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Simulating Well-Being Dynamics: A Stochastic Model of Interacting Life Domains and Happiness Trajectories

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Abstract

This paper presents a stochastic model for simulating the dynamic evolution of individual well-being, or happiness. Happiness is conceptualized as an emergent property of an interconnected system of key life domains, including income, mental health, physical health, stress, coping strategies, relationships, and voluntary actions. Each domain is modeled as a stochastic differential equation (SDE), capturing inherent randomness, mean-reversion, and specific influences such as expectation gaps and inter-domain feedback. We develop a composite SDE for overall happiness, driven by the states and changes in these underlying domains. The model incorporates fractional Brownian motion for smooth, persistent randomness and jump processes for significant life events. We discuss the formulation of these SDEs, focusing on clarity and psychological plausibility. Finally, we present simulated happiness trajectories under illustrative scenarios, demonstrating the model's capacity to generate rich, dynamic patterns of well-being and highlight the complex interplay between various life factors and individual happiness over time.

Keywords

Subjective Well-Being, Stochastic Differential Equations, Happiness, Stochastic Model

1. Introduction

The study of happiness, or subjective well-being, is a central concern in psychology, economics, and public policy, reflecting a fundamental human aspiration. While early investigations often focused on static determinants, it is increasingly recognized that happiness is a dynamic, multifaceted phenomenon, constantly evolving in response to internal states and external life circumstances [1] [2]. We articulate, happiness can be conceptualized as an interconnected system influenced by key variables including income, mental and physical health, stress, coping strategies, relationships, voluntary actions, and the pivotal concept of "expectation gaps"—the difference between an individual's expectations and their experienced reality.

Understanding the intricate interplay between these factors and their collective impact on well-being over time requires a framework capable of capturing both structured dynamics (like mean reversion) and inherent uncertainty. Stochastic differential equations (SDEs) offer a robust mathematical tool for modeling such time-dependent phenomena, allowing for the incorporation of randomness, feedback mechanisms, and the continuous evolution of system components [3] [4].

This paper aims to develop and simulate a system of SDEs to model the evolution of individual happiness. We focus on the dynamic interactions between several key life domains, each represented by its own SDE. These domains are influenced by inherent stochasticity, often modeled using fractional Brownian motion for persistent, smooth randomness [5], and by specific mechanisms such as the impact of expectation gaps. A composite SDE for overall happiness is then formulated, driven by the state and evolution of these underlying domains.

The novelty of this work lies in the explicit simulation of these coupled SDEs to generate and analyze happiness trajectories. We aim to provide insights into how interventions or life events might propagate through the system and ultimately affect an individual's path of well-being. This paper will first detail the SDEs for each life domain and for overall happiness. We will then describe the simulation methodology and present illustrative trajectories under various scenarios, highlighting the model's capacity to capture the rich, emergent dynamics of human happiness.

This investigation seeks to provide a novel lens to understand and quantify the dynamics of happiness as a system of interacting and entangled components in an uncertain world, bridging psychology, economics, and simulation science.

2. Literature Review

The pursuit of understanding and enhancing human happiness, or subjective well-being, has a long and rich history across philosophy, psychology, and economics [2] [6]. Early research often focused on identifying static correlates of happiness, such as income, health, and social relationships [7]. However, the dynamic and fluctuating nature of well-being has increasingly become a focal point, recognizing that happiness is not a fixed state but rather a process that unfolds over time.

Indeed [1] famously highlighted the "stochastic phenomenon" of happiness, suggesting that daily and momentous life events cause happiness levels to fluctuate around a genetically influenced set point. This perspective opened the door for models that incorporate randomness and adaptation. The concept of hedonic adaptation, or the "hedonic treadmill", posits that individuals tend to return to a baseline level of happiness despite major positive or negative life events [8] [9], particularly concerning income [10].

More recent approaches have sought to model the complex interplay of various life domains. For instance, the influence of income on well-being is recognized as non-linear, often exhibiting diminishing returns [11] [12]. Similarly, the robust connections between mental health, physical health, social relationships, and overall life satisfaction are well-documented [13] [14]. Stress and coping mechanisms are also critical dynamic components influencing an individual's experienced well-being [15].

The use of mathematical and computational models to capture these dynamics is a growing field. While some models utilize network approaches [16] or explore specific psychological phenomena like the peak-and-end rule with differential equations [17], the application of systems of stochastic differential equations (SDEs) offers a powerful framework for representing continuous-time dynamics, inherent randomness, and feedback loops within a multifaceted system [3] [4]. We propose such an SDE-based framework, conceptualizing happiness as influenced by income, mental and physical health, stress, coping, relationships, voluntary actions, and crucial "expectation gaps". This present work builds upon that conceptual foundation to simulate and analyze the emergent trajectories of happiness.

3. A Stochastic Model of Interacting Life Domains

The foundation of our model is a system of coupled stochastic differential equations (SDEs), where each SDE describes the evolution of a key life domain influencing overall happiness. This approach allows us to capture the inherent randomness and interdependencies within the human experience.

The guiding principles for the SDE formulation for each domain are:

- **Mean Reversion:** Most life domains exhibit a tendency to revert towards a baseline, equilibrium, or target level over time.
- **Stochasticity:** Each domain is subject to random fluctuations, representing unmodeled influences, inherent variability, and unexpected events. We primarily utilize fractional Brownian motion (fBm) to model these random components, capturing potential long-range dependence and smoother paths than standard Brownian motion. Significant, abrupt changes are modeled via jump processes where appropriate.
- **Interdependencies:** The domains are not isolated; the state or change in one domain can directly influence the dynamics of others, creating feedback loops.
- **Expectation Gaps:** The discrepancy between an individual's idealized expectations and their experienced reality in a particular domain can significantly impact that domain's state and other related domains.

3.1. Expectation Gaps: A Domain-Specific Adaptive Approach

A pivotal concept in the framework is that of expectation gaps, representing the discrepancy between an individual's aspirations or ideals and their perceived reality within specific life domains. These gaps are posited to significantly influence various aspects of well-being.

For each relevant life domain i (e.g., mental health, stress, coping ability), we define the following components:

• **Experienced Reality** $\boldsymbol{E_{\text{real},i}(t)}$: This variable reflects the individual's current state or perception within domain i. It is typically directly linked to the state variable of the corresponding domain. For example, for mental health, $E_{\text{real,hm}}(t) = H_m(t)$, and for coping, $E_{\text{real,c}}(t) = C(t)$. For stress, $E_{\text{real,s}}(t) = \text{Stress}(t)$, representing the currently experienced stress level.

• **Idealized Expectation** $\boldsymbol{E_{\text{ideal},i}(t)}$: This represents the individual's desired state, aspiration, or ideal level for domain i. Crucially, $E_{\text{ideal},i}(t)$ is not static but adapts over time based on the persistence and magnitude of the expectation gap $G_i(t)$. The adaptation rules are as follows:

1) $E_{\text{ideal},i}(t)$ generally remains constant from one time step to the next.

2) **If** $\boldsymbol{G_i(t) \approx 0}$ (*i.e.*, the current expectation gap is negligible, meaning reality meets or exceeds the ideal): $E_{\text{ideal},i}(t)$ may increase via a "Capped Random Increment". This reflects that when expectations are met, aspirations might rise, albeit cautiously.

$$E_{\text{ideal},i}(t+\Delta t) = E_{\text{ideal},i}(t) + \min\left(\Delta E_{\text{max\_inc}}, \max(0, Z_i)\right) \tag{1}$$

where Z_i is a small, non-negative random increment (e.g., drawn from a normal distribution with a small positive mean and capped at zero from below), and $\Delta E_{\text{max\_inc}}$ is a cap on this upward adjustment.

3) **If** $\boldsymbol{G_i(t) > 0}$ **and persists for a duration** $\boldsymbol{T_{\text{threshold},i}}$: $E_{\text{ideal},i}(t)$ may decrease through a "Historical Reduction". This signifies that if reality consistently falls short of ideals for a prolonged period, the individual might adjust their ideals downwards to align them more closely with achievable reality.

$$E_{\text{ideal},i}(t+\Delta t) = E_{\text{ideal},i}(t) - \eta_i \cdot \bar{G}_{i,\text{hist}}(t) \tag{2}$$

where η_i is an adjustment rate, and $\bar{G}_{i,\text{hist}}(t)$ is an average of the gap $G_i(s)$ over a recent historical window (e.g., the past $T_{\text{threshold},i}$ time steps during which $G_i(s) > 0$). If $G_i(t) > 0$ but the persistence is less than $T_{\text{threshold},i}$, $E_{\text{ideal},i}(t)$ remains unchanged.

The specific interpretation of $E_{\text{ideal},i}(t)$ and the direction of these adjustments depend on the nature of the domain. For instance, for stress, $E_{\text{ideal,s}}(t)$ might represent a desired *low* level of stress. If actual stress $E_{\text{real,s}}(t)$ is consistently above this ideal (*i.e.*, $G_s(t) = \max\left(0, E_{\text{real,s}}(t) - E_{\text{ideal,s}}(t)\right) > 0$), then $E_{\text{ideal,s}}(t)$ might increase (*i.e.*, the individual becomes accustomed to or accepts a higher baseline stress).

• **Expectation Gap** $\boldsymbol{G_i(t)}$: The gap for domain i is defined algebraically as the non-negative difference between the idealized expectation and the experienced reality. The precise formulation depends on whether a higher ideal is "good" or "bad" for the domain:

- For domains like mental health or coping ability, where higher is better:

$$G_i(t) = \max\left(0, E_{\text{ideal},i}(t) - E_{\text{real},i}(t)\right) \tag{3}$$

A positive gap here means ideals are not being met.

- For domains like stress, where lower is better, and $E_{\text{ideal},i}(t)$ represents a desired *low* level:

$$G_i(t) = \max\left(0, E_{\text{real},i}(t) - E_{\text{ideal},i}(t)\right) \tag{4}$$

A positive gap here means current reality (e.g., high stress) exceeds the desired low ideal.

These domain-specific gaps $G_i(t)$ then act as inputs to the SDEs of their respective domains (e.g., $G_{\text{hm}}(t)$ affects $H_m(t)$) and potentially cross-domain influences, typically exerting a negative influence on well-being when the gap signifies an unmet aspiration or an undesirable state. This adaptive mechanism for $E_{\text{ideal},i}(t)$ allows the model to capture phenomena like aspiration adjustment and the psychological impact of persistent discrepancies between desires and outcomes.

- **Expectation Gap Dynamics:** Ideal expectation levels (E_{ideal}) for Mental Health (H_m), Stress, and Coping (C) were dynamically updated based on the persistent discrepancy (gap, G) between the ideal and realized states, following the rules defined in the update_expectation_gaps function (governed by parameters like eta_ideal_adj, T_thresh_days, delta_E_max_inc).

- **Standard Gap Influence (NoFD Scenarios):** In scenarios without fractional dynamics (NoFD), the current gap $G(t)$ directly influences the relevant domain's SDE (e.g., $H_m(t)$ is affected by $G_{H_m}(t)$ through a coefficient like $\alpha_{G_m H_m}$).

- **Fractional Derivative Gap Influence (FD Scenarios):** For scenarios where fractional dynamics (FD) were enabled, an additional term incorporating a fractional derivative of the historical gap sequence was introduced into the drift component of the SDEs for $H_m(t)$, $\text{Stress}(t)$, and $C(t)$. This term aims to model memory effects, where the past trajectory of the gap (not just its current value) influences the domain's evolution.

* **Fractional Derivative Calculation:** The fractional derivative of order α for a gap history $G_t, G_{t-\Delta t}, \cdots$ was approximated using the Grünwald-Letnikov definition:

$$D_t^{\alpha} G(t) \approx \frac{1}{(\Delta t)^{\alpha}} \sum_{j=0}^{M} (-1)^j \binom{\alpha}{j} G(t - j\Delta t) \tag{5}$$

where $\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}$ are generalized binomial coefficients, M is the memory length (memory_steps_..._frac_gap), and Δt is the time step. In the implementation, this was calculated using the calculate_fractional_derivative_gl function which utilizes a deque to store the gap history.

* **Parameterization of FD Effect:** For each domain with FD enabled (e.g., H_m), two parameters controlled this effect:

• alpha_..._frac_gap_order: The order α of the fractional derivative (e.g., alpha_Hm_frac_gap_order).

• 'zeta_..._frac_gap_effect': A coefficient ζ determining the strength and direction of the fractional derivative's influence on the domain's drift (e.g., 'zeta_Hm_frac_gap_effect').

The term added to the drift was typically of the form $\zeta \cdot D_t^{\alpha} G(t)$.

* **Rationale:** This introduction of fractional derivatives allows the model to capture long-range memory and non-local effects in how individuals adapt to or are influenced by persistent discrepancies between their ideal and actual states in key psychological domains. For example, a prolonged history of a large gap in mental well-being, even if the current gap is small, might still exert a lingering influence on $H_m(t)$'s tendency to change.

3.2. Income Dynamics

Income is modeled as a **drift-dominated process**, reflecting the fact that, on average, income tends to increase over time in a predictable manner but can still exhibit short-term fluctuations. We assume that income follows a **Fractional Brownian motion** (fBM), where the drift term represents the long-term growth rate of income and the diffusion term captures short-term volatility.

The model incorporates deterministic income (monthly salary), fractional random variations, and jump processes to account for unexpected expenditures and rare income events. The stochastic differential equation (SDE) for income is defined as:

$$\mathrm{d}I(t) = \theta_I(t)\left(\mu_I(t) - I(t)\right)\mathrm{d}t + \sigma_I \mathrm{d}B_H(t) - \mathrm{d}J_{\exp}(t) + \mathrm{d}J_{\mathrm{inc}}(t) + S_{\mathrm{periodic}}(t)\mathrm{d}t \quad (6)$$

where:

- θ: Mean-reversion rate, determining how quickly income is pulled back to its running average ($\mu_I(t)$). Capturing the natural reluctance, simulating an attempt to "save" and return towards the running average.
- $\mu_I(t)$: Running average tracks the overall trend of the income process and is defined as $\mu_I = \frac{1}{t}\int_0^t I(s)\mathrm{d}s$ or in discrete time, for the numerical solution, as $\mu_I(t) = \mu_I(t-1) + \frac{1}{t}\left(I(t) - \mu_I(t-1)\right)$.
- $\sigma_I \mathrm{d}B_H(t)$: Fractional Brownian motion term that introduces smooth random fluctuations with long-range dependence, where $B_H(t)$ is fractional Brownian motion with Hurst exponent $H > 1/2$.
- $-\mathrm{d}J_{\exp}(t)$: Jump process for unexpected expenditures, modeled using a Poisson process with exponentially distributed jump sizes.
- $\mathrm{d}J_{\mathrm{inc}}(t)$: Jump process for unexpected income, modeled using a Poisson process with lognormally distributed jump sizes.
- $\lambda_{\exp}$ and λ_{inc} are the expected rates of jumps for expenditure and income(normally set quite low and with $\lambda_{\exp} > \lambda_{\mathrm{inc}}$).
- $S_{\mathrm{periodic}}(t)$ is a periodic cash injection function, defined as

$$S(t) = \begin{cases} S(t) & \text{if } t \equiv 0 \bmod 30 \\ 0 & \text{otherwise} \end{cases}$$

3.2.1. Explanation of Each Term

1) *Mean-Reversion* ($\theta_I(\mu_I - I(t))\mathrm{d}t$): - The Ornstein-Uhlenbeck (OU) process ensures that income fluctuates around its the running average $\mu_I(t)$. The mean-reversion rate θ_I controls how quickly deviations from the base income are corrected. Incorporating the *mean-reversion* term to reflect natural tendency to save or return to a previous financial "comfort zone" aligns well with real-world behavior.

2) *Fractional* Brownian Motion ($\sigma_I \mathrm{d}B_H(t)$): - We use fractional Brownian motion (fBm) with Hurst exponent $H > 1/2$ to model persistent, smooth variations in income. Unlike standard Brownian motion, which has independent increments, fBm introduces memory effects where increments are positively correlated over time. This captures gradual, small-scale fluctuations in income that are realistic for most individuals.

3) *Jump Process* for Expenditures ($-J_{\exp}(t)$): - Expenditures are modeled as negative jumps in income, occurring randomly according to a Poisson process. The sizes of these jumps are sampled from an exponential distribution, reflecting rare but potentially significant costs such as unexpected bills or repairs. The jump rate for expenditures ($\lambda_{\exp}$) is set slightly higher than for income jumps, to account for the general tendency of unexpected costs to outweigh unexpected income.

4) *Jump Process for Income* ($J_{\mathrm{inc}}(t)$): - Positive jumps represent rare sources of unexpected income, such as bonuses, refunds, or windfalls. These jumps are modeled using a Poisson process with lognormally distributed jump sizes to account for the fact that large income events are less common than small ones. The rate of these income jumps (λ_{inc}) is set lower than the rate for expenditure jumps.

3.2.2. How the Model Can Be Adapted

The proposed model provides a flexible framework for simulating income dynamics over time. It can be adapted to fit specific scenarios by modifying the parameters or processes:

- *Scaling for Different Income Levels*: The base income (μ_I) can be adjusted to represent varying salary levels for different individuals or demographics.
- *Customizing Jump Processes*: The distributions for expenditure and income jumps (e.g., exponential, lognormal) can be replaced or re-parameterized to capture specific real-world patterns (e.g., heavy-tailed distributions for larger expenses).
- *Adjusting Time Scales*: The model is currently designed for daily simulations over one month but can easily be adapted for longer periods (e.g., annual income dynamics) by adjusting the time step ($\mathrm{d}t$) and total simulation time (T).
- *Including Seasonal Effects*: Additional terms can be added to account for periodic changes in income or expenditure, such as holiday spending or tax refunds.

3.3. Relationship Dynamics $\mathbf{R}(\mathbf{t})$

The quality and state of relationships, $R(t)$, are dynamic and influenced by several factors including income, mental health, and personality traits, as well as inherent randomness. The SDE is:

$$\begin{aligned}\mathrm{d}R(t) = \theta_R\left(\mu_R - R(t)\right)\mathrm{d}t + \alpha_{I_R} f_{I_R}\left(I(t)\right)\mathrm{d}t + \alpha_{M_R} H_m(t)\mathrm{d}t \\ + \alpha_{P_R} P_{\text{pers}}\mathrm{d}t + \sigma_R \mathrm{d}B_{H_R}(t)\end{aligned} \tag{7}$$

where:

- $\theta_R\left(\mu_R - R(t)\right)\mathrm{d}t$ describes mean reversion to a baseline relationship quality μ_R.
- $\alpha_{I_R} f_{I_R}\left(I(t)\right)\mathrm{d}t$ represents the non-linear contribution from income. $f_{I_R}\left(I(t)\right) = -\beta_1\left(I_{\text{low}} - I(t)\right)^2 + \beta_2\left(I(t) - I_{\text{high}}\right)^2$, indicating that income deviations from certain thresholds (both low and high) can impact relationships.
- $\alpha_{M_R} H_m(t)\mathrm{d}t$ reflects the positive influence of mental health $H_m(t)$ on relationship quality.
- $\alpha_{P_R} P_{\text{pers}}\mathrm{d}t$ is a contribution from personality traits P_{pers} (e.g., a parameter representing introversion/extroversion influencing the tendency or quality of relationships).
- $\sigma_R \mathrm{d}B_{H_R}(t)$ introduces smooth, memory-based random fluctuations in relationship dynamics.

This formulation emphasizes the interplay between material resources, mental well-being, and intrinsic personality in shaping an individual's relational life.

3.4. Mental Health Dynamics $\mathbf{H}_\mathbf{m}(\mathbf{t})$

Mental health, $H_m(t)$, is a core component of well-being, significantly influenced by expectation gaps, stress, and the quality of relationships. The SDE is simplified here by focusing on the impact of a general mental well-being gap and other key interactors:

$$\begin{aligned}\mathrm{d}H_m(t) = \theta_m\left(\mu_m - H_m(t)\right)\mathrm{d}t + \sigma_m \mathrm{d}B_{H_{Hm}}(t) - a_{G_m} G_{\text{mental}}(t)\mathrm{d}t \\ - a_{S_m}\text{Stress}(t)\mathrm{d}t + a_{R_m} R(t)\mathrm{d}t\end{aligned} \tag{8}$$

where:

- $\theta_m\left(\mu_m - H_m(t)\right)\mathrm{d}t$ is the mean-reversion to a baseline mental health level μ_m.
- $\sigma_m \mathrm{d}B_{H_{Hm}}(t)$ captures random fluctuations in mental health using fBm.
- $a_{G_m} G_{\text{mental}}(t)\mathrm{d}t$ represents the negative impact of expectation gaps specific to mental well-being or overall life satisfaction, $G_{\text{mental}}(t)$ (derived as per Section 0). a_{G_m} is the sensitivity.
- $a_{S_m}\text{Stress}(t)\mathrm{d}t$ models the detrimental effect of current stress levels $\text{Stress}(t)$ on mental health.
- $a_{R_m} R(t)\mathrm{d}t$ reflects the supportive role of positive relationships $R(t)$ in bolstering mental health.

This model positions mental health as a dynamic state, constantly buffered or eroded by life's expectations, stressors, and social supports.

3.5. Physical Health Dynamics $H_p(t)$

Physical health, $H_p(t)$, evolves based on baseline health, lifestyle choices, random fluctuations, and susceptibility to external shocks. Its SDE is:

$$\mathrm{d}H_p(t)=\theta_p\left(\mu_p-H_p(t)\right)\mathrm{d}t+L_{\text{lifestyle}}\left(H_p(t)\right)\mathrm{d}t+\sigma_p\mathrm{d}B_{H_{Hp}}(t)+\mathrm{d}J_{\text{health\_shocks}}(t) \quad (9)$$

where:

- $\theta_p\left(\mu_p-H_p(t)\right)\mathrm{d}t$ represents mean reversion to a baseline physical health μ_p, influenced by factors like age and genetics.
- $L_{\text{lifestyle}}\left(H_p(t)\right)\mathrm{d}t$ encapsulates the deterministic inputs from lifestyle factors, which are themselves influenced by the current health state $H_p(t)$: $L_{\text{lifestyle}}\left(H_p(t)\right)=\beta_E E\left(H_p(t)\right)+\beta_D D\left(H_p(t)\right)+\beta_S S_{\text{sleep}}\left(H_p(t)\right)$. Each term $E(\cdot),D(\cdot),S_{\text{sleep}}(\cdot)$ (Exercise, Diet, Sleep) is a sigmoidal function: e.g., $E\left(H_p(t)\right)=E_0\Big/\left(1+\exp\left(-\alpha_E\cdot\left(x\left(H_p(t)\right)-x_0\right)\right)\right)$, where $x\left(H_p(t)\right)=x_0+\alpha_H\cdot\left(H_p(t)-H_{\min}\right)$ reflects that the effectiveness or engagement in these activities depends on current health.
- $\sigma_p\mathrm{d}B_{H_{Hp}}(t)$ models smooth random variations in health.
- $\mathrm{d}J_{\text{health\_shocks}}(t)$ is a jump process representing sudden adverse health events like illness or injury, typically modeled as negative jumps.

This SDE captures the feedback loop where health status influences health behaviors, which in turn affect health outcomes, alongside random and acute health events.

3.6. Stress Dynamics $\mathrm{Stress}(t)$

Stress, $\mathrm{Stress}(t)$, is modeled as a mean-reverting process influenced by expectation gaps and the effectiveness of coping strategies. Higher stress levels are detrimental to overall well-being. The SDE is:

$$\mathrm{dStress}(t)=\theta_S\left(S_{\min}-\mathrm{Stress}(t)\right)\mathrm{d}t+a_{G_S}G_{\text{stress}}(t)\mathrm{d}t-a_{C_S}C(t)\mathrm{d}t+\sigma_S\mathrm{d}B_{H_{\text{Stress}}}(t) \quad (10)$$

where:

- $\theta_S\left(S_{\min}-\mathrm{Stress}(t)\right)\mathrm{d}t$ represents mean reversion towards a baseline or minimum achievable stress level $S_{\min}$.
- $a_{G_S}G_{\text{stress}}(t)\mathrm{d}t$ models the increase in stress due to expectation gaps $G_{\text{stress}}(t)$ related to perceived manageability of life demands or achievement of stress-related goals. a_{G_S} is the sensitivity.
- $a_{C_S}C(t)\mathrm{d}t$ signifies that effective coping strategies $C(t)$ reduce stress levels.
- $\sigma_S\mathrm{d}B_{H_{\text{Stress}}}(t)$ introduces smooth random fluctuations to the stress level via fBm.

This formulation captures stress as a dynamic balance between environmental/internal pressures (via gaps) and the individual's capacity to manage them (via coping).

3.7. Coping Strategy Dynamics $C(t)$

Coping strategies, $C(t)$, are adaptive mechanisms individuals use to manage stress and are influenced by perceived stress, expectation gaps related to coping efficacy, relationships, and available resources like income. The SDE for coping is:

$$
\begin{aligned}
\mathrm{d}C(t) = {} & \theta_C\left(\mu_C - C(t)\right)\mathrm{d}t + \frac{\alpha_{S_{\text{coping}}}}{1+\beta_{S_{\text{coping}}}\text{Stress}(t)}\mathrm{d}t - a_{G_C} G_{\text{coping}}(t)\mathrm{d}t \\
& + a_{R_C} R(t)\mathrm{d}t + a_{I_C} I(t)\mathrm{d}t + \sigma_C \mathrm{d}B_{H_C}(t)
\end{aligned}
\tag{11}
$$

where:

- $\theta_C\left(\mu_C - C(t)\right)\mathrm{d}t$ describes mean reversion to a baseline level of coping strategies μ_C.
- $\frac{\alpha_{S_{\text{coping}}}}{1+\beta_{S_{\text{coping}}}\text{Stress}(t)}\mathrm{d}t$ is the contribution from stress. Higher stress reduces coping ability (or that coping effectiveness is inversely related to stress levels). $\alpha_{S_{\text{coping}}}$ and $\beta_{S_{\text{coping}}}$ are parameters modulating this non-linear effect.
- $a_{G_C} G_{\text{coping}}(t)\mathrm{d}t$ represents the negative impact of expectation gaps $G_{\text{coping}}(t)$ concerning the perceived effectiveness of one's coping mechanisms.
- $a_{R_C} R(t)\mathrm{d}t$ indicates that supportive relationships $R(t)$ improve coping strategies.
- $a_{I_C} I(t)\mathrm{d}t$ signifies that income $I(t)$ enhances coping by providing resources.
- $\sigma_C \mathrm{d}B_{H_C}(t)$ models smooth random variations in the deployment or effectiveness of coping strategies.

This model highlights that coping is not static but dynamically adapts based on demands (stress, gaps) and available supports (relationships, income).

3.8. Voluntary Action Dynamics $V(t)$

Voluntary actions, $V(t)$, encompass self-driven activities that contribute to personal well-being, such as hobbies, learning, or community engagement. These are influenced by income, physical and mental health, and available free time. The SDE is:

$$
\begin{aligned}
\mathrm{d}V(t) = {} & \theta_V\left(\mu_V - V(t)\right)\mathrm{d}t + \alpha_{I_V} f_{I_V}\left(I(t)\right)\mathrm{d}t + \alpha_{P_h V} H_p(t)\mathrm{d}t \\
& + \alpha_{M_h V} H_m(t)\mathrm{d}t + \alpha_{T_V} T_{\text{free}}(t)\mathrm{d}t + \sigma_V \mathrm{d}B_{H_V}(t)
\end{aligned}
\tag{12}
$$

where:

- $\theta_V\left(\mu_V - V(t)\right)\mathrm{d}t$ represents mean reversion to a baseline level of voluntary actions μ_V.
- $\alpha_{I_V} f_{I_V}\left(I(t)\right)\mathrm{d}t$ is the contribution from income, where $f_{I_V}\left(I(t)\right) = I(t)/\left(1+\beta_{I_V} I(t)\right)$ models diminishing returns of income on voluntary actions.
- $\alpha_{P_h V} H_p(t)\mathrm{d}t$ signifies that better physical health $H_p(t)$ enables more voluntary actions.

- $\alpha_{M_h V} H_m(t)\mathrm{d}t$ indicates that better mental health $H_m(t)$ also promotes engagement in voluntary actions.
- $\alpha_{T_V} T_{\text{free}}(t)\mathrm{d}t$ is the contribution from available free time $T_{\text{free}}(t)$, which itself can be modeled as $T_{\text{free}}(t) = T_{\max} - \gamma_{\text{workload}} W(t)$, where $W(t)$ is workload (potentially related to income generation efforts).
- $\sigma_V \mathrm{d}B_{H_V}(t)$ introduces smooth random fluctuations in the level or intensity of voluntary actions.

This model captures the idea that engaging in fulfilling voluntary activities depends on having the necessary resources, health, and time.

4. The Stochastic Happiness Equation $\mathrm{H}(\mathrm{t})$

Following the conceptualization of happiness as a dynamic state influenced by the collective status of various life domains, we propose a refined stochastic differential equation for overall happiness, $H(t)$. Instead of happiness being a direct sum of the *changes* in its components, we model happiness as a variable that mean-reverts to a target level, $\mu_{H_{\text{target}}}(t)$. This target level is itself a dynamic function of the current states of the underlying life domains discussed in Section 3. This approach allows for a more intuitive understanding of how life circumstances set an "equilibrium" for happiness, around which momentary fluctuations occur.

The proposed SDE for overall happiness $H(t)$ is:

$$\mathrm{d}H(t) = \theta_H \left(\mu_{H_{\text{target}}}(t) - H(t)\right)\mathrm{d}t + \sigma_{H_{\text{own}}} \mathrm{d}B_{H_H}(t) \tag{13}$$

where:

- θ_H is the speed of mean reversion of happiness towards its current target level. A higher θ_H implies that happiness adjusts more quickly to changes in underlying life circumstances.
- $\mu_{H_{\text{target}}}(t)$ is the dynamic target happiness level, determined by the current state of the individual's life domains. It is formulated as a weighted combination of these domains:

$$\begin{aligned} \mu_{H_{\text{target}}}(t) = \lambda_{\text{const}} + \lambda_I f_{I_H}\left(I(t)\right) + \lambda_{Hm} H_m(t) + \lambda_{Hp} H_p(t) \\ - \lambda_S \text{Stress}(t) + \lambda_C C(t) + \lambda_R R(t) + \lambda_V V(t) \end{aligned} \tag{14}$$

- λ_{const} is a baseline constant for happiness.
- $f_{I_H}\left(I(t)\right)$ represents the contribution of income to the happiness target. To capture diminishing returns and hedonic adaptation effects, a logarithmic function is often suitable, e.g., $f_{I_H}\left(I(t)\right) = \ln\left(1 + I(t)/I_{\text{scale}}\right)$, where I_{scale} is a scaling parameter. λ_I is the weight for this income contribution.
- $\lambda_{Hm}, \lambda_{Hp}, \lambda_S, \lambda_C, \lambda_R, \lambda_V$ are the respective sensitivity coefficients (weights) indicating how strongly each domain (Mental Health, Physical Health, Stress, Coping, Relationships, Voluntary Actions) influences the target happiness level. Note that the sign for λ_S is negative, as higher stress is expected to lower the happiness target.
- These λ coefficients are not necessarily normalized to sum to one; they

represent the magnitude of impact of each (appropriately scaled) domain on the target happiness.

- $\sigma_{H_{\text{own}}} \mathrm{d}B_{H_H}(t)$ represents the intrinsic volatility of happiness, capturing mood swings, unmodeled short-term influences, or inherent randomness in subjective experience, modeled using fractional Brownian motion for smooth, persistent fluctuations.

Justification for this $H(t)$ Formulation

This formulation of the happiness SDE offers several advantages for modeling and interpretation:

- **Intuitive Dynamics:** Happiness striving towards an equilibrium defined by current life conditions aligns well with psychological theories of well-being and adaptation.
- **Separation of Influences:** It clearly distinguishes between the factors that determine the *target* level of happiness (the state of life domains) and the *momentary fluctuations* around that target (intrinsic randomness and the speed of adjustment).
- **Interpretability of Parameters:** The λ coefficients directly indicate the strength and direction of influence of each life domain on the potential happiness level, while θ_H quantifies the resilience or responsiveness of happiness itself.
- **Flexibility in Domain Contributions:** The function $\mu_{H_{\text{target}}}(t)$ can accommodate various non-linear relationships (like the logarithmic income effect) for how each domain contributes to overall happiness.

This model provides a robust yet interpretable framework for simulating how an individual's happiness might evolve over time as a complex interplay of their life circumstances and internal dynamics.

5. Simulation Methodology

Simulating the interconnected system of stochastic differential equations (SDEs) outlined in Sections 3 and 4 is essential for understanding the emergent dynamics of happiness trajectories. This section details the approach taken for these simulations, addressing the inherent complexity, the numerical scheme employed, the setup of simulation parameters, and the software environment.

5.1. Complexity of the Simulation

The simulation of this happiness model presents several layers of complexity:

1) **Coupled System of SDEs:** The model consists of eight primary stochastic processes ($I(t)$, $H_m(t)$, $H_p(t)$, $\text{Stress}(t)$, $C(t)$, $R(t)$, $V(t)$, and $H(t)$), plus potentially several auxiliary processes for expectation gaps ($E_{\text{ideal},i}(t)$ if modeled stochastically). These SDEs are tightly coupled, meaning the drift or volatility term of one SDE often depends on the current state of one or more other SDEs. This requires careful handling of the update order within each time step or the use of values from the previous time step for contemporaneous dependencies.

2) **Fractional Brownian Motion (fBm):** Most SDEs incorporate fBm ($B_H(t)$

with $H > 1/2$) for their stochastic components. Unlike standard Brownian motion, fBm has correlated increments (long-range dependence). Generating fBm paths or their increments is more computationally intensive than generating standard Wiener process increments. It requires methods like the Davies-Harte algorithm (for exact generation of a full path then differencing) or Hosking's method (for sequential generation of increments), each with its own computational trade-offs.

3) **Jump Processes:** The income SDE includes compound Poisson processes for expenditures and unexpected income, and the physical health SDE includes jumps for health shocks. Simulating these requires generating Poisson arrival times and then sampling jump sizes from their respective distributions (exponential, lognormal).

4) **Non-Linearities:** Many SDEs feature non-linear terms in their drift components. For instance, the income effect on relationships ($f_{I_R}\left(I(t)\right)$), the effect of stress on coping, the lifestyle contributions to physical health ($L_{\text{lifestyle}}\left(H_p(t)\right)$ involving sigmoidal functions), and the logarithmic income effect on the happiness target ($f_{I_H}\left(I(t)\right)$). These non-linearities are crucial for realistic dynamics but can make analytical solutions intractable, necessitating numerical simulation.

5) **Expectation Gap Dynamics:** While simplified in this sub-paper (Section 3), the calculation of $G_i(t)$ still involves updating $E_{\text{ideal},i}(t)$ (possibly stochastically) and $E_{\text{real},i}(t)$, then applying the $\max(0,\cdot)$ function, before $G_i(t)$ feeds into other SDEs.

6) **Parameter Space:** The model involves a large number of parameters (mean-reversion rates, baseline levels, volatilities, sensitivities between domains, Hurst exponents, jump parameters, etc.). Managing, calibrating (even illustratively), and conducting sensitivity analyses across this space is a significant undertaking.

Despite these complexities, numerical simulation provides a viable path to explore the model's behavior.

5.2. Numerical Scheme

Given the nature of the SDEs, including those with non-additive noise (if fBm is considered generally) and path-dependent components, the Euler-Maruyama scheme is a common and relatively straightforward method for numerical approximation [18]. For a generic SDE of the form:

$$\mathrm{d}X(t) = a\left(X(t),t\right)\mathrm{d}t + b\left(X(t),t\right)\mathrm{d}W(t) \tag{15}$$

where $a\left(X(t),t\right)$ is the drift term and $b\left(X(t),t\right)$ is the diffusion term, and $\mathrm{d}W(t)$ is the increment of the stochastic process (e.g., fBm or standard BM for auxiliary processes), the discrete-time update rule is:

$$X(t_{k+1}) = X(t_k) + a\left(X(t_k),t_k\right)\Delta t + b\left(X(t_k),t_k\right)\Delta W_k \tag{16}$$

where $\Delta t = t_{k+1} - t_k$ is the time step, and $\Delta W_k = W(t_{k+1}) - W(t_k)$ is the increment of the stochastic process over that time step.

For the SDEs involving fractional Brownian motion, ΔW_k becomes $\Delta B_{H,k} = B_H(t_{k+1}) - B_H(t_k)$. These increments are not independent and must be generated accordingly. For jump processes, at each time step Δt, the probability of one or more jumps occurring is determined (e.g., $1 - e^{-\lambda \Delta t} \approx \lambda \Delta t$ for small $\lambda \Delta t$). If a jump occurs, its size is sampled from the specified distribution and added to the state variable.

5.3. Simulation Setup

The simulation for this paper is configured as follows:

- **Time Horizon (T_{final}):** The total duration for which the trajectories are simulated (e.g., representing several years in daily or weekly steps). For instance, $T_{\text{final}} = 1095$ days (3 years).
- **Time Step (Δt):** A sufficiently small time step is chosen to ensure numerical stability and accuracy of the Euler-Maruyama scheme. For example, $\Delta t = 0.1$ (if units are days, this allows for sub-daily resolution) or $\Delta t = 1$ day. The choice depends on the characteristic time scales of the SDEs.
- **Initial Conditions:** Each state variable ($I(0), H_m(0), H_p(0), \text{Stress}(0), C(0), R(0), V(0), H(0)$, and $E_{\text{ideal},i}(0)$ for all relevant gaps) must be initialized. These can be set to their respective baseline levels (μ_i) or other chosen starting values to represent a particular individual profile.
- **Parameter Values:** A consistent set of illustrative parameter values is used for all SDEs. These include:
  - Mean-reversion rates (θ_i).
  - Baseline/target levels ($\mu_i, S_{\min}$).
  - Volatilities (σ_i).
  - Hurst exponents (H_i) for each fBm process (typically $0.5 < H_i < 1$).
  - Interaction coefficients ($\alpha_{ij}, a_{G_i}, \lambda_j$, etc.).
  - Jump process parameters (rates λ_{jump}, distribution parameters for jump sizes).
  - Parameters for expectation gap dynamics ($\kappa_{\text{ideal},i}, \sigma_{\text{ideal},i}$).

A table of key parameters used in the illustrative simulations will be provided in Section 8. It is acknowledged that these parameters are chosen for demonstration and are not empirically calibrated in this study.

- **Number of Trajectories:** While this sub-paper may focus on presenting a few illustrative individual trajectories, for statistical analysis or ensemble averages (not the primary focus here), multiple trajectories (N_{ensemble}) would be generated.

5.4. Implementation and Software

The simulations are implemented in Python 3.x, leveraging several scientific computing libraries:

- **NumPy:** For numerical operations, array management, and random number generation (for standard normal deviates needed for fBm and jump processes).

• **SciPy:** Potentially for statistical distributions (e.g., lognormal, exponential for jump sizes) and other scientific functions.

• **fbm package (or similar):** A dedicated Python library such as 'fbm' (by Christopher Flynn and collaborators) or 'stochastic' is used for generating fractional Brownian motion paths or increments with a specified Hurst exponent. Alternatively, custom implementations of algorithms like Hosking's method can be used.

• **Matplotlib/Seaborn:** For visualizing the simulated trajectories.

The simulation proceeds iteratively: at each time step t_k, the values of all state variables $X(t_k)$ are used to calculate the drifts. Stochastic increments $\Delta B_{H,k}$ and jump occurrences are generated. Then, all state variables are updated to $X(t_{k+1})$ using the Euler-Maruyama scheme. Dependencies between SDEs are handled by using the values from t_k to compute all drifts and updates for t_{k+1}.

6. Psychological Consistency of the Model

The stochastic model of well-being dynamics presented in this paper, while employing mathematical formalism, is deeply rooted in and seeks to operationalize several core concepts from psychological science. This section details the consistency between the model's architecture and established psychological theories, thereby underscoring its plausibility as a representation of human happiness dynamics.

6.1. Holistic and Multi-Component Nature of Well-Being

• **Psychological Theories:** Contemporary psychology views subjective well-being (SWB) not as a monolithic entity, but as a multifaceted construct encompassing life satisfaction, positive affect, and the absence of negative affect [2]. More specific theories further delineate key dimensions:

- **Ryff's Model of Psychological Well-Being (PWB)** [19] [20] identifies six core dimensions: self-acceptance, personal growth, purpose in life, environmental mastery, autonomy, and positive relations with others.

- **Self-Determination Theory (SDT)** [21] posits that well-being arises from the satisfaction of three basic psychological needs: autonomy (feeling volitional), competence (feeling effective), and relatedness (feeling connected).

- **Seligman's PERMA Model** [22] conceptualizes flourishing through five pillars: Positive emotion, Engagement, Relationships, Meaning, and Accomplishment.

• **Model Consistency:** The model reflects this multi-component view by defining overall happiness $H(t)$ (Equation 11) as dynamically influenced by the states of several key life domains: income (resource for mastery/accomplishment), mental and physical health (foundational), stress (detriment), coping strategies (mastery), relationships (relatedness, positive relations), and voluntary actions (engagement, purpose, autonomy, growth). Each domain (Section 3) is modeled with its own SDE, acknowledging its unique dynamics while also being interconnected. For example, Relationship Dynamics $R(t)$ directly map to "positive relations" and "relatedness", while Voluntary Action Dynamics $V(t)$ can encompass "engagement", "purpose", and opportunities for "personal growth".

6.2. Adaptation, Set-Points, and Homeostasis

- **Psychological Theories:**

- **Set-Point Theory** [1] [23] suggests that individuals have a genetically influenced baseline level of happiness. Life events may cause temporary deviations, but individuals tend to revert to this set-point.

- **Hedonic Adaptation** [8] [9] describes the psychological process by which individuals' affective responses to positive or negative events diminish over time. We adapt to new circumstances, and their initial emotional impact fades.

- **Model Consistency:** The model incorporates these concepts through:

- **Mean Reversion:** Each life domain $X(t)$ (e.g., $H_m(t)$, $R(t)$) includes a mean-reversion term (e.g., $\theta_m(\mu_m - H_m(t))\mathrm{d}t$ in Equation 6), pulling the domain towards a baseline level μ.

- **Dynamic Happiness Target:** The overall happiness $H(t)$ mean-reverts towards a *dynamic target level* $\mu_{H_{\text{target}}}(t)$ (Equation 11), which itself is a function of the current states of the life domains (Equation 12). This offers a nuanced view of the set-point: while there's a tendency towards an equilibrium, this equilibrium is not static but adapts to sustained changes in life circumstances, aligning with findings that set-points can, to some extent, be shifted [24].

- **Income Adaptation:** The logarithmic function for income's contribution to $\mu_{H_{\text{target}}}(t)$ ($f_{IH}(I(t)) = \ln(1 + I(t)/I_{\text{scale}})$) inherently models diminishing returns, a form of adaptation where each additional unit of income provides less additional happiness.

6.3. The Role of Expectation Gaps and Discrepancies

- **Psychological Theories:**

- **Discrepancy Theories of Happiness** (e.g., [25] [26] – Self-Discrepancy Theory) posit that satisfaction and emotional states are influenced by the perceived gap between one's current state and a standard of comparison (e.g., ideal self, ought self, aspirations, past self, others' states).

- **Aspiration Level Theory** suggests that individuals adjust their aspirations based on past successes and failures.

- **Goal Pursuit & Well-Being:** The process of setting and striving for goals, and the feedback from goal attainment (or non-attainment), significantly impacts well-being [27].

- **Model Consistency:** The "Expectation Gaps" $G_i(t)$ (subsection on page 5, Equations 3 and 4) are central to the model and directly operationalize discrepancy theories.

- $G_i(t)$ is defined as the difference between an "Idealized Expectation" $E_{\text{ideal},i}(t)$ and "Experienced Reality" $E_{\text{real},i}(t)$ for relevant domains.

- These gaps directly impact domain dynamics (e.g., $G_{\text{mental}}(t)$ negatively affects $H_m(t)$ in Equation 6; $G_{\text{stress}}(t)$ increases $\text{Stress}(t)$ in Equation 8).

- The adaptive nature of $E_{\text{ideal},i}(t)$ (Equations 1 and 2), where ideals may rise cautiously with success ("Capped Random Increment") or lower after prolonged unmet expectations ("Historical Reduction"), mirrors psychological processes of aspiration adjustment and coping with dissonance [28].

6.4. Stress, Coping, and Resource Theories

- **Psychological Theories:**

- **Transactional Model of Stress and Coping** [15]: Stress is viewed as an outcome of an appraisal process where an individual evaluates environmental demands (stressors) against their perceived resources to manage them. Coping strategies are the cognitive and behavioral efforts to manage these demands.

- **Conservation of Resources (COR) Theory** [29] [30]: Stress occurs when there is a threat of resource loss, actual resource loss, or a failure to gain resources following significant resource investment. Resources can be objects, conditions, personal characteristics, or energies.

- **Model Consistency:** The SDEs for $\text{Stress}(t)$ (Equation 8) and Coping Strategy Dynamics $C(t)$ (Equation 9) embody these principles:

- $\text{Stress}(t)$ increases due to expectation gaps $G_{\text{stress}}(t)$ (perceived unmanageability of demands or failure to achieve stress-related goals) and is reduced by effective coping $C(t)$.

- Coping $C(t)$ is influenced by factors like income $I(t)$ (material resource), relationships $R(t)$ (social support resource), and is also affected negatively by high stress levels (reflecting how overwhelming stress can impair coping efficacy, as seen in the denominator of the stress term in Equation 9).

- The interplay between stress, coping, and other domains like mental health ($\alpha_{sm}\text{Stress}(t)\text{d}t$ in Equation 6) reflects the systemic impact of stress and the resources available to manage it.

6.5. Agency, Purposeful Activity, and Intrinsic Motivation

- **Psychological Theories:**

- **Self-Determination Theory (SDT)** [21] emphasizes that activities pursued out of intrinsic motivation (for inherent satisfaction) or well-internalized extrinsic motivation contribute more to well-being than controlled or amotivated behaviors.

- **Flow Theory** [31] highlights the state of optimal experience achieved when individuals are fully immersed in challenging activities that match their skill level, often leading to feelings of enjoyment and accomplishment.

- **Purpose in Life:** Having a sense of purpose or meaning is consistently linked to higher well-being [19] [32].

- **Model Consistency:** The "Voluntary Action Dynamics" $V(t)$ (Equation 10) represents engagement in self-driven activities like hobbies, learning, or community involvement.

- $V(t)$ contributes positively to the happiness target $\mu_{H_{\text{target}}}(t)$ ($\lambda_V V(t)$ in Equation 12).

- The ability to engage in $V(t)$ is supported by enabling factors such as physical health $H_p(t)$, mental health $H_m(t)$, income $I(t)$ (providing resources and opportunity), and available free time $T_{\text{free}}(t)$, consistent with the idea that basic needs and resources are necessary to pursue higher-order growth activities.

6.6. Validation of Economic Consistency

The proposed income model, while stylized, incorporates several features that align well with established economic theories and observed financial behaviors:

1) Mean Reversion and Adaptive Expectations ($\theta_I(t)(\mu_I(t)-I(t))\mathrm{d}t$ with $\mu_I(t)$ as a running average)

- *Consumption Smoothing* & *Habit Formation*: The mean-reversion mechanism, where income (or more accurately, liquid funds represented by $I(t)$) tends to revert to a running average $\mu_I(t)$, is consistent with individuals attempting to smooth their consumption. When $I(t)$ is significantly above $\mu_I(t)$, individuals may increase spending or transfer funds to less liquid savings, effectively pulling $I(t)$ down. Conversely, if $I(t)$ falls below $\mu_I(t)$, they might curtail discretionary spending or draw on short-term credit/buffers, pulling $I(t)$ up. This reflects a "financial comfort zone" or habit persistence.

- *Adaptive Expectations*: The use of a running average $\mu_I(t)$ as the reversion target implies that individuals adapt their perception of "normal" income based on past experience. This is a common assumption in models of expectation formation.

2) Drift-Dominated Process and Periodic Income ($S_{\text{periodic}}(t)\mathrm{d}t$)

- *Lifecycle Income Profile*: The model's ability to be "drift-dominated" primarily through periodic salary injections ($S_{\text{periodic}}(t)\mathrm{d}t$) reflects the fundamental reality of most individuals' income streams, which are characterized by regular, predictable payments (salaries, wages). Over the long term, these injections typically lead to an upward trend in cumulative income, consistent with career progression and general economic growth.

3) Stochastic Fluctuations (Fractional Brownian Motion $\sigma_I \mathrm{d}B_H(t)$)

- *Persistent Shocks* & *Autocorrelation*: The use of Fractional Brownian Motion (fBm) with $H > 1/2$ introduces persistence or long-range dependence in the income fluctuations. This acknowledges that real-world income streams are often not memoryless; periods of slightly higher or lower income (or spending patterns affecting net cash flow) can persist beyond what a standard Brownian motion would suggest. This can be due to unobserved factors, minor changes in earning capacity, or ingrained spending habits.

- *Smoothness*: fBm provides smoother paths than standard Brownian motion, which can be more realistic for daily fluctuations in available funds, barring discrete events.

4) Jump Processes for Shocks ($-\mathrm{d}J_{\exp}(t)+\mathrm{d}J_{\mathrm{inc}}(t)$)

- *Unexpected Events* & *Lumpy Expenditures/Income*: Economic life is punctuated by unexpected, significant financial events. The Poisson jump processes directly model these:

* $\mathrm{d}J_{\exp}(t)$: Captures lumpy, unforeseen expenditures (e.g., medical emergencies, urgent repairs) which are a common feature of household finance. The exponential distribution for jump sizes is plausible, reflecting many small unexpected costs and fewer very large ones.

* $\mathrm{d}J_{\mathrm{inc}}(t)$: Models rare positive windfalls (e.g., bonuses, small inheritances, tax refunds). The lognormal distribution allows for a skewed distribution where large positive jumps are possible but less frequent, a common characteristic of such income events.

- *Asymmetry of Shocks:* The typical parameterization ($\lambda_{\exp} > \lambda_{\mathrm{inc}}$) reflects the common experience that unexpected negative financial shocks tend to be more frequent than unexpected positive windfalls for most individuals.

In summary, the model combines a deterministic component (salary) with various stochastic elements (mean-reversion to an adaptive average, persistent smooth fluctuations, and discrete jumps) that mirror key aspects of economic theory concerning income dynamics, expectation formation, consumption behavior, and the impact of uncertainty and rare events on individual financial well-being.

6.7. Stochasticity and Dynamic Nature of Human Experience

• **Psychological Theories:** Human lives are not static; they are characterized by continuous change, unexpected events, and internal fluctuations. [1] explicitly described happiness as a "stochastic phenomenon". Psychological states are rarely stable and are subject to numerous, often unobservable, influences.

• **Model Consistency:** The model's foundation on stochastic differential equations (SDEs) intrinsically captures this dynamism.

- **Fractional Brownian Motion (fBm):** The use of $\mathrm{d}B_H(t)$ terms in most SDEs, where H represents a Hurst parameter typically > 0.5, models persistent randomness—where past fluctuations have a lingering influence on future ones, creating smoother, more correlated paths than standard Brownian motion. This can represent mood persistence or slowly evolving unobserved factors.

- **Jump Processes ($\mathrm{dJ}(\mathrm{t})$):** The inclusion of jump processes (e.g., $\mathrm{d}J_{\mathrm{health\_shocks}}(t)$ in Equation 7 for physical health) allows for the modeling of significant, abrupt life events (e.g., sudden illness, job loss, unexpected windfall), which are known to have immediate and sometimes lasting impacts on well-being.

In conclusion, the proposed stochastic model of interacting life domains and happiness trajectories integrates multiple key insights from psychological theories of well-being, adaptation, stress, motivation, and the inherent dynamism of human life. By translating these psychological constructs into a system of SDEs, the model offers a quantitative framework to explore the complex, emergent patterns of happiness over time, grounded in established psychological understanding.

7. Optimizing Happiness: A Stochastic Gradient Descent Approach to Life Domain Weighting

While the preceding sections have focused on simulating well-being dynamics given a set of parameters, a natural extension is to explore how an individual might, metaphorically, "optimize" their long-term happiness. This section proposes a computational approach using Stochastic Gradient Descent (SGD) to find a set of sensitivity coefficients (λ_i in Equation 17) that maximize average simulated happiness over an extended period. The goal is not to prescribe a universal formula for happiness, but rather to use this optimization framework as an analytical tool to understand which life domains, under various simulated conditions and initial states, tend to contribute most significantly to sustained well-being within the model's structure.

7.1. Conceptual Framework for Optimization

The target happiness level, $\mu_{H_{\text{target}}}(t)$, is defined as a weighted sum of various life domains (Equation):

$$\mu_{H_{\text{target}}}(t) = \lambda_{const} + \lambda_I f_{IH}(I(t)) + \lambda_{Hm} H_m(t) + \lambda_{Hp} H_p(t) - \lambda_S \text{Stress}(t) + \lambda_C C(t) + \lambda_R R(t) + \lambda_V V(t) \tag{17}$$

The core idea is to treat the weights $\boldsymbol{\lambda} = (\lambda_I, \lambda_{Hm}, \lambda_{Hp}, \lambda_S, \lambda_C, \lambda_R, \lambda_V)$ as learnable parameters. We aim to find a $\boldsymbol{\lambda}^*$ that maximizes the expected long-term average happiness, $E[\bar{H}]$, where $\bar{H} = \frac{1}{T_{\text{final}}}\int_0^{T_{\text{final}}} H(t)\,\mathrm{d}t$.

Since $H(t)$ is a stochastic process driven by the SDE system, its evolution depends on $\boldsymbol{\lambda}$ through $\mu_{H_{\text{target}}}(t)$. The inherent randomness and path-dependency make direct analytical optimization intractable, motivating the use of simulation-based stochastic optimization.

7.2. Conceptual Alignment with Well-Being Enhancement Literature

The endeavor to identify and prioritize factors that enhance human well-being and fulfillment is a central theme across psychology, particularly within positive psychology and intervention research [33] [34]. While individuals do not typically engage in formal mathematical optimization of "happiness coefficients", their choices and efforts to improve their lives often reflect an implicit process of prioritizing certain life domains or activities believed to yield greater well-being.

Our proposed optimization of the λ coefficients, which weight the contribution of various life domains to the overall happiness target $\mu_{H_{\text{target}}}(t)$, is conceptually aligned with these broader efforts in several ways:

1) **Focus on Key Life Domains:** The model's structure, based on distinct yet interconnected life domains (e.g., health, relationships, income, purposeful engagement), mirrors multi-component theories of well-being [19] [21] [2]. These theories inherently suggest that nurturing these domains is crucial for flourishing.

2) **Implicit Weighting in Interventions:** Psychological literature abounds with evidence for interventions targeting specific domains—such as fostering social connections [35], cultivating gratitude, or engaging in meaningful activities [32]—as effective means to boost happiness. This implicitly assigns a significant "weight" or leverage to these factors. Our optimization seeks to make such weightings explicit within the model's framework.

3) **Resource Allocation and Goal Pursuit:** Individuals continuously make decisions about allocating their finite resources (time, energy, attention) towards goals they believe will lead to a more satisfying life [36]. The optimization of λ coefficients can be viewed as a computational analogue to an idealized process of learning which areas of life (domains) offer the greatest return in terms of sustained well-being, given the system's dynamics.

4) **Personalization and Context-Dependence:** The strategy of running the optimization under different initial conditions (e.g., varying levels of income or relationship quality) resonates with the "person-activity fit" model in positive psychology [37]. This model posits that the effectiveness of well-being strategies can vary depending on individual characteristics and circumstances. Our approach allows for exploring how the "optimal" (model-derived) focus might shift based on an individual's starting point or simulated life context.

Thus, while the methodology of using Stochastic Gradient Descent with simplex projection to find optimal λ coefficients is a formal computational abstraction, the underlying goal—to understand how different life factors contribute to and can be leveraged for enhanced well-being—is deeply consonant with established psychological inquiry. The optimization provides a quantitative, model-based lens to explore questions of balance, prioritization, and the pursuit of a fulfilling life.

7.3. Methodology: Stochastic Gradient Descent (SGD) with Simplex Projection

We employ a simulation-based Stochastic Gradient Descent (SGD) approach to find the vector of sensitivity coefficients $\boldsymbol{\lambda} = \left(\lambda_I, \lambda_{Hm}, \lambda_{Hp}, \lambda_S, \lambda_C, \lambda_R, \lambda_V\right)$ that maximizes the expected long-term average happiness, $J(\boldsymbol{\lambda}) = E\left[\bar{H}(\boldsymbol{\lambda})\right]$. The coefficients λ_i (for $i \in \{I, Hm, Hp, C, R, V\}$) represent the positive contributions of their respective domains, while λ_S represents the magnitude of the negative impact of stress. To ensure these coefficients are interpretable as relative positive contributions and to maintain stability, we impose constraints:

1) Non-negativity: $\lambda_i \geq 0$ for all i.

2) Sum-to-one for positive contributors: $\sum_{i \in \{I, Hm, Hp, C, R, V\}} \lambda_i = 1$.

The coefficient λ_S (magnitude of stress impact) is optimized as a non-negative value but is not included in the sum-to-one constraint of the positive contributors. The constant term λ_{const} is typically kept fixed or optimized separately.
The SGD algorithm iteratively updates the parameters $\boldsymbol{\lambda}$ in the direction of an estimated gradient of $J(\boldsymbol{\lambda})$:

$$\boldsymbol{\lambda}'_{k+1} = \boldsymbol{\lambda}_k + \eta \nabla_{\boldsymbol{\lambda}} \bar{H}\left(\boldsymbol{\lambda}_k; \omega_k\right) \tag{18}$$

where $\boldsymbol{\lambda}_k$ is the vector of weights at iteration k, η is the learning rate, and $\nabla_{\lambda} \bar{H}\left(\boldsymbol{\lambda}_k; \omega_k\right)$ is an estimate of the gradient of the average happiness $\bar{H}$ obtained from one or more simulation runs (using stochastic path(s) ω_k) with the current parameters $\boldsymbol{\lambda}_k$.

The gradient $\nabla_{\boldsymbol{\lambda}} \bar{H}$ is estimated using a finite difference method. For each component λ_j in $\boldsymbol{\lambda}$:

$$\frac{\partial \bar{H}}{\partial \lambda_j} \approx \frac{\bar{H}\left(\boldsymbol{\lambda}_k \text{ with } \lambda_j + \delta; \omega'_k\right) - \bar{H}\left(\boldsymbol{\lambda}_k; \omega''_k\right)}{\delta} \tag{19}$$

where δ is a small perturbation, and ω'_k, ω''_k represent (potentially different) stochastic paths for the perturbed and baseline simulations. To improve gradient stability, $\bar{H}$ at each point can be averaged over multiple simulation runs.

To enforce the constraints, the updated vector $\boldsymbol{\lambda}'_{k+1}$ from Equation 18 is then projected onto the feasible set. Specifically:

1) The sub-vector $\boldsymbol{\lambda}'_{pos,k+1}$ corresponding to the positively contributing domains ($\lambda_I, \lambda_{Hm}, \lambda_{Hp}, \lambda_C, \lambda_R, \lambda_V$) is projected onto the standard probability simplex (denoted Δ). This operation, $\text{Proj}_{\Delta}(\cdot)$, ensures these components are non-negative and sum to 1.

$$\boldsymbol{\lambda}_{pos,k+1} = \text{Proj}_{\Delta}\left(\boldsymbol{\lambda}'_{pos,k+1}\right) \tag{20}$$

2) The updated stress magnitude coefficient, $\lambda'_{S,k+1}$, is ensured to be non-negative:

$$\lambda_{S,k+1} = \max\left(0, \lambda'_{S,k+1}\right) \tag{21}$$

The vector $\boldsymbol{\lambda}_{k+1}$ is then formed by combining $\boldsymbol{\lambda}_{pos,k+1}$ and $\lambda_{S,k+1}$. The projection onto the simplex is a standard operation in optimization, often achieved efficiently using sorting-based algorithms (see **Appendix** for details). This iterative process of gradient estimation, update, and projection is repeated until convergence or for a fixed number of iterations. The average overall happiness $\bar{H}$ using the optimized $\boldsymbol{\lambda}^*$ is then evaluated by averaging over multiple independent simulation runs.

7.4. Interpreting Optimized Coefficients and Implications

The resulting optimized $\boldsymbol{\lambda}^*$ vector provides insights into the relative importance of different life domains for maximizing long-term happiness *within the confines of the model and the specific simulation setup (initial conditions, other parameters)*.

- **Identifying Key Drivers:** Domains with larger (positive for positive influences like H_m, R, V, C, I ; negative for Stress) magnitudes in $\boldsymbol{\lambda}^*$ would be identified by the model as more critical levers for achieving higher average happiness.
- **Sensitivity to Initial Conditions/Scenarios:** The optimization can be run

starting from different initial states for the life domains (e.g., low income, poor mental health vs. high income, good mental health) or under different persistent environmental conditions (e.g., high baseline societal stress).

- For an individual starting with low income, the optimized λ_I^* might be relatively higher, suggesting that, from that starting point, focusing on improving income (as modeled) yields significant happiness gains.

- For an individual with chronic health issues, λ_{Hp}^* might be less prominent if improvements are difficult to achieve within the model, and perhaps λ_C^* (coping) or λ_{Hm}^* (mental health resilience) might become more important.

- **Guidance for a "Well-Balanced Life":** The optimized $\boldsymbol{\lambda}^*$ does not prescribe direct actions but can highlight areas of focus. If, for instance, λ_R^* (relationships) consistently emerges as very high across various scenarios, it reinforces the psychological importance of social connections. A "well-balanced" set of λ_i values (where no single λ_i is zero, and several have significant magnitudes) would suggest that attention to multiple life domains is crucial for optimal well-being, aligning with holistic views of a fulfilling life.
- **What to "Pay Attention To":** The optimization can reveal non-obvious interactions. For example, if investing in $V(t)$ (Voluntary Actions) strongly boosts $H_m(t)$ (Mental Health), which in turn has a high λ_{Hm}^*, then the indirect path through $V(t)$ might be more important than its direct λ_V^* might suggest. The SGD process implicitly captures these systemic effects.

7.5. Caveats and Limitations

It is crucial to interpret the results of such an optimization with caution:

- **Model Dependence:** The "optimal" $\boldsymbol{\lambda}^*$ is entirely dependent on the model's structure, assumptions, and the chosen parameter values (other than $\boldsymbol{\lambda}$). It reflects what is optimal *within the simulated world*, not necessarily in reality.
- **Simplification of Choice:** Real-life decisions are far more complex than adjusting abstract weights. Individuals don't directly "choose" their λ_i; they make choices that affect the *state* of their life domains. This optimization is a high-level abstraction.
- **Ethical Considerations:** The notion of "optimizing happiness" can be misconstrued. The goal here is analytical insight, not a reductionist recipe for living.
- **Computational Cost**: Estimating gradients through multiple simulations per SGD step can be extremely computationally expensive.
- **Local Optima**: SGD can get stuck in local optima, especially in complex, non-convex landscapes.
- **Definition of "Average Happiness"**: Maximizing average happiness might neglect other important aspects like minimizing volatility or avoiding extremely low happiness states.

Despite these limitations, performing this optimization can provide valuable insights into the model's behavior and generate hypotheses about the relative importance of different life factors under various simulated circumstances, prompting reflection on pathways to a more fulfilling life as conceptualized by the model. The resulting "optimized average happiness" serves as a benchmark achievable within the model's framework given a flexible weighting scheme.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendices, Simulations and Python Codes

See Simulations: View Simulation Results (PDF)
https://drive.google.com/file/d/1Ne1G8EWy5PYszZtfkgkf3QdT-mUQRtY2n/view?usp=sharing