

Practical Implications of Ignoring Slope When Conducting Timber Appraisals of Longleaf Pine Plantations

Curtis L. Vander Schaaf 

Central Mississippi Research and Extension Center, Mississippi State University, Raymond, MS, USA
Email: clv127@msstate.edu

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Abstract

Horizontal point sampling is commonly applied during forest inventories. A problem widely known is the error associated with probabilities of selection when projecting horizontal angles along the slope of a site. However, to date, the impacts of not accounting for slope when using horizontal point sampling on timber appraisals has been minimal. This study quantified the potential impacts on timber appraisals when failing to account for slope for two common English Basal Area Factors (BAF) of 10 (2.296 metric) and 20 (4.592 metric). When percent slope exceeds 15% the impacts can become meaningful and hence foresters should make sure to address this issue when conducting appraisals. For stands of ages 20 and 30, and for optimistic stumpage prices of \$20, \$30, and \$45 per ton of pulpwood, chip-n-saw, and sawtimber, respectively, projecting angles along 20% slopes were found to reduce appraisals ranging from around 3.6% to 3.9%. When projecting angles along 30% slopes appraisal reductions ranged from around 7.6% to 8.2%.

Keywords

Variable-Radius, Stumpage, Forest Inventory, *Pinus palustris*, Sampling

1. Introduction

Variable-radius sampling is widely used during forest inventories. Many texts provide explanations of the theory behind sampling probabilities and application in the field (e.g. Burkhart et al., 2019, Chapter 12; Bell & Dilworth, 2002: 181-251; Kershaw et al., 2017: 287-296, 362-376; Iles, 2003, Chapter 12; Shiver & Borders, 1996, Chapter 4; West, 2004: 72-77). The common application of variable-radius sampling is the projection of horizontal angles, sometimes referred to as horizon-

tal point sampling. A frequently addressed and widely known problem is that of projecting horizontal angles along a slope (e.g. Burkhart et al., 2019: 269-270; Shiver & Borders, 1996: 90-91). Not accounting for the impacts of slope when projecting horizontal angles produces incorrect sampling probabilities.

If not correctly addressed, many texts state that slope ranging from 10% to 15% and greater has a meaningful impact on the probability of selection (e.g. Burkhart et al., 2019: 269-270; Shiver & Borders, 1996: 91). **Figure 1** shows the impact of different slopes on the probability of selection for a tree of 10 inches (25.4 cm) when using a 10 BAF (2.296 metric) prism and a 20 BAF (4.592 metric) prism. BAFs of 10 (2.296 metric) and 20 (4.592 metric) are commonly used in the southeastern USA (Burkhart et al., 2019, Chapter 12; Shiver & Borders, 1996, Chapter 4). Slopes of 15% and less have minimal impact on sampling probabilities. **Figure 2** shows the impact of different slopes on the probability of selection if a tree's center is directly in line with the sampling point parallel with the slope, assuming the slope is constant throughout the entire population (or stand). If a tree's center is directly in line with the sampling point perpendicular to the slope, assuming the slope is constant throughout the entire population (or stand), then there is no reduction in the probability of selection (**Figure 1**). These opposing behaviors lead to the oval in the "imaginary" variable-radius tree plot if the impact of slope is ignored.

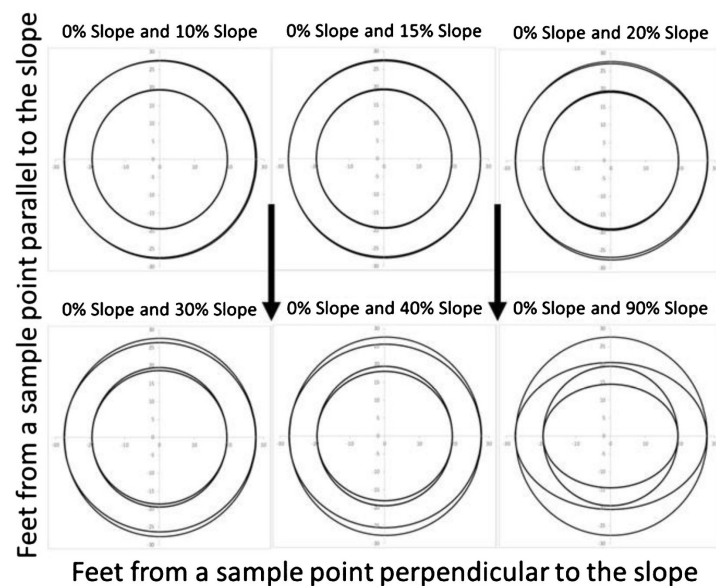


Figure 1. Impacts of percent slope on the "imaginary" variable-radius plot area for a 10 inch (25.4 cm) tree when using a 10 BAF (2.296 metric) prism and a 20 BAF (4.592 metric) prism. The arrows indicate the direction of the slope. Due to the error associated with projecting a non-horizontal angle, the probability of selection has been reduced. All graphics are to relative scale. If not corrected for, the reduction in the plot areas due to the slope can lead to excluding trees that should be sampled and/or incorrect per acre/hectare tree expansion factors. The external circle and oval are for a 10 BAF (2.296 metric) prism and the internal circle and oval are for a 20 BAF (4.592 metric) prism. A 90% slope is included to clearly demonstrate the impacts of slope.

For instance, when using a 20 BAF (4.592 metric) prism and projecting a horizontal angle for a diameter at breast height (4.5 feet (1.37 meters), dbh) of 10 inches (25.4 cm) on flat ground, the probability of selection is 0.0273, calculated as:

$$\begin{aligned} \text{Probability of selection} &= \frac{\pi * (10 \text{ inches } (25.4 \text{ cm}) \times 1.944)^2}{43,560 \text{ sq feet/acre } (4,047 \text{ sq m})} \\ &= \frac{1,187 \text{ sq feet } (110 \text{ sq m})}{43,560 \text{ sq feet/acre}} \end{aligned} \quad (1)$$

where:

1.944—plot radius factor for a 20 BAF (4.592 metric), producing a horizontal limiting distance of 19.44 feet (5.93 meters).

But when projecting the horizontal angle along a slope of 40%, the probability of selection is reduced to 0.0235. If not corrected for the slope, this error will produce incorrect volume, weight, biomass, carbon, etc., estimates that will ultimately impact timber appraisals.

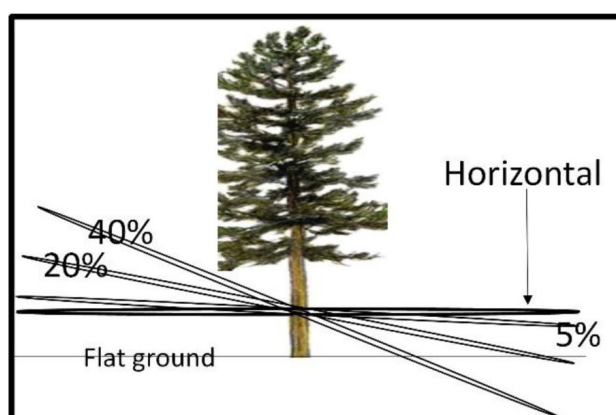


Figure 2. Impacts of percent slope on the “imaginary” variable-radius plot area if the tree’s center is directly parallel with the sampling point along the slope. Due to the error associated with projecting a non-horizontal angle, the probability of selection has been reduced. All graphics are to relative scale. If not corrected for, the reduction in the plot areas due to the slope can lead to excluding trees that should be sampled and/or incorrect per acre/hectare tree expansion factors.

Shiver and Borders, 1996: 91, present an equation to calculate the modified critical/limiting distance along the slope:

$$MCD = \sqrt{1 + \left(\frac{SL}{100}\right)^2} \times R \quad (2)$$

where:

MCD—modified critical/limiting distance,

SL—percent slope, and

R—limiting distance on flat-ground.

Equation (2) can be rearranged to give a modified limiting distance on flat-

ground assuming the angle (and hence the original limiting distance) is erroneously projected parallel with the slope without being corrected:

$$R_l = LD / \sqrt{1 + \left(\frac{SL}{100}\right)^2} \quad (3)$$

where:

R_l —modified limiting distance along flat-ground assuming the correct limiting distance (LD) is incorrectly used parallel with the slope, and

LD—correct limiting distance along flat-ground assuming the angle is projected horizontally.

Equation (3) allows for the determination of what trees should be selected if the angle was incorrectly projected along the slope, resulting in some trees being incorrectly excluded from the sample. For example, Equation (4) shows R_l for the 10-inch (25.4 cm) tree when using a 20 BAF (4.592 metric) prism along a 40% slope:

$$R_l = LD / \sqrt{1 + \left(\frac{SL}{100}\right)^2} = 19.44 / \sqrt{1 + \left(\frac{40}{100}\right)^2} = 18.05 \text{ feet (5.50 m)} \quad (4)$$

The correct limiting distance on flat-ground is 19.44 feet (5.93 meters) but since the angle is incorrectly projected parallel with the slope the limiting distance on flat-ground is shortened to 18.05 feet (5.50 meters). Hence, the probability of selection has been reduced from 0.0273 to 0.0235. This is a 14% reduction in probability of selection. R_l will differ to some extent for a particular dbh, BAF, and percent slope depending on where the tree's center is located parallel with the slope—thus producing the oval in **Figure 1**. Therefore, Equation (5) was ultimately used to estimate R_l :

$$R_l = \sqrt{LD^2 - \left[HD * \left(\frac{SL}{100}\right)\right]^2} \quad (5)$$

where:

HD—horizontal distance from plot center (or the point) parallel in direction with the slope.

Values of Equation (5) for a particular percent slope range from those modified limiting distances calculated directly parallel to the slope (e.g. Equation (4)) to those calculated perpendicular to the slope (e.g. the correct limiting distance when angles are projected horizontally). For example, for the 10-inch (25.4 cm) tree when using a 20 BAF (4.592 metric) prism along a 40% slope R_l ranges from 18.05 feet (5.50 meters) to 19.44 feet (5.93 meters).

Figure 3 shows the relative scale of 20% and 40% slopes to various tree heights with the intent of demonstrating visually what slopes look like that have an impact on sampling probabilities. Knowledge wanted to be gained about the potential impacts of ignoring different percent slopes when conducting timber appraisals. Longleaf pine (*Pinus palustris* Mill.) plantations in the Western Gulf (Lohrey & Bailey, 1977), USA were used as an example.

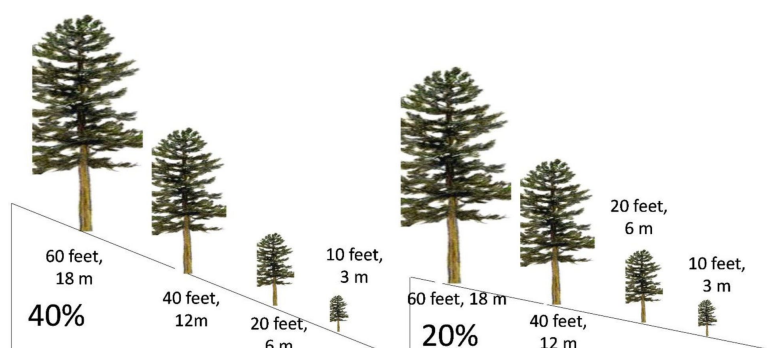


Figure 3. Percent slope relative to tree heights.

2. Methods

To determine potential economic impacts across a range of initial and current stand conditions, observations (**Table 1**) from a yield table (Lohrey & Bailey, 1977) were used to estimate diameter distributions, from which trees are established within square 0.4047 hectare (one-acre) virtual-world plantations. For simplicity, all plantations are pure planted longleaf pine, spatial correlation among individual trees was ignored (which could impact the dbh's of neighboring trees in the virtual-world plantations), all tree diameters are assumed to be perfect circles and are centered around the spatial location of a tree center, and all spacings are assumed square. It is assumed that a sample point is established in the center of each square 0.4047 hectare (one-acre) thus eliminating any potential edge bias. For simplicity, it is assumed that spatial locations of tree centers were established on flat-ground and thus are constant across all percent slopes, implying that planting distances varied parallel to slopes to maintain the constant planting distance on flat-ground.

Table 1. Stand-level variables and three-parameter Weibull distribution parameter estimates as obtained from Lohrey and Bailey, (1977) to conduct simulations. Where: D_q is quadratic mean diameter and BAA is basal area per acre/hectare.

Site Index 50 Base Age 25 Years (15.2 meters)								
Age	Site index	Planting density	Trees per	D _q	BAA	<i>a</i>	<i>b</i>	<i>c</i>
Years	ft/m	per acre/Surviving at age 15	acre/trees per ha	in./cm	(sq ft/acre)/ (sq m/ha)	Location	Scale	Shape
20	50/15.2	1,075/200	190/469	6.3/16.0	40.9/9.4	0	6.66	3.61
30			163/403	8.7/22.1	67.9/15.6	0	9.28	3.90
Site Index 70 Base Age 25 Years (21.3 meters)								
Age	Site index	Planting density	Trees per	D _q	BAA	<i>a</i>	<i>b</i>	<i>c</i>
Years	ft/m	per acre/Surviving at age 15	acre/trees per ha	in./cm	(sq ft/acre)/ (sq m/ha)	Location	Scale	Shape
20	70/21.3	460/400	372/919	6.6/16.8	88.6/20.3	0	7.01	3.69
30			295/729	8.8/22.4	125.6/28.8	0	9.38	3.87
20	70/21.3	913/700	630/1,557	6.0/15.2	123.3/28.3	0	6.33	3.26
30			448/1,107	8.3/21.1	170.1/39.0	0	8.85	3.59

2.1. Stand Conditions

For the sake of brevity, only two site indices (SI) were used (50 feet (15.2 meters) and 70 feet (21.3 meters) at base age 25 years). A SI of 50 feet (15.2 meters) is indicative of average site quality for plantations established from 1970 to 2010 while a SI of 70 feet (21.3 meters) is a high quality site and could be considered as providing inference about the maximum financial impacts. On the SI 50 (15.2 meters) site, observations were obtained for combinations of planting density per acre (~1,075 seedlings per acre [2,656 seedlings per hectare]) and age (20 and 30 years). This planting density corresponds to surviving trees per acre at age 15 of 200 within the yield tables presented in [Lohrey and Bailey, \(1977\)](#). Planting densities near 1,075 seedlings per acre [2,656 seedlings per hectare] were more commonly established in the 1970s and 1980s ([South, 2006](#); [Hausle et al., 2023](#)), in part because of concerns about seedling mortality ([South, 2006](#)), and these virtual plantations may be indicative of older plantations still existing today. On the SI 70 feet (21.3 meters) site, observations were obtained for combinations of planting density per acre (460 [1,137 per hectare] and 913 [2,256 per hectare]) and age (20 and 30 years). These planting densities correspond to surviving trees per acre at age 15 of 400 (988 per hectare) and 700 (1,730 per hectare), respectively, within the yield tables presented in [Lohrey and Bailey, \(1977\)](#). Given declines in the value of pulpwood across much of the southeastern USA over the past 20 years ([VanderSchaaf, 2023](#), [Lamichhane, 2026](#)) and more emphasis on wildlife habitat and associated ecosystem benefits and available cost-shares, planting densities near 460 seedlings per acre [1,137 per hectare] have become more common for longleaf pine ([Demers et al., 2000](#), [South, 2006](#), [Hausle et al., 2023](#)). Planting densities near 913 [2,256 per hectare]) seedlings per acre were more commonly established in the 1990s ([Demers et al., 2000](#), [South, 2006](#), [Hausle et al., 2023](#)), especially when pine straw production was an objective. Hence average planted tree survival per hectare, quadratic mean diameter at breast height, and parameter estimates of the three parameter Weibull diameter distribution were obtained for each combination of age and density.

An individual tree dbh was then assigned using the predicted Weibull distribution to each grid point, and survival of that tree was determined by comparing the percent survival rate to a uniformly distributed random variable ([SAS Institute, 2016](#)). For example, if the yield table reported percent survival as 35% for a particular planting density and age combination, a tree assigned a uniformly distributed random variable less than or equal to 0.35 indicated the tree survived. However, any tree assigned a uniformly distributed random variable greater than 0.35 indicated the tree died and the diameter was removed from consideration for sampling.

For surviving trees, individual tree height was estimated using an equation obtained from [VanderSchaaf et al., \(2018\)](#) that predicts height as a function of dbh:

$$H_t = \exp^{[2.6575+0.5666 \ln dbh+0.00938/2]} \quad (6)$$

where:

Ht—predicted total tree height (feet), and
dbh—diameter at breast height in inches.

After establishing a virtual plantation, a sample point was placed at the center of each square 0.4047 hectare (one-acre) and sampling was conducted using two English BAF's (10 (2.296 metric) and 20 (4.592 metric) square feet per acre).

Five percent slopes were examined; 10%, 15%, 20%, 30%, and 40%. To determine the impacts of projecting horizontal angles to dbh along a slope, Equation (5) was used for each percent slope to calculate a modified limiting distance on flat ground (R_f) assuming the horizontal angle was incorrectly projected along the slope.

Based on the location of a tree within a plantation and the particular slope, the three-dimensional Euclidean distance from the sample point to each tree was determined and compared to the tree's modified limiting distance (R_f) for a particular slope. If the modified limiting distance (R_f) for each tree was equal to or greater than the tree's Euclidean distance, the tree was sampled for that particular slope. However, the tree per hectare expansion factor was calculated assuming the horizontal angle was projected along flat ground (or 0% slope).

Thus, for angles projected along slopes, the probability of inclusion in the sample is based on the slope distance but the expansion factor is based on assuming the horizontal angle was projected along flat ground—leading to inconsistencies among the probability of sampling and the expansion factor. Recall, the concept of probability proportional to size (PPS) is that smaller trees have a lower probability of inclusion. Thus, if the probability of inclusion for a particular tree size is small, yet a tree of that size is sampled, there must be many of them in the sampled population, leading to greater expansion factors for smaller trees (or in this case lower probabilities of sampling for the same dbh because of the slope) when using the same BAF. Hence, since the angle is projected along the slope, the “imaginary” plot area and thus the probability of inclusion is smaller than what it should actually be (e.g. **Figure 1** and **Figure 2**). Therefore, the expansion factor should be greater than what it is calculated to be because the expansion factor is incorrectly calculated assuming the angle is truly horizontal and hence, a greater, but incorrect probability is used, resulting in the incorrect smaller expansion factor.

To examine variability among repeated cruises for a particular percent slope, for survival amount at age 15 (200 trees per acre (494 per hectare) for SI 50 (15.2 meters) and 400 (988 per hectare) or 700 (1,730 per hectare) trees per acre for SI 70 [21.3 meters]), age (20 and 30 years), and BAF (10 (2.296 metric) and 20 [4.592 metric]), a total of 500 separate virtual plantations were established and a cruise was conducted in each virtual plantation. For each of the 500 cruises, a sample size of $n = 50$ sample points was established. Each sample point is assumed to be spatially independent of the other 49 sample points and no tree was sampled at more than one point. Thus, it can be thought that for a particular combination of factors, 500 separate cruises were conducted.

Individual tree weight and product weights (pulpwood, chip-n-saw, and sawtimber) were then estimated using equations found in Baldwin and Saucier, (1983):

$$Wt_{Tot} = 10^{\left[-0.66098 + 0.96407 \log[dbh2Ht]\right]} \quad (7)$$

$$Wt_d = 10^{Wt_{Tot} * \left(1 - 6.80342 \left[\frac{dm^{2.97184}}{(dbh2Ht)^{1.04751}} \right] \right)} \quad (8)$$

where:

Wt_{Tot} = total bole green weight, outside bark in pounds,

Wt_d = outside bark green weight to upper stem diameter merchantability limit (e.g. 2 inches [5.1 cm], 6 inches [15.2 cm]), in pounds,

dm = upper stem diameter outside bark (dob) merchantability limit in inches, and all other variables as previously defined.

Assumed merchantability standards for three product classes are presented in **Table 2**. For simplicity, any tree meeting minimum merchantability standards for a particular product class was placed in that product class (thus no cull trees and no product degrades). If portions of trees met sawtimber merchantability standards, that tree was assigned to that product class, if portions of trees did not meet sawtimber merchantability standards but met chip-n-saw merchantability standards the tree was assigned to that product class, and so forth for the pulpwood class. Sawtimber and chip-n-saw trees were considered mutually exclusive. However, upper-stem portions of sawtimber and chip-n-saw trees were assigned to the pulpwood product class, sometimes referred to as topwood.

Table 2. Product merchantability limits and two scenarios of product revenues per ton, where dbh is diameter at breast height (4.5 feet (1.37 meters) above the ground) and dm is the upper stem diameter outside bark (dob) merchantability limit.

	Merchantability limits (inches, cm)			Revenues per ton (\$)	
	Min dbh	Max dbh	dm	Poor	Optimistic
Pulpwood	4.5/11.4	9.5/24.1	2/5.1	10	20
Chip-n-saw	9.5/24.1	11.5/29.2	6/15.2	18	30
Sawtimber	11.5/29.2	.	6/15.2	23	45

2.2. Economic Impacts

To determine the economic value of an individual tree, all weight estimates were divided by 2,000 lbs to convert to tons, and then multiplied by two different sets of stumpage prices based on 2022 (Guo, 2023) market conditions for the Western Gulf region of the southeastern USA (**Table 2**). Revenues of all sampled trees (“in” trees) were then summed by sample point and averaged across sample points ($n = 50$) for each of the 500 cruises. Mean values from each of the 500 cruises for a particular percent slope were then averaged to determine the average economic value per hectare for a particular survival at age 15, age (20 or 30 years), and BAF (10 (2.296 metric) or 20 [4.592 metric]) combination.

3. Results and Discussion

3.1. Survival of 200 Trees per Acre (494 Per Hectare) at Age 15 on Site Index 50 (15.2 Meters) Sites (Base Age 25)

At age 20, based on basal area estimates, on average, for both a 10 (2.296 metric) and 20 BAF (4.592 metric), when projecting horizontal angles along slopes of 20% and greater errors begin to become more meaningful. When using a 10 BAF (2.296 metric) on a 20% slope one tree would be missed every 7 points while for a 40% slope one tree would be missed every two points (Table 3). If a 20 BAF (4.592 metric) was being used along a 40% slope one tree would be missed roughly every 5 points when projecting angles.

Table 3. Impacts on basal area and economic value per hectare estimates when incorrectly projecting horizontal angles along percent slopes of 10, 15, 20, 30, and 40 for two sets of product revenues at age 20 and 30 years when 200 trees per acre (494 per hectare) are surviving at age 15 years. Where: BAH is basal area per hectare. Site index (base age 25 years) is 50 feet (15.2 meters).

Age	% Slope	Average BAH estimate	Average revenue per hectare (\$)							
			Poor price				Optimistic price			
			Pulp	Chip	Saw	Total	Pulp	Chip	Saw	Total
BAF 10 (2.296 metric)										
20	0	9.52	574.65	81.45	4.87	652.39	1149.30	135.75	9.53	1277.77
	10	9.43	569.33	80.75	4.78	645.96	1138.66	134.58	9.36	1265.16
	15	9.32	562.46	79.59	4.67	637.42	1124.91	132.66	9.13	1248.48
	20	9.18	553.65	78.54	4.54	626.82	1107.30	130.89	8.88	1227.68
	30	8.81	531.18	75.99	4.37	600.46	1062.36	126.64	8.56	1175.86
	40	8.37	505.65	72.48	4.11	568.89	1011.29	120.80	8.05	1113.99
BAF 20 (4.592 metric)										
20	0	10.59	640.20	94.26	5.05	655.68	1280.39	157.10	9.88	1283.33
	10	10.51	634.93	93.90	5.06	648.84	1269.86	156.50	9.89	1269.84
	15	10.42	628.88	93.41	4.95	641.08	1257.75	155.68	9.69	1254.54
	20	10.30	621.24	92.38	4.75	630.32	1242.48	153.96	9.30	1233.46
	30	9.99	601.75	89.44	4.54	602.23	1203.51	149.07	8.89	1178.52
	40	9.63	580.11	83.82	4.52	567.58	1160.22	139.71	8.84	1111.31
BAF 10 (2.296 metric)										
30	0	15.52	614.99	664.64	603.42	1881.04	1229.98	1107.73	1180.61	3514.57
	10	15.36	609.09	657.53	597.52	1862.08	1218.18	1095.88	1169.07	3479.27
	15	15.18	602.08	649.35	590.72	1839.97	1204.17	1082.25	1155.75	3438.09
	20	14.93	591.89	638.58	582.47	1810.57	1183.78	1064.30	1139.61	3383.27
	30	14.29	565.30	612.19	557.60	1732.48	1130.60	1020.32	1090.96	3237.00
	40	13.52	534.31	584.06	524.10	1638.71	1068.61	973.44	1025.41	3060.44
BAF 20 (4.592 metric)										

Continued

	0	16.06	634.58	690.14	630.01	1897.30	1269.16	1150.23	1232.62	3544.71
	10	15.92	628.63	685.55	623.74	1879.20	1257.26	1142.58	1220.36	3510.50
30	15	15.75	621.15	680.11	616.51	1856.63	1242.30	1133.51	1206.21	3467.82
	20	15.53	611.96	674.23	604.62	1827.60	1223.93	1123.72	1182.94	3412.55
	30	14.95	586.66	657.51	575.04	1749.52	1173.31	1095.85	1125.08	3264.20
	40	14.23	560.13	624.15	546.46	1652.37	1120.26	1040.25	1069.16	3083.42

As discussed in several texts (e.g. [Burkhart et al., 2019: Chapter 12](#); [Shiver & Borders, 1996: 91](#)), incorrectly projecting angles along slopes of 15% or less has minimal impact on basal area per hectare estimates and timber appraisals. For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 8.0% to 13.4%.

At age 30, most likely because the trees are larger in diameter and greater in value, the impacts of projecting angles along the slope on basal area per hectare estimates are greater in magnitude relative to age 20. For a 10 BAF (2.296 metric) on a 20% slope close to one tree every four points would be missed while on a 40% slope close to one tree would be missed every point, while for a 20 BAF (4.592 metric) on a 20% slope close to one tree every nine points would be missed while on a 40% slope close to one tree every two points would be missed.

For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 7.9% to 13.0%.

Table 4 reports the standard deviation among the 500 separate virtual plantation estimates of revenue per hectare for each age (20 or 30 years) and BAF (10 (2.296 metric) or 20 [4.592 metric]) combination. For a particular BAF and age, due to decreases in average revenue per hectare, standard deviations are lower for greater percent slopes and the Poor set of product revenues. However, the coefficient of variation actually increases slightly as percent slope increases for each set of product revenues. Similar results are observed for the two site index 70 (21.3 meters) sites as well.

Table 4. Standard deviation of economic value per hectare estimates when incorrectly projecting horizontal angles along percent slopes of 10, 15, 20, 30, and 40 for two sets of product revenues at age 20 and 30 years when 200 trees per acre (494 per hectare) are surviving at age 15 years [site index (base age 25 years) is 50 feet (15.2 meters)], 400 trees per acre (988 per hectare) are surviving at age 15 years [site index (base age 25 years) is 70 feet (21.3 meters)], and when 700 trees per acre (1,730 per hectare) are surviving at age 15 years [site index (base age 25 years) is 70 feet (21.3 meters)].

		200 Trees at Age 15		400 Trees at Age 15		700 Trees at Age 15	
Age	% Slope	Poor	Optimistic	Poor	Optimistic	Poor	Optimistic
BAF 10 (2.296 metric)							
20	0	47.400	90.982	57.315	107.460	63.566	121.768
	10	47.444	90.991	57.188	107.221	63.113	120.984

Continued

	15	46.818	89.868	56.671	106.139	62.059	119.026
	20	46.493	89.334	56.508	105.830	61.849	118.733
	30	45.598	87.690	55.910	104.564	61.589	118.406
	40	44.309	85.235	54.292	101.601	59.881	114.922
	0	103.693	193.851	129.516	244.190	150.157	278.974
	10	102.743	192.093	129.142	243.324	149.238	277.343
30	15	102.253	191.212	128.042	241.067	147.852	274.806
	20	100.685	188.409	126.997	239.216	145.448	270.686
	30	98.105	183.557	122.588	230.971	142.585	265.685
	40	97.162	181.772	122.030	230.005	139.018	259.238
BAF 20 (4.592 metric)							
	0	67.539	130.369	81.670	152.448	89.826	171.901
	10	67.233	129.789	80.705	150.495	89.858	172.049
20	15	66.614	128.533	80.285	149.665	89.165	170.727
	20	66.034	127.379	78.809	146.712	88.991	170.486
	30	64.036	123.528	77.808	144.856	86.968	166.221
	40	61.299	118.142	72.917	135.983	85.710	163.656
	0	151.298	282.944	180.784	340.858	219.582	409.367
	10	150.556	281.376	179.304	337.792	218.598	407.657
30	15	150.350	281.043	179.046	336.940	217.923	406.336
	20	148.901	277.910	176.740	332.439	218.820	407.752
	30	145.920	272.356	173.291	325.487	213.336	397.423
	40	139.808	261.182	169.830	318.967	209.733	390.668

3.2. Survival of 400 Trees per Acre (988 Per Hectare) at Age 15 on Site Index 70 (21.3 Meters) Sites (Base Age 25)

At age 20, based on basal area estimates, on average, for both a 10 (2.296 metric) and 20 BAF (4.592 metric), when projecting horizontal angles along slopes of 15% and greater errors begin to become more meaningful (Table 5). When using a 10 BAF (2.296 metric) on a 15% slope one tree would be missed every 5 points while for a 40% slope at least one tree would be missed every point. If a 20 BAF (4.592 metric) was being used along a 40% slope at least one tree would be missed every two points when projecting angles. For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 7.7% to 13.1%.

At age 30, most likely because the trees are larger in diameter, the impacts of projecting angles along the slope on basal area per hectare estimates are greater in magnitude relative to age 20. For a 10 BAF (2.296 metric) on a 15% slope close to one tree every four points would be missed while on a 40% slope close to two trees would be missed every point, while for a 20 BAF (4.592 metric) on a 20% slope

close to one tree every four points would be missed while on a 40% slope close to one tree every point would be missed. For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 7.7% to 12.9%.

Table 5. Impacts on basal area and economic value per hectare estimates when incorrectly projecting horizontal angles along percent slopes of 10, 15, 20, 30, and 40 for two sets of product revenues at age 20 and 30 years when 400 trees per acre (988 per hectare) are surviving at age 15 years. Where: BAH is basal area per hectare. Site index (base age 25 years) is 70 feet (21.3 meters).

Age	% Slope	Average BAH estimate	Average revenue per hectare (\$)							
			Poor price				Optimistic price			
			Pulp	Chip	Saw	Total	Pulp	Chip	Saw	Total
BAF 10 (2.296 metric)										
20	0	20.30	1,220.15	268.10	25.19	1,513.44	2,440.29	446.84	49.29	2,936.42
	10	20.11	1,208.28	264.96	24.99	1,498.22	2,416.56	441.60	48.89	2,907.04
	15	19.87	1,193.71	261.57	24.49	1,479.77	2,387.42	435.95	47.92	2,871.29
	20	19.54	1,173.31	257.37	23.88	1,454.56	2,346.62	428.95	46.73	2,822.30
	30	18.69	1,121.30	246.28	22.80	1,390.32	2,242.59	410.46	44.61	2,697.56
	40	17.67	1,058.33	235.15	21.68	1,315.11	2,116.66	391.92	42.41	2,550.90
BAF 20 (4.592 metric)										
20	0	20.32	1,210.16	276.31	25.94	1,510.23	2,420.32	460.52	50.75	2,927.37
	10	20.12	1,197.26	275.58	25.50	1,496.00	2,394.52	459.30	49.89	2,899.18
	15	19.87	1,181.90	273.74	24.95	1,478.29	2,363.80	456.23	48.82	2,864.38
	20	19.56	1,163.74	270.01	24.30	1,455.49	2,327.48	450.02	47.55	2,820.08
	30	18.72	1,112.88	261.75	23.46	1,395.31	2,225.76	436.25	45.91	2,702.52
	40	17.71	1,060.98	237.79	22.59	1,317.36	2,121.96	396.32	44.21	2,554.72
BAF 10 (2.296 metric)										
30	0	28.92	1,093.31	1,245.98	1,280.47	3,619.75	2,186.61	2,076.63	2,505.26	6,768.50
	10	28.64	1,082.74	1,233.34	1,268.47	3,584.54	2,165.48	2,055.56	2,481.78	6,702.82
	15	28.30	1,070.32	1,219.98	1,252.25	3,542.55	2,140.63	2,033.31	2,450.06	6,623.99
	20	27.85	1,052.27	1,203.04	1,229.47	3,484.78	2,104.54	2,005.06	2,405.48	6,515.09
	30	26.64	1,007.06	1,151.21	1,174.02	3,332.29	2,014.12	1,918.69	2,296.99	6,229.80
	40	25.19	949.22	1,090.60	1,114.44	3,154.26	1,898.44	1,817.67	2,180.42	5,896.53
BAF 20 (4.592 metric)										
30	0	28.94	1,094.31	1,241.36	1,288.45	3,623.51	2,188.63	2,068.93	2,520.89	6,777.30
	10	28.66	1,081.32	1,235.47	1,272.89	3,589.08	2,162.64	2,059.12	2,490.44	6,711.07
	15	28.33	1,066.66	1,227.90	1,254.60	3,548.43	2,133.32	2,046.50	2,454.65	6,633.11
	20	27.87	1,048.10	1,213.39	1,230.87	3,491.65	2,096.20	2,022.32	2,408.23	6,525.42
	30	26.68	996.28	1,173.51	1,180.15	3,348.99	1,992.57	1,955.85	2,308.99	6,255.63
	40	25.23	951.85	1,090.37	1,120.04	3,160.71	1,903.69	1,817.29	2,191.38	5,909.48

At ages 20 and 30, for the Optimistic prices and when using both 10 (2.296 metric) and 20 BAF (4.592 metric) s to project angles along a 20% slope, “missing” trees resulted in underestimating economic value close to \$114 per hectare and \$253 per hectare, respectively. The reductions ranged from 3.7% to 3.9% of the correct economic value at breast height. These economic underestimates may not appear serious, but if a 30 yr-old stand is 20 hectares (50 acres), economic value could be underestimated by about \$5,150 (based on the yield tables used). Across 100 stands this would be \$515,000. Thus, since the simulated timber valuation/appraisal estimates are biased downward, consistently projecting assumed horizontal angles along 20% slopes across time, for example, can result in significant losses to the landowner and potentially gains to the buyer.

3.3. Survival of 700 Trees per Acre (1,730 Per Hectare) at Age 15 on Site Index 70 (21.3 Meters) Sites (Base Age 25)

At age 20, based on basal area estimates, on average, for both a 10 and 20 BAF, when projecting horizontal angles along slopes of 15% and greater errors begin to become more meaningful. When using a 10 BAF (2.296 metric) on a 15% slope one tree would be missed every four points while for a 40% slope close to two trees per point would be missed (Table 6). If a 20 BAF (4.592 metric) was being used along a 40% slope close to one tree would be missed every point. For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 7.6 to 13.1%.

Table 6. Impacts on basal area and economic value per hectare estimates when incorrectly projecting horizontal angles along percent slopes of 10, 15, 20, 30, and 40 for two sets of product revenues at age 20 and 30 years when 400 trees per acre (988 per hectare) are surviving at age 15 years. Where: BAH is basal area per hectare. Site index (base age 25 years) is 70 feet (21.3 meters).

Age	% Slope	Average BAH estimate	Average revenue per hectare (\$)							
			Poor price				Optimistic price			
			Pulp	Chip	Saw	Total	Pulp	Chip	Saw	Total
BAF 10 (2.296 metric)										
20	0	28.38	1,653.01	226.12	19.58	1,898.70	3,306.01	376.86	38.31	3,721.18
	10	28.10	1,636.47	224.12	19.46	1,880.05	3,272.94	373.53	38.07	3,684.54
	15	27.76	1,616.07	221.15	19.29	1,856.51	3,232.15	368.58	37.73	3,638.46
	20	27.31	1,588.56	217.81	19.04	1,825.42	3,177.12	363.02	37.26	3,577.41
	30	26.12	1,517.88	209.34	18.56	1,745.79	3,035.76	348.91	36.32	3,420.99
	40	24.68	1,432.37	201.07	17.58	1,651.02	2,864.74	335.12	34.39	3,234.25
BAF 20 (4.592 metric)										
20	0	28.34	1,661.84	217.87	21.03	1,900.14	3,323.68	363.12	41.15	3,726.77
	10	28.05	1,645.40	215.67	20.93	1,881.40	3,290.80	359.44	40.95	3,690.02
	15	27.73	1,627.34	213.92	20.59	1,861.26	3,254.69	356.53	40.28	3,650.34
	20	27.28	1,602.13	210.80	20.35	1,832.69	3,204.26	351.33	39.81	3,594.26
	30	26.09	1,532.91	204.65	19.16	1,755.94	3,065.82	341.08	37.48	3,442.86
	40	24.67	1,448.10	198.70	17.65	1,663.40	2,896.19	331.17	34.53	3,259.84

Continued

		BAF 10 (2.296 metric)								
	0	39.18	1,647.74	1,554.20	1,324.80	4,526.74	3,295.49	2,590.33	2,592.01	8,477.82
	10	38.79	1,631.65	1,537.04	1,312.77	4,481.46	3,263.31	2,561.73	2,568.47	8,393.51
30	15	38.31	1,612.17	1,515.89	1,298.58	4,426.64	3,224.35	2,526.48	2,540.71	8,291.53
	20	37.68	1,584.82	1,488.72	1,280.61	4,354.16	3,169.64	2,481.21	2,505.54	8,156.39
	30	36.05	1,514.36	1,421.10	1,233.95	4,169.41	3,028.71	2,368.50	2,414.25	7,811.46
	40	34.10	1,426.47	1,355.52	1,166.51	3,948.51	2,852.94	2,259.20	2,282.31	7,394.46
		BAF 20 (4.592 metric)								
	0	39.02	1,658.75	1,503.40	1,342.44	4,504.59	3,317.51	2,505.66	2,626.51	8,449.68
	10	38.62	1,642.63	1,486.91	1,328.59	4,458.13	3,285.27	2,478.18	2,599.42	8,362.87
30	15	38.16	1,624.04	1,466.75	1,313.55	4,404.34	3,248.09	2,444.58	2,569.99	8,262.66
	20	37.55	1,599.68	1,440.28	1,294.36	4,334.32	3,199.36	2,400.47	2,532.44	8,132.28
	30	35.95	1,531.34	1,381.25	1,237.24	4,149.83	3,062.69	2,302.09	2,420.68	7,785.46
	40	33.99	1,438.21	1,331.95	1,157.96	3,927.80	2,876.42	2,219.91	2,265.57	7,361.31

At age 30, most likely because the trees are larger in diameter and greater in value, the impacts of projecting angles along the slope on basal area per hectare estimates are greater relative to age 20. For a 10 BAF (2.296 metric) on a 15% slope close to one tree every three points would be missed while on a 40% slope close to two trees would be missed every point, while for a 20 BAF (4.592 metric) on a 20% slope close to one tree every three points would be missed while on a 40% slope at least one tree every point would be missed. For slopes of 30% and 40%, and for the optimistic prices, percent reductions in average revenue per hectare ranged from 7.9% to 12.9%.

At ages 20 and 30, for the Optimistic prices and when using both 10 (2.296 metric) and 20 BAF (4.592 metric) s to project angles along a 20% slope, “missing” trees resulted in underestimating economic value close to \$144 per hectare and \$321 per hectare, respectively. The reductions ranged from 3.6% to 3.9% of the correct economic value at breast height. These economic under-estimates may not appear serious, but if a 30 yr-old stand is 20 hectares (50 acres), economic value could be underestimated by about \$6,500 (based on the yield tables used). Across 100 stands this would be \$650,000. Thus, since the simulated timber valuation/appraisal estimates are biased downward, consistently projecting assumed horizontal angles along 20% slopes across time, for example, can result in significant losses to the landowner and potentially gains to the buyer.

In actuality, most experienced foresters are aware of the impacts of substantial slope on variable-radius estimates. However, young foresters or other inexperienced foresters may not realize the seriousness of such measurement errors. Additionally, percent slopes are never constant across an entire tract. However, this study shows that in some cases, the impacts can be meaningful and hence foresters

should certainly take seriously the potential errors associated with not addressing slope.

Differences in stand-level estimates among the percent slopes occur 1) because some trees that should be sampled are not, due to projecting an angle along a particular percent slope, and 2) because for those trees that are sampled when projecting horizontal angles along slopes the corresponding trees per hectare expansion factor is incorrect. Due to smaller horizontal “imaginary” plots, and hence incorrect expansion factors since all expansion factors in the sample are based on assuming angles are projected on flat-ground, the expansion factors are smaller than what they should be based on the probability of sampling along sloping ground (resulting in underestimates). Recall that the concept of probability proportional to size (PPS) is that trees with a lower probability of inclusion, but when still sampled, implies there must be more of them in the sampled population relative to the larger trees with higher probabilities of being sampled. Thus, theoretically, this leads to greater expansion factors for those trees of lower probabilities of being sampled.

Figure 1 and **Figure 2** show that when projecting assumed horizontal angles along a 40% slope that the “imaginary” plot associated with a particular BAF is smaller than what it theoretically should be. Hence, the probability of sampling is less than the theoretical probability of sampling assuming a horizontal angle. Since the probability of sampling is reduced when projecting the angle along a 40% slope, some trees will be incorrectly excluded from being sampled and for the trees that are sampled the expansion factor per hectare should be greater - these are the errors.

Longleaf pine plantations were chosen because they are relatively easy to project into the virtual world. Other factors such as stand densities, plantation rectangularity, the presence of hardwoods or wildling pines, as well as the level of “clumpiness” in stands, will also impact sampling on slopes. Western Gulf, USA longleaf pine plantations are not commonly established along slopes as great as 40% but these high sloping grounds do exist. Certainly slopes of 15% and 20% more commonly occur. However, longleaf pine is likely more commonly established on higher slope sites in the Piedmont, Ridge and Valley, and Mountain provinces of the USA, e.g. “montane” longleaf pine (Varner et al., 2003). Results from this study can provide some inference about mixed-species longleaf pine stands and irregular terrain but the impacts of percent slopes on appraisal value in these stands will depend on the species mixtures, stumpage values of each species, and factors such as variability in slopes across the terrain.

Although high slopes are not as common in the Western Gulf, USA as in more mountainous regions of the World, it may in fact be a larger issue here because foresters may not know to account for slope or may not take the impacts of slope serious enough to account for it due to their relative rarity. The potential economic impacts will likely be greater in stands that contain a significant amount of sawtimber and veneer (e.g. age 30). Impacts will likely be even more serious if

stands contain a significant amount of relatively highly valuable utility poles, assuming these stands are sampled and are not inventoried using a 100% tally.

4. Conclusion

This present study demonstrates that failing to identify what trees to sample when conducting point sampling can have a substantial impact on the valuation/appraisal of stands. As percent slope increased timber valuation/appraisal estimates were consistently biased downward. These errors can lead to underbidding on timber tracts, can result in incorrect decisions about the economic feasibility of conducting various management practices, and can lead to making poor management decisions in general. For slopes up to around 15%, and particularly when conducting inventories of relatively low-value timber, the impacts are minimal. But as the timber becomes more valuable even on slopes of 15% or 20% the economic impacts can be meaningful.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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