Algorithm

The above modeled problem is a nonlinear and complex optimization problem. Let us state the Hamilton function:

$$\begin{aligned} H(t,y,p,w) &= p(t,y,w)\,q\big(t,\tau(t),\lambda(t),\mu_i(t),\sigma_i(t),u_i(t),D_i(t),E_i(t),W(t)\big) \\ &\quad + p(t,y,w)\,f(t,y,w) - A_a(t,y,w) \end{aligned}.$$

To solve this problem, it is a question of making a discretization of Runge Kutta combined with a symplectic partitioned method within the framework of a dynamic programming of which the resulting algorithm is as follows:

**Symplectic partitioned Runge Kutta algorithm (SPRK)**

1) By discretizing the time $[t_0, t_f]$ in $N$ step $h = t_{n+1} - t_n = \frac{t_f - t_0}{N}$, $N$ is the maximum number of iterations.

2) Pour $0 \le n \le N$,

$$\begin{aligned} & H_w\left(w_{ki}, y_{ki}, p_{ki}\right) = 0 \\ & D_{i,n+1} = D_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p\left(w_{n_k}, y_{n_k}\right), D_i(t_0) = D_{i,0}, \\ & D_{i,n_k} = D_{i,n} + h\sum_{j=1}^{s} a_{kj} \mathcal{H}_p\left(w_{n_j}, y_{n_j}\right), k = 1,\cdots,s \\ & y_{n+1} = y_n + h\sum_{k=1}^{s} b_k \mathcal{H}_p\left(w_{n_k}, y_{n_k}\right), y(t_0) = y_0, \\ & y_{n_k} = y_n + h\sum_{j=1}^{s} a_{kj} \mathcal{H}_p\left(w_{n_j}, y_{n_j}\right), k = 1,\cdots,s \\ & p_{n+1} = p_n - h\sum_{k=1}^{s} \tilde{b}_k \mathcal{H}_y\left(p_{n_k}, y_{n_k}\right), p_N = \phi'(t_f), \\ & p_{n_k} = p_n - h\sum_{j=1}^{s} \tilde{a}_{kj} \mathcal{H}_y\left(p_{n_j}, y_{n_j}\right), \\ & \mathcal{H}\left(y_{n_k}, p_{n_k}\right) = H\left(\psi\left(y_{n_k}, p_{n_k}\right), y_{n_k}, p_{n_k}\right), \\ & \tilde{a}_{kj} := b_j - \frac{b_j}{b_k} a_{jk}, b_j = \tilde{b}_j, A_a\left(y_n, w_{n_k}\right) \le 0, \\ & \textit{Display } t_{n+1}, y_{n+1}, w_{n_k}, p_{n+1}. \end{aligned} \tag{19}$$

3) Stop.

In this algorithm, a digitization of the state variables and the co-state variables as well as the control variables emerges.

### 3.3. Details of the Equations of the Symplectic Partitioned Runge Kutta Algorithm

Consider that the variable index $i = 1,\cdots,N$ and that $n$ is this time the iterative index, the system (19) becomes explicitly:

$$H_{wi}\left(w_{ki}, y_{ki}, p_{ki}\right) = 0, i = 1, \cdots, N$$

$$D_{i,n+1} = D_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p\left(w_{n_k}, y_{n_k}\right), D_i\left(t_0\right) = D_{i,0}$$

$$D_{i,n_k} = D_{i,n} + h\sum_{j=1}^{s} a_{kj} \mathcal{H}_p\left(w_{n_j}, y_{n_j}\right), k = 1, \cdots, s$$

$$y_{i,n+1} = y_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p\left(w_{i,n_k}, y_{i,n_k}\right), y_i\left(t_0\right) = y_0, i = 1, \cdots, N$$

$$y_{i,n_k} = y_{i,n} + h\sum_{j=1}^{s} a_{i,kj} \mathcal{H}_p\left(w_{i,n_j}, y_{i,n_j}\right), k = 1, \cdots, s; i = 1, \cdots, N$$

$$p_{i,n+1} = p_{i,n} - h\sum_{k=1}^{s} \tilde{b}_k \mathcal{H}_{yi}\left(p_{i,n_k}, y_{i,n_k}\right), p_{i,N} = \phi'\left(t_f\right), i = 1, \cdots, N \quad (20)$$

$$p_{i,n_k} = p_{i,n} - h\sum_{j=1}^{s} \tilde{a}_{kj} \mathcal{H}_y\left(p_{i,n_j}, y_{i,n_j}\right), i = 1, \cdots, N$$

$$\mathcal{H}\left(y_{i,n_k}, p_{i,n_k}\right) = H\left(\psi\left(y_{i,n_k}, p_{i,n_k}\right), y_{i,n_k}, p_{i,n_k}\right),$$

$$\tilde{a}_{kj} := b_j - \frac{b_j}{b_k} a_{jk}, b_j = \tilde{b}_j, i = 1, \cdots, N$$

$$A_{i,a}\left(y_{i,n}, w_{i,n_k}\right) \le 0, i = 1, \cdots, N$$

$$\textit{Afficher } t_{n+1}, y_{i,n+1}, w_{i,n_k}, p_{i,n+1}.$$

An AMPL programming language will be coupled with Gurobi, KNITRO SNOPT solvers for the extraction of solutions. The following section is devoted to the discussion and interpretation of the solutions as well as the optimization and convergence criteria.

## 4. Data Capturing and Description

Considering complexity constraints matrix, questionnaire survey of administrative real data is used in order to complete the optimal modeling problem. This administrative data concerns statistical stratifications of Burundian pilot province. The data consist of the state and control variables representing a non analytical abstract optimal modeling of the poverty problem. Details of this Data are included in the numerical process programming.

## 5. Numerical Output and Convergence Process

By using the following numerical scheme, we demonstrate the commutation of the two RK4 and SPRK4 numerical methods.

**Figure 3** shows the solving method of this problem by demonstrating the Commutation Process of RK4 and SPRK4 Numerical Methods, to run through a robust algorithm implementation under an AMPL programming language A S-treamlined Modeling for Real Optimization combined with Gurobi & SNOPT solvers.

AMPL integrates its modeling language with a command language for analysis

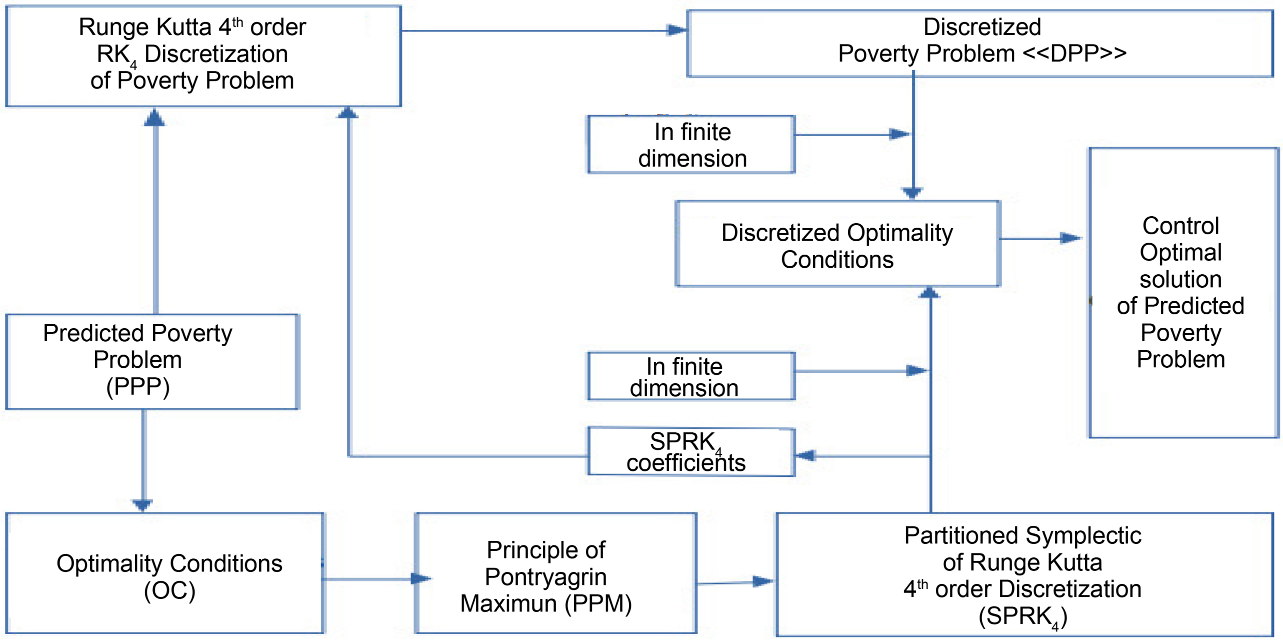


**Figure 3.** The commutation process of RK4 and SPRK4 numerical methods.

and debugging, and a scripting language for manipulating data and implementing optimization strategies. All use the same concepts to promote streamlined model-building. When you reach the stage of embedding your optimization models into applications and deploying them in enterprise systems, AMPLs APIs for C++, C#, Java, MATLAB, Python, and R insure that you have a reliable and maintainable implementation.

The AMPL system is a sophisticated modeling tool that supports the entire optimization modeling lifecycle: development, testing, deployment, and maintenance. By using a high-level representation that represents optimization models in the same ways that people think about them, AMPL promotes rapid development and reliable results. AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for browsing models and analyzing results; and a scripting language for gathering and manipulating data and for implementing iterative optimization schemes. All use the same concepts and syntax for streamlined application-building.

**AMPL Powerful modeling language features:**

- Broad support for sets and set operators. AMPL models can use sets of pairs, triples, and longer tuples; collections of sets indexed over sets; unordered, ordered, and circular sets of objects; and sets of numbers;
- General and natural syntax for arithmetic, logical, and conditional expressions; familiar conventions for summations and other iterated operators;
- Automatic handling of linear and convex quadratic problems in continuous and integer variables;
- Nonlinear programming features such as initial primal and dual values, user-

defined functions, fast automatic differentiation, and automatic elimination of "defined" variables;

- Convenient alternative notations for network flows, piecewise-linearities, complementarity conditions, and logical implications.

**Valuable modeling support features**

- Interactive command environment with batch processing options. Powerful display commands let you view any model component or expression, browsing on-screen or writing to a file, using automatic formatting or your own preferences;
- Powerful scripting language including looping and if-then-else commands. Programs in the AMPL command language can define sophisticated iterative schemes that process input data, repeatedly adjust and solve instances of multiple models, and prepare results for analysis;
- Separation of model and data. AMPL models remain concise even as sets and data tables grow. Models may incorporate many kinds of conditions for validity of the data;
- Data input and output connections. Concise statements relate the model data and results to the contents of relational data tables;

In order to demonstrate the switching and the relevance of the solution, an algorithmic implementation under a Matlab programming language with global optimization toolboxes Simulink will also be carried out.

## 6. Analysis Relationship for Consumption versus Production

The daily intake varies according to age, physical expenditure and sex for a moderately active person, per day. The following Table 1 shows the Burundi Population Necessary Consumption Nutritive Value in Calories.

Figure 4 shows the population size when considering the following categories and their nutritive value needs. Those categories are so very meaningful and they are considered as Decision-maker indicators. By modeling the optimal control poverty problem, the population consumption needs are used to canalize poverty control variables with the production system.

**Table 1.** The Burundi population necessary consumption nutritive value in calories.

| Population type | Population size | Person daily consumption (Ca) | Year consumption (Ca) |
|---|---|---|---|
| Children: 0 - 17 | 5,905,136 | 2700 | 5.8354553952e12 |
| Man: 18 - 65 years | 2,956,808 | 2500 | 2.70547932e12 |
| Women: 18 - 65 years | 3,611,520 | 2100 | 2.775814272e12 |
| Old person: 65 years and over | 357,713 | 2000 | 2.61845916e11 |
| Total | 12,831,177 | - | 9.1436635152e12 |

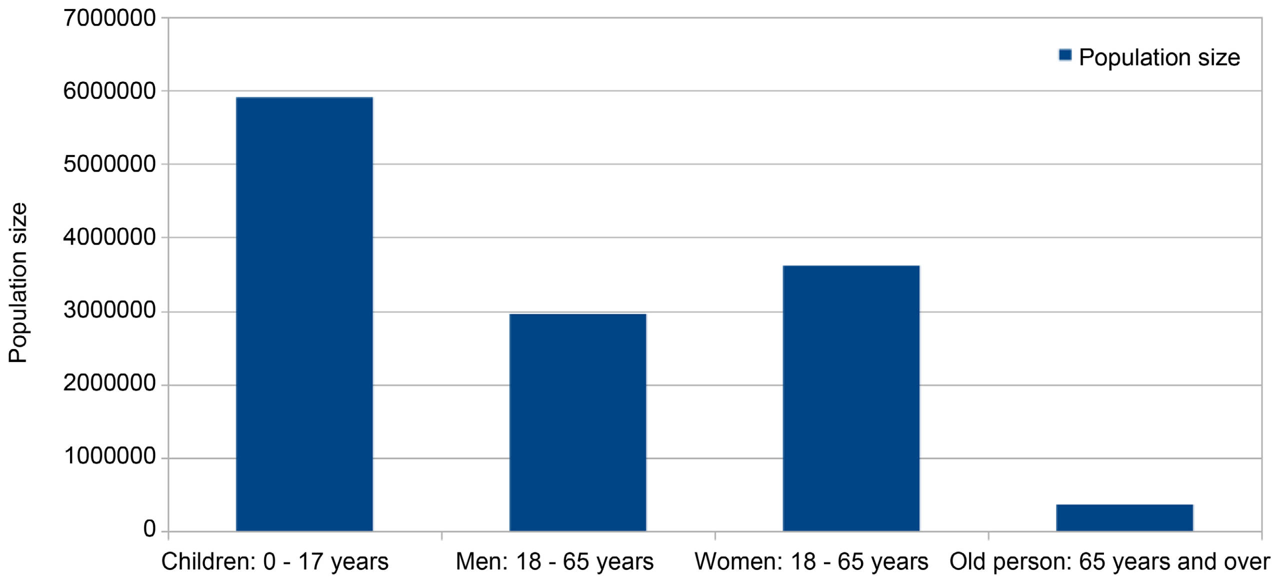


**Figure 4.** The Burundi population size.

Figure 5 shows the matching between population size categories and necessary Consumption Nutritive Value in Calories.

Table 2 and Table 3 show relationship between population Necessary Consumption Nutritive Value in Calories and the national system Nutritive Value.

In fact of eradicating poverty problem, state variables show quantity of production food when constraints applied are consumption, resource availability and production.

Table 4 and Table 5 characterize the food groups by their main nutrient intakes. They are called food constituents and their composition is different. All foods from a menu can be placed in any of them.

We show you the types of foods to control in your diet. These elements are meats, fish and eggs, Dairy products, fats, Cereals and Starches, Fruits and vegetables, Drinks and Sweet products. It is important to respect the balance between all these elements to enjoy their benefits, in bodybuilding or in all other sports.

Analyzing Tables 4-7, the comparing of the annual food production and the required annual consumption shows an imbalance between different types of food. Proteins (1.2075834566e12 Ca), minerals (5.9566475835e11 Ca) and vitamins (1.363975366e12 Ca) produced in Burundi are sufficient when considering their consumption being divided concretely into (1.12631534844e12 Ca) for proteins, (3.98401163088e11 Ca) for minerals, (1.2635893068e11 Ca) for vitamins as required by the entire Burundian population. This positive contribution for the latter comes from the fact that some cows, goats, fishes, ⋯, slaughtered in Burundi come from neighboring countries. Real production remains in deficit. The lipids (2.6453448701e12 Ca), acids (5.80712e9 Ca), calcium (6.385756e10 Ca), fibers (7.066505032e10 Ca) and carbohydrates (9.321735891e11 Ca) produced in Burundi are insufficient for consumption. Indeed, their necessary annual consumption

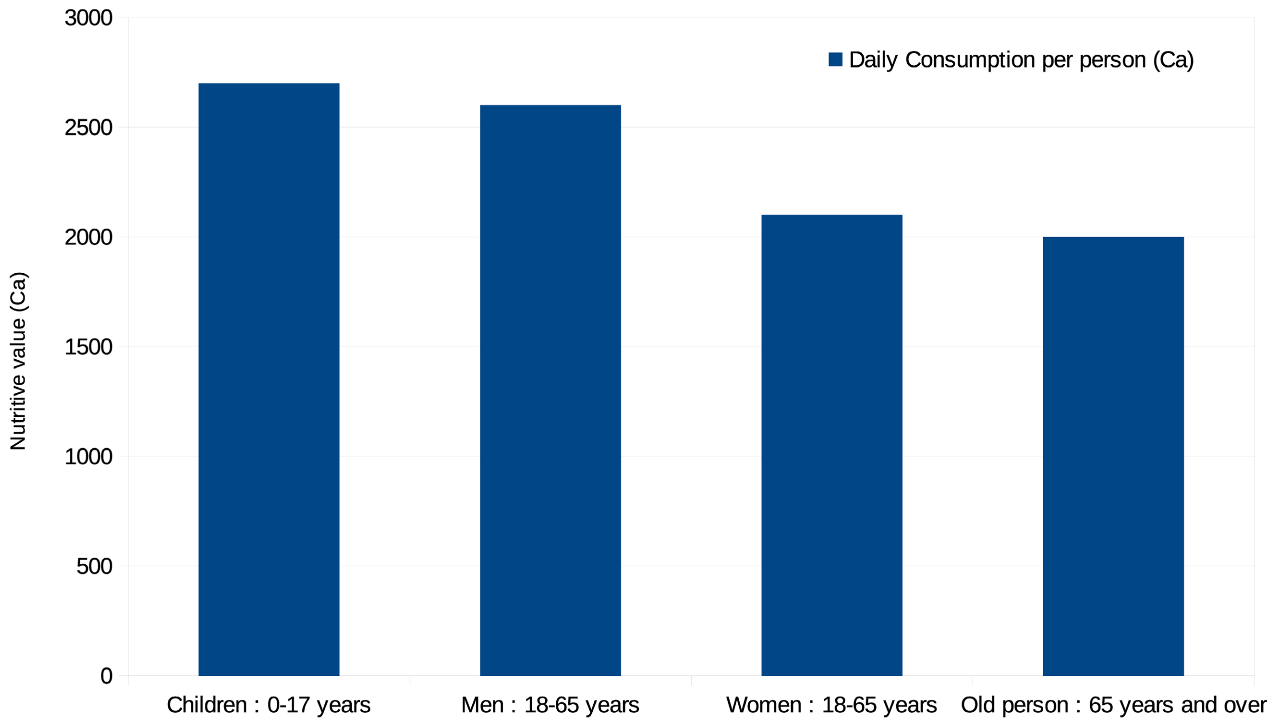


**Figure 5.** The Burundi population necessary consumption nutritive value in calories.

is distributed (3.4242924132e12 Ca) for lipids, (4.3706226354e10 Ca) for acids, (6.750047598e10 Ca) for fibers, (1.9680209241e12 Ca) for calciums and (1.5022572297e12 Ca) for carbohydrates. This negative contribution proves a Burundian food deficit. It is a decision-making indicator for the design and updating of agricultural policy and implementation programs and projects. Investment and economic growth are only possible when food security is mastered. The capital allocated to food investment must be revised upwards. Demographic control is also a relevant indicator to establish Burundi in the list of Emerging Countries in 2040 [7].

**Figure 6** shows the real simulation of necessary national year nutritive value consumption by type of food. Considering the categories of population, it is noted that the consumption is very high in carbohydrates and lipids. This means that national production must take this indicator of decision support into account in planning for optimal food production. The deficient consumption of acids and calcium remains minimal, which does not put much emphasis on production measurements.

The optimal results will be obtained using AMPL (A Mathematical Programming Modeling Language) programming and the KNITRO solver. We must test also the Algorithmic implementation under a Matlab programming language with global optimization toolboxes & Simulink. The results must demonstrate the commutation process.

The expected solutions are the decision indicators on the quantity of production, consumption, monetary and non-monetary yield in relation to the identified

**Table 2.** The national production nutrition value in calories (1).

| Variables | Year production (Ton) | Production (Ca)/(100g) | Year production (Ca) |
|---|---|---|---|
| Fish | 2.7706e4 | 0.208e3 | 5.762848e10 |
| Vegetables | 3.71892e5 | 0.33e3 | 1.2272436e12 |
| Cucumber | 0.18e4 | 0.13e2 | 2.34e8 |
| Squash | 0.1875e4 | 0.30e2 | 5.625e8 |
| Zucchini | 0.1875e4 | 0.30e2 | 5.625e8 |
| Tea | 1.1382e4 | 0.8e2 | 9.1056e9 |
| Coffee | 1.4058e4 | 0.1e4 | 1.4058e11 |
| Mushrooms | 0.393e3 | 0.28e2 | 1.1004e8 |
| Bean | 6.19151e5 | 0.12e3 | 7.429812e11 |
| Maize | 2.80813e5 | 0.356e3 | 9.9969428e12 |
| Wheat | 0.5628e4 | 0.149e3 | 8.38572e9 |
| Sorghum | 0.8851e4 | 0.328e3 | 2.903128e10 |
| Eleusine | 0.3084e4 | 0.151e3 | 4.65684e9 |
| Palm | 0.18e3 | 0.56e2 | 1.008e8 |
| Bananas | 1.179759e6 | 0.9e2 | 1.0617831e12 |
| Potato | 3.7644e5 | 0.9e2 | 3.38796e11 |
| Sweet potato | 1.023458e6 | 0.11e3 | 1.1258038e12 |
| Cassava | 2.114848e6 | 0.14e3 | 2.9607862e12 |
| Rice | 2.52853e5 | 0.9e2 | 2.275677e11 |
| Soja | 1.2436e4 | 0.446e3 | 5.546456e10 |
| Cabbage | 4.4e4 | 0.28e2 | 1.232e10 |
| Cauliflower | 0.45e3 | 0.3e2 | 1.35e8 |
| Onions | 2.12e5 | 0.46e2 | 1.35e8 |
| Leek | 0.48e4 | 0.42e2 | 9.752e10 |
| Carrots | 0.45e4 | 0.38e2 | 1.71e9 |
| Ananas | 0.2e5 | 0.51e2 | 1.02e10 |
| Mangoes | 0.3e4 | 0.62e2 | 1.86e9 |
| Oranges | 0.35e4 | 0.42e2 | 1.47e9 |
| Lemons | 0.28e4 | 0.4e2 | 1.12e9 |
| Papayas | 0.24e4 | 0.44e2 | 1.056e9 |
| Strawberries | 0.175e4 | 0.4e2 | 0.7e9 |
| Avocados | 3.6e5 | 0.2e3 | 7.2e11 |
| Watermelons | 0.42e4 | 0.3e2 | 1.26e9 |
| Sunflower oil | 1.469e4 | 0.9e3 | 1.3221e11 |
| Peanut oil | 1.469e4 | 0.9e3 | 1.3221e11 |
| Palm oil | 1.9216e4 | 0.56e2 | 1.076096e10 |

**Continued**

| | | | |
|---|---|---|---|
| Sugar cane | 2.18115e5 | 0.387e4 | 8.4410505e11 |
| Small weight | 1.2389e4 | 0.7e2 | 8.6723e9 |
| Butters | 0.1e5 | 0.76e3 | 7.6e10 |
| Pepper | 2.12e5 | 0.22e2 | 4.664e10 |
| Tomatoes | 8.75e4 | 0.2e2 | 1.75e10 |
| Coconut | 0.1e2 | 0.371e3 | 3.71e7 |
| Guava | 0.2e4 | 0.64e2 | 1.28e9 |
| Cow meat | 4.7071e4 | 0.25e3 | 1.176775e11 |
| Yam | 0.108e3 | 0.1e3 | 1.08e8 |
| Goat meat | 8.1456e4 | 0.3e3 | 2.44368e11 |
| Pork meat | 1.2655e5 | 0.3e3 | 3.7965e11 |
| Mutton | 6.8376e4 | 0.265e3 | 1.811964e11 |
| Rabbit meat | 0.15e4 | 0.16e3 | 2.4e9 |
| Duck meat | 7.03428e5 | 0.25e3 | 1.75875e12 |
| Turkey | 0.12e4 | 0.26e3 | 3.12e9 |
| Pigeon | 0.8e3 | 0.108e3 | 8.64e8 |
| Chicken | 8.1456e4 | 0.3e3 | 2.44368e11 |

**Table 3.** The national production nutrition value in calories (2).

| Variables | Year production (Ton) | Production (Ca)/(100g) | Year production (Ca) |
|---|---|---|---|
| Chicken Meat | 8.1456e4 | 0.15e3 | 1.22184e11 |
| Eggs | 0.5e5 | 0.28e2 | 1.4e10 |
| Yogurt | 0.2e5 | 0.55e2 | 1.1e10 |
| Cow cheese | 0.2e5 | 0.8e2 | 1.6e10 |
| Goat cheese | 0.5e4 | 0.8e2 | 0.4e10 |
| Primus Beer | 9.45562e5 | 0.8e2 | 7.564496e11 |
| Amstel Beer | 9.45562e5 | 0.76e2 | 7.1862712e11 |
| Lemonades | 9.45562e5 | 0.37e2 | 3.4985794e11 |
| Akezamutima juice | 0.8e3 | 0.48e2 | 3.84e8 |
| Maracuja juice | 0.7e3 | 0.48e2 | 3.36e8 |
| Ananas juice | 0.8e3 | 0.54e2 | 4.32e8 |
| Carrots | 2.12e5 | 0.38e2 | 8.056e10 |
| Carrots juice | 0.7e3 | 0.24e2 | 1.68e8 |
| Céleri | 0.48e4 | 0.2e2 | 9.6e8 |
| Beetroot | 0.4e4 | 0.4e2 | 1.6e9 |
| Aubergine | 0.35e4 | 0.29e2 | 1.4e9 |
| Wheat flour | 0.5628e4 | 0.364e3 | 2.048592e10 |
| Maize flour | 2.70813e5 | 0.37e3 | 1.0020081e12 |
| Cassava flour | 2.1665e5 | 0.35e3 | 7.58275e11 |
| Bred | 2.5e5 | 0.255e3 | 6.375e11 |
| Sugar | 2.0428e4 | 0.398e3 | 8.130344e10 |
| Water | - | 0 | 0 |

**Table 4.** The national year production by type of food (calories (1)).

| Type of food | Nutritious food composition | Ca/100g | Year production/100g | Year production by type of food (Ca) | Total by type of food |
|---|---|---|---|---|---|
| | Beans | 16.64 | 6.19151e9 | 1.03026726400e11 | |
| | Soja | 153.87 | 1.2436e9 | 1.91352732e11 | |
| | Cow cheese | 6.4 | 0.2e9 | 1.28e9 | |
| | Goat cheese | 6.4 | 0.5e7 | 3.2e8 | |
| | Maize | 10.68 | 2.80813e9 | 2.99908284e10 | |
| | cow meat | 61.5 | 4.7071e8 | 2.8948665e10 | |
| | Yam | 1.69 | 1.08e8 | 1.8252e6 | |
| | Potatoes | 17.73 | 3.7644e9 | 6.6742812e10 | |
| Proteins | Goat meat | 143 | 8.1456e8 | 1.1648208e11 | |
| | Pork meat | 75 | 1.2655e9 | 9.49125e10 | 1.2075834566e12 |
| | Mutton meat | 88.51 | 6.8376e8 | 6.05195976e10 | |
| | Rabbit meat | 32.8 | 1.5e8 | 4.92e9 | |
| | Duck meat | 58.25 | 7.03428e9 | 4.0974681e11 | |
| | Turkey meat | 24.19 | 1.2e7 | 2.9028e8 | |
| | Pigeon meat | 25.812 | 0.8e7 | 2.06496e8 | |
| | Chicken | 76.20 | 8.1456e8 | 6.206472e10 | |
| | Chicken meat | 45.15 | 8.1456e8 | 3.6777384e10 | |

**Table 5.** The national year production by type of food (calories (2)).

| Type of food | Nutritious food composition | Ca/100g | Year production/100g | Year production by type of food (Ca) | Total by type of food |
|---|---|---|---|---|---|
| | Butter | 62.472 | 0.1e9 | 6.2472e9 | |
| | Avocado | 41.2 | 5.2776e10 | 2.1743712e12 | |
| | Pork meat | 31 | 1.2655e9 | 3.92305e10 | |
| | Mutton meat | 14.6 | 6.8376e10 | 9.982896e10 | |
| | Rabbit meat | 9.2 | 1.5e7 | 1.38e8 | |
| | Peas | 3.5 | 1.2389e8 | 4.33615e9 | |
| Lipids | Duck meat | 30.5 | 7.03428e9 | 2.1454554e11 | |
| | Turkey meat | 4.74 | 1.2e7 | 5.688e9 | |
| | Pigeon meat | 11.35 | 0.8e7 | 9.08e9 | |
| | Chicken | 10 | 8.1456e8 | 8.1456e9 | 2.6453448701e12 |
| | Chicken meat | 9.8 | 8.1456e8 | 7.982688e9 | |
| | Cow meat | 2.5 | 4.7071e8 | 1.176775e9 | |
| | Fish | 5.65 | 2.7706e8 | 1.565389e10 | |

**Continued**

| | | | | | |
|---|---|---|---|---|---|
| | Palm oil | 100 | 1.9216e8 | 1.9216e6 | |
| | Peanut oil | 96.7 | 1.469e8 | 1.420523e10 | |
| | Sunflower oil | 100 | 1.469e8 | 1.469e6 | |
| | Corn | 1.5 | 2.80813e9 | 4.212195e10 | |
| | Bananas | 0.35 | 1.179759e10 | 4.1291565e9 | |
| Minerals | Corn | 51 | 2.80813e9 | 1.4321463e11 | 5.9566475835e11 |
| | Bananas | 83 | 1.179759e10 | 9.7919997e10 | |
| | Primus beer | 23 | 9.45562e9 | 2.1747926e11 | |
| | Amstel beer | 23 | 9.45562e9 | 2.1747926e11 | |
| | Ananas juice | 139 | 0.8e7 | 1.112e10 | |
| | Akezamutima juice | 25 | 0.8e7 | 0.2e9 | |
| | Maracuja juice | 8 | 0.7e7 | 5.6e8 | |
| | Carrot juice | 1.85 | 0.7e7 | 1.295e7 | |
| | Coffee | 15 | 1.4058e8 | 2.1087e9 | |
| | Yogurt | 7 | 0.5e9 | 3.5e9 | |
| | Sweet potatoes | 4 | 1.023458e10 | 4.093832e10 | |
| Acids | Cow cheese | 0.1 | 0.2e9 | 0.2e10 | 5.80712e9 |
| | Goat cheese | 0.4 | 0.5e8 | 0.2e9 | |
| | Peas | 0.2 | 1.2389e8 | 2.4778e9 | |
| | Tomatoes | 0.3 | 3.7644e9 | 1.12932e9 | |
| Calcium | Cow cheese | 78 | 0.2e9 | 1.56e10 | 6.385756e10 |
| | Goat cheese | 13 | 0.5e8 | 6.5e9 | |
| | Eggs | 9 | 0.5e9 | 4.5e9 | |
| | Beans | 6 | 6.19151e9 | 3.714906e10 | |
| | Orange | 0.031 | 0.35e8 | 0.1085e9 | |
| Vitamins | Eggs | 197 | 0.5e9 | 9.85e10 | 1.363975366e12 |
| | Fish | 254 | 2.7706e8 | 7.037324e10 | |
| | Yogurt | 56 | 0.2e9 | 1.12e10 | |
| | Mangoes | 91 | 0.3e8 | 2.73e9 | |
| | Sweet potatoes | 69 | 1.023458e10 | 7.0618602e11 | |
| | Lemon | 95 | 0.28e8 | 2.66e9 | |
| | Orange | 94 | 0.35e8 | 3.29e9 | |
| | Ananas | 44 | 0.2e9 | 8.8e9 | |
| | Papayas | 0.0653 | 0.24e8 | 1.56106e6 | |
| | Avocado | 127 | 3.6e9 | 4.572e11 | |
| | Carrot | 64 | 0.45e8 | 2.88e9 | |

**Table 6.** The national year production by type of food (calories (3)).

| Type of food | Nutritious food composition | Ca/100g | Year production/100g | Year production by type of food (Ca) | Total by type of food |
|---|---|---|---|---|---|
| Fibers | Lemon | 1.5 | 0.28e8 | 4.2e9 | 7.066505032e10 |
| | Orange | 1.5 | 0.35e8 | 5.25e9 | |
| | Ananas | 1.2 | 0.2e9 | 2.4e10 | |
| | Papayas | 1.3 | 0.24e8 | 3.12e9 | |
| | Avocado | 4 | 3.6e9 | 1.44e10 | |
| | Soja | 15.1 | 1.2436e8 | 1.877836e10 | |
| | Small weight | 5.8 | 1.2389e8 | 7.18562e8 | |
| | Beans | 3.2 | 6.19151e9 | 1.9812832e8 | |
| Carbohydrates | Rice | 28.8 | 2.52853e9 | 7.2821664e10 | 9.321735891e11 |
| | Bread | 44.3 | 2.5e9 | 1.1075e7 | |
| | Carrot | 9.58 | 0.45e9 | 4.311e9 | |
| | Wheat flour | 69.3 | 0.5628e8 | 3.900204e9 | |
| | Maize flour | 80 | 2.70813e9 | 2.166504e11 | |
| | Cassava flour | 35 | 2.1665e9 | 7.58275e10 | |
| | Potatoes | 16.2 | 3.7644e9 | 6.098328e10 | |
| | Orange | 8.03 | 0,35e8 | 2.8105e8 | |
| | Leeks | 4.9 | 0.48e8 | 2.352e8 | |
| | Cabbage | 4.8 | 4.4e8 | 2.112e9 | |
| | Sugar | 96.7 | 2.0428e8 | 1.9753876e10 | |
| | Primus beer | 1.6 | 9.45562e9 | 1.5128992e10 | |
| | Amstel beer | 4.6 | 9.45562e9 | 4.3495852e10 | |
| | Ananas juice | 13.1 | 0.8e7 | 1.048e8 | |
| | Akezamutima juice | 10 | 0.8e7 | 0.8e8 | |
| | Maracuja juice | 10.2 | 0.7e7 | 7.14e7 | |
| | Carrot juice | 9.58 | 0.7e7 | 6.706e7 | |
| | Wheat | 27.4 | 0.5628e8 | 1.542072e9 | |
| | Sorghum | 93.7 | 0.8851e8 | 8.293387e9 | |
| | Beans | 15.01 | 6.19151e9 | 9.29345651e10 | |
| | Sweet potatoes | 12.2 | 1.023458e10 | 1.24861876e11 | |
| | Maize | 67.2 | 2.80813e9 | 1.88706336e11 | |

**Table 7.** The necessary national year consumption by type of food (Ca).

| Population | Size | Proteins | Lipids | Minerals | Acids | Calcium | Vitamin | Fibers | Carbohydrates |
|---|---|---|---|---|---|---|---|---|---|
| Man: 18 - 65 years | 2.956808e6 | 200 | 700 | 87 | 13 | 20 | 30 | 450 | 1000 |
| Women: 18 - 65 years | 3.61152e6 | 180 | 650 | 80 | 10 | 10 | 20 | 350 | 800 |
| Children: 0 - 17 years | 5.905136e6 | 300 | 800 | 87 | 13 | 20 | 30 | 450 | 100 |
| Old person: 65 years and over | 3.57713e5 | 180 | 600 | 80 | 10 | 10 | 20 | 350 | 750 |
| Total per day | - | 3.077364340e9 | 9.3559902e9 | 1.088527768e9 | 1.19415919e8 | 1.8442753e8 | 3.4524298e8 | 5.377106350e9 | 4.10452795e9 |
| Total per year | - | 1.12631534844e12 | 3.4242924132e12 | 3.98401163088e11 | 4.3706226354e10 | 6.750047598e10 | 1.2635893068e11 | 1.9680209241e12 | 1.5022572297e12 |

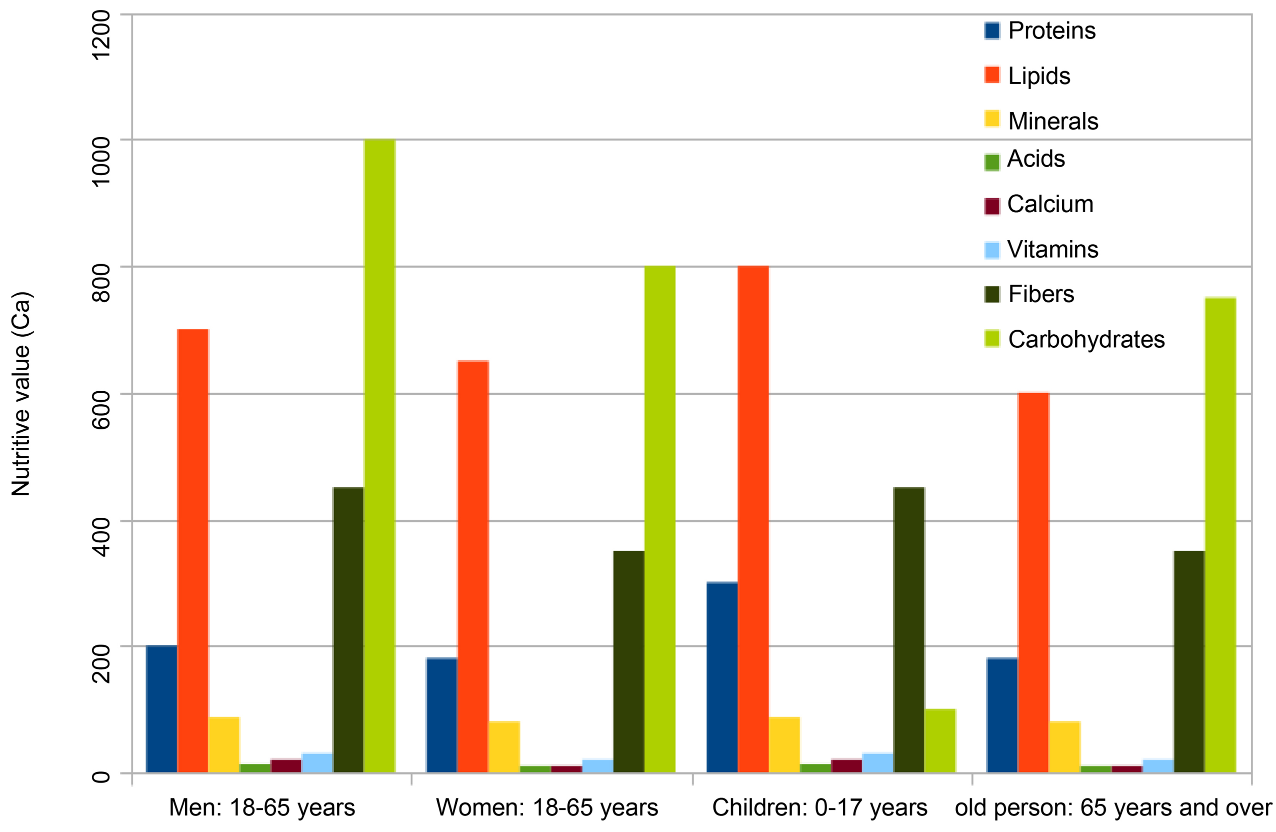


**Figure 6.** The necessary national year consumption by type of food (Ca).

project variables. The technical variables of the control show the mastery of poverty in Burundi. This solving problem can be extended to big production area as other country and continent with some complexity and cost considerations because of the big number of states and control variables.

## 7. Conclusions

This paper develops the predictive mathematical and statistical modeling of the dynamic poverty problem in Burundi. It is an innovative economic optimization system which determines the dynamic and optimal control problem when considering the poverty problem.

The numerical program had been coded with the Ubuntu Linux system. The programming software combines dynamic programming technic for mathematical computing and optimization system, whose are run under Ampl and KNITRO Solver.

High running performance is demonstrated with results giving feasible trajectories and decision support indicators with a robust optimiztion of the national monetary yield objective function. The national food production and the necessary annual consumption in terms of calories per person per day are determined.

By comparing the annual food production and the required annual consumption, there is an imbalance between different types of food. Proteins, minerals and vitamins produced in Burundi are sufficient when considering their consumption as required by the entire Burundian population. This positive contribution for the latter comes from the fact that some cows, goats, fishes, ⋯, slaughtered in Burundi come from neighboring countries. Real production remains in deficit. The lipids, acids, calcium, fibers and carbohydrates produced in Burundi are insufficient for consumption. This negative contribution proves a Burundian food deficit. It is a decision-making indicator for the design and updating of agricultural policy and implementation programs as well as projects. Investment and economic growth are only possible when food security is mastered. The capital allocated to food investment must be revised upwards. Demographic control is also a relevant indicator to push forward Burundi among the emerging countries in 2040.

## 8. Future Perspective

Challenges are so many in an innovative economic optimization. The perspective future is to establish the decision support indicators for all the economic areas. This problem can be extended to regional area. This is the process of setting up a reference center on the incubation of this research results, the knowledge transfer process to the community, the process of perpetuating this technology, the process of predicting the economic future of Burundi, the innovation in the community. A responsible and enlightened elite is the future of the nation and wealth begins with \og the thinking design \fg.

## Acknowledgements

This paper was developed in Laboratoire Universitaire de la Recherche en Modélisation et en Ingéniérie de la Statistique Appliquée LURMISTA, Institute of Applied Statistic, University of Burundi and supported by the Government of Burundi through the Ministry of Higher Education and Scientific Research Project.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

[1] Waais, I.O. (2018) Analyse multidimensionnelle de la pauvreté: Le cas de Djibouti. Université Bourgogne Franche-Comté, Dijon.

[2] Daouda, Y.H. (2011) Dynamiques de pauvreté, ingalité et croissance économique en Afrique Subsaharienne: Une investigation appliquée au cas du Niger. Université Montesquieu-Bordeaux IV, Pessac.

[3] André, L.-A.V. (1969) Fonctions de production et formules de productivité. *Revue économique*, **20**, 1-36. https://doi.org/10.3406/reco.1969.407851

[4] Cogneau, D. and Robilliard, A.-S. (2001) Croissance, distribution et pauvreté: Un modèle de microsimulation en équilibre général appliqué à Madagascar. IRD (Inland Revenue Department) et DIAL, Wellington.

[5] Couper, M.P., Kalton, G., Lyberg, L., Rao, J.N.K., Schwarz, N., Skinner, C., Robert, M. (2016) Groves in Analysis of Poverty Data by Small Area Estimation. Wiley Series in Survey Methodology, Monica Pratesi University of Pisa, Pisa.

[6] United Nations and Banque Centrale Européenne (2015) Production, flux et stocks financiers dans le Système de Système nationale. United Nations, New York.

[7] Institute of Statistics and Economic Studies of Burundi (ISTESBU) (2017, April) Projections Dmographiques 2010-2050. Institute of Statistics and Economic Studies of Burundi, Bujumbura.

[8] Institute of Statistics and Economic Studies of Burundi (ISTESBU) (2020, December). Annuaire Statistique du Burundi 2019. Institute of Statistics and Economic Studies of Burundi, Bujumbura.

[9] Paredes-López, O. (Ed.) (2021) Plant Food for Human Nutrition. Vol. 76, Springer, Berlin.

[10] DAAF La Réunion (2016) Vegetable Crop Practices Survey.

[11] Projet d'Appui à la Gestion Economique (PAGE) (2007) Rapport intgrateur soutenant le diagnostic de la chane de valeur et llaboration de la stratgie de l'horticulture au Burundi.

[12] Republic of Burundi (2012) Strategic Framework for Growth and Poverty Reduction-CSLP II. United Nations Development Program (UNDP) in Burundi.

[13] République du Burundi—Ministère de lagriculture et de lélevage (2011) Plan national d'investissement agricole (PNIA) 2012-2017. Banque Mondiale, Bujumbura.

[14] République du Burundi—Ministère de l'agriculture et de l'élevage (2008) Stratégie Agricole Nationale (SAN) 2008-2015. FIDA, Bujumbura.

[15] Instituto de Salud de Bucaramanga (ISABU) (2014) Bulletin de la Recherche Agronomique, 2015. Bulletin trimestriel No. 5, Instituto de Salud de Bucaramanga, Santander.

[16] Instituto de Salud de Bucaramanga (ISABU) (2014) Bulletin de la Recherche Agronomique, 2014. Bulletin trimestriel, No. 3, Instituto de Salud de Bucaramanga, Santander.

[17] République du Burundi—Ministère de l'Agriculture et de l'Elevage (2013) Cadre stratégique pour le développement de lhorticulture au Burundi.

[18] Thibault, N., Dévote, N., Cyrille, M. and N. Evariste (2019) Guide to Good Production

Practices for Fruits and Vegetables. Collectif 21 Agriculture urbaine, Fiche pratique N°4, le potager en planche et la rotation des cultures, Bujumbura.

[19] Food and Agriculture Organization of the United Nations (FAO) (1998) Coopération rgionale pour le développement des productions horticoles en Afrique, Les pépinières marachères en Afrique soudano sahélienne.

[20] Institut des Techniques Agricoles du Burundi (2015) Produire des légumes biologiques: Fiches techniques par legumes, Karusi.

[21] République du Niger, Ministère de l'agriculture (2012, September) Guide de Bonnes pratiques de production, stockage et conservation de l'oignon. 1ère edition, Association Internationale pour le Développement, Niger.

[22] Institute of Statistics and Economic Studies of Burundi (ISTESBU) (2017, September) Annuaire Des Statistiques Agricoles Du Burundi 2016. Institute of Statistics and Economic Studies of Burundi, Bujumbura.

[23] Word Food Program (2017) Analyse de la sécurité alimentaire en situation d'urgence au Burundi, Bujumbura.