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Design of a Device for Measuring the Thermal Diffusivity of Granular Materials Using the Method of Numerical Solution of the Heat Equation

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Abstract

In West Africa, controlling the thermal behavior of pavements is a major challenge. This study presents the design and calibration of an economical device for measuring the thermal diffusivity of granular materials. The method is based on the numerical resolution of the one-dimensional heat equation using temperature data acquired by three sensors. The device enables temperature to be measured as a function of time for a minimum duration of 3600 seconds per test. Calibration, carried out on dry gully sand, yielded an average thermal diffusivity value of 3.35×10^{-7} m^2/s, with a relative error of 5.4% compared with the documented reference value of 3.17×10^{-7} m^2/s.

Keywords

Pavement Base Course, Thermal Diffusivity, Granular Material, Temperature, Relative Error

1. Introduction

In tropical countries in general, and in West Africa in particular, the effects of solar radiation are one of the factors contributing to the deterioration of road infrastructure [1]. At certain times of the day, the temperature at the pavement surface peaks at between 63.60˚C and 69.30˚C, exceeding the laboratory reference of

60˚C. This temperature, stored in the surface layer, is transmitted throughout the structure by conduction, weakening the granular sub-base layers and sometimes causing cracks [2].

As a result, thermal diffusivity characterizes the rate at which heat propagates, by conduction, in a body, and becomes a critical parameter to pin down [3].

In 2017, Bendahir F. & Elfodda K. [4] determined the thermal diffusivity of a clay concrete and studied the influence of the degree of saturation on thermal diffusivity, noting that the thermal diffusivity of clay concrete increases as the degree of saturation increases. In 2017, Vianou *et al.* [5] determined the thermal diffusivity of cement-stabilized laterite using the transient thermal field and induced surface stress, assuming a constant temperature inside the samples. Thermal diffusivity values indicated the ability of stabilized laterite to provide thermal comfort in buildings. It should be noted that very few studies have been carried out and published on the thermal characteristics of laterite gravel in Benin, which means that its behavior under the effect of high temperatures in the base layers is not really under control. In fact, for its exploitation, we just check compliance with specifications based on identification tests for the most part, but the thermal aspect is not taken into account. According to Berraha Y. [6], a good knowledge of the thermal properties of the material is, in fact, important when dimensioning roads to cope with high temperatures, in order to limit degradation linked to the effects of heat.

With this in mind, it is becoming imperative to overcome the lack of suitable equipment in our laboratories, due to their very high acquisition cost. The aim of this study is to set up a device capable of acquiring data for calculating the thermal diffusivity of granular road materials.

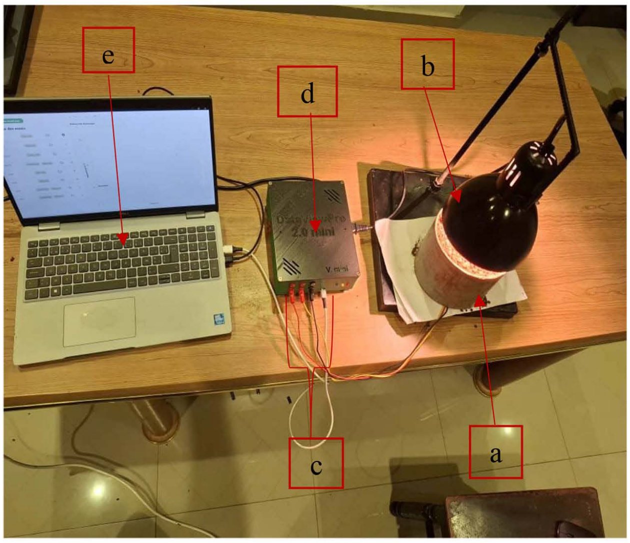
This equipment will be used to measure the transmission of heat flow through the various pavement base layers, in order to assess thermal diffusivity using the method of numerical resolution of the heat equation in the case of transient, one-dimensional heat flow proposed by Mrawira and Luca [7].

2. Experimental Set-Up

The data acquisition system used to determine thermal diffusivity consists of:

- A DataViewPro 2.0 mini system, an advanced temperature acquisition system designed to determine the thermal diffusivity of materials. It comprises a physical part for experimental measurements and a software part accessible via the computer to which it is connected [8].
- Three DS18B20 type temperature sensors with very negligible error and accuracy of ±0.5˚C for a measurement range from −55˚C to 125˚C. They transmit the observed temperatures to the software via a USB port. The software uses a Python script to store the data collected in a CSV (.csv) format file [8]:
  - A 200 W heating lamp (heating source);
  - A computer thermal reading unit, designed to read the temperature in ˚C;

The above elements are recorded in **Figure 1**.



Legend:

- a- CBR mould in PVC;
- b- Heat lamp;
- c- Three thermocouples;
- d- Acquisition box;
- e- One computer

Figure 1. Equipment for determining thermal diffusivity.

- **A drill bit (Figure 2)**: An instrument capable of drilling a straight vertical hole with a diameter as close as possible to that of the probe and to a depth at least equal to the length of the needle;
- Figure 2 shows a PVC cylinder with the following characteristics:
  - PVC wall thickness: 2.5 mm;
  - Cylinder height: 112 mm;
  - Cylinder diameter: 155 mm;

  giving a volume of 2114.2 cm^3 (same volume as CBR molds).

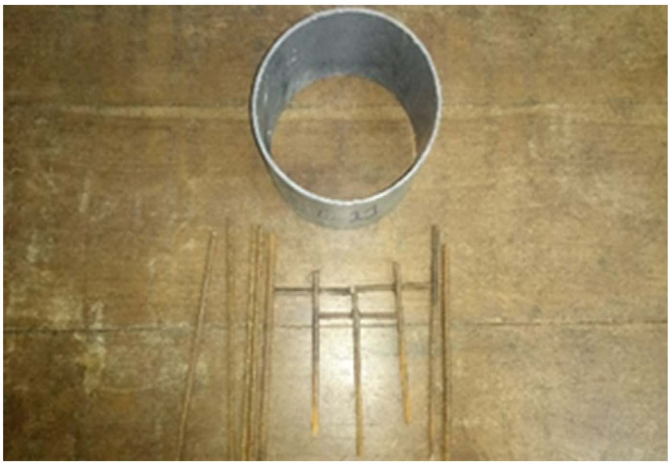


Figure 2. PVC mold and drilling equipment.

- Multimeter (Figure 3): Device capable of measuring voltage and current;
- **A stopwatch** for measuring operating times, to enable calculation of the times allocated to range operations;
- **A precision balance**, type ABDPRO with a capacity of 30 kg and accuracy of 1 g, in the counting balance category.

Figure 3. Multimeter.

3. Method

3.1. Experimental Procedure

The aim of this test method is to determine the thermal diffusivity of soils under quasi-steady-state conditions, using the finite-difference method. In chronological order, the experimental procedure is as follows:

- Prepare the thermocouples so that they are 21 mm apart;
- Position the cylindrical PVC mold;
- Compact the material in successive layers, for a total of 5 layers (applicable to coherent materials);
- Drill a hole of the same diameter as the thermocouple in the center of the specimen (applicable to coherent materials);
- Insert the thermocouples into the drilled hole (applicable to coherent materials);
- Turn the test piece over;
- Position thermocouples in empty mold from below (applicable to powdered materials);
- Fill cylindrical mold with material to be tested (applicable to powdered materials);
- Position the heating lamp 3 cm above the test tube;
- Connect thermocouples to acquisition box;
- Connect the acquisition system to the computer;
- Switch on the device by simultaneously switching on the lamp and the acquisition box;
- Simultaneously switch off the lamp and the acquisition box after one hour,
- Recover data;
- Process recovered data;
- Plot the temperature vs. time curves for the three thermocouples;
- Choose the time range to be considered for calculating thermal diffusivity, ensuring that there is a variation in temperature at the three measurement points at each instant of the range: *i.e.*, consider temperature values beyond a certain time;
- Choose a time step.

Recommendation: The choice of time step must be made in accordance with a stability condition specific to the method chosen to solve the problem posed. Stability is the property that ensures that the difference between the numerical solution obtained and the exact solution of the discretized equations is bounded [9]. For temporal evolution problems, some schemes are stable provided that the time step is less than a certain critical value depending on the space step. In the present case, this is written as:

$$\alpha \frac{\Delta t}{\Delta x^2} < 0.5 \tag{1}$$

- Calculate the thermal diffusivity, starting from the beginning of the time range under consideration and per chosen time step, as described below:

The calculation of thermal diffusivity is based on the solution of the heat equation in the one-dimensional case according to Mrawira and Luca [7]. The heat equation in the one-dimensional case is written:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, 0 \le x \le L, t \ge 0 \tag{2}$$

with $T = T(x,t)$ the temperature at time t and position x, and α the thermal diffusivity of the medium. Relation 1 is written for a semi-infinite medium of thickness L (**Figure 4**). Initial conditions (at $t = 0$) and boundary conditions (for $x = 0$ and $x = L$) are such that:

$$T(0,t) = T_0, T(L,t) = T_L, T(x,0) = f_0(x) \tag{3}$$

with $f_0(x)$ the initial relative temperature distribution as $f_0(x) = T(x,t) - T(x,0)$. Equation (2) is a partial differential equation whose numerical solution requires discretization of the study domain under consideration, in both time and space. For this purpose, it is assumed that the time interval is divided into a grid with a finite number of points and intervals (**Figure 4**). The solution is calculated only at these points. If we note Δt the time interval (assumed constant) of the temporal discretization grid, and Δx the space step (assumed constant) of the spatial discretization grid, then the calculation of the solution will take place at points $x = i\Delta x$ and $t = m\Delta t$ with $i = 1, 2, \cdots, N$ and $m = 1, 2, \cdots, MN$ et M being respectively the total number of space and time nodes.

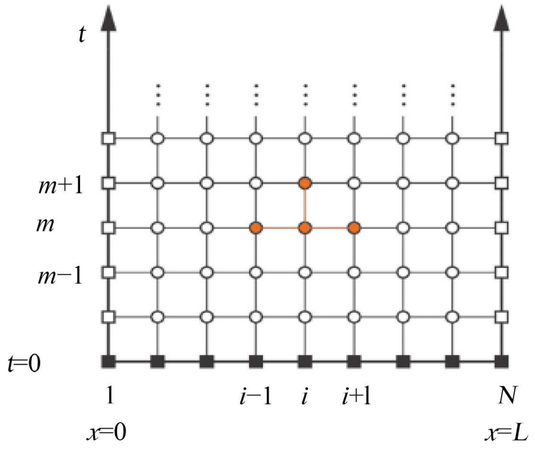


Figure 4. Mesh of the semi-infinite domain used to calculate the solution of the one-dimensional heat equation [10].

By replacing δx by the time step Δt in relation 2, and by the space step Δx in relation 2, then Equation (2) becomes:

$$\frac{T(x,t+\Delta t)+T(x,t)}{\Delta t}=\alpha\frac{T(x+\Delta x,t)-2T(x,t)+T(x-\Delta x,t)}{\Delta x^2} \tag{4}$$

Referring to **Figure 4**, if:

$T(x,t)=T_i^m \quad T(x,t+\Delta t)=T_i^{m+1}$

$T(x-\Delta x,t)=T_{i-1}^m \quad T(x+\Delta x,t)=T_{i+1}^m$

then thermal diffusivity is given by the following relationship:

$$\alpha=\frac{T_i^{m+1}-T_i^m}{T_{i-1}^m+T_{i+1}^m-2T_i^m}x\frac{\Delta x^2}{\Delta t} \tag{5}$$

This method takes into account certain physical and mechanical parameters such as density, water content, mineralogy, grain size and grain arrangement [11] [12].

Obtain the thermal diffusivity by averaging the different thermal diffusivities obtained.

- Determine the dry density of the material at the end of the test.
- Determine the water content at the end of the test.

Figure 5 shows the device used to measure thermal diffusivity.

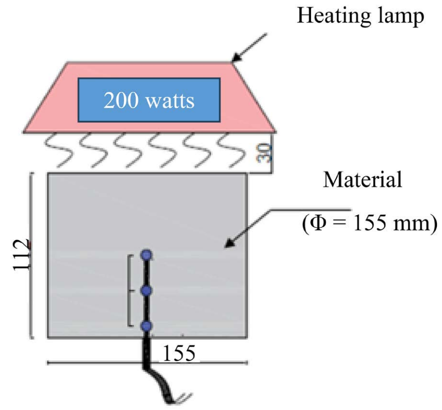


Figure 5. Schematic representation of the thermal diffusivity measurement method (Dimensions in millimeters).

3.2. Calibration

The aim of the calibration phase is to determine the correct configuration and materials to be used for the set-up [8]. Calibration tests were carried out on dry sand (gully sand) with a thermal diffusivity of 3.17×10^{-7} m²/s and a density of 1600 kg/m³.

The results obtained enabled us to evaluate and correct several parameters concerning the assembly, the most important being:

- Spacing between thermocouples;
- The position of the thermocouples along the cylinder's main axis;
- The spacing between the heating lamp and the top surface;

- The nature of the container (PVC).

The relative error $\delta\alpha_r$, defined by relation 6, is used to assess the accuracy of the measurement.

$$\delta\alpha_r = \left|\frac{\alpha_m - \alpha_c}{\alpha_c}\right| \tag{6}$$

The calibration phase also made it possible to determine the correct approach to take when analyzing the results.

In order to assess the accuracy of measurements obtained using thermocouples, calibration tests were carried out on dry gully sand. Once the gully sand had been sampled, it was placed in an oven and heated to 105 ± 5˚C for over 48 hours, in order to dry it to constant mass. The target thermal diffusivity is. 3.17 × 10^{-7} m^2/s.

Dry gully sand, being the material chosen as the reference for defining the relative error, was placed under the same conditions as the material under study.

4. Results and Discussion

4.1. Graphical Representation

Figure 6 shows the variation in temperature as a function of time for dry gully sand.

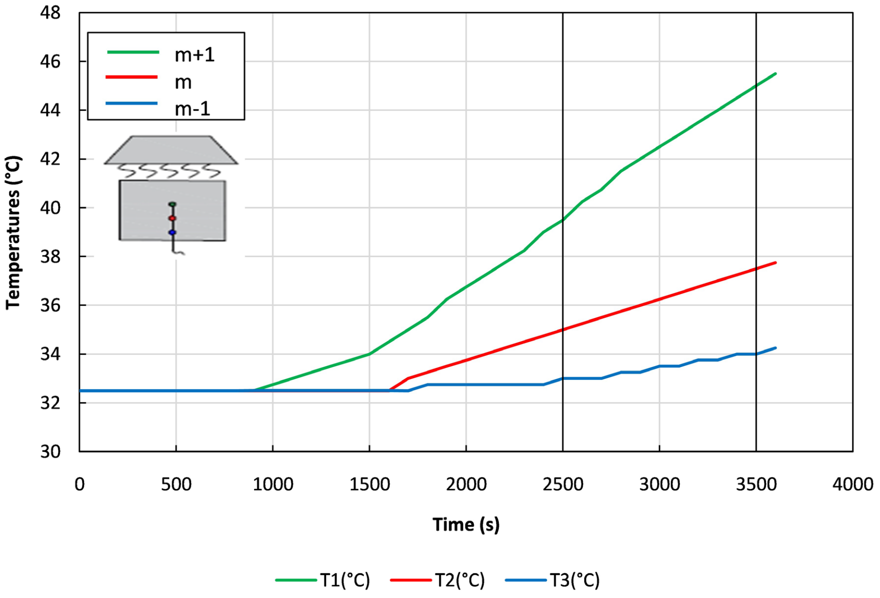


Figure 6. Temperature variation at three different depths as a function of time.

4.2. Diffusivity Calculation α

As described above, the discretization of space is given by the spacing between thermocouples. Hence, according to Figure 6, we have:

$$\Delta x = 21 \times 10^{-3}\ \text{m}$$

Thus, the condition expressed by equation 1 in §3.1 is verified in all the tests carried out as part of this research project. The choice of time step was therefore made by considering the evolution of the relative error of calibration test results as a function of time step, and the value chosen is:

$$\Delta t = 100\ \text{s}$$

Since the calculation of thermal diffusivity requires a variation in temperature at all three measurement points, it is natural to consider temperature values only after a certain time has elapsed. According to Mrawira and Luca [7], the observation period to be used for thermal diffusivity evaluation is determined from the curves, which must be included in the ascending range common to all three curves. It is delimited on **Figure 6** by the two vertical segments. This portion of the curve begins at $t_d = 2500\ \text{s}$ and ends at $t_f = 3500\ \text{s}$.

Given the delicacy of the temporary range to be retained when evaluating a material's thermophysical parameter, Gustafsson [13] explains that for the theoretical model to be valid, the duration of the experiment must be chosen carefully. Thus, the time must be long enough:

- Long enough for the heat to have had time to penetrate the material significantly, so as to minimize the influence of thermal contact resistance between the heating probe and the sample. This contact effect is predominant at very short times.
- This contact effect predominates at very short times, so that the heat flow is not disrupted by edge effects, *i.e.*, the heat has not yet reached the sample boundaries.

This double constraint means we must select a "time window" for analysis that excludes the very first instants (dominated by contacts) and longer times (where the semi-infinite model is no longer valid).

Let's take a look at $t = 2800\ \text{s}$, then we have:

$$T(x,t) = T(m,2800) = 35.75^\circ\text{C}$$

$$T(x,t+\Delta t) = T(m,2900) = 36^\circ\text{C}$$

$$T(x-\Delta x,2800) = T(m-1,2800) = 33.25^\circ\text{C}$$

$$T(x+\Delta x,2800) = T(m+1,2800) = 41.5^\circ\text{C}$$

Thus, we have:

$$\frac{36-35.75}{100} = \alpha\frac{41.5-2\times35.75+33.25}{\left(21\times10^{-3}\right)^2}$$

$$\alpha = \frac{36-35.75}{41.5-2\times35.75+33.25}\times\frac{\left(21\times10^{-3}\right)^2}{100}$$

$$\alpha = 3.39\times10^{-7}\ \text{m}^2/\text{s}$$

Keeping the same time step, the previous calculation can be repeated at different positions along the time axis. In our case, a calculation is performed every 100 seconds (let's note this translation time t_{dt}). In other words, at:

- $t = 2500\text{ s}$ $\alpha = 4.41\times10^{-7}\text{ m}^2/\text{s}$;
- $t = 2600\text{ s}$ $\alpha = 4.01\times10^{-7}\text{ m}^2/\text{s}$
- $t = 2700\text{ s}$ $\alpha = 4.01\times10^{-7}\text{ m}^2/\text{s}$
- $t = 2800\text{ s}$ $\alpha = 3.39\times10^{-7}\text{ m}^2/\text{s}$
- $t = 2900\text{ s}$ $\alpha = 3.39\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3000\text{ s}$ $\alpha = 3.15\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3100\text{ s}$ $\alpha = 3.15\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3200\text{ s}$ $\alpha = 2.94\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3300\text{ s}$ $\alpha = 2.94\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3400\text{ s}$ $\alpha = 2.75\times10^{-7}\text{ m}^2/\text{s}$
- $t = 3500\text{ s}$ $\alpha = 2.75\times10^{-7}\text{ m}^2/\text{s}$

These results are shown in Figure 7, which also shows the target value 3.17 × 10^{-7} m^2/s and the arithmetic mean of the values obtained. This figure, therefore, shows the thermal diffusivity values calculated over the time interval chosen previously $\left(\left[t_d;t_f\right]\right)$ with a space between each measurement, which is $t_{dt} = 100\text{ s}$.

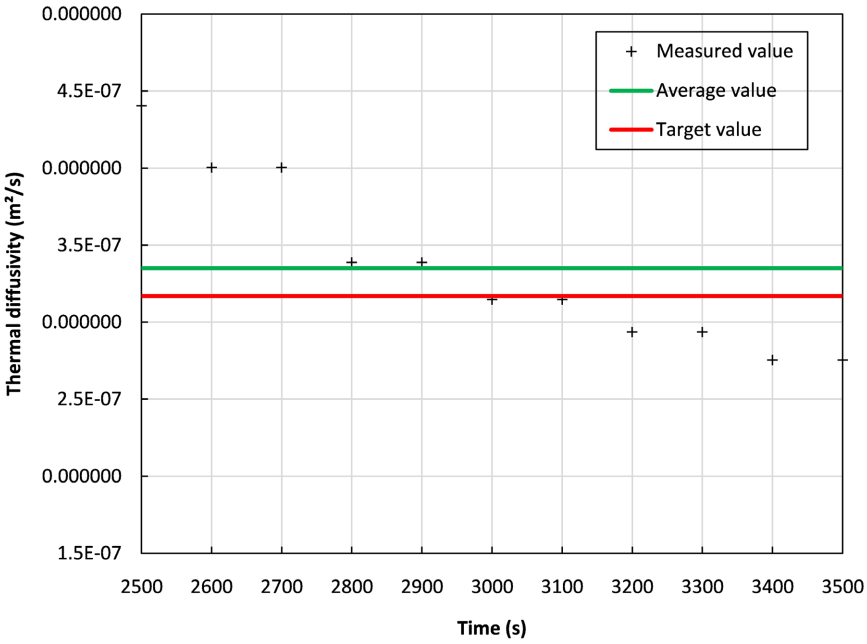


Figure 7. Variation of thermal diffusivity as a function of time.

In summary, the parameters chosen in the example calculation above are:

$\Delta x = 21\times10^{-3}\text{ m}$, $t_d = 2500\text{ s}$, $t_f = 3500\text{ s}$, $\Delta t = 100\text{ s}$, $t_{td} = 100\text{ s}$.

The mean thermal diffusivity value obtained is 3.35×10^{-7} m²/s, giving a relative error $\delta\alpha_r$ of 5.4%. The relative error obtained lies within the range 3.2% to 5.9% determined by Hay *et al.* [14]. In this study, Hay *et al.* [14] showed that it is difficult to obtain relative errors below 3% when measuring the thermal diffusivity of opaque, homogeneous, and isotropic materials using the partial time-moment method. It is generally accepted that measurements are accurate when the relative error is less than 10%.

5. Conclusions

The present study has enabled us to develop a device for measuring the thermal diffusivity of unbound granular materials. It is a simple and highly practical device for laboratory and in situ testing. The device consists of an acquisition box, a 200 watts lamp, three temperature sensors and a computer. It is essential to emphasize that the present study constitutes the first stage in the validation of our device, focusing on its calibration with a reference material, dry sand. This has enabled us to verify the accuracy of the device, which has a relative error of 5.4%. The device cost 400,000 CFA francs (around US$690). Equipment marketed by manufacturers via Ali Baba costs between US$6200 and US$9000, *i.e.*, at least 9 times the cost of the present device. As recommended by Houanou *et al.* [8], the DS18B20 sensors need to be checked or recalibrated periodically, as despite their good accuracy (±0.5˚C within the specified range) they are subject to self-heating, leading to a possible time lag.

The system thus set up will be used to provide the thermal diffusivity data required for thermomechanical modelling of granular pavements, enabling their design to be optimized by incorporating the high thermal stresses typical of tropical climates.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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