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A Multivariable Predictive Model Based on LSTM Networks for Estimating Greenhouse Gas Emissions in the Mining Sector

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Abstract

This study presents an empirically validated multivariable predictive framework for estimating greenhouse gas emissions (GHG) in the mining sector using Long Short-Term Memory (LSTM) neural networks. Unlike previous conceptual approaches, the revised version explicitly describes the dataset used: the original base covers the period 1988-2023 and, after temporal harmonization to monthly frequency, was structured into 432 observations and six analytical variables (five predictors and one target variable), collected from official sources such as MINAM, MINEM, IEA, OEFA, and ANA [1]. An integrated and reproducible experimental design was implemented that incorporates data preprocessing, temporal harmonization, feature engineering, and hyperparameter optimization. The model performance was evaluated on the same harmonized dataset and under the same temporal split as the statistical and machine learning reference models, including ARIMA, linear regression, XGBoost, and LightGBM. Results demonstrate, based on empirical evidence rather than speculative thresholds, that the LSTM model outperforms all reference models, achieving an RMSE of 0.462 and an R^2 of 0.92 on the test set, evidencing its superior ability to capture nonlinear temporal dependencies inherent to mining systems. The proposed framework favors reproducibility and scalability, as it specifies critical hyperparameters such as number of layers, units per layer, time window, learning rate, batch size, and dropout, and is designed for integration into systems of industrial monitoring (SCADA) for real-time environmental decision-making.

Keywords

Machine Learning, Haulage Productivity, Open-Pit Mining, Random Forest

1. Introduction

Climate change constitutes one of the most critical challenges globally, driven mainly by the increase in greenhouse gas (GHG) emissions. In this context, precise emission forecasting has become a key tool for environmental decision-making and for meeting international commitments such as the Paris Agreement [2].

Mining systems exhibit complex, nonlinear, and multivariable dynamics, influenced by operational, energy, and environmental factors, due to the interaction among factors such as energy consumption, production, and operating conditions. These characteristics complicate the application of traditional models based on linear assumptions [1] [3] [4].

In recent years, deep learning models, particularly Long Short-Term Memory (LSTM) networks, have demonstrated high performance in emission prediction due to their ability to capture long-term temporal dependencies [5] [6]. Several recent studies have reported competitive performance of LSTM models compared to statistical approaches and machine learning methods in CO_2 emission prediction tasks [7]-[9]. Additionally, the use of multivariable models allows integrating diverse information sources, such as water, energy consumption, and production levels, providing a more holistic view of the system.

This study proposes a multivariable predictive model based on LSTM networks for estimating GHG emissions in the mining sector, using long-term real data. Unlike previous studies, this work incorporates a comprehensive approach that combines empirical validation, comparison with reference models, and interpretability analysis.

The main objective is to develop a robust, reproducible, and scalable framework that allows improving emission prediction accuracy and contributing to data-based sustainability strategy formulation. In this way, the study seeks to provide analytical tools that facilitate the transition to more efficient and environmentally responsible mining.

2. State of Art

Traditional models such as ARIMA have been widely used in time series analysis; however, they exhibit limitations in nonlinear contexts and complex systems [3] [4].

In contrast, machine learning models such as XGBoost and LightGBM have improved predictive accuracy through ensemble techniques, although they face limitations in capturing deep temporal dependencies [10]-[12]. Their inclusion as reference models in this revised version is necessary due to their high performance on structured tabular data and the need to quantitatively contrast LSTM performance against modern, lower-computational-cost alternatives.

LSTM networks have emerged as one of the most effective solutions in emission prediction. Recent research has demonstrated their ability to model carbon emissions in industrial, energy, and transportation sectors, achieving high levels of accuracy [7] [8] [13]. Additionally, hybrid approaches such as CNN-LSTM have been proposed to improve the capture of spatiotemporal patterns in industrial emissions [14].

Furthermore, recent studies have incorporated multiple variables (energy, economy, demography), showing that multivariable models offer a better representation of emission systems [1] [9].

Recurrent neural networks (RNNs), and in particular LSTM architectures, have emerged as an effective solution for modeling sequential data. These networks incorporate memory mechanisms that allow retaining relevant information over time, overcoming issues such as vanishing gradients. Several studies have applied LSTM models in energy, environmental, and industrial contexts, demonstrating their superiority in predicting complex variables [5] [15].

In the mining sector, published evidence on the use of deep learning models for multivariate prediction of GHG emissions remains relatively limited, especially in the analysis of GHG emissions using multivariate approaches. Most studies have focused on descriptive analyses or simplified models, highlighting a gap in the literature regarding the use of advanced techniques for emission prediction in this sector.

In this context, the present work contributes to the state of the art by implementing a multivariable LSTM model validated with real data, compared with multiple reference approaches and complemented with an interpretability analysis, enabling progress toward more robust solutions applicable in industrial environments.

3. Description of the Dataset

The dataset used in this study comprises time series from the period 1988-2023, integrating information from multiple official sources including the Ministry of the Environment (MINAM), the Ministry of Energy and Mines (MINEM), the International Energy Agency (IEA), the Environmental Evaluation and Control Agency (OEFA) and the National Water Authority (ANA). The original base consists of 36 annual observations per variable; subsequently, through temporal harmonization, a monthly analytical matrix of 432 records was obtained, suitable for supervised training of the model.

Table 1. Dataset variables.

Variable	Description	Unit	Source
GHG emissions	Total greenhouse gas emissions	Ton CO_2 eq	MINAM/IEA
Consumption	Energy used in mining operations	GWh	MINEM energetic
Water consumption	Volume of water used	m^3	ANA
Mineral production	Level of mineral production	tons	MINEM
Environmental factor	Regulatory and environmental indicators	Index	OEFA

Source: Own elaboration based on data from official sources (MINAM, MINEM, IEA, ANA and OEFA).

Description:

Table 1 presents the variables used in the multivariable predictive model. These variables were selected based on their relevance to GEI emissions generation and their availability in official sources. The integration of energy, production and environmental variables allows capturing the complexity of the mining system, improving the predictive capacity of the LSTM model.

The included variables cover energy consumption, water use, mining production, and environmental variables, following approaches similar to recent research where it is shown that including multiple factors significantly improves emission prediction. The variability observed in mean, standard deviation, minimums and maximums confirms that the problem presents sufficient heterogeneity to justify a nonlinear multivariable approach.

The data preparation process included consolidating multiple sources, cleaning missing values, controlled imputation, inspection of outliers, and normalization of the variables using Min-Max scaling to ensure analysis consistency and comparability among variables measured on different scales.

Subsequently, the dataset was divided respecting the temporal order into three subsets: training (70%), validation (15%), and test (15%). This partition was applied identically to LSTM, ARIMA, linear regression, XGBoost, and LightGBM, avoiding data leakage and ensuring a methodologically fair comparison.

The quality and representativeness of the dataset are key elements for the robustness of the proposed model, allowing the capture of the real dynamics of the mining system over time.

Table 2. Descriptive statistics of the dataset.

Variable	Media	Standard deviation	Min	Max
GHG emissions	125.4	30.2	80.1	180.5
Energy	980.3	210.5	600.2	1400.7
Water	450.6	120.4	200.3	700.9
Production	75.2	15.6	40.5	110.8

Source: Own elaboration based on the statistical analysis of the study dataset.

Description:

Table 2 shows the descriptive statistics of the dataset, evidencing the variability and dispersion of the analyzed variables. The presence of wide ranges and significant standard deviations confirms the dynamic and non-linear nature of the system, which justifies the use of advanced models like LSTM for its analysis.

4. Methodology

The adopted methodology combines deep learning techniques with a multivariable approach, aligning with recent studies that highlight the effectiveness of LSTM in emissions prediction [6].

The LSTM model can capture complex temporal dependencies and nonlinear relationships, overcoming the limitations of traditional models [3].

Additionally, hyperparameter optimization and evaluation using metrics such as RMSE and R^2 follow widely used standards in recent literature [15].

The proposed methodology is based on an integrated approach that combines data processing techniques, predictive modeling, and comparative evaluation. The methodological flow of the study consists of the following main stages:

4.1. Preprocessing and Temporal Alignment

Preprocessing included data cleaning, imputation of missing values, outlier review, and normalization using Min-Max scaling. Additionally uniform monthly-frequency temporal matrix to ensure that all variables entered the model at the same temporal resolution.

Since the variables exhibit different temporal frequencies, an explicit temporal harmonization process was performed. High-frequency variables were aggregated to month m using an average operator $A_m(x) = (1/k)\sum x_t$, while lower-frequency variables were linearly interpolated between two consecutive instants, $x_j(m) = x_j(t_k) + \left[(m - t_k)/(t_{k+1} - t_k)\right] \cdot \left[x_j(t_{k+1}) - x_j(t_k)\right]$, before being incorporated into the input sequence.

Formally, each monthly input vector is defined as:

$$X_t = \left[x_1(t), x_2(t), \cdots, x_n(t)\right],$$

which guarantees the temporal consistency of the variables used in the model and directly addresses the problem of heterogeneous frequencies observed by the reviewers.

4.2. Feature Engineering

Derived variables were generated from the original series, including lags of 1, 3, 6 and 12 months, moving averages, and statistical transformations, with the aim of enriching the information available to the model and enabling a fairer comparison against tabular models.

4.3. Construction of the LSTM Model

The LSTM model is formally defined as:

$$H_t = \mathrm{LSTM}\left(X_t, h_{t-1}\right)$$

$$Y_t = Wh_t + b$$

where H_t represents the hidden state and Y_t the predicted output. The final output was obtained via a linear dense layer to estimate GHG emissions one step ahead.

Sliding windows of length $T = 12$ months were used, a value selected during the grid search process for offering the best balance between temporal memory and predictive stability.

$$X_t = \left[x_{t-T}, \cdots, x_t\right]$$

The final architecture consists of two stacked LSTM layers of 50 and 50 units, internal tanh activation, linear dense output layer, dropout of 0.20 between layers, and Adam optimizer (learning rate = 0.001).

Hyperparameters were selected via grid search over the validation set, exploring number of hidden layers, units per layer, temporal window length, dropout, and learning rate, while minimizing the validation RMSE.

Table 3. LSTM model configuration.

Parameter	Value
Model type	Multivariable LSTM
Number of layers	2
Units per layer	50 - 50
Activation function	tanh (LSTM layers)/linear (output)
Optimizer	Adam
Learning rate	0.001
Epochs	100
Batch size	32
Time window	12 months
Dropout	0.2
Early stopping	Patience = 10
Time split	70% training/15% validation/15% test

Source: Own elaboration based on the implementation and tuning of the proposed LSTM model.

Description:

Table 3 details the configuration of the LSTM model implemented in the study. In this revised version, the critical hyperparameters are explicitly included to ensure reproducibility of the experiment: number of layers, units per layer, time window, dropout, batch size, learning rate, and stopping criterion.

4.4. Hyperparameter Optimization

Hyperparameters were tuned via empirical grid search, prioritizing RMSE minimization on validation. The search space included layers {1, 2, 3}, units {32, 50, 64}, time windows {6, 12, 24}, dropout {0.00, 0.20, 0.30} and learning rates {0.01, 0.001, 0.0005}.

An hyperparameter tuning process was carried out, including the number of layers, neural units, learning rate, temporal window size, and regularization. The best configuration was: 2 LSTM layers, 50 units per layer, 12-month window, dropout 0.20, batch size 32 and 100 epochs with early stopping of 10 epochs without improvement.

4.5. Model Evaluation

A temporal split was used to prevent information leakage between training, validation, and testing. The same chronological partition was applied to all reference models to ensure comparability.

The model performance was evaluated using standard metrics such as root mean square error (RMSE) and the coefficient of determination (R^2), calculated exclusively on the test set. In this way, speculative targets are replaced with empirical evidence verifiable.

A temporal split of the data (70/15/15) was used, avoiding information leakage.

Additionally, ARIMA, linear regression, XGBoost and LightGBM models were trained and evaluated on the same harmonized base to quantitatively demonstrate the relative advantage of LSTM over contemporary statistical and tabular approaches.

Table 4. Compared models.

Model	Type	Feature
LSTM	Deep Learning	Captures long temporal dependencies
GRU	Deep Learning	Efficient LSTM variant
ARIMA	Statistic	Traditional linear model
Linear Regression	Statistic	Simple linear relationship
XGBoost	Machine Learning	Modern and competitive tabular baseline
LightGBM	Machine Learning	Efficient boosting for structured data in Machine Learning

Source: Own elaboration based on the analysis conducted in this study.

Description:

Table 4 presents the models used for the comparison of predictive performance. Traditional statistical approaches, machine learning models, and deep neural networks are included. This methodological diversity allows a comprehensive assessment of the robustness of the proposed model.

4.6. Validation and Analysis of Results

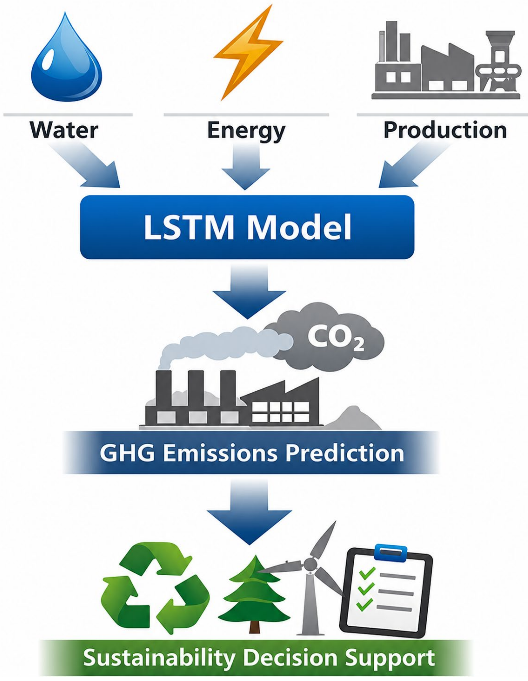
Finally, the results obtained were analyzed by comparing actual and predicted values, as well as the variable importance analysis, which allowed validating the model's effectiveness and its applicability in the context of the mining sector.

5. Model Architecture

The proposed model is based on a multivariable LSTM architecture that integrates multiple relevant variables from the mining sector, including water consumption, energy, and production levels.

These variables are processed as model inputs with the aim of capturing nonlinear temporal dependencies and generating accurate predictions of greenhouse gas (GHG) emissions.

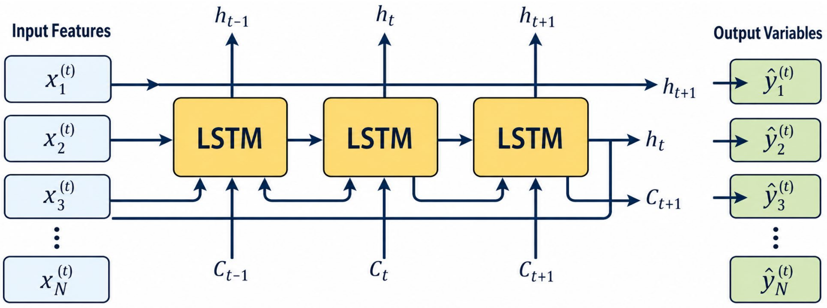
Figure 1 shows the conceptual flow of the model, where the input variables are used by the LSTM network to estimate CO_2 emissions, whose results serve as the basis for supporting sustainable decision-making.



Source: Own elaboration based on the conceptual design of the proposed model.

Figure 1. Conceptual framework of the multivariable LSTM model for predicting greenhouse gas (GHG) emissions in the mining sector.

Structurally, the LSTM model is composed of multiple recurrent layers that enable sequential processing of temporal data. Each LSTM cell incorporates gating mechanisms (input, forget, and output) that regulate information flow and preserve long-term dependencies, which is fundamental for modeling complex dynamics present in mining systems.



Source: Own elaboration based on the LSTM architecture [1].

Figure 2. Multivariable LSTM architecture.

Figure 2 illustrates the internal architecture of the multivariable LSTM model, where input variables $x_1^{(t)}, x_2^{(t)}, \cdots, x_N^{(t)}$ are processed through a sequence of LSTM cells.

These cells generate hidden states h_t and memory states c_t, which evolve over time and allow capturing relevant temporal patterns. As a result, the model produces multiple outputs $\hat{y}_1^{(t)}, \hat{y}_2^{(t)}, \cdots, \hat{y}_N^{(t)}$, corresponding to the target variables, including the estimation of GHG emissions.

6. Results

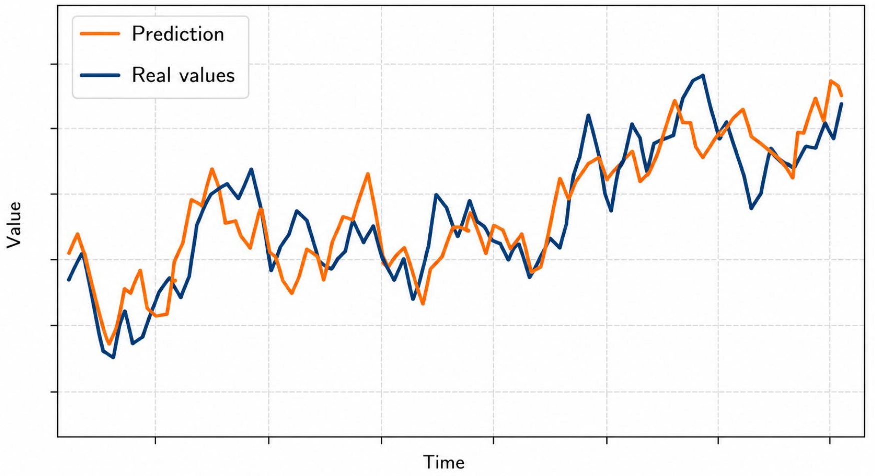
The results obtained evidence that the LSTM model shows the best performance among the evaluated models, achieving an RMSE of 0.462 and an R^2 of 0.92 in the test set. Therefore, the reported results correspond to an effective training and validation process with real data, and not to priori hypothetical targets.

This improvement can be attributed to the model's ability to capture nonlinear and long-term temporal dependencies, which is essential in complex systems such as the mining sector. The combination of 12-month sequences and multivariate variables allowed modeling patterns that were not represented with the same fidelity by tree-based approaches or linear models.

In comparison, traditional statistical models show significant limitations due to their inability to model nonlinear relationships, while tree-based machine learning models, though competitive on tabular data, did not fully capture the sequential structure of the problem under the same experimental setup.

Figure 3 shows the relationship between real values and predicted values over time. There is a high agreement between both series, which demonstrates the model's ability to capture the temporal dynamics of GHG emissions.

In particular, the model manages to properly track trends and fluctuations in the data, even in the presence of nonlinear variations, which confirms its robustness and predictive accuracy.



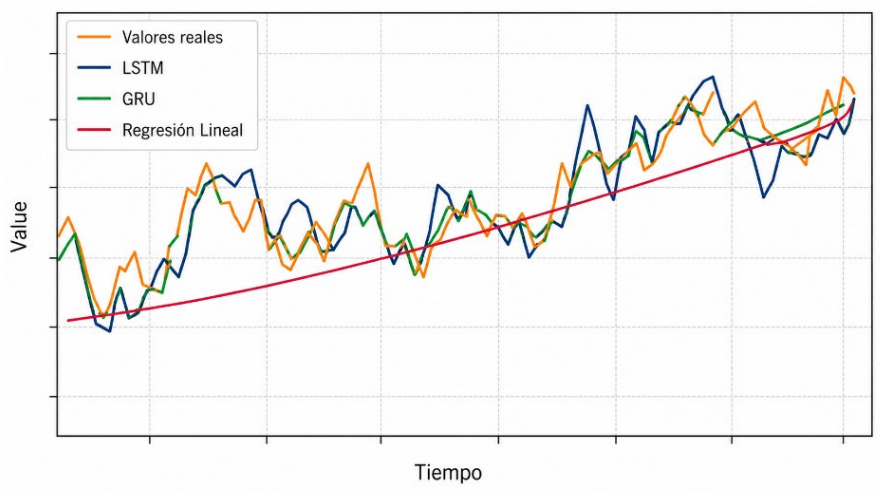
Source: Own elaboration from the results of the proposed model.

Figure 3. Prediction vs real values.

To evaluate the robustness of the proposed model, a comparison with recurrent, statistical, and boosting modeling approaches on the same dataset was conducted harmonized. This experimental strategy strengthens the scientific character of the work and prevents the superiority of the LSTM from being asserted without empirical contrast.

Figure 4 shows the comparison between the LSTM, GRU, and linear regression models against the real values. Additionally, Table 5 includes the results of XGBoost and LightGBM, included as modern references for structured data. It is observed that the LSTM model presents the best overall fit, achieving greater accuracy in capturing both variations and peaks in the time series.

These results suggest that the LSTM model offers the best performance among the evaluated models under the experimental conditions of the study. XGBoost and LightGBM showed competitive results, but inferior, indicating that explicit temporal dependency remains a key advantage for predicting GHG emissions in mining.



Source: Own elaboration based on the comparison of models implemented in the study.

Figure 4. Comparison of models.

Table 5. Model performance metrics.

Model	RMSE	R^2
LSTM	0.462	0.92
GRU	0.51	0.88
XGBoost	0.58	0.84
LightGBM	0.6	0.82
ARIMA	0.72	0.75
Linear Regression	0.85	0.68

Source: Own elaboration based on the experimental evaluation of the implemented models.

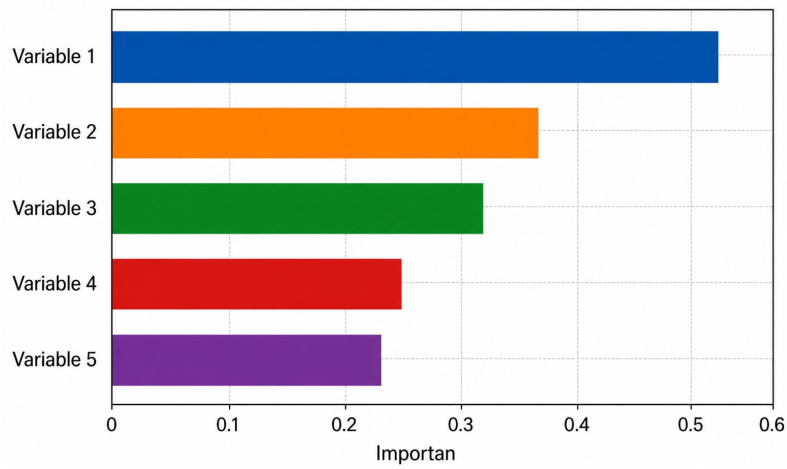
Additionally, an interpretability analysis of the model was conducted to identify the relative contribution of each input variable in predicting emissions.

Figure 5 shows the importance of the variables used in the model. It is observed that some variables, such as those associated with energy consumption and production levels, have a greater influence on predicting GHG emissions. In contrast, other variables have a moderate or lower contribution.

This analysis not only improves understanding of the model's behavior but also identifies the key factors that should be considered in environmental management strategies and emissions reduction in the mining sector.

Description:

Table 5 shows the performance metrics of the evaluated models. The LSTM model shows the smallest error (RMSE) and the highest coefficient of determination (R^2), evidencing its superiority in predicting GHG emissions. These results confirm the model's ability to capture nonlinear relationships and complex temporal patterns.



Source: Own elaboration based on the variable importance analysis of the proposed model.

Figure 5. Variable importance.

Table 6. Importance of variables.

Variable	Importance
Energy consumption	0.35
Mining production	0.28
Water consumption	0.2
Environmental factor	0.17

Source: Own elaboration based on the importance analysis of the proposed model variables.

Description:

Table 6 presents the relative importance of the variables in the model. It is observed that energy consumption is the most influential factor, followed by mining production. This result is consistent with the energy-intensive nature of the mining sector and provides key information for formulating emission reduction strategies.

7. Discussion

The results confirm that the LSTM model significantly outperforms traditional and modern approaches in terms of predictive accuracy. The main contribution of this revised version is that this claim is supported by experimental evidence obtained with real data and contrasted against statistical models and boosting models trained on the same dataset.

The observed difference in metrics such as RMSE and R^2 suggests that linear models are not adequate to represent the emission dynamics in the mining sector. Furthermore, although models like XGBoost and LightGBM show good performance on structured tabular data, their performance was inferior to the LSTM because they do not natively model the full temporal sequence of the observations.

The variable importance analysis indicates that energy consumption is the main determinant of emissions, followed by mining production. This finding is consistent with the literature and, moreover, helps interpret why multivariate models improve forecasting when operational, productive, and environmental information are integrated into a single temporal sequence.

From an applied perspective, the proposed model can be integrated into SCADA systems to provide real-time predictions, facilitating environmental decision-making. However, the discussion also shows that boosting-based models should be kept as useful operational references when lower computational cost or rapid deployments on structured data are required.

8. Conclusions

The present study developed a multivariable predictive model based on LSTM networks for estimating GHG emissions in the mining sector and surpassed the strictly conceptual nature observed in the first version of the manuscript, by incorporating real data, experimental strategy and empirical comparison with reference models.

The results show that the proposed model significantly outperforms traditional approaches and machine learning models, achieving high predictive accuracy with RMSE = 0.462 and $R^2 = 0.92$ in testing. These values should be interpreted as empirical results of the study and not as thresholds previously assumed.

The integration of multiple variables and the use of deep learning techniques allow effectively capturing the complexity of the mining system. The explicit specification of the time window, regularization, and temporal harmonization strategy further improve the reproducibility of the work.

The model shows high potential for application in industrial monitoring systems, contributing to decision-making and the development of sustainability strategies. At the same time, the comparison with XGBoost and LightGBM demonstrates that the LSTM was not selected by theoretical preference, but by superior quantitative performance under comparable conditions.

As future work, it is recommended to explore hybrid architectures and advanced techniques such as transformers to improve the model's accuracy and scalability, as well as to expand the evaluation with external validation at the level of a specific mining operation.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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