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Modelling of Daily Long-Term Urban Road Traffic Flow Distribution: A Poisson Process Approach

Jojo D. Lartey

Department of Natural Science, Heritage Christian University Ghana, Accra, Ghana
Email: jdl015a@hcuc.edu.gh

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Abstract

Road traffic flow forecasting provides critical information for the operational management of road mobility challenges, and models are used to generate the forecast. This paper uses a random process to present a novel traffic modelling framework for aggregate traffic on urban roads. The main idea is that road traffic flow is random, even for the recurrent flow, such as rush hour traffic, which is predisposed to congestion. Therefore, the structure of the aggregate traffic flow model for urban roads should correlate well with the essential variables of the observed random dynamics of the traffic flow phenomena. The novelty of this paper is the developed framework, based on the Poisson process, the kinematics of urban road traffic flow, and the intermediate modelling approach, which were combined to formulate the model. Empirical data from an urban road in Ghana was used to explore the model's fidelity. The results show that the distribution from the model correlates well with that of the empirical traffic, providing a strong validation of the new framework and instilling confidence in its potential for significantly improved forecasts and, hence, a more hopeful outlook for real-world traffic management.

Keywords

Poisson Process, Macroscopic Traffic Flow, Urban Road, Long-Term Forecast, Multiple Entries-Exits Dynamics

1. Introduction

Road traffic flow forecasts are crucial information for any travelling community, and both individual members and organisations depend on such details for travel decisions and operational management of road mobility [1]. Individuals use the information to plan their travels for socioeconomic endeavours such as work, shopping, and entertainment. Organisations also use the information for proactive interventions for potential road travel challenges such as road capacity management, including congestion and signal controls [2] [3]. Increasing demand for transportation of passengers, goods, and services between communities correlates with increased vehicular population. The correlation demands critical support from traffic forecasts to mitigate efficient passenger and freight movement and increase economic development for the communities. For instance, in Ghana, road traffic alone handles 94 % of freight annually and another 97% of passenger traffic, contributing nearly 9% Gross Domestic Product (GDP) [4] [5]. Socio-economic factors in Accra are reportedly expected to grow by more than 58% over the next ten years while the human population is also expected to see a 26.5% increase per annum over the next five years, and this will increase the current daily 129,678 passengers and freight transport in Accra. [6] [7]. With the current traffic deteriorating congestion situation in Accra, [2] [5] [8] [9] much research is needed into improved traffic forecast for efficient travel decisions and proactive operational management of traffic flow in the context of urban road mobility in Accra.

Long-term traffic forecast, as opposed to short-term estimates for up to 1 hour ahead of current time, provides the traffic pattern for days ahead of current observed traffic state information and is therefore preferred for proactive operational traffic management as well as advanced travel intention [3] [10]. The daily long-term traffic flow forecast is crucial for Accra traffic forecast to understand the pattern of the morning rush hours for weeks ahead into the future. Current traffic flow theory employs two main modelling approaches, classical statistics and machine learning, to describe the traffic phenomenon and subsequently predict the future patterns as a forecast [3] [11]-[15]. Meanwhile, actual traffic flow is reportedly complex and random, even for the recurrent rush hour conditions [16]-[18]. For aggregate traffic flow consideration, partial differential equations lumped up with stochastic forcing functions are used to formulate the structure of the model [13] [19]-[22]. Suppose the random characteristics in the real traffic are to be considered. In that case, the model should have a stochastic basis in the partial differential equation and not only the forcing function. An alternative statistical approach worth considering identifying the actual random dynamics in the traffic flow is an empirical experiment to determine the pattern in the traffic flow. The primary contribution of this paper addresses the gap in the stochastic model structure by applying a novel framework grounded on stochastic macroscopic traffic flow. The framework encapsulates the essential variables as a Poisson process in an intermediate modelling approach for traffic flow on urban road traffic. The simulation result from the model is given in comparison to empirical data for actual traffic flow in the morning rush hours to validate the model fidelity. The rest of the paper is structured as follows: the next section highlights the gap in the literature and then provides a theoretical basis for the new framework used to develop the model for urban road traffic. Section 3 discusses the data and method employed in the study. In Section 4, the results of the model simulation and actual traffic flow data are presented. Section 5 then provides the discussion of the results and in contrast, Section 6 concludes the paper by giving insights gained from the results and suggestions for future work to be carried out.

2. Gaps & Adopted Framework

Since the pioneering works of [23]-[26] that provided road traffic research with modelling techniques that facilitate prediction of the level of traffic on the road, deterministic models have dominated the field, the pivotal [23] studies established the concept of fundamental relation which identifies the interplay between speed, density, and flow rate of vehicles to constitute the variables of the model [11] [15] [17]. Meanwhile, [27] provided the field with pivotal statistical traffic flow studies. Other studies in statistical settings include [28]-[36]. Current understanding admits road traffic flow is random, at least according to the studies of [13] [19]-[22], among others. These studies, by their respective efforts, looked at elements of random description of the macroscopic traffic dynamics. However, most of the descriptions do not depart from deterministic considerations, and it appears that the process of developing complete random dynamics in macroscopic road traffic flow is not well known, at least according to the available literature. Meanwhile, the real case scenario suggests elements of randomness in the empirical traffic flow data [12] [16] [37]-[40]. This study, with a focus on relevance, used the N6 urban road at Accra, indicated on the map in **Figure 1**, for a random experiment to collect traffic flow data, also illustrated in **Figure 2**. The sources of the illustrated data are among the urban roads in the Greater Accra region severely predisposed to daily traffic congestion, especially during rush hours [2] [9]. Consequently, a relaxed model incorporating random dynamics will be suitable and ideal for meeting the traffic flow descriptions on urban roads. Such a model is required to warrant the long-time forecast of congestion on the urban roads in Accra, like the N6.

The identified limitations in the literature regarding macroscopic traffic flow models formulated entirely from a stochastic basis demonstrate the need for stochastic models that account for the observed random dynamics in authentic traffic to ensure accurate model prediction. The implication for macroscopic traffic description is a stochastic formulation that departs from deterministic bases and yet is structurally comparable to the widely referenced deterministic model known as the LWR [24] [25] and [26] and its higher order improvements. The current study addresses these gaps by employing a framework based on the Poisson process coupled with an intermediate road traffic modelling approach, which allows the description of heterogeneous aggregate vehicular traffic dynamics on urban road configuration. Furthermore, this study adopts time-varying random variables for the sources and the sinks to traffic flow on a segment of the urban road uniquely characterised by junctions as access ways to traffic exits and entry to the roadway. Given its constituent and context, the framework is consistent with the empirical experiment on N6. Hence, using the framework provides an improved formulation and accurate understanding of the random macroscopic traffic dynamics and distribution, with direct implications for urban traffic management and planning.

3. Data and Method

The stochastic process is the widely used approach for describing the behaviour of random systems and has been employed in this study. In particular, the Birth and Death Poisson process [41]-[43], and the intermediate traffic flow modelling approach [15] [17] [18] were used to develop the modelling framework christened Multiple Entries and Exits Dynamics (MEED), which was then simulated in Python programming language. Statistical experiment was conducted to collect empirical data on the traffic at test site to validate the model.

3.1. Data

This section provides the scope and context of the utilised data to inform the study's level of generalisability. The data represents the traffic flow data collected from the N6 road at the dual lane segment between the Amasaman and Pokuase catchments, and in the north-south direction. **Figure 1** provides the map of the test site for data collection. This segment of N6 is situated in the Ga-West Districts of the Greater Accra region of Ghana. The chosen segment was used for the standard random experiment to collect traffic data consistent with recurrent traffic and among locations in the district reportedly predisposed to severe uncertainty in the traffic flow states, including the morning rush-hour traffic. The experiment was therefore conducted between the hours of 6 am to 11 am daily for four continuous weeks during the period July-August over four weeks. This period coincided with the warm, dry season typical of the country's tropical climate. The investigation involved counting the daily number n of heterogeneous traffic flow across the test site every 60 seconds t across encompassing six junctions to the road catchments. The heterogeneity of the traffic includes different types of road vehicles by size, wheels, and axle load, such as bikes, cars, lorries, and trucks.

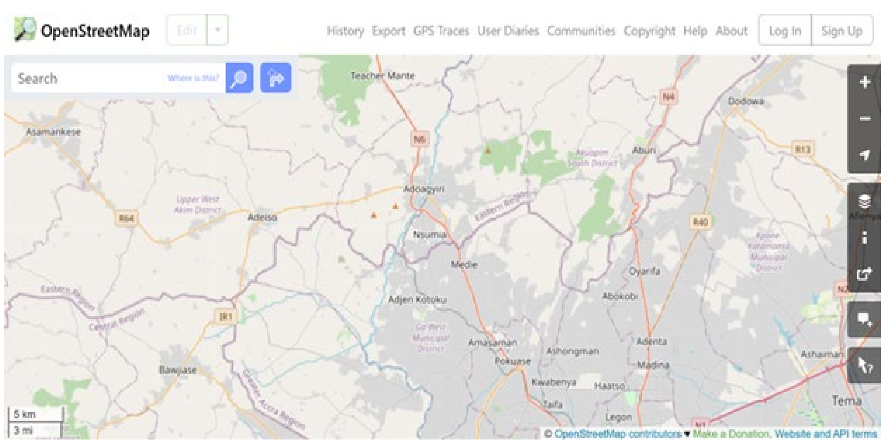


Figure 1. Map of test site—N6 urban road catchments of Amasaman and Pokuase.

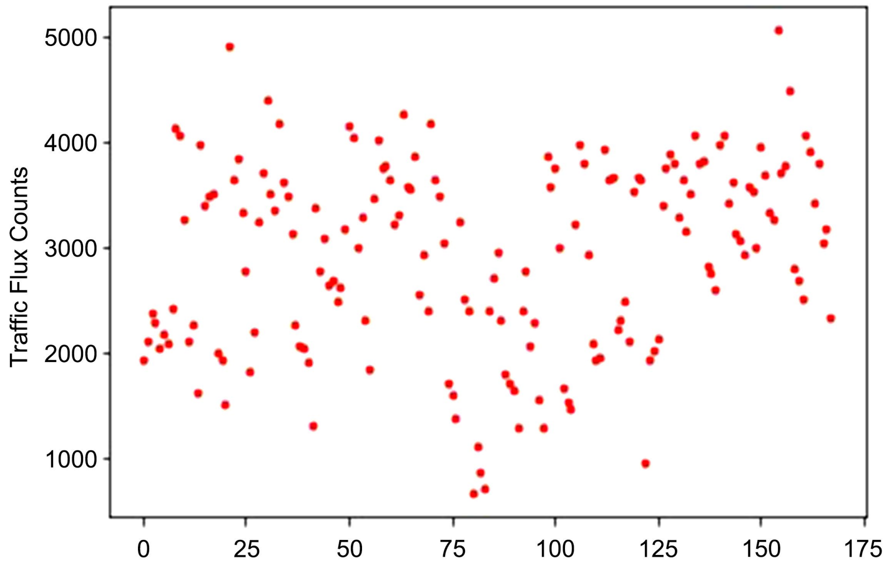


Figure 2. A plot of daily morning rush-hour traffic flux readings on M6.

Given that vehicles enter and exit on the segment, like any typical urban road in Ghana and many countries alike around the globe, the observation was that the junctions serve as sources and sinks to traffic build-up on the segment, per the respective studies of [44]-[46]. The simultaneous recordings from the six sources and sinks, as well as the main entry and exit to the road stretch were tabulated at $t = 60$ seconds. The flux $Q(t)$ of the homogenous traffic was then calculated as the net sum of the sources and sinks. The main roadway was not signalised, so the only factor to the traffic builds up and composition is the contributions from the sources and sinks. The assumption here is that every vehicle that enters the segment exits it at some access point and that the observable traffic build-up is the net aggregate of the sources and sinks onto the main road segment and is consistent with the macroscopic conserved traffic flow [47] [48].

3.2. Method

The stochastic process, a widely used approach for describing the behaviour of random systems, was utilised in this study. In particular, the Birth and Death Poisson process [41]-[43], and the intermediate traffic flow modelling approach [15] [17] [18] were used to develop the MEED model, which was then simulated. A statistical experiment was conducted to collect empirical data on traffic flow dynamics at the test site and validate the model.

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \begin{cases} 0; & \text{for homogeneous flow} \\ g(\rho(x,t),x,t) & \text{for non-homogeneous flow} \end{cases} \tag{1}$$

For comparative purposes, the classical deterministic MEED model structure is reproduced in Equation (1) as a first-order, quasi-linear partial differential equation (PDE). The LWR describes a conserved macroscopic traffic flow in a continuum, hydrodynamic, and speed-density relationships [24] [26] [45] [49] [50]. The essential variables of the LWR model are the distance x and time t for the road segment A to B of length L, such that $x \in [0, L]$; and $t \in [0, T]$ for time elapsed during which the macroscopic traffic flow was observed. The flow rate $q(x,t)$ is the observed net number of entry and exit vehicles that traversed x in time t. Also, flow density $\rho(x,t)$ describes the number of vehicles within the segment AB at time t and that $\rho(x,0) = 0$ for the homogeneous and non-homogeneous cases. However, the available literature reports that both the homogeneous and the non-homogeneous forms in Equation (1) show the defects of diffusivity and inertial. The defects are at variance with actual vehicular traffic flow, as observed on any road [19] [21] [51]. Meanwhile, extensive studies have been done to improve the LWR, and its higher orders variants thereof, and this paper also contributes to the positive direction, however from the perspective of stochastic process.

3.2.1. Multiple Entry-Exits Dynamics

The novelty of this paper is the proposed MEED framework. Consider a heterogeneous stream of vehicular traffic flowing on a segment AB of urban road with multiple access points, such as junctions, for vehicles to enter and exit the road segment. Also, suppose AB is on a one-dimensional straight line with the access points numbered $0, 1, 2, \cdots, m$ on the line. Such configuration of the road segment A to B length L, such that 0 and m correspond to the extreme A and B; and numbers $1, 2, \cdots, m-1$ correspond to the other access points. Given that each access point serves as a source and sink to traffic build-up on AB, and A is a source-only and B is sink-only to the total traffic on AB, then within the time interval t, the k access point contributed to the traffic build-up. Now suppose that $E_k(t)$ and $Y_k(t)$ are the counted vehicles that:

$$X_k(t) = E_k(t) - Y_k(t) \tag{2}$$

respectively entered and exited the k access point, then the net traffic $X_k(t)$ contributed by the k access point to the traffic build up on AB is given by Equation (2). For all values of k then $X_k(t) > 0$ for $E_k(t) > Y_k(t)$ and $X_k(t) < 0$ for $E_k(t) < Y_k(t)$. Hence, $X_k(t)$ generates a random variable for each of the finite m access points that may contribute simultaneously but with different counts of entries and exits at the different time intervals t. Table 1 illustrates the MEED framework for the road configuration at the various access points in the experiment at time interval $t \in [0, \infty]$, such that $X_k(t)$ provides a discrete random variable, for distinct individual values on $\mathbb{Z}$.

Table 1. Variables of traffic flow statistical experiment.

Time (T)	Roadway access points (K)								Flux
	0	1	2	3	4	5	…	m	
t	$X_0(t)$	$X_1(t)$	$X_2(t)$	$X_3(t)$	$X_4(t)$	$X_5(t)$	…	$X_m(t)$	$Q(t)$

3.2.2. Poisson Process Specification

Definition 1 (Poisson Distribution) The discrete random variable X with non-negative integer support $R_X \in \mathbb{Z}^+$ and a mean $\lambda \in (0,\infty)$, has a Poisson distribution $X \sim Poisson(\lambda)$ if X has a probability mass function that satisfies the following [42]:

$$p_X(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{if } x \in R_X \\ 0 & \text{if } x \notin R_X \end{cases} \tag{3}$$

Well known for modelling problems involving random arrivals of the number of customers at a supermarket and telephone calls at a switch [42], the Poisson distribution is also satisfied by the definitions of E_k and Y_k for the respective arrivals and departures of vehicles to and from *AB*. The constants λ and ν such that $\lambda \neq \nu$ correspond to the arrival and departure rates. Given their basis in Poisson distribution, then $X_k(t)$ also possess a distribution, and the accumulation of all the $X_k(t)$ for $k = \{0,1,2,\cdots,m\}$ observed across the m finite access points within the segment AB generated the traffic flux $Q(t)$ at the end of time $t \in [0,\infty]$, according to Equation (4).

$$Q(t) = \sum_{k=0}^{m} X_k(t) \tag{4}$$

Hence, the possible observable values of counts at time t is $Q(t) = n_t$, where $n_0 \geq 0$ is the initial counted flux at time $t = 0$. The sum across the row in **Table 1** gives $Q(t)$ that satisfies the statistical experiment of counting a population of n vehicles by the end of time $t \in [0,\infty]$ within *AB*. For the flux $Q(t) = n_t$ at time t, let $p_i(t) = Pr(Q(t)) = n_i)$, be the corresponding probability, then the sequence of $\{Q(t_i)\}$ for $i \in 0,1,2,\cdots,\infty$ has the Markov property given by Equation (5).

$$\begin{aligned} &Pr(X(t_n) = j \mid X(t_1) = j_1, X(t_2) = j_2, \cdots, X(t_n) = j_n) \\ &= Pr(X(t_n) = j \mid X(t_{n-1})) \end{aligned} \tag{5}$$

and that the future values of $Q(t_i)$ depends on only the current counted values and no other [41] [43].

The transition in the flux, $Q(t_i)$, is based on the observation that individual vehicles entered and exited the road segment at the rates of λ and ν. Respectively. Given that the entries and exits satisfy the Poisson distribution, then neither λ nor ν show systematic trends in time and are therefore considered constants [42] [43].

Definition 2 (Poisson Process) A counting process $N(t): t \geq 0$ with stationary and independent increments is said to be a Poisson process if $N(t)$ counts the total number of events within any interval of time t and has a Poisson distribution with mean λt such that $\lambda \geq 0$, and $\forall s,t > 0$. Therefore $\forall s,t > 0$ then $N(t)$ satisfies the following equation according to [43]:

$$P[X(s+t) - X(s) = n] = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \quad n = 0,1,2,3,\cdots$$

Now since the contributions $X_k(t)$ from the access point k generated the flux $Q(t)$, then $Q(t)$ satisfies proposition 3.

Proposition 3 (Poisson Process) The stochastic process $Q(t): t \geq 0$ is a Poisson process given that $Q(t)$ counted the total net volume of events of vehicles that accumulated within the road segment in a fixed time when the events occurrences were observed to have a Poisson distribution with a constant rate.

Now since $Q(t)$ describes a process for the point of events on the real line due to t and that $Q(t, t+\delta t)$ provides the number of events in the half-opened time interval $(t, t+\delta t]$.

$$Pr\{Q(t, t+\delta t) = 0\} = 1 - \lambda\delta t + o(\delta t) \tag{6}$$

$$Pr\{Q(t, t+\delta t) = 1\} = \lambda\delta t + o(\delta t) \tag{7}$$

$$Pr\{Q(t, t+\delta t) > 1\} > o(\delta t) \tag{8}$$

Then given the positive constants λ and ν such that $\delta t \to 0$, then equation (6) to (8) establishes the random variable $Q(t)$ as a Poisson process, while $\lambda \neq \nu$, the following Poisson process conditions governed the transition of vehicle population [42] [43] for both the entries and exits in an infinitesimal time $\delta t > 0$:

1) The flux can increase or decrease at any time t but not both simultaneously, since each access point serves the dual purpose of entry and exits.

2) When the population of vehicles in the flux has n number of vehicles at time t, then:

a) $Pr\{\text{a unit vehicle increase occurs in the time interval } (t, t+\delta t)\}$
$= \lambda_n\delta t + o(\delta t)$

b) $Pr\{\text{a unit vehicle decreases occurs in the time interval } (t, t+\delta t)\}$
$= \nu_n\delta t + o(\delta t)$

c) $Pr\{\text{more than a unit change occurs in the time interval } (t, t+\delta t)\} = o(\delta t)$

3) Given conditions (2a) to (2c) above, then when the flux has size of n vehicles:

4) $Pr\{\text{no change in the flux in the time interval } (t, t+\delta t)\}$
$= 1 - (\lambda_n + \nu_n)\delta t + o(\delta t)$

By conditions (1) to (4), the occurrence of the points of events of vehicle entries and exits, captured by the flux, and the corresponding probabilities are provided in Equation (9).

$$Pr\{X(t+\delta t) = j \mid X(t) = n\} = \begin{cases} \lambda_n\delta t + o(\delta t) & j = n+1 \\ \nu_n\delta t + o(\delta t) & j = n-1 \\ 1 - (\lambda_n + \nu_n)\delta t + o(\delta t) & j = n \\ o(\delta t) & |j-1| > n \end{cases} \tag{9}$$

Consequently, for the counting process $N(t)$, the number of individual vehicles in the flux $Q(t)$ at time t, have the probability probability $Pr\{N(t) = n\}$. As such the possible values of j in Equation (9) leads to Equation (10) as the forward equation associated with the counting process.

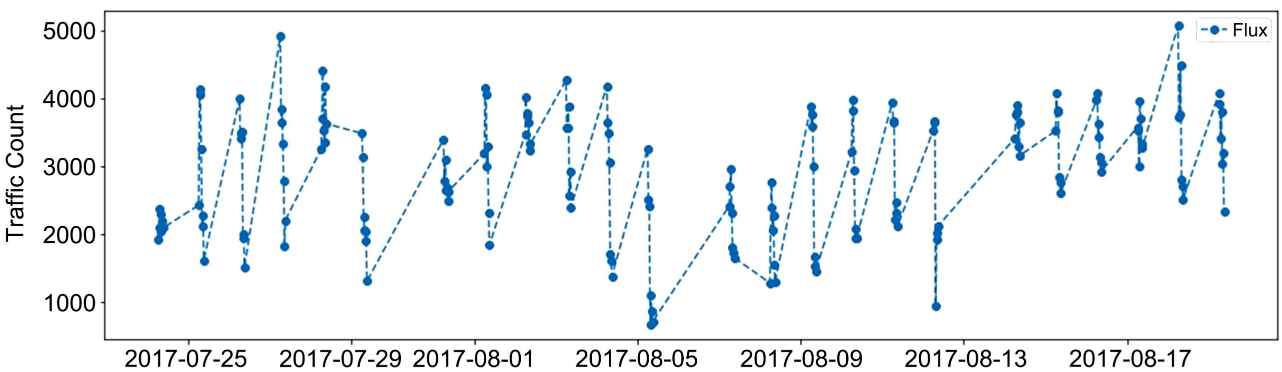
$$p_n'(t) = -(\lambda + n\nu) p_i(t) + (n+1)\nu p_{n+1}(t) + \lambda p_{n-1}(t) \quad (n = 0,1,2,\cdots) \tag{10}$$

Observe that for $n = 0$ the function p_{n-1} is identically equal to zero [41] [43], and rewriting Equation (10) resulted in Equation (11), where $G(z,t)$ is the probability generated function that describes the population $Q(t)$ at time t.

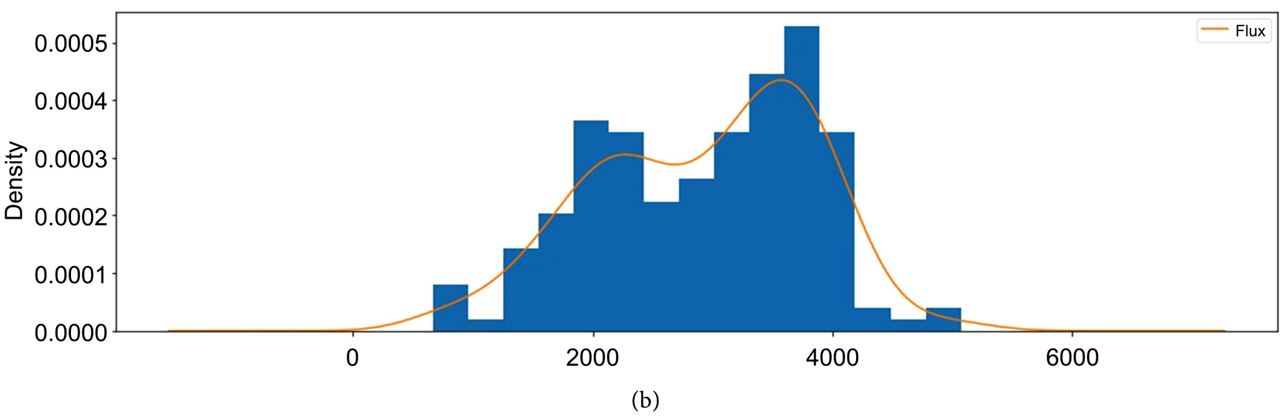
$$\frac{\partial G(z,t)}{\partial t} + \nu(z-1)\frac{\partial G(z,t)}{\partial z} = \lambda(z-1)G(z,t) \tag{11}$$

4. Results

This section presents the findings of the study in the form of figures and graphs obtained from both the N6 empirical traffic data and the simulation of the model. **Figure 3** gives two sets of graphs for the empirical traffic data and that of the simulated model. **Figure 3(a)** and **Figure 3(b)** are the empirical data graphs. While **Figure 3(a)** illustrates the line plot for the daily traffic flux during the morning hours, the illustration in **Figure 3(b)** displays the corresponding histogram with the kernel density. Similarly, the graphs from the model simulation are presented in **Figure 3(c)** and **Figure 3(d)** for which **Figure 3(c)** displays the simulated line graph. The corresponding histogram with kernel density is displayed om in **Figure 3(d)** for three different time step simulation runs that used initial flux size $n_0 = 1975$, obtainable from the empirical data. Also, the same parameter values of $\lambda = 370$ and $\nu = 90$ were used in each of the three different time steps of 10, 100 and 1000.



(a)



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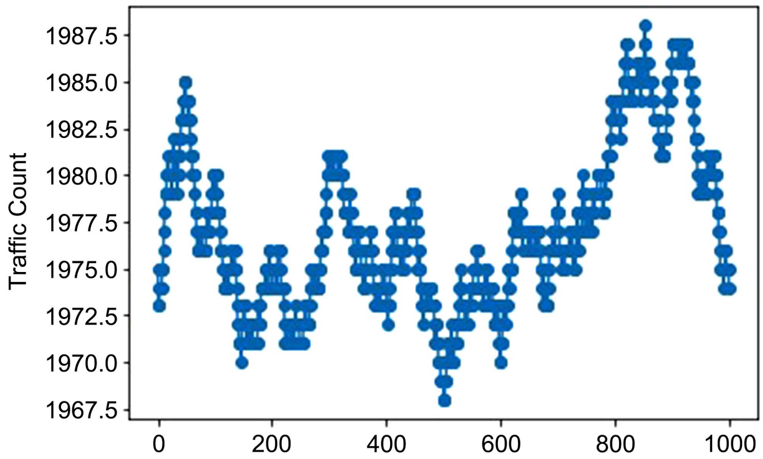
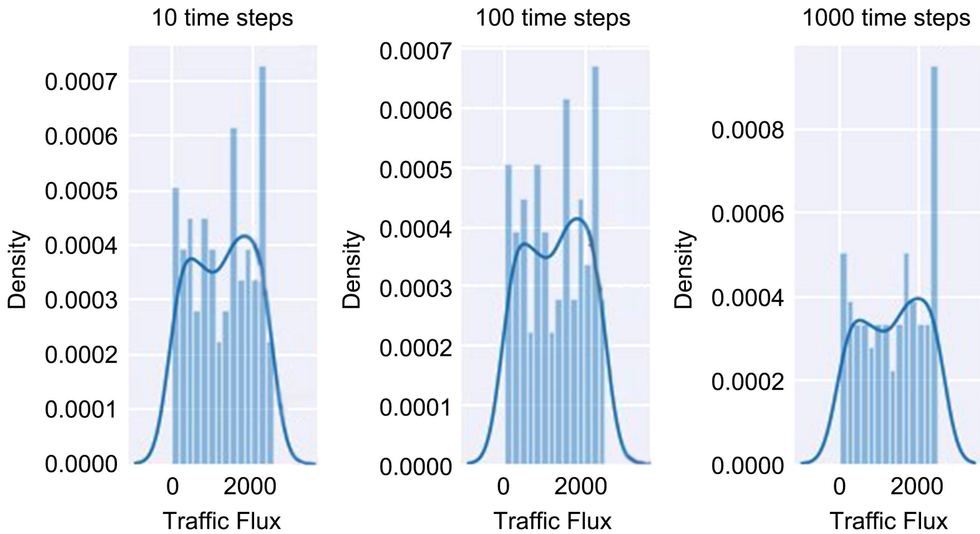


(c)



(d)

Figure 3. Daily morning hours traffic flow distribution on N6 road.

5. Discussion

The study explored MEED framework to model the daily long-term urban road traffic distribution. The new framework is rooted in random variables observable in the real traffic phenomenon [37] [43]. The obtained sets of results, respectively, for the empirical and the simulated MEED, appear relatively identical qualitatively. The graphs also show volatilities in the traffic flow pattern, evidenced by the erratic and extremities in **Figure 3(a)**. Again, the pattern shown in **Figure 3(b)** suggests the level of randomness in the experimental data. The corresponding pairs of graphs generated by the MEED model, as shown in **Figure 3(c)** and **Figure 3(d)**, appear identical to that of the empirical. The shape of the kernel density distribution and the histograms are characteristic of the theoretical probability distribution for a random variable and in agreement with the multi-modal distributions in the individual studies by [28]-[30] [32] [52] on road traffic flow distribution. The results of this study show the bi-modal distribution, which is a multi-modal distribution consistent with previous studies in the existing literature. Hence, both the empirical and theoretical distributions confirmed the presence of the characteristic multi-modal distribution for the traffic flow.

The results of the study reveal a novel traffic flow model for urban road infrastructure that significantly enhanced the predictive performance of the daily long-term traffic flow forecaster. The innovative approach, which align with previous research, underscoring the effectiveness of stochastic process approaches for describing macroscopic traffic flow [13] [19] [21] [22] [53] [54], has the potential to revolutionise urban road traffic flow modelling research outcomes. The positive impact observed across the different simulation runs suggests that the modelling framework design effectively addresses forecasting needs for the stochastic contexts of the target urban road traffic flow dynamics.

The LWR, which structure is given in Equation (12), has vehicle density ρ and vehicle flow rate q. These variables are all functions of distance, x and time t—the basic variables utilised in MEED framework. The LWR,

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = g\left(\rho(x,t), x, t\right) \quad \text{and} \quad q = \rho v \tag{12}$$

a pivotal kinematic macroscopic traffic flow description in the field, is identified with the defects of inertial and diffusivity. Both defects are inconsistent with any real observed road traffic flow [15] [40] [55]. First, regarding the inertial defect, the model assume local equilibrium thereby neglecting the place of inertia to the vehicle interactions. The implication is that moving vehicles adjust their speed instantaneously to changing traffic movement and density. However, the real case scenario is that a vehicle may be delayed in responding to such changes. For instance, delay reactions to braking or acceleration are observable in real traffic dynamics [15] [50] [55] [56]. Second, diffusivity effect is about the spread of the traffic density over time, but the MEED fails to capture such diffusivity in macroscopic. For instance, complex traffic interactions and variations in factors responsible for traffic density observable in real traffic flow [55].

Incremental improvement to the LWR, such as the [57] model, has the structural form of the:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{C_0^2}{\rho}\frac{\partial \rho(x,t)}{\partial x} = \frac{v_e(\rho) - v}{\tau} \tag{13}$$

Equation (13). Known as the PW, the model introduced C_0^2 as parameter for driver spatial adjustment, and τ for the reaction time, into a second order partial differential equations, to account for the defects. Other studies like [51] [58] [59], and most recently [60] have considered various improvements to the PW model. The incremental improvements notwithstanding, shortcomings in the PW and its variants have been criticised by [61]-[63] for inconsistencies in conserved macroscopic flow as observed in real road traffic phenomenon.

Given that the MEED model contributes to stochastic description, it serves as an incremental improvement on existing forms of stochastic macroscopic continuum models. This observation is that existing models are partly deterministic in the partial differential equations describing the traffic flow phenomenon. (See [21] [22] studies for instance).

1) Gunnarson's model [21]

$$\frac{\partial \rho(x,t)}{\partial t}+\frac{\partial Q(\rho(x,t))}{\partial x}=\frac{\xi}{2}\Delta\rho+f(\rho,x)+\sigma(\rho,x)\mathbf{W}(dx,dt) \tag{14}$$

2) Ngoduy's model [22]

$$\begin{aligned}&\frac{\partial V(x,t)}{\partial t}+V(x,t)\frac{\partial V(x,t)}{\partial x}\\&=-\alpha(r,V)\frac{\partial r}{\partial x}-\beta(r,V(x,t))\frac{\partial V(x,t)}{\partial x}+\frac{U(r,V,r_a,V_a)-V(x,t)}{\tau}\end{aligned} \tag{15}$$

3) MEED model

$$\frac{\partial G(z,t)}{\partial t}+v(z-1)\frac{\partial G(z,t)}{\partial z}=\lambda(z-1)G(z,t) \tag{16}$$

The models have structures in Equations (14) and (15) respectively in comparison to that of the MEED also in Equation (16). All three models have the fundamental variables of space x and time t as well as their derivatives such as velocity, acceleration and density. Again, all three models have the form of quasi-linear partial differential equations. However, both Equation (14) and Equation (15) did not depart from deterministic components and accounted for the stochastic character in the forcing functions only – the right-hand sides of the respective equations. On the contrary, the MEED in Equation (16) has all its functions and components grounded entirely on a stochastic basis. The existing models therefore appear limited in their stochastic character, which may affect their effectiveness. The MEED therefore addresses the identified limitations and hence serves as an incremental improvement and contribution that advances stochastic macroscopic road traffic flow modelling with an approach which is at variance with existing ones.

A notable strength of this study is the empirical investigation of the urban traffic flow for the long-term forecast. By incorporating both empirical and modelling measures, the study was able to capture a holistic view of the impact of stochastic traffic flow dynamics, considering all aspects of traffic flow. The significantly closed comparison in the graphs of the two sets of results underscores the model's potential to generate a more informative forecast. However, this study also encountered limitations, including the relatively small sample size, the lack of diverse weather conditions as external factors, and the short duration, all of which may limit the generalizability of the findings. The limitations might have influenced the results. As such, there is a need for more robust evidence in this field. Future research should address the limitations by conducting studies with more extensive and diverse samples, incorporating other road types as control groups, and extending the duration of the study to provide robust evidence for the needed fidelity of the model.

Despite the identified limitations, the findings carry significant implications for long-term road traffic flow forecast practices and policy for urban road infrastructure. The new model's success highlights the need for stochastic frameworks that are at variance with deterministic components and instils hope for the future of road traffic flow research. Traffic managers and the travelling public should consider incorporating similar forecasting strategies into their decision-making tools to enhance the accuracy of the forecast information, fostering a sense of optimism for the potential positive impact on road travel.

6. Conclusion

In conclusion, this study contributes to the growing evidence supporting stochastic traffic flow modelling approaches in road traffic research. The model shows promise in improving the daily long-term road traffic forecast outcomes. However, further research is needed to refine and expand the implementation of the framework. By continuing to explore and innovate in this area, traffic research can work towards more accurate and reliable forecast information for reduced-risk travel experiences for the travelling public. This research has the potential to significantly impact the field, underscoring the importance of ongoing research in the field.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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