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Quantum Cosmology: Cosmology Directly Linked to the Planck Scale in General Relativity Theory and Newton Gravity: The Link between Microcosmos and Cosmos

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Abstract

In this paper, we will demonstrate that there is a link between cosmology and the Planck scale. It has, in recent years, been shown that the Planck length can be determined independently of G, $\hbar$, and c, and that a series of cosmological predictions can be derived solely from two constants, namely the Planck length and the speed of gravity. The speed of gravity can be easily determined without knowledge of the speed of light [1] [2]. This provides a new perspective on cosmology and demonstrates that there is a link between the Planck scale and cosmology. This is fully consistent with a recent quantization of general relativity theory that links general relativity to the Compton frequency and the Planck scale. We examine both the Friedmann cosmology and the recently introduced cosmology based on the extremal solution of the Reissner-Nordström, Kerr, and Kerr-Newman metric.[1]

Keywords

Hubble Constant, Hubble Radius, Universe Equation, Freedman Universe, Extremal Universe, Planck Length, Compton Length

1. Background

Quantum cosmology has received increased attention among researchers; see, for example, [4]-[12]. However, it remains an unsolved problem whether there is a link between cosmology and the Planck scale, just as it is considered an unsolved

[1]This paper is a significantly improved version based on an early pre-print on Hal archive on 10 November 2021 [3].

problem to develop a quantum gravitational theory. Here, we will demonstrate that such a relation exists and that common cosmological units can be predicted and described through quantum units, particularly in relation to the Planck scale. What we will show is fully consistent with general relativity theory, as general relativity has recently been quantized and linked to the Planck scale; see [13]-[16].

Max Planck [17] [18] assumed there were three important universal constants, namely G, $\hbar$, and c, and, based on this and dimensional analysis, derived what is today known as the Planck length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, the Planck time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, the Planck mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and the Planck temperature $T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}}$. Already in 1984, Cahill [19] [20] suggested that the gravitational constant could be expressed in terms of the Planck mass in the form:

$$G = \frac{\hbar c}{m_p^2}, \tag{1}$$

which is simply the Planck mass formula solved with respect to G. However, as pointed out by Cohen [21] in 1987, this seems to lead to a circular problem: one needs to know G to determine the Planck units. Thus, expressing G in terms of Planck units is of little or no practical use if this is the case—a view still widely held. McCulloch [22], in 2016, also highlighted the same formula as Cahill and Cohen for G, emphasizing that G must be known to calculate the Planck units. In 2016, Haug [23] [24] suggested that the gravitational constant is merely a composite constant that can be expressed as:

$$G = \frac{l_p^2 c^3}{\hbar}. \tag{2}$$

This is simply solving the Planck length formula with respect to G, rather than the Planck mass formula, so it leads to the same circular problem as previously mentioned. However, in 2017, Haug demonstrated for the first time how the Planck length can be determined without any knowledge of G (see, in particular, the appendix in that paper). Later, Haug [2] [25] [26] showed how to derive the Planck length and Planck time without requiring knowledge of either G or $\hbar$, in a practically feasible way.

In addition to this composite view of G, we will utilize the fact that the rest mass in kilograms for any object, whether small or large, can be expressed by the formula:

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}, \tag{3}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. This formula is simply the result of solving the Compton [27] wavelength formula with respect to m. While this is mathematically straightforward, expressing the mass in terms of the Compton wavelength, rather than deriving the Compton wavelength from the mass, was perhaps first suggested in 2016; see [24] [28].

Elementary particle masses larger than the Planck mass will have a reduced Compton wavelength shorter than the Planck length, which seems impossible if, as many physicists assume, the Planck length is the shortest possible length. However, we propose that masses larger than the Planck mass must be composite masses, where any composite mass consists of many elementary particles. Similarly, masses smaller than the Planck mass can also be composite masses, such as the proton. The reduced Compton wavelength of the elementary particles comprising a composite mass can be aggregated using the formula:

$$\bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots + \frac{1}{\bar{\lambda}_n}}, \tag{4}$$

where $\bar{\lambda}$ now represents the aggregate of all the reduced Compton wavelengths of the elementary particles comprising the mass. This is fully consistent with, and can even be derived from, the standard mass aggregation rule for non-bound components. For now, we are excluding binding energy, and the rule is given by:

$$m = m_1 + m_2 + m_3 + \cdots + m_n$$

$$\frac{\hbar}{\bar{\lambda}}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c} + \cdots + \frac{\hbar}{\bar{\lambda}_n}\frac{1}{c}$$

$$\bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots \frac{1}{\bar{\lambda}_n}}. \tag{5}$$

However, we will also consider masses consisting of bound elements. In such cases, the simple mass addition described above will slightly overestimate the total mass due to the exclusion of nuclear binding energy; see, for example, Walker, Halliday, and Resnick [29]. Plain hydrogen has no nuclear binding energy, as it consists only of a proton and an electron. For known nucleons beyond plain hydrogen, the nuclear binding energy ranges from approximately 2.23 MeV for hydrogen-2 to 8.79 MeV for nickel-62, which corresponds to 0.24% to 0.94% of the total observed mass. Ignoring binding energy can therefore result in up to a 0.94% overestimation of the mass when simply counting protons and neutrons, and up to a 0.94% underestimation of the reduced Compton wavelength.

In addition, there are other binding energies, such as molecular bond energy; however, these are extremely small compared to nuclear binding energy.

We can also treat energy as mass equivalent, as is often done in physics, since $m = \frac{E}{c^2}$. Thus, any type of binding energy in a nucleus or a clump of matter can be treated as mass equivalent in this formula, with a corresponding Compton wavelength (and reduced Compton wavelength). This allows us to generalize Equation (5) to also account for binding energy and other types of energy, as we can write:

$$m = m_1 + m_2 + m_3 + \cdots + m_n + \frac{E_1}{c^2} + \frac{E_2}{c^2} + \cdots + \frac{E_n}{c^2}, \tag{6}$$

where E_1 to E_n represent binding energies (these are typically negative since binding energy is usually released from the mass when elements bind), as well as other types of potentially relevant energy converted into a mathematically equivalent rest mass. This must also hold true for pure electromagnetic energy, as we must have:

$$E = hf = h\frac{c}{\lambda_\gamma}$$

$$\frac{E}{c^2} = \frac{h\frac{c}{\lambda_\gamma}}{c^2}$$

$$m_\gamma = \frac{h}{\lambda_\gamma}\frac{1}{c}$$

$$m_\gamma = \frac{\frac{h}{2\pi}}{\frac{\lambda_\gamma}{2\pi}}\frac{1}{c}$$

$$m_\gamma = \frac{\hbar}{\bar{\lambda}_\gamma}\frac{1}{c}, \tag{7}$$

where λ_γ is the photon wavelength. The equivalent mass derived from pure energy is identical to a mass that has a reduced Compton wavelength equal to the reduced photon wavelength (see [30] for an in-depth discussion of the importance of the Compton wavelength). The reduced Compton wavelength corresponds to the photon wavelength divided by 2π, and this formulation incorporates the reduced Planck constant.

Alternatively, we could write $m_\gamma = \frac{h}{\lambda_\gamma}\frac{1}{c}$, which yields exactly the same mass value, as it simply involves multiplying by 2π in both the numerator and the denominator. Thus, we have:

$$m_\gamma = \frac{\hbar}{\bar{\lambda}_\gamma}\frac{1}{c} = \frac{h}{\lambda_\gamma}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}, \tag{8}$$

when $\bar{\lambda}_\gamma = \frac{\lambda_\gamma}{2\pi}$ and $\bar{\lambda}_\gamma = \bar{\lambda}$. While it is not practically possible to perform Compton scattering on photons (at least not with current knowledge), it is also not feasible to observe the mass of a photon in the standard way. Nevertheless, the mass (or mass equivalent) of a photon has been used in numerous calculations across various studies by assuming $E/c^2 = m$. Here, we are referring to the mass equivalence of energy and the Compton equivalent wavelength of a photon. As shown above, the Compton equivalent wavelength of a photon must be identical to the reduced wavelength of the photon if $m = E/c^2$ holds true.

Some may be critical and rightly point out that the photon wavelength and the Compton wavelength are entirely different concepts, arguing that this interpretation contradicts the basic understanding of these terms. The photon wavelength is generally associated with the frequency of light, while the Compton wavelength is related to the wavelength derived from Compton scattering of particles such as electrons. However, treating energy as mass in this way is fully consistent with $m = E/c^2$, as we can recover the correct energy and frequency by simply multiplying the equivalent photon mass by c^2.

The relationship $m = E/c^2$ is well-known, as is $E = h\frac{c}{\lambda_\gamma}$. But what happens to the Planck constant and the wavelength of light when we treat it as an equivalent theoretical mass? Typically, the equation is stopped at $\frac{h\frac{c}{\lambda_\gamma}}{c^2} = m$. We argue that the deeper interpretation, unsurprisingly, is $\frac{hf}{c^2} = \frac{h\frac{c}{\lambda_\gamma}}{c^2} = m = \frac{\hbar}{\bar{\lambda}_\gamma}\frac{1}{c}$. This suggests that the wavelength of light can indeed be treated as an equivalent Compton wavelength. In our view, this should not be seen as more controversial than treating massless photons as equivalent to mass, a concept frequently employed in physics.

However, it is important to note that this approach would not apply when using the de Broglie [31] [32] wavelength. For instance, the de Broglie wavelength $\lambda_B = \frac{h}{mv\gamma}$ does not have a mathematically defined value for a particle at rest, as discussed in [30]. Furthermore, as the velocity of the mass approaches zero, the de Broglie wavelength approaches infinity, which has led to a variety of interpretations regarding the de Broglie wavelength.

In this analysis, we will primarily focus on mass at the cosmological scale, where hydrogen and helium are estimated to comprise roughly 74% and 24% of all baryonic matter in the observable universe, respectively. Hydrogen consists only of one proton and one electron and has no nuclear binding energy, though it does have smaller binding energies in the form of gravitational and molecular binding energies. There is also hydrogen-2, known as deuterium, which consists of one proton, one neutron, and one electron. However, estimates indicate that there are only about 26 atoms of deuterium per million hydrogen atoms, so its contribution to corrections involving nuclear binding energy is negligible.

Helium-4, on the other hand, constitutes nearly 24% of the baryonic mass in the universe and has a relatively high nuclear binding energy of 28,295.7 keV, which is almost 0.8% of the total mass of helium-4. Nevertheless, since helium-4 accounts for only 24% of the baryonic mass in the universe, neglecting nuclear binding energy would result in an error of approximately 0.2% in the total baryonic mass. This error would also propagate into the Compton wavelength, which is linearly proportional to the mass. While an error of 0.2% may be significant in certain nuclear physics experiments, it is relatively insignificant when working at cosmological scales, such as estimating the mass of the universe or determining the Hubble constant, where observational errors are typically on the order of percentage points.

Thus, the main baryonic components of the universe have relatively low binding energies. In standard cosmology, the baryonic density is estimated to be between 1% and 15% of the critical density; see, for example, Craig, Schramm, and Turner [33]. Since up to 95% of the critical density may be attributed to dark energy, the binding energy's contribution to the total equivalent mass of the universe becomes less significant. Nevertheless, it should still be accounted for in our model, as we do not differentiate between contributions from energy and mass at the cosmological scale.

While we personally remain skeptical of the dark energy hypothesis, this topic lies outside the scope of this paper. The main point is that in the following analysis, it is unnecessary to distinguish between the proportions of energy and mass.

It is experimentally confirmed that mass affects energy, for example, by bending light (deflection). It is generally accepted that energy in the form of electromagnetic waves also has gravitational effects similar to its equivalent mass, though this has likely not yet been exclusively experimentally confirmed. However, interest in this topic has grown as experimental techniques advance, bringing us closer to being able to measure gravitational effects from electromagnetic waves; see [34]-[39]. If the hypothesis that the critical density of the universe primarily consists of energy is correct, then our quantum cosmological model indirectly provides a means to test whether energy has its own gravitational field.

We will use the Compton wavelength and the Planck length to derive new quantum relations for phenomena such as the Hubble scale and the mass of the universe. As will be evident from the formulas we derive, there is a linear relationship between, for instance, the Hubble constant and the reduced Compton wavelength of the mass in the universe. We propose that even nuclear binding energy and gravitational binding energy are likely embedded in our calculations after calibration to observed phenomena such as cosmological redshift. This is because the cosmological models we examine do not distinguish between energy and mass at the scale at which we are working. Energy is treated as equivalent to mass through the relation $E = mc^2$, meaning the mass defect is implicitly accounted for.

For example, when considering the mass of the observable universe in the Friedmann model, the model does not specify how much of this is energy versus mass. Instead, it treats energy and mass as equivalent, allowing us to choose whether to express the output in units of mass or energy. For other purposes, there is a significant body of literature exploring the proportions of total energy in the universe that are mass, energy, and dark energy. However, this is beyond the scope of this paper. The main focus here is to establish the connection between the Planck scale and the cosmological scale. The methodology presented here not only establishes this link but also appears to work effectively in practice.

By 'practice,' we mean that it appears possible to determine the Planck length and the Compton wavelength of the observable universe (without requiring knowledge of G or h). Using these values, along with the speed of light (or gravity), we can predict quantities such as the Hubble constant, the Hubble time, and the mass (or energy) of the observable universe.

If we know the mass of an object in kilograms and the Planck constant, we can calculate the reduced Compton wavelength using the Compton wavelength formula:

$$\bar{\lambda} = \frac{\hbar}{mc}. \tag{9}$$

This formula can also be used for any mass size. However, we aim to rely on as few constants as possible, and we may not necessarily know the kilogram mass of large objects. So, is there a way to determine the Compton wavelength of any object without knowing the Planck constant or the object's mass in kilograms? We can begin by considering Compton scattering of an electron—that is, shooting photons at an electron. In this case, the Compton wavelength is given by:

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$$

$$\lambda_2 - \lambda_1 = \frac{h}{\frac{h}{\lambda_e}\frac{1}{c}c}(1-\cos\theta)$$

$$\lambda_e = \frac{\lambda_2 - \lambda_1}{1-\cos\theta}, \tag{10}$$

where λ_1 and λ_2 are the wavelengths of the photon before and after its collision with the electron, and θ is the angle between the photon's paths before and after the collision. In other words, we can determine the Compton wavelength of the electron without needing to know its mass or the Planck constant.

To find the Compton wavelength of the proton, we can utilize the fact that the ratio of the Compton wavelengths of the proton and the electron is identical to the cyclotron frequency ratio:

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{\bar{\lambda}_e}{\bar{\lambda}_P} \approx 1836.15 \tag{11}$$

So, one can simply divide the electron Compton wavelength by 1836.15 to obtain the proton Compton wavelength. Interest in the proton Compton wavelength dates back to at least 1958, in a paper by Levitt [40], and in recent years, there has been renewed attention to this topic [41]. After determining the Compton wavelength of a proton, it is possible to calculate the Compton wavelength of larger macroscopic masses by counting the number of atoms in the mass. This is feasible, at least for uniform macroscopic masses of manageable size, such as hand-sized objects. Counting atoms in such masses is challenging but entirely possible; see, for example, [42]-[46]. For the first macroscopic mass analyzed, adjustments must be made for nuclear binding energy to achieve high precision.

One can measure the gravitational effect of a mass for which the number of atoms has been counted, for instance, using a Cavendish apparatus. From this, the Compton wavelength of larger masses, such as the Earth, can be determined, as the ratio of Compton wavelengths is identical to:

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1}, \tag{12}$$

g_1 and g_2 represent the gravitational acceleration for mass one and mass two, respectively. We [47] [48] have recently demonstrated that the Compton wavelength of the universe's mass can be extracted in a similar manner, using cosmological redshift, the CMB temperature, or the Hubble constant.

Solving any of the Planck unit formulas with respect to G, or the Compton wavelength formula with respect to m, is straightforward. However, combining several straightforward steps can sometimes lead to breakthroughs. When G is multiplied by M and expressed in this form, it becomes apparent that the Planck constant always cancels out:

$$GM = \frac{l_p^2 c^3}{\hbar} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}}. \tag{13}$$

This means that to know GM, we do not need to know $\hbar$, but only c and l_p. However, to determine G and m in their individual composite forms, we need to know $\hbar$, c, and l_p. As demonstrated in the papers mentioned earlier, the Planck length and the Compton wavelength[2] can be determined without any knowledge of G or $\hbar$. **Table 1** presents a series of gravitational phenomena, showing that all observable gravitational phenomena depend on GM, not GMm.

For two-body problems where the gravitational effects of the second mass m are also significant, the gravity parameter becomes $\mu = GM + Gm$, not GMm. This implies that the Planck constant is never needed for gravitational predictions, even at the quantum level. Moreover, the gravitational constant G is also unnecessary. As shown in the results on the far-right side of **Table 1**, all that is required are the constants l_p and c, along with a variable representing the size of the gravitational object, which is the Compton wavelength of the object. Additionally, we naturally need to know the distance to the gravitational object where predictions are made or where the model is being tested against observations.

Thus, we argue that only two constants are needed for gravitational predictions: the Planck length and the speed of light, in addition to relevant variables. As can be seen from **Table 1**, in gravitational phenomena typically associated with general relativity, the only constant required is the Planck length.

Note that in all formulas, we essentially encounter the term $\frac{l_p}{\bar{\lambda}_M}$, which is the Planck length divided by the reduced Compton wavelength. This represents the reduced Compton frequency per Planck time for the mass in question and is what provides quantization in gravity. As we will see, this concept also extends to

[2]Or the reduced Compton wavelength.

Table 1. The table shows that any observable gravity phenomena contain GM and not GMm and further than when assuming G is a composite, then we end up that we can predict all observable gravity phenomena only from l_p and c.

Mass	$M=\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}(\text{kg})$
Non observable (contains GMm)	
Gravitational constant	$G,\left(G=\frac{l_p^2c^3}{\hbar}\right)$
Gravity force	$F=G\frac{Mm}{R^2}\left(\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\right)$
Observable predictions: (contains only GM)	
Gravity acceleration	$g=\frac{GM}{R^2}=\frac{c^2}{R^2}l_p\frac{l_p}{\bar{\lambda}_M}$
Orbital velocity	$v_o=\sqrt{\frac{GM}{R}}=c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Orbital time	$T=\frac{2\pi R}{\sqrt{\frac{GM}{R}}}=\frac{2\pi\sqrt{R^3}}{c\sqrt{l_p\frac{l_p}{\bar{\lambda}_M}}}$
Periodicity pendulum[a] (clock)	$T=2\pi\sqrt{\frac{L}{g}}=2\pi R\sqrt{\frac{L}{GM}}=\frac{2\pi R}{c}\frac{\sqrt{L}}{\sqrt{l_p\frac{l_p}{\bar{\lambda}_M}}}$
Frequency Newton spring	$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi R}\sqrt{\frac{GM}{x}}=\frac{c}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\bar{\lambda}_M}}$
Velocity ball Newton cradle[b]	$v_{out}=\sqrt{2\frac{GM}{R^2}H}=\frac{c}{R}\sqrt{2Hl_p\frac{l_p}{\bar{\lambda}_M}}$
Observable predictions (from GR): (contain only GM)	
Advance of perihelion	$\sigma=\frac{6\pi GM}{a\left(1-e^2\right)c^2}=\frac{6\pi l_p}{a\left(1-e^2\right)}\frac{l_p}{\bar{\lambda}_M}$
Gravitational redshift	$z=\frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}}-1=\frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\bar{\lambda}_M}}}-1$
Time dilation	$T_R=T_f\sqrt{1-\sqrt{\frac{2GM}{R}}^2/c^2}=T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Deflection	$\delta=\frac{4GM}{c^2R}=\frac{4l_p}{R}\frac{l_p}{\bar{\lambda}_M}$
Microlensing	$\theta_E=\sqrt{\frac{4GM}{c^2}\frac{d_S-d_L}{d_Sd_L}}=2\sqrt{l_p\frac{l_p}{\bar{\lambda}_M}\frac{d_S-d_L}{d_Sd_L}}$

[a]The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for full circle, see [49]. [b]Where H is the height of the ball drop.

cosmology. Both gravity and cosmology can thus be referred to as Planck quantized.

For a Planck mass, the term $\frac{l_p}{\bar{\lambda}}=\frac{l_p}{l_p}=1$. Observable reduced Compton frequencies occur in integers, meaning they are quantized. For masses smaller than the Planck mass, the frequency will be less than one, and no frequency below one can be directly observed. However, such frequencies can be interpreted as the probability of being in a Planck mass state within a given Planck time observational window.

Interestingly, the Planck constant does not appear in these formulas, leading some to question whether this approach can be classified as quantum gravity. Furthermore, the Planck constant seems unnecessary in many aspects of quantum mechanics; see [30] for further discussion. The Planck constant is only required when expressing quantities in kilograms or joules. Notably, the kilogram is related to an arbitrary clump of matter that has been designated as a kilogram by convention.

2. Cosmology and the Planck Scale

Gravity theory is also closely linked to cosmology. One of the most studied cosmological models is the Friedmann [50] model, which comes from Einstein's [51] general relativity theory, but also can be derived from Newton theory. However, there has been considerable effort in trying to link cosmology to the Planck scale, without much success. In this paper, we will demonstrate how the Friedmann equations can be expressed in Planck unit form. We will also do the same with a new cosmological model recently derived from general relativity theory, based on extremal solutions.

One of the central Friedmann equations is expressed as:

$$H_0^2=\frac{8\pi G\rho+\Lambda c^2}{3}, \tag{14}$$

where Λ is the cosmological constant and ρ is the critical mass density in the Friedmann universe. Furthermore, H_0 represents the Hubble constant. By setting the cosmological constant to zero and solving for the mass, we obtain:

$$M_c=\frac{c^3}{2GH_0}. \tag{15}$$

It is worth noting that to include the cosmological constant in the Friedmann equation, it must be introduced ad hoc into Einstein's field equation, as Einstein [52] first did in 1917, referring to it as his extended field equation.

Recently, in our work [53], we demonstrated that a new cosmological equation emerges from the extremal solutions of the Reissner-Nordström [54] [55], Kerr [56], and Kerr-Newman [57] metrics:

$$H_0^2=\frac{8\pi G\rho-\Lambda c^2}{3}, \tag{16}$$

where $\Lambda=3\left(\frac{H_0}{c}\right)^2=\frac{3}{R_H^2}$. The same cosmological equation can also be derived from the Haug and Spavieri metric [58] [59]. The extremal universe model is based on the extremal solutions of Einstein's 1916 field equation. In the 1916 field equation, there was no explicit inclusion of a cosmological constant, yet it arises naturally from straightforward derivations of the extremal solutions of the Reissner-Nordström, Kerr, Kerr-Newman metrics and the Haug-Spavieri metric.

In 1917, when Einstein first introduced the cosmological constant, he proposed it as $\Lambda = \frac{2}{r^2}$, where r represented some type of horizon. At that time, the Hubble constant and cosmological redshift had not yet been suggested or observed. It is significant that in our new cosmological model, Λ directly emerges from the derivation of the extremal solution of the 1916 field equation. In contrast, in Einstein's 1917 proposal, the Friedmann model, and the Λ-CDM model, the cosmological constant is somewhat ad hoc in its inclusion.

Solving Equation (16) for mass yields:

$$M_u = \frac{c^3}{GH_0}, \tag{17}$$

which is twice the mass of the Friedmann critical universe: $M_u = 2M_c$. This also implies $H_0 = \frac{c^3}{GM_u}$.

Table 2 compares the Friedmann model and the extremal universe cosmological model, expressed in what we can refer to as the Planck form or quantum form. Note that the Friedmann model is slightly more complex than the extremal model, as seen in [60]. The table reports only the Friedmann model for a critical universe, meaning that the cosmological constant and the k parameter are set to zero—in other words, for a flat universe.

It is important to highlight that this is much more than merely rewriting the gravitational constant in Planck units. The Planck length and Planck time can be determined relatively easily without any knowledge of G, and the Compton wavelength of the total mass (and energy) in the observable universe can also be found without knowledge of G; see [47]. Furthermore, the universal constants G, $\hbar$, and c can be replaced with only c and the Planck length l_p, both for observable gravitational phenomena (as shown in **Table 1**) and for cosmological predictions (as shown in **Table 2** and **Table 3**).

This new approach to predicting cosmological phenomena using just two constants—the Planck length and the speed of light—provides strong support for the idea that this theory or perspective is on the right track. It is fully consistent with the recent quantization of general relativity theory.

This means that we now have multiple models of the cosmos directly linked to the Planck scale and the reduced Compton frequency of matter. How should this be interpreted? In our view, this suggests that quantum gravity is already embedded within Newtonian and Einsteinian gravity—not by design, but as a result of understanding gravity at a deeper, more fundamental level. This interpretation aligns with the recent reformulation of Einstein's field equations and metrics into

Table 2. There are alternative ways to express cosmological equations rooted in the Planck scale and the reduced Compton wavelength. However, linking cosmological observations to the Planck mass rather than the Planck length appears to add unnecessary complexity, as it requires introducing an additional constant—the Planck constant. This is why we have not done that here.

	Critical Friedmann universe	Haug extremal universe and Haug-Spavieri universe
Universe equation	$H_0^2=\frac{8\pi G\rho}{3}=\left(\frac{c}{2l_p\frac{l_p}{\bar{\lambda}_c}}\right)^2$	$H_0^2=\frac{8\pi G\rho-\Lambda c^2}{3}=\frac{4\pi G\rho}{3}=\left(\frac{c}{l_p\frac{l_p}{\bar{\lambda}_u}}\right)^2$
Cosmological constant	$\Lambda=0$	$\Lambda=3\left(\frac{H_0}{c}\right)^2=3\left(\frac{\bar{\lambda}_u}{l_p^2}\right)^2$
Universe kilogram mass density	$\rho_c=\frac{3H_0^2}{4\pi G}=\frac{3\hbar\bar{\lambda}_c^2}{16\pi c l_p^6}$	$\rho_u=\frac{3H_0^2}{4\pi G}=\frac{3\hbar\bar{\lambda}_u^2}{4\pi c l_p^6}$
Universe energy Joule density	$\rho_c=\frac{3H_0^2c^2}{4\pi G}=\frac{3\hbar\bar{\lambda}_c^2c}{16\pi l_p^6}$	$\rho_u=\frac{3H_0^2c^2}{4\pi G}=\frac{3\hbar\bar{\lambda}_u^2c}{4\pi l_p^6}$
Universe mass kilogram	$M_c=\frac{c^3}{2GH_0}=\frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$	$M_u=\frac{c^3}{GH_0}=\frac{\hbar}{\bar{\lambda}_u}\frac{1}{c}$
Hubble constant	$H_o=\frac{c}{2l_p\frac{l_p}{\bar{\lambda}_c}}=\frac{1}{2t_p\frac{l_p}{\bar{\lambda}_c}}$	$H_o=\frac{c}{l_p\frac{l_p}{\bar{\lambda}_u}}=\frac{1}{t_p\frac{l_p}{\bar{\lambda}_u}}$
Hubble Radius	$R_H=\frac{c}{H_0}=2l_p\frac{l_p}{\bar{\lambda}_c}$	$R_H=\frac{c}{H_0}=l_p\frac{l_p}{\bar{\lambda}_u}$
Hubble Circumference	$C_H=2\pi\frac{c}{H_0}=4\pi l_p\frac{l_p}{\bar{\lambda}_c}$	$C_H=2\pi\frac{c}{H_0}=2\pi l_p\frac{l_p}{\bar{\lambda}_u}$
Hubble volume	$V_H=\frac{4}{3}\pi R_H^3=\frac{32}{3}\pi l_p^3\frac{l_p^3}{\bar{\lambda}_c^3}$	$V_H=\frac{4}{3}\pi R_H^3=\frac{4}{3}\pi l_p^3\frac{l_p^3}{\bar{\lambda}_u^3}$
Age Universe	$T_H=\frac{R_H}{c}=\frac{1}{H_0}=2l_p\frac{l_p}{\bar{\lambda}_c c}=2t_p\frac{l_p}{\bar{\lambda}_c}$	$T_H=\frac{R_H}{c}=\frac{1}{H_0}=l_p\frac{l_p}{\bar{\lambda}_u c}=t_p\frac{l_p}{\bar{\lambda}_u}$
Hubble frequency	$f_H=\frac{1}{T_H}=\frac{1}{2t_p}\frac{\bar{\lambda}_c}{l_p}$	$f_H=\frac{1}{T_H}=\frac{1}{t_p}\frac{\bar{\lambda}_u}{l_p}$
Mass to radius ratio relation	$\frac{M_c}{R_H}=\frac{m_p}{2l_p}$	$\frac{M_u}{R_H}=\frac{m_p}{l_p}$
Compton wavelength universe mass	$\bar{\lambda}_c=\frac{\hbar}{cM_c}=2l_p\frac{l_p}{R_H}$	$\bar{\lambda}_u=\frac{\hbar}{cM_u}=l_p\frac{l_p}{R_H}$
Cosmological red-shift $z\ll 1$:	$z\approx\frac{2DH_0}{c}=\frac{D}{l_p\frac{l_p}{\bar{\lambda}_c}}$	$z\approx\frac{2DH_0}{c}=\frac{2D}{l_p\frac{l_p}{\bar{\lambda}_u}}$
Cosmological red-shift	$z=\sqrt{\frac{\bar{\lambda}_{c,t}}{\bar{\lambda}_c}}-1$	$z=\sqrt{\frac{H_t}{H_0}}-1=\sqrt{\frac{\bar{\lambda}_{c,t}}{\bar{\lambda}_c}}-1$

Continued

Compton wavelength and z:	$\bar{\lambda}_{c,t}=\bar{\lambda}_c(1+z)^2$	$\bar{\lambda}_{c,t}=\bar{\lambda}_c(1+z)^2$
Planck length from Cosmological red-shift	$l_p\approx\sqrt{\frac{d\bar{\lambda}_c}{2z_c}}$ when $z\ll 1$	$l_p\approx\sqrt{\frac{d\bar{\lambda}_u}{z_u}}$ when $z_u\ll 1$
Planck length from H_0 and T_0:	$l_p=\frac{H_0}{T_0^2}\frac{\hbar^2 c}{k_b 32\pi^2}$	$l_p=\frac{H_0}{T_0^2}\frac{\hbar^2 c}{k_b 32\pi^2}$
CMB temperature	$T_0=\frac{1}{32\pi k_b}\frac{c}{l_p}\sqrt{\frac{\bar{\lambda}_c}{l_p}}$	$T_0=\frac{1}{32\pi k_b}\frac{c}{l_p}\sqrt{\frac{\bar{\lambda}_c}{l_p}}$
CMB temperature from distance	$T_t=T_0\sqrt{\frac{\bar{\lambda}_{c,t}}{\bar{\lambda}_c}}=\frac{1}{32\pi k_b}\frac{c}{l_p}\sqrt{\frac{\bar{\lambda}_{c,t}}{l_p}}$	$T_t=T_0\sqrt{\frac{\bar{\lambda}_{u,t}}{\bar{\lambda}_u}}=\frac{1}{16\pi k_b}\frac{c}{l_p}\sqrt{\frac{\bar{\lambda}_{u,t}}{2l_p}}$

Table 3. Cosmology expressed in terms of its relation to the Planck scale and the reduced Compton wavelength.

	Friedmann critical universe	Extremal universe
Universe equation	$H_0^2=\frac{2c^2l_p^2}{\bar{\lambda}_c R_H^3}$	$H_0^2=\frac{c^2l_p^2}{\bar{\lambda}_u R_H^3}$
Universe equation	$H_0^2=\frac{\bar{\lambda}_c^2c^2}{4l_p^4}$	$H_0^2=\frac{\bar{\lambda}_u^2c^2}{l_p^4}$
Hubble constant from t_p	$H_0=\frac{\bar{\lambda}_c}{2t_p^2c}$	$H_0=\frac{\bar{\lambda}_u}{t_p^2c}$
Hubble constant from m_p	$H_0=\frac{\bar{\lambda}_c m_p^2c^3}{2\hbar^2}$	$H_0=\frac{\bar{\lambda}_u m_p^2c^3}{\hbar^2}$
Hubble constant from l_p and t_p	$H_0=\frac{\bar{\lambda}_c}{2t_pl_p}$	$H_0=\frac{\bar{\lambda}_u}{t_pl_p}$
Hubble constant from m_p and t_p	$H_0=\frac{\bar{\lambda}_c m_pc}{2t_p\hbar}$	$H_0=\frac{\bar{\lambda}_u m_pc}{t_p\hbar}$
Radius universe from t_p	$R_H=\frac{c}{H_0}=\frac{2t_p^2c^2}{\bar{\lambda}_u}$	$R_H=\frac{c}{H_0}=\frac{t_p^2c^2}{\bar{\lambda}_u}$
Radius universe from m_p	$R_H=\frac{c}{H_0}=\frac{2\hbar^2}{\bar{\lambda}_c m_p^2c^2}$	$R_H=\frac{c}{H_0}=\frac{\hbar^2}{\bar{\lambda}_u m_p^2c^2}$
Radius universe from m_p and t_p	$R_H=\frac{c}{H_0}=\frac{2t_p\hbar}{\bar{\lambda}_c m_p}$	$R_H=\frac{c}{H_0}=\frac{t_p\hbar}{\bar{\lambda}_u m_p}$

a Planck-scale form (see [14] [16] [61]).

In our view, all observable gravitational phenomena, are indirect detections of the Planck scale. This is why we can now extract the Planck length from most gravitational phenomena and even from cosmological redshift, without requiring any knowledge of G or $\hbar$. This perspective is in stark contrast to the prevailing view, which considers Newton and general relativity theory to be distinct from the quantum scale.

While not everyone agrees with our perspective, we encourage other researchers to carefully investigate and study this approach before forming their conclusions.

The cosmological redshift equation $z=\sqrt{\frac{\bar{\lambda}_{c,t}}{\bar{\lambda}_c}}-1$ provides the deepest quantum-level interpretation of the cosmological redshift $z=\sqrt{\frac{R_h}{R_t}}-1$. Here, $\bar{\lambda}_c$ represents the reduced Compton wavelength of the current mass in the universe, and $\bar{\lambda}_{c,t}$ represents the reduced Compton wavelength of the mass in the $R_h=ct$ type Haug-Tatum universe at earlier cosmic epochs. We have:

$$\bar{\lambda}_{c,t}=\frac{\hbar}{M_{c,t}c}=\frac{\hbar}{\frac{c^3}{2H_tG}c}=\frac{2H_tl_p^2}{c} \tag{18}$$

where H_t is the Hubble parameter at earlier cosmic epochs. Haug and Tatum [62]-[65] were the first to propose this type of redshift: $z=\sqrt{\frac{R_h}{R_t}}-1$, where $R_h=\frac{c}{H_0}$ and $R_t=\frac{c}{H_t}$ represent the Hubble radius now and at earlier cosmic epochs, respectively, in a manner consistent with the $R_h=ct$ cosmological principle. Their model appears to resolve the Hubble tension. Naturally, there is currently no consensus on this, and only time will tell whether it will be accepted after further investigation by multiple researchers over time.

This cosmological redshift can also naturally be expressed as $z=\sqrt{\frac{H_t}{H_0}}-1=\sqrt{\frac{t_0}{t_t}}-1=\sqrt{\frac{f_{H,t}}{f_{H,0}}}-1$, where $f_{H,t}=\frac{c}{R_{h,t}}=\frac{1}{2t_p}\frac{\bar{\lambda}_{c,t}}{l_p}$ is the Hubble sphere frequency at time t, and $f_{H,0}=\frac{c}{R_h}=\frac{1}{2t_p}\frac{\bar{\lambda}_c}{l_p}$ is the Hubble frequency now. The Hubble constant is, in fact, identical to the Hubble frequency.

Furthermore, $T_0=\frac{1}{32\pi k_b}\frac{c}{l_p}\sqrt{\frac{\bar{\lambda}_c}{l_p}}$ represents the deeper quantum-level interpretation of the cosmological temperature, first heuristically suggested by Tatum *et al.* [66] in the form $T_0=\frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}}=\frac{\hbar c}{k_b 4\pi\sqrt{R_s 2l_p}}$, and later proven to be derivable from the Stefan-Boltzmann law by Haug and Wojnow [48] [67].

Once again, note that in all formulas, we frequently encounter the term $\frac{l_p}{\bar{\lambda}_c}$ (or $\frac{\bar{\lambda}_c}{l_p}$), which is the Planck length divided by the reduced Compton wavelength of the critical universe in the Friedmann model, and $\frac{l_p}{\bar{\lambda}_u}$, which is the Planck length divided by the reduced Compton wavelength of the mass (or equivalent mass) in the extremal universe. This term corresponds to the reduced Compton frequency per Planck time, representing the quantization of mass and energy, and thus provides Planck quantization of gravity and cosmology.

Table 3 presents several additional ways to express different aspects of the cosmos using Planck units. When the Planck mass (in kilogram terms) is used instead of the Planck length or Planck time, knowledge of the Planck constant is also required. However, as shown in Table 2, using the Planck mass is unnecessary, as these cosmological phenomena can be predicted directly from the Planck length or Planck time. Therefore, only the two constants l_p and c are needed for cosmology predictions. Furthermore, the Planck mass itself can be determined without requiring any knowledge of G.

It is worth noting that the extremal universe model can also be derived from the new exact solution of the Einstein field equation recently derived by Haug and Spavieri [58].

3. Relativistic Newton Theory and Conservation of Space-time

In addition, we emphasize that the same cosmological model, which can be derived from both the extremal solution and the Haug-Spavieri metric, can surprisingly also be obtained from the special relativistic modified Newtonian gravitational theory [68] and its corresponding quantum gravity theory [69]. We believe it is important to highlight that the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics, as well as the Haug-Spavieri metric within general relativity, correspond to many cosmological and black hole-related derivations that can also be obtained from the relativistic modified Newtonian gravitational theory.

The Schwarzschild solution is calibrated to Newton's weak field limit in the final step before it can be used for predictions. However, even in a strong gravitational field, it predicts several formulas identical to those derived from the Newtonian weak field limit, such as the escape velocity and the radius of a black hole-like object, $r_s = \frac{2GM}{c^2}$. Here, we challenge the consensus by arguing that calibrating a model to one that is valid only in a weak field may inadvertently cause the calibrated model to also remain valid only in weak gravitational fields.

In contrast, the extremal solutions and the Haug-Spavieri metric align with the relativistic modified Newtonian theory in both weak and strong fields. However, there is one notable exception: while relativistic Newtonian theory predicts the bending of space and time, it assumes flat space-time. From this perspective, the relativistic modified Newtonian theory is, in our view, superior.

How is it possible that the Λ-CDM model, based on general relativity, assumes almost infinitely curved space-time at the beginning of the Big Bang, while simultaneously predicting the universe will end in a "cold death" with near-flat space-time, all while maintaining the assumption of energy conservation? This implies getting something from nothing, which violates the principle of energy conservation. In contrast, in relativistic Newtonian theory, the space-time interval always remains flat, yet it still allows for the bending of space and time.

4. It Requires Much Less Information to Determine $l_p \frac{l_p}{\bar{\lambda}}$ than to Find l_p and $\bar{\lambda}$ Separately

In addition to the speed of light (gravity), some cosmological predictions require knowledge of the term $l_p \frac{l_p}{\bar{\lambda}}$. This term is embedded in all cosmological predictions. Remarkably, we can determine both the Planck length and the Compton wavelength of the universe entirely independently of the Planck constant and the gravitational constant G. For the entire cosmos, we simply need to measure the Hubble constant. Then we have:

$$l_p \frac{l_p}{\bar{\lambda}_u} = \frac{c}{H_0} \tag{19}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of all the mass and energy in the observable universe, derived from the extremal solution of Einstein's field equations. Alternatively, we have:

$$l_p \frac{l_p}{\bar{\lambda}_c} = \frac{c}{2H_0} \tag{20}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical Friedmann universe. To find the reduced Compton wavelength of the observable critical universe or the extremal universe, we simply need to determine the Planck length, which can be done independently of knowledge of G and $\hbar$, as demonstrated in [25] [26].

Not only is it easier to determine $l_p \frac{l_p}{\bar{\lambda}}$ than l_p and $\bar{\lambda}$ separately, but it also provides much higher precision. Finding l_p and $\bar{\lambda}$ separately is necessary only when studying the quantization of gravity. For predicting macroscopic gravitational phenomena, we only need the term $l_p \frac{l_p}{\bar{\lambda}}$.

5. Conclusion

We have demonstrated how both the critical Friedmann model and a recent cosmological model, based on the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics, as well as the cosmological model derived from the Haug-Spavieri metric, can be expressed using the Planck length, the speed of light (gravity), and the Compton wavelength of the relevant mass—in this case, the mass of the universe. Notably, we can determine the Planck length and the Compton wavelength of the universe without any prior knowledge of G or $\hbar$.

This represents a significant breakthrough, as it enables, for the first time, a direct link between the smallest scale—the Planck scale—and the largest scales in the universe, namely cosmic scales.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix, Some Derivations

The Friedmann Universe

Just to demonstrate some derivations of the results given in the tables. The mass in the Freedman universe is given by

$$M_c = \frac{c^3}{2GH_0}. \tag{21}$$

Further, the Hubble constant is then given by

$$H_0 = \frac{c^3}{2GM_c}. \tag{22}$$

Next we can replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and the universe mass with $M_c = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$ which gives

$$H_0 = \frac{\bar{\lambda}_u c}{2l_p^2}, \tag{23}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the critical mass in the Freedman universe. This means the (reduced) Compton wavelength of the critical mass in the Friedmann universe is given by

$$H_0^2 = \frac{8\pi G\rho}{3}$$

$$\left(\frac{\bar{\lambda}_c c}{2l_p^2}\right)^2 = \frac{2GM_c}{R_H^3}$$

$$\frac{\bar{\lambda}_u^2 c^2}{4l_p^4} = 2\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_c}\frac{1}{R_H^3}$$

$$\frac{\bar{\lambda}_c^3}{4l_p^4} = 2\frac{l_p^2}{R_H^3}$$

$$\bar{\lambda}_c^3 = \frac{8l_p^6}{R_H^3}$$

$$\bar{\lambda}_c = \left(\frac{8l_p^6}{R_H^3}\right)^{1/3}$$

$$\bar{\lambda}_c = \frac{2l_p^2}{R_H}, \tag{24}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the mass in the critical Friedmann universe.

Further since $R_H = \frac{c}{H_0}$ we can also write this as

$$\bar{\lambda}_c = \frac{2l_p^2 H_0}{c} = \frac{2l_p^2}{R_H}. \tag{25}$$

The Extremal Universe

The mass in the extremal universe is given by

$$M_u = \frac{c^3}{2GH_0}. \tag{26}$$

Next we can replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and the universe mass with $M_u = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$ which gives

$$H_0 = \frac{\bar{\lambda}_u c}{l_p^2}, \tag{27}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the extremal universe. The mass in this extremal universe is twice that of the Friedmann universe. The reduced Compton wavelength of the mass in this universe we can express as

$$H_0^2 = \frac{4\pi G\rho}{3}$$

$$\left(\frac{\bar{\lambda}_u c}{l_p^2}\right)^2 = \frac{GM_u}{R_H^3}$$

$$\frac{\bar{\lambda}_u^2 c^2}{l_p^2} = \frac{l_p^2 c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_u}\frac{1}{R_H}$$

$$\frac{\bar{\lambda}_u^3}{l_p^4} = l_p \frac{l_p}{\bar{\lambda}_u R_H^3}$$

$$\bar{\lambda}_u^3 = \frac{l_p^6}{R_H^3}$$

$$\bar{\lambda}_u = \left(\frac{l_p^6}{R_H^3}\right)^{1/3}$$

$$\bar{\lambda}_u = \frac{l_p^2}{R_H}, \tag{28}$$

and since $R_H = \frac{c}{H_0}$ we can also re-write this as

$$\bar{\lambda}_u = \frac{l_p^2 H_0}{c} = \frac{l_p^2}{R_H}. \tag{29}$$