Open Journal of Applied Sciences, 2026, 16(5), 1901-1915
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Research on Stock Volatility Based on Investor Sentiment and Two-Dimensional Ising Model

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How to cite this paper: Zhang, J.Y. and Mu, N.G. (2026) Research on Stock Volatility Based on Investor Sentiment and Two-Dimensional Ising Model. *Open Journal of Applied Sciences*, **16**, 1901-1915.
https://doi.org/10.4236/ojapps.2026.165105

Received: April 10, 2026
Accepted: May 24, 2026
Published: May 27, 2026

Abstract

Integrating the two-dimensional Ising model with behavioral finance theory, this study uses investor sentiment to explain financial-market volatility. Specifically, spin flips in the Ising model are employed to simulate investors' buy-sell switching behavior in financial markets, thereby constructing a quantifiable investor-sentiment variable, which is then embedded in a GARCH-X model to analyze stock-market volatility. The empirical results show that the price data simulated by the Ising model successfully reproduce typical statistical properties of financial markets, including sharp peaks, fat tails, volatility clustering, and long memory. Meanwhile, during periods of elevated investor sentiment, the fat-tail effect becomes significantly stronger. Compared with the benchmark GARCH model, the GARCH-X model that incorporates the sentiment variable exhibits a markedly better goodness of fit, confirming that investor sentiment has significant explanatory power for volatility. These findings indicate that investor sentiment drives changes in stock volatility and provide a new perspective for volatility research in financial markets.

Keywords

Two-Dimensional Ising Model, Volatility, Econophysics, Behavioral Finance

1. Introduction

Financial markets are a key sector of the national economy, and research on financial-market volatility has gradually become one of the central topics in finance. Vagif and Rustamov [1] pointed out that volatility is not only a crucial parameter for risk control but also a key clue for identifying market opportunities. At present, traditional research mainly relies on econometric methods for forecasting, such as ARCH- and GARCH-type models [2] [3]. Deep learning methods are also

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increasingly used to improve predictive performance [4] [5].

However, traditional models display clear limitations in predicting extreme events such as the 2008 subprime mortgage crisis and the 2020 U.S. stock-market circuit breakers. The root cause is that they do not adequately characterize the impact of investors' aggregate trading behavior—the foundation of the market—on market volatility. This paper uses an agent-based Ising-model system to characterize the market effect generated by investors' trading behavior, namely investor sentiment, and projects this effect onto the irrational component of market volatility [6]. Such cross-scale research remains challenging for much of modern finance built on non-systemic approaches.

Many studies have introduced the perspective of behavioral finance, emphasizing the important influence of investors' psychological factors on stock-market volatility [7]-[9]. For example, Akin and Akin [10] examined the impact of behavioral finance on U.S. stock-market volatility and revealed the link between investor behavior and stock-market volatility behind market anomalies. This suggests that behavioral finance offers a possible explanation for market fluctuations by identifying investors' psychological biases and market anomalies. Although the explanatory role of behavioral finance in volatility has been recognized to some extent, some scholars argue that integrating behavioral finance with traditional financial theory remains difficult [11].

Against this background, this paper develops a system-modeling approach with investor networks as the micro-foundation. In addition to endogenously characterizing market phenomena such as herding, bubbles, and crashes [12] [13], the resulting composite model also makes it possible to explore the irrational component of market volatility [14]. In recent years, scholars have begun to combine behavioral-finance concepts and theories with the Ising model [15]-[18] to explain financial-market "anomalies." Fang *et al.* [15], for instance, even used a three-dimensional Ising model to study stock-market volatility and its underlying mechanism. In domestic research on China's financial markets, one-dimensional models have been used to simulate return characteristics [19]-[21], but in-depth discussion of volatility based on a stochastic two-dimensional Ising model remains rare [22]. By using a two-dimensional Ising model, this paper better captures the dynamic characteristics of complex investor networks and offers a useful attempt to explain behavioral-finance anomalies and model volatility.

The remainder of this paper is organized as follows. Section 1 introduces the financial-market model and the construction of the sentiment variable. Section 2 presents the modeling results and simulation analysis. Section 3 evaluates the performance of the GARCH-X specification. Section 4 concludes and discusses future research directions.

2. Financial Market Modeling

2.1. Simulating Market Behavior at the Microscopic Scale: The Ising Model

In this study, the financial market is viewed as a complex system composed of investors. A stochastic two-dimensional Ising model is used to construct the investor network and to characterize how interactions among investors affect stock-price volatility. In this model, each investor is mapped onto a spin on a two-dimensional lattice of size $L \times L$, yielding L^2 spins in total, and each spin is emotionally coupled only with its four nearest neighbors (up, down, left, and right).

The total energy of the system is described by the following Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} s_i s_j - h_t \sum_i s_i$$

where $s_i \in \{-1, +1\}$ denotes the decision state of investor *i*, with $s_i = +1$ indicating a buy decision and $s_i = -1$ indicating a sell decision; $J > 0$ denotes the interaction strength among investors and sets the scale of system energy and temperature, following Fang *et al.* [15]; $\langle i, j \rangle$ denotes nearest-neighbor sites. *h* represents the external field. In the first-stage experiment, h_t is set to 0 in order to focus on the endogenous mechanism through which investor interactions affect prices [18]. In the second stage of the real-market empirical experiment, to capture the feedback effect of historical prices on investor sentiment, we further introduce a dynamic external field based on historical returns, specified as:

$$h_t = \gamma_m r_{t-1}$$

where h_t is the dynamic external field at time *t*; $\gamma_m > 0$ is the external field sensitivity coefficient, which calibrates the impact intensity of historical returns on investor decisions; based on historical returns, in which the return r_{t-1} is the log return of the HS300 Index at time $t-1$.

The probability of a given spin configuration is given by the Boltzmann distribution:

$$P(\{s_i\}) = Z^{-1} \mathrm{e}^{-\beta H}, \ \beta = \frac{1}{k_B T}$$

where $Z = \sum_{\{s_i\}} \mathrm{e}^{-\beta H}$ is the partition function, k_B is the Boltzmann constant, and T is the system temperature, which links investor sentiment fluctuations by governing the competition between energy and entropy. The critical temperature $T_c = \frac{2J}{k_B \ln(1+\sqrt{2})} \approx \frac{2.26918J}{k_B}$ is the point at which the system undergoes a phase transition [23].

The degree of investor irrationality is defined by the inverse temperature $\beta = \frac{1}{k_B T}$. When the system is at low temperature ($\beta \gg 1$), low-energy configurations dominate, investor sentiment becomes highly excited, and irrationality rises; investors are therefore more likely to be influenced by their neighbors and to herd blindly, generating extreme states such as market bubbles or crashes. When the system is at high temperature ($\beta \ll 1$), the probabilities of different configurations become similar, investor sentiment is more stable, rational investors dominate, decisions become more independent, and the market remains in a state of normal fluctuation in which prices follow a random walk.

This paper uses the Metropolis algorithm [13] [15] [24] to conduct Monte Carlo simulation of the two-dimensional Ising model. In the numerical simulation, the acceptance probability for spin flipping is:

$$P_{\text{accept}} = \min\left(1, \mathrm{e}^{-\beta \Delta E}\right)$$

where ΔE denotes the energy change induced by a spin flip. At low temperature (large β), only energetically favorable flips are likely to be accepted, so spins tend to stabilize and form an ordered structure. By contrast, at high temperature (small β), flips are easily accepted, spins change frequently, and the system becomes disordered. Thus, temperature T directly controls the flexibility of spin changes.

2.2. Sentiment-Driven Price Formation Mechanism

Based on the above sentiment-evolution process and drawing on the logic of composite sentiment-index construction in behavioral finance [25], this paper constructs a investor-sentiment variable for the system. In essence, it quantifies the degree of consensus in investors' opinions and thus represents the consistency of aggregate sentiment:

$$m_t = \frac{1}{L^2} \sum_{i,j} s_{ij}$$

Based on the spin configuration, we define the trading imbalance variable $L(t)$, which quantifies the difference between long and short forces in the market at time t, with the exact calculation formula:

$$L(t) = \left|L_+(t) - L_-(t)\right| = \left|\sum_{i=1}^{L} s_i(t)\right|$$

where $L_+(t)$ is the number of investors with spin state $s_i = +1$ (buy decision) at time t, $L_-(t)$ is the number of investors with spin state $s_i = -1$ (sell decision) at time t, and $L = L_+(t) - L_-(t)$ is the total number of investors in the market. $L(t)$ ranges from 0 to N, with a larger value indicating a more severe imbalance between long and short forces and stronger consistency in investors' trading behavior.

When the absolute value of the sentiment variable is large, investor sentiment is highly synchronized and market investment behavior becomes strongly aligned; when it is close to zero, market sentiment is dispersed and bullish and bearish forces are broadly balanced.

Building on the Metropolis algorithm, this study follows the Ising-model price-evolution framework proposed by Lan and Fang [18], incorporates the sentiment variable into the model, and establishes a stock-price dynamics model at time t so as to characterize the effect of sentiment on prices more precisely. The specific form is:

$$S_t = \mathrm{e}^{\gamma_p \xi(t)|m_t|} S_{t-1}$$

$$S_t = S_0 \cdot \exp\left(\sum_{k=1}^{t} \gamma_p \xi(t)|m_t|\right)$$

where S_t is the price at time t, S_0 is the initial price, m_t is the investor-sentiment variable, and $\xi(t) \in \{-1, +1\}$ is the random news-shock term, with $\xi(t) = +1$ indicating favorable news and negative values unfavorable news. $\gamma_p > 0$ is the market-depth parameter used to control the magnitude of daily return changes. To reflect China's daily price-limit mechanism, $\gamma_p = 0.1$ is set so that the simulated return fluctuations fall within the corresponding range. By mapping microscopic interactions onto macroscopic prices, the model effectively captures the price dynamics induced by sentiment transmission.

Accordingly, the logarithmic return of the stock calculated by the Ising model at time t is given by:

$$r_t^{\text{sim}} = \ln\left(\frac{S_t}{S_{t-1}}\right) = \gamma_p \xi(t)|m_t|$$

2.3. Macroscopic Volatility Modeling: The GARCH-X Model

The volatility of asset returns characterizes short-term uncertainty and risk, and particularly reflects price turbulence caused by the behavior of market participants. Traditional ARCH family models have shown good statistical forecasting performance in volatility modeling. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, proposed by Bollerslev (1986) [26], is the most important extension of the ARCH family model. A large body of existing financial empirical research shows that the GARCH (1, 1) model is sufficient to characterize the volatility clustering and heteroskedasticity of most stock return series, and is the most concise and robust benchmark specification for financial volatility modeling. Therefore, this paper selects GARCH (1, 1) as the benchmark model, with the specific form:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sigma_t \cdot z_t \\ \sigma_t^2 &= \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 \end{aligned}$$

where r_t is the logarithmic return at time t, μ is the mean return term, ε_t is the residual term at time t, σ_t is the conditional volatility at time t, z_t is the standardized residual, and the remaining terms ω, α, β are GARCH-model parameters.

To further quantify the dynamic influence of investor sentiment on volatility, the sentiment variable constructed from the Ising model is introduced into the GARCH model, yielding the behaviorally enhanced GARCH-X specification:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sigma_t \cdot z_t \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma\, m_{t-1} \end{aligned}$$

The sentiment variable m_t characterizes the degree of consistency in market sentiment and reflects the driving effect of microscopic investor sentiment on macroscopic volatility. This is an endogenous, self-organizing dynamic process that helps reveal the internal mechanism behind market fluctuations and enhances the explanatory power of the model.

3. Experimental Design and Simulation Results of the Ising Financial Model

3.1. Experimental Design and Analytical Methods

To ensure comparability between the simulation results and the empirical data, this study sets the trading horizon of the simulated system to match that of the empirical sample, and then compares statistical characteristics and model-fitting performance on that basis. The real-market data consist of daily observations for the CSI300 Index from January 2, 2014 to December 31, 2024, comprising 2675 valid trading days. The actual return series of the CSI300 Index is defined as:

$$r_t^{\text{real}} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

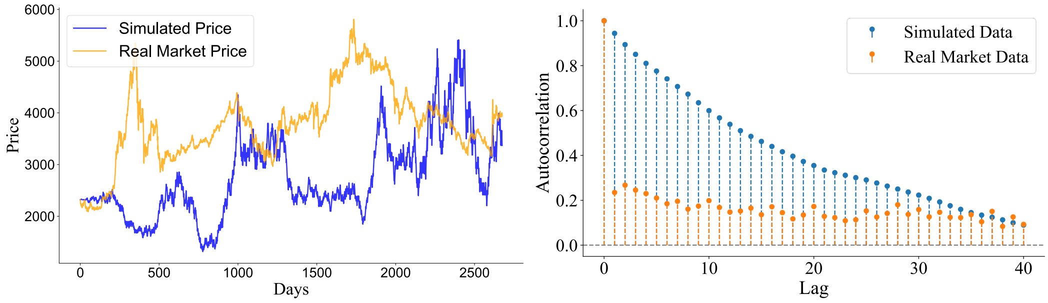
In the model setting, each trading day is divided into 240 time steps, corresponding to the scenario where the daily trading time in the real Chinese stock market is 4 hours. It is assumed that investors transact once per minute. At each minute step, all participants undergo one Monte Carlo update, including random site selection, calculation of the energy change, and spin flipping according to the Metropolis criterion. The daily price change is computed only after all 240 minute-level updates for a given trading day have been completed.

3.2. Simulation Analysis of the Ising Model

To verify whether the Ising model can spontaneously generate financial-market features in the absence of external feedback, this section provides a visual analysis of the statistical properties of the prices and returns generated by the model, as well as the nonlinear relationship between investor sentiment and stock prices.

3.2.1. Statistical Properties of Prices and Returns Generated by the Ising Model

Figure 1 compares the data generated by the Ising model with those of the CSI 300 Index. First, in Figure 1(a), although the two price patCSI do not coincide exactly, they are highly consistent in structural characteristics: the simulated path exhibits typical nonlinear volatility, trend reversals, and multi-stage peaks, and is macroscopically similar to the real market, indicating that even without introducing actual price feedback, the model is capable of generating a realistic-market-like price series. Second, the comparison of the autocorrelation function (ACF) of the absolute return series reveals volatility clustering, meaning that high-volatility periods tend to be followed by high-volatility periods and low-volatility periods tend to be followed by low-volatility periods. Figure 1(b) shows that the ACF curve of the simulated data decays slowly rather than dropping back to zero immediately, indicating strong volatility memory. The ACF of the real market shows a similar pattern, though with slightly lower values, suggesting that market volatility also exhibits clustering but contains stronger background noise and nonstructural components. In particular, at lags 1 - 10, the correlation structure of the simulated data is highly consistent with that of the real data, indicating that the model can capture short-run volatility inertia.



(a) (b)

Figure 1. Comparison between simulated data from the Ising model and real stock-market data. (a) Price path comparison; (b) Comparison of the ACF of the absolute return series.

To further verify whether the model reproduces the long-memory characteristics commonly observed in real market price fluctuations, this paper calculates the Hurst exponent and the detrended fluctuation analysis (DFA) exponent for the simulated price series and the real-market return series. The results are reported in **Table 1**.

Table 1. Long-memory indicators of simulated data and real-market return series.

Data source	Hurst exponent	DFA exponent
Ising-model simulated data	0.556	0.508
Real-market data	0.498	0.533

The Hurst exponent is a classic indicator used to characterize whether a time series exhibits long memory. $H > 0.5$ indicates persistence, meaning that the current trend is likely to continue in the future; $H < 0.5$ indicates anti-persistence, meaning that the series is prone to reversal; and $H = 0.5$ corresponds to a pure random walk. **Table 1** shows that the Hurst exponent of the simulated data is 0.556, indicating mild persistence, which is close to the value of 0.498 for the real-market data. This suggests that even in the absence of feedback from the external field, the model can still generate volatility behavior with a certain degree of trend persistence.

The DFA exponent does not directly assume stationarity and is more robust than the traditional Hurst estimate, especially for real financial data with trends or noise. When $\alpha > 0.5$, the series exhibits long memory; the closer the value is to 1, the stronger the memory. In this study, the DFA exponents of both the simulated data and the real-market data are greater than 0.5, indicating that the model successfully reproduces the long-range correlation characteristics of real financial markets and confirming that collective behavior induced by sentiment transmission has persistent effects.

Taken together, the Hurst and DFA results indicate that the Ising model can dynamically reproduce the long-memory characteristics of real-market price fluctuations.

3.2.2. Nonlinear Relationship between Investor Sentiment and Stock Prices

Figure 2 shows the evolution of the sentiment variable m_t simulated by the Ising model. The series fluctuates mainly around zero and exhibits upward or downward deviations in several stages, reflecting bullish or bearish sentiment tendencies among market participants during different periods, which in turn drive corresponding price fluctuations. This collective behavioral deviation is precisely the modeled manifestation of herding and trend-following behavior in financial markets.

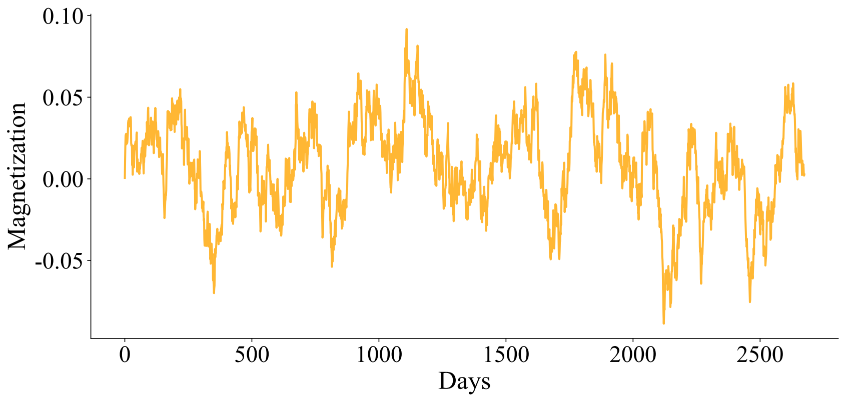


Figure 2. Magnetization series simulated by the Ising model.

Figure 3 presents the evolution of spin configurations under different degrees of investor irrationality. Yellow spins denote $s_i = +1$, meaning that investors are bullish and choose to buy, whereas blue spins denote $s_i = -1$, meaning that investors are bearish and choose to sell. The figure clearly shows that when the degree of investor irrationality $\beta < 0.44$, the numbers of buyers and sellers are relatively balanced and spins of the same color do not form large clusters. As irrationality approaches the critical point $\beta = 0.44$, same-color spins begin to cluster and a phase transition emerges. When irrationality $\beta > 0.44$, the system forms multiple clusters with the same orientation, and the aggregation of same-color spins becomes pronounced. Thus, the critical point $\beta_c \approx 0.44$ serves as the system threshold near which local investor structures begin to display sentiment convergence. The larger the degree of investor irrationality, the more elevated investor sentiment becomes, the easier it is for investors to herd, and the stronger the clustering effect.

Figure 4 shows the nonlinear relationship between stock trading imbalance $L(t)$ and the parameter β representing the degree of investor irrationality. As shown in the figure, in the subcritical region $\beta < \beta_c \approx 0.44$ that below the critical point, trading imbalance remains low and changes smoothly, indicating that the market as a whole stays in a relatively rational equilibrium state and that supply and demand tend to be stable. However, as investor irrationality increases, stock trading imbalance displays a clear nonlinear growth pattern. As the system approaches the critical point, the imbalance exhibits an accelerating accumulation effect, and its growth becomes significantly stronger than in the preceding phase. Once the system enters the supercritical region $\beta > \beta_c \approx 0.44$ that above the critical point, the imbalance rises rapidly and displays a typical phase-transition feature—that is, the system undergoes an abrupt transition from order to disorder.

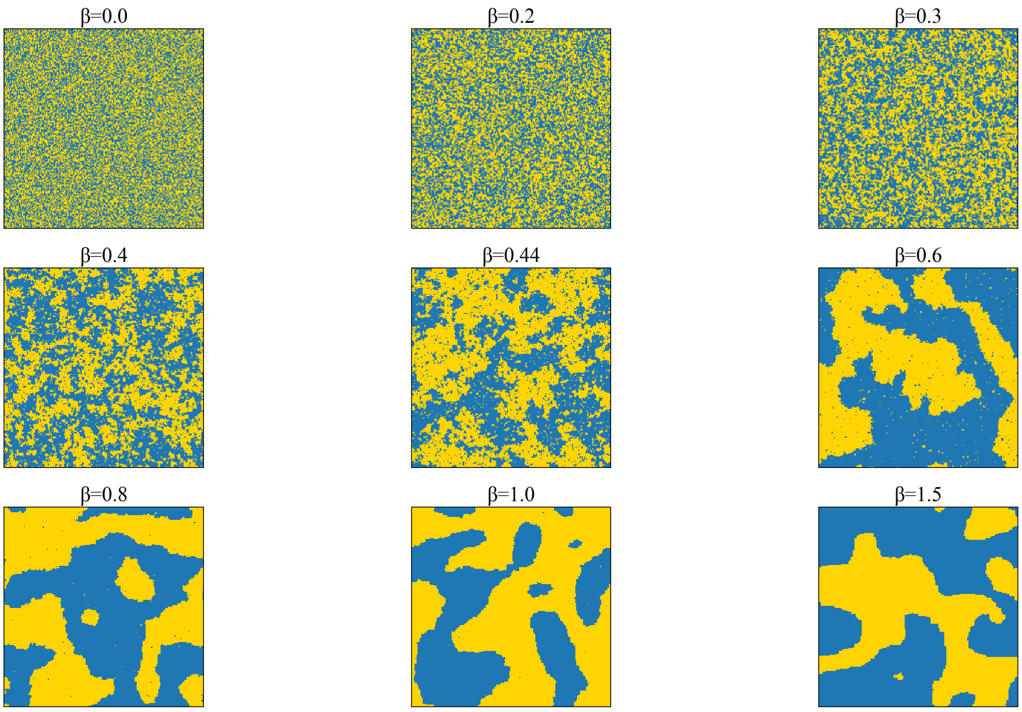


Figure 3. Evolution of spin configurations under different degrees of investor irrationality.

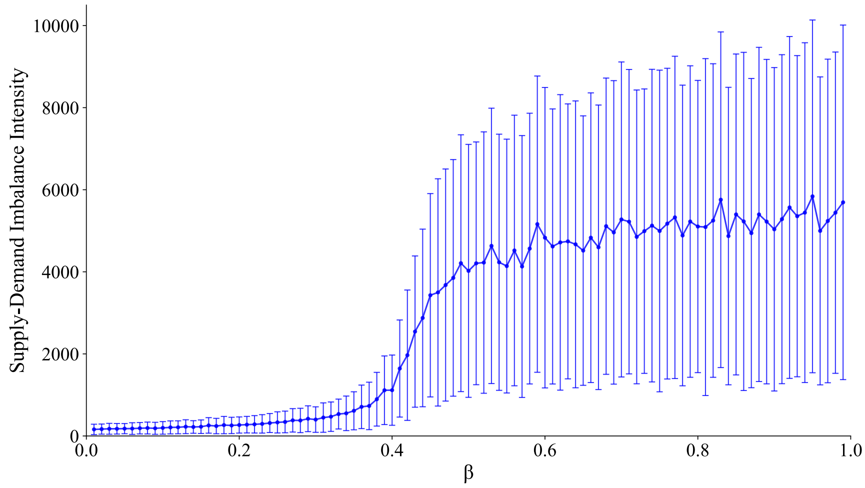


Figure 4. Relationship between stock trading imbalance and the degree of investor irrationality.

This critical point $\beta_c \approx 0.44$ can be regarded as the threshold at which the market shifts from rational dominance to irrational dominance. Near the threshold, investors' emotional behavior triggers a positive-feedback mechanism that intensifies collective trend chasing and panic selling, thereby inducing systemic imbalance. Notably, in the supercritical region, the imbalance continues to rise and its volatility increases significantly, indicating that under a highly irrational environment the market becomes more imbalanced and far less stable, making extreme market episodes or sharp price turbulence more likely. This phenomenon is highly consistent with the critical phenomenon in complex-systems theory and suggests that the microstructure of financial markets may exhibit phase-transition characteristics similar to those of physical systems: small disturbances in investor irrationality can be amplified near the critical point into a violent response of market structure, thereby revealing a potential source of market fragility.

Figure 5 compares the probability density functions of logarithmic returns under different degrees of investor irrationality with the Gaussian distribution. Because the scale changes discontinuously across the critical point, the results are displayed separately in **Figure 5(a)** and **Figure 5(b)** for visual clarity, with the critical point $\beta_c \approx 0.44$ repeated in both panels to connect the phase-transition process. As shown in the figure, at the critical point $\beta_c \approx 0.44$ the probability density function displays both a sharp peak and fat tails, which is highly consistent with the statistical regularities of real markets. When investor irrationality is low ($\beta < 0.44$), the distribution is sharply peaked but has relatively thin tails, implying a low probability of extreme events. When irrationality is high ($\beta > 0.44$), the curve exhibits fat tails but lower kurtosis, and the concentration of volatility weakens.

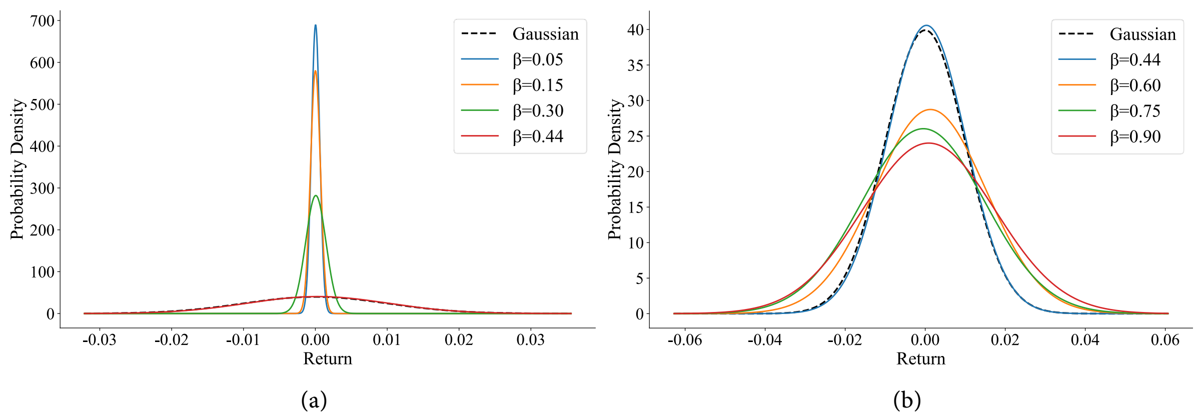


Figure 5. Comparison of probability density functions of logarithmic returns simulated under different degrees of investor irrationality. (a) Probability density function of $\beta \le 0.44$; (b) Probability density function of $\beta \ge 0.44$.

This indicates that the critical point $\beta_c \approx 0.44$ is the key threshold at which the system shifts from rationality-dominated ordered fluctuations to irrationality-dominated disordered fluctuations. Through a positive-feedback mechanism, interactions among investors dynamically couple volatility concentration with the probability of extreme events, thereby accurately reproducing the coexistence of sharp peaks and fat tails observed in real markets. By contrast, regions away from the critical point display only a single characteristic and cannot match the complex volatility pattern of real markets.

Figure 6 further shows the evolution of the complementary cumulative distribution function (CCDF) of absolute returns under different degrees of investor irrationality. The vertical axis of the CCDF represents the probability that fluctuations exceed a given magnitude. If the tail of the CCDF curve is high, large fluctuations are more likely to occur, implying higher market risk. The figure shows that as investor irrationality increases from 0.05 to 0.35 within the subcritical region, the tail of the distribution remains relatively low in $\left(10^{-3}, 10^{-2}\right)$. Once the critical point $\beta_c \approx 0.44$ is reached, however, the tail rises abruptly, indicating an exponential increase in the probability of extreme returns. This confirms that when investors are in a highly irrational state—namely, during periods of elevated sentiment—the market's fat-tail effect is significantly strengthened.

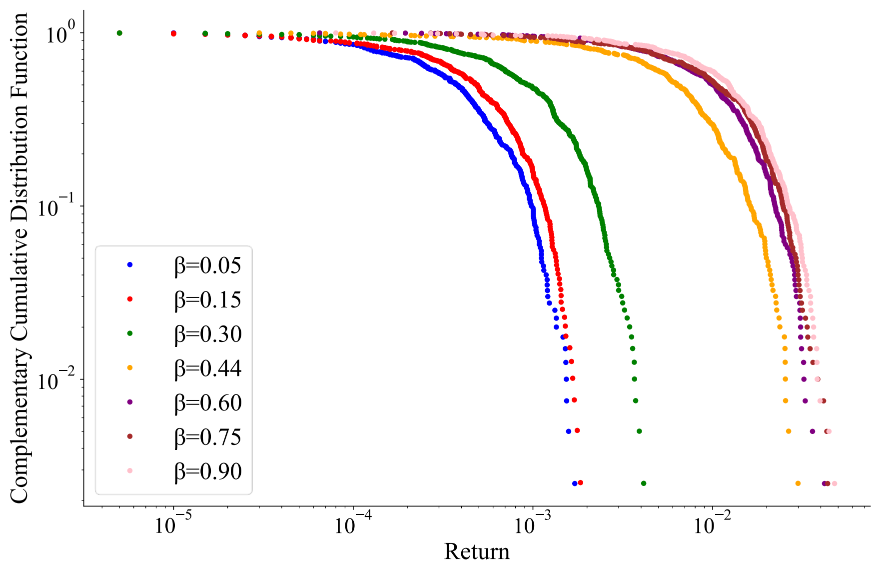


Figure 6. CCDF of absolute returns simulated under different degrees of investor irrationality.

4. Verification of GARCH-X Modeling Performance

4.1. GARCH-X Fit Based on Simulated Data

To further verify the validity of the sentiment variable under the endogenous mechanism of the model, this paper first constructs logarithmic returns r_t^{sim} from the price series simulated by the Ising model and extracts the corresponding magnetization series as the sentiment variable m_t. It then estimates the standard GARCH (1, 1) model and the GARCH-X model proposed in this study, and compares them using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC), so as to test whether the sentiment variable possesses explanatory power in a closed system. The results are reported in **Table 2**.

Table 2. Model-fitting analysis based on simulated data.

Model	AIC	BIC
GARCH (1, 1)	−7083.50	−7059.93
GARCH-X	−7193.46	−7164.00

The results show that the GARCH-X model that incorporates the sentiment variable has lower AIC (−7193.46) and BIC (−7164.00) values. Relative to the GARCH (1, 1) model, the AIC decreases by about 109.96 and the BIC by about 104.07, both of which constitute very strong evidence in favor of the augmented model. Therefore, the GARCH-X model outperforms the conventional GARCH (1, 1) model on both information criteria and exhibits better overall fitting performance, indicating that the sentiment variable has explanatory power in a closed system.

4.2. Empirical GARCH-X Analysis Based on Real-Market Data

To verify the effectiveness of the sentiment variable constructed by the Ising model in the real market, this paper uses the daily closing-price data of the CSI 300 Index (2675 observations) to construct a logarithmic return series, which is then used as the external-field feedback to drive the Ising model and generate a synchronized sentiment series. Subsequently, the standard GARCH (1, 1) model and the GARCH-X model containing the sentiment variable are estimated separately to test their explanatory power in volatility modeling. The results are presented in Table 3.

Table 3. Model-fitting analysis based on real-market data.

Model	AIC	BIC
GARCH (1, 1)	−16324.49	−16300.92
GARCH-X	−16334.83	−16305.37

The empirical results show that, relative to the benchmark model, the AIC declines by about 10.34 (strong evidence) and the BIC by about 4.45 (moderate evidence), indicating that the sentiment variable also has significant explanatory power for volatility in the real-market environment and can improve the overall fit of the model.

4.3. Comparative Analysis of Models and Mechanism Transfer

Taken together, the results from the purely simulated stage and the real-market-driven stage show that after incorporating the sentiment variable generated by the Ising model, the GARCH-X model outperforms the benchmark GARCH (1, 1) model in both environments.

In the purely simulated stage, prices are driven entirely by the internal mechanism of the Ising system, and the sentiment variable directly reflects the collective state formed by local investor interactions. Its relationship with price volatility is therefore highly consistent. Accordingly, the GARCH-X model achieves a substantial improvement in fit at this stage ($\Delta AIC \approx 109.96$; $\Delta BIC \approx 104.07$), indicating that the sentiment variable can adequately explain changes in volatility within a closed system.

In the real-market-driven stage, the sentiment variable is generated using actual market returns as the external field and can therefore reflect the dynamic evolution of investor sentiment in the market. In this more complex environment, although the information captured by the sentiment variable is constrained by the model setting and the diversity of external shocks, the GARCH-X model constructed in this paper still outperforms the benchmark model in terms of goodness of fit ($\Delta AIC \approx 10.34$; $\Delta BIC \approx 4.45$), indicating that the sentiment mechanism continues to have significant explanatory power in real markets.

5. Conclusions

This paper constructs an investor-sentiment variable on the basis of a two-dimensional Ising model and introduces it into the GARCH-X framework to characterize the generation mechanism of market volatility. In a purely simulated environment without external intervention, the resulting price series naturally exhibits typical statistical properties of financial markets, including sharp peaks, fat tails, volatility clustering, and long memory. At the same time, when investors are in a highly irrational state—that is, during periods of elevated investor sentiment—the market's fat-tail effect becomes significantly stronger and market risk rises. Moreover, in GARCH-X estimation based on the purely simulated stage, the sentiment variable significantly improves the goodness of fit, demonstrating its structural explanatory power for volatility in a closed system.

After introducing real-market external-field feedback, the generated sentiment variable still enables the GARCH-X model to outperform the benchmark GARCH model, and the improvement in fit is statistically meaningful, demonstrating the feasibility and robustness of transferring the mechanism from a purely simulated environment to the real market. This indicates that a sentiment variable generated from microscopic interaction mechanisms can retain important explanatory power for volatility even in more complex real-market settings.

Theoretically, this study strengthens the reliability of the proposition that microscopic investor sentiment can affect macroscopic stock volatility, and it validates the effectiveness and transferability of the sentiment variable through multiple statistical indicators. In practical terms, the variable can serve as an explanatory enhancement factor for risk early warning and sentiment-index construction, and it has the potential to be extended further to multi-market and multi-asset environments. Future research may proceed by incorporating investor heterogeneity, refining sentiment-measurement methods, and exploring cross-market transfer, thereby improving market-modeling approaches driven by investor behavior.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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