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Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) with Mobility of Certain Customers

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Abstract

Our study is based on modelling the Vehicle Routing Problem with Time Windows (VRPTW) and mobility of certain customers. The distribution network consists of a depot, a uniform fleet of vehicles and several customers, some of whom have mobility. A mobile customer is a customer that has two alternative positions, both with distinct time windows, distinct requests, and distinct service times between positions. The model chooses exactly one position per mobile customer. A mathematical formulation of this problem with a vehicle index is given in the form of a Linear Mixed Integer Program (LMIP). The mathematical model of this problem is solved using the Branch and Cut (B&C) and also using a two-phase decomposition heuristic method for mobile customers (TPDH-FMC). The tests are performed using data from Solomon's literature adapted to our problem. The results of these two methods were compared. The tests were carried out on 12 instances. For each instance, we obtained 16 results. We had a total of 192 results. The results obtained through these tests show that out of the 192 results obtained, the heuristic gave a better cost for 10 results (5.20%) compared to the (B&C) algorithm. Furthermore, for 15 results (7.81%), the heuristic and the (B&C) algorithm had the same cost.

Keywords

Mobile Customer, Two-Phase Decomposition Heuristic Method for Mobile Customers, Time Window, Branch-and-Cut Algorithm

1. Introduction

The issue of transport has become very important in society because a large part of the economy is based on transport. For a transport company, knowing how to manage the deployment of its vehicle fleet can bring significant savings. This is what drives us to conduct our research on vehicle routing problems (VRP). Several studies have been conducted on VRP. It should be noted that the VRP has had several variants depending on the specificities of transport problems related to reality. One such variant is Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). A CVRPTW consists of minimising the transport cost of vehicle routes departing from a depot, visiting each customer once to satisfy their requests, within a time frame called a time window, while respecting the capacities of the vehicles. We are going to work on a very interesting variant of the CVRPTW, which we have named CVRPTW with mobility of certain customers. In some cases, the customer may be mobile, meaning they may have two different geographical positions where they can receive their order. In this new problem, the constraints of the CVRPTW found in [1] have been modified. To solve this problem, we drew inspiration from the work of [2]. We have outlined the various variants of the VRP in the state of the art. We have organised our work into several sections for a clear understanding of the work: In Section 2, a literature review on VRP variants. Section 3 describes our problem. In Section 4, we propose a mathematical model of the problem. Our two-phase decomposition heuristic method is described in Section 5. Section 6 describes the experiment, and Section 7 presents the results and analysis of these results. Finally, Section 8 allows us to draw conclusions about all the work that has been done.

2. Literature Review

The VRP was introduced by Dantzig and Ramsès in 1959 to ensure optimal petrol delivery using a fleet of vehicles. It should be noted that the VRP is an extension of the travelling salesman problem (TSP). For more information on the TSP, see [3], in which David Johnson explains TSP modelling and his work on the symmetric TSP, giving its application in fields such as VLSI chip manufacturing and X-ray crystallography. He also spoke about the complexity of TSP. As for the complexity of the VRP, Lenstra and Rinnooy addressed it in [4] and demonstrated the complexity of other types of tour problems. They showed that these problems are NP-hard. The VRP has had several variations, taking into account the realities of life. We can mention the VRP with vehicle capacities (CVRP). Indeed, Fisher et al. added a condition to the VRP whereby the depot has a fixed capacity stock, each vehicle has a fixed capacity, and each customer has a fixed order within the vehicle's capacity. For the CVRP, refer to [5]. We have dynamic vehicle routing problems (DVRP) where, after starting the vehicle route, certain parameters related to the problem may change, such as a change in customer demand or the unavailability of a road on that route. [6] gives us some ideas on this. We also have the VRP with backhaul (VRPB). First, customer requests are satisfied, followed by the collection of requests to replenish the depot, see [7]. There is also the stochastic VRP (SVRP) in which there may be a random element in relation to customer demand or the availability of a certain route with a given probability, see [8]. We have the VRP with Time Windows (VRPTW) where each customer has a time window in which they must receive their order, read [9]. We will look at a new and very interesting variant of the VRPTW. Several variants of this problem have emerged, namely the VRP with multiple time windows (VRPMTW). In this problem, each customer has several time intervals in which they can receive their order, see [10]. In our article, we will focus on a new variant of the VRPTW called CVRPTW with mobility for certain customers. Here, we consider some customers as fixed and others as mobile, which prompts us to add a new constraint to the VRPTW that allows us to manage the mobility of certain customers. This problem can be considered an extension of the VRPMTW. Indeed, Eduardo Bogue et al. in 2019 used column generation and a post-optimisation VNS (Variable Neighbourhood Search heuristic) to solve the VRPMTW. For the data sets, they used the 56 Solomon instances of the VRPTW with 100 customers, and as these instances differ in terms of customer time windows while keeping the same geographical positions, they were able to assign several time windows to each customer and then carry out their test [10]. In our case, each customer may have a second position with a different time window. We can view our problem as a combination of the VRPTW for a certain number of customers and the VRPMTW, where each new position for the same customer is assigned a new time window for that customer. The problem is formulated with a vehicle index in the form of a linear integer programme. For more information on VRP indices, see [5] [11]. To solve this problem, we used the Branch and Cut (B&C) method as an exact method, see [12], then we used the two-phase decomposition heuristic method for mobile customers, which borrows the nearest neighbour method to solve this problem in an approximate manner. To learn more about the nearest neighbour method, see [13].

3. Description of the Problem

Figure 1 and **Figure 2** illustrate the distribution network in our problem.

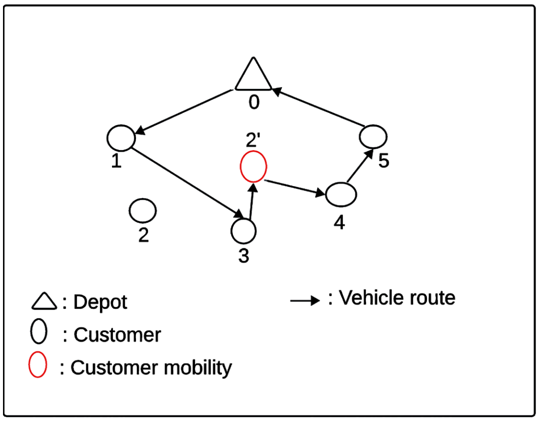

Figure 1. The route of a vehicle passing through the mobile position of customer 2.

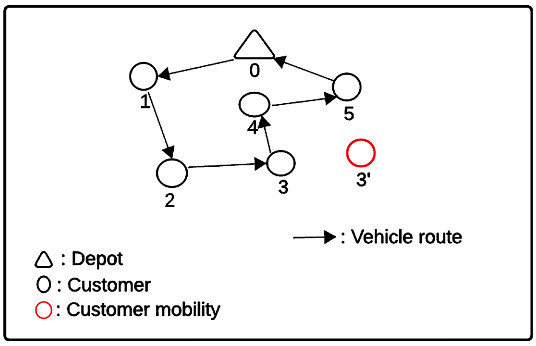

Figure 2. The route of a vehicle that does not pass through a mobile position.

Figure 1 shows an example of a route obtained by a vehicle after solving the problem, and Figure 2 also shows an example of a route obtained by a vehicle after solving the problem. In the first figure, Figure 1, it is advantageous for the vehicle to move to the mobile position of customer 2 to minimise the length of the journey while respecting the constraints of the problem, whereas in the second figure, Figure 2, it is preferable for the vehicle not to move to any mobile position in order to minimise the length of the journey. In both cases, we see that the vehicle does not pass through the two positions of the customer who is mobile.

4. Mathematical Formulation

Let $G=(N,A)$ be a complete graph in which N represents nodes formed by the depot and cuctomers with index $i\in\{0,1,\cdots,n,n+1,\cdots,n+p\}$ and $A(N)=\{(i,j):i,j\in N,i\neq j\}$ the set of arcs in N. The depot is indexed by 0. Each mobile customer i is indexed by $n+i$ at its second position, where n represents the number of customers. The set of customers with all indexes is represented by $\{1,\cdots,n,n+1,\cdots,n+p\}$ where p represents the number of customers with two different positions.

Sets

We set:

$N=\{0\}+N_{2p}+F$ the set of customers with their mobility and the depot;

$N_{2p}=\{1,\cdots,p\}$ the set of mobile customers;

$F=\{p+1,\cdots,n\}$ the set of fixed customers;

$K=\{1,\cdots,m\}$ the set of vehicles.

Indices

i,j represent the indices for the nodes of the set N;

k is the index for each vehicle in a homogeneous fleet of vehicles.

Parameters

c_{ij}: transportation cost for a vehicle travelling directly from node i to node j;

d_i: the customer's request i;

s_{ik}: this is the service time of vehicle k at customer i;

t_{ij}: this is the time taken by the vehicle from node i to node j;

$[a_i, b_i]$: the time window of client i;

M_{ij}: a symbolic and very large constant. It can be taken as $\max\{b_i + t_{ij} - a_j\}$, $(i, j) \in A$;

q: maximum vehicle capacity.

Decision variables

x_{ijk}: binary variable, equal to 1 if vehicle k travels directly from node i to node j;

T_{ik}: This is the arrival time of vehicle k at node i.

$$Z = \min \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ijk} \tag{1}$$

$$\sum_{k\in K} \sum_{j\in N} x_{ijk} = 1 \quad \forall i \in F \tag{2}$$

$$\sum_{j\in N} \sum_{k\in K} x_{ijk} + \sum_{j\in N} \sum_{k\in K} x_{n+i,jk} = 1, \quad \forall i \in N_{2p} \tag{3}$$

$$\sum_{i\in C} d_i \sum_{j\in N} x_{ijk} \le q \quad \forall k \in K \tag{4}$$

$$\sum_{j\in N} x_{0jk} \le 1 \quad \forall k \in K \tag{5}$$

$$\sum_{i\in N} x_{ihk} - \sum_{j\in N} x_{hjk} = 0 \quad \forall h \in C, \quad \forall k \in K \tag{6}$$

$$\sum_{i\in N} x_{i0k} \le 1 \quad \forall k \in K \tag{7}$$

$$a_i \le T_{ik} \le b_i \quad \forall i \in N \quad \forall k \in K \tag{8}$$

$$T_{ik} + s_{ik} + t_{ij} - M_{ij}\left(1 - x_{ijk}\right) \le T_{jk} \quad \forall i, j \in N, \quad \forall k \in K \tag{9}$$

$$x_{ijk} \in \{0,1\} \quad i, j \in N, \quad \forall k \in K \tag{10}$$

Constraint (1) minimises the total travel cost. This (2) ensures that each fixed customer is visited exactly once, and (3) ensures that a mobile customer cannot be delivered to both positions during the tour. Furthermore, customer i at its alternative position is called customer $n + i$. The service times, time windows, requests and routes differ from one position to another for this mobile customer. The service times, time windows and routes are known in advance. We also have constraint (4), which ensures that no vehicle can be loaded beyond its capacity; (5) ensures that a vehicle leaves the depot when necessary; (6) ensures that when a vehicle arrives at a node, the vehicle departs from that node; (7) ensures that each vehicle that leaves the depot returns to the depot; (8) ensures that the arrival time of vehicle k at customer i is within the time window of customer i. (9) ensures consistency between the arrival time at customer j and the departure time at customer i when a vehicle k travels from customer i to customer j. In addition, (9) allows for the elimination of sub-tours because the arrival time at customer j is greater than or equal to the arrival time at customer i plus the service time at customer i plus the travel time from customer i to customer j, given that time increases along the route. Also, any vehicle k is allowed to wait when it arrives before a_i, and in this case, the arrival time at customer i is taken as a_i, and constraint (10) is an integrity constraint.

5. Description of the Two-Phase Decomposition Heuristic Method for Mobile Customers (TPDH-FMC)

Our resolution method consists of two phases:

The generation of the initial solution followed by **the improvement of the solution**.

1) Phase 1. **Generation of the initial solution**

Each mobile customer i and its alternative position $n + i$ are defined in advance.

- all vehicles are empty at the outset.
- Each route contains the depot from the outset.
- The first customer for the first vehicle is selected at random from all customers, including those with alternative positions, and is added to the depot.
- Starting from the last customer i inserted, the closest and most feasible customer j is selected according to the following constraints:
  - time windows are respected.
  - vehicle capacity is respected.
  - the alternative position of customer j is not already selected in a route.
- If no customer j can be added, the vehicle returns to the depot and another vehicle repeats the same process as the first vehicle until all fixed customers have been visited and mobile customers have also been visited at exactly one alternative position.

This construction ensures that each customer is visited exactly once and that all constraints are satisfied from the initial solution generation onwards. Also, exactly one alternative position of each mobile customer is chosen from the set of routes.

Figure 3 illustrates the construction of an initial solution. Here we have considered 7 customers with 2 mobility levels, where customer 1 has a mobility level of 8 and customer 2 has a mobility level of 9. We can see that the vehicle follows the route 0-1-6-4-8-0, and each customer visited is marked with a cross so that the next vehicle cannot visit a customer who has already been visited. It should be noted that customer 1 was chosen at random.

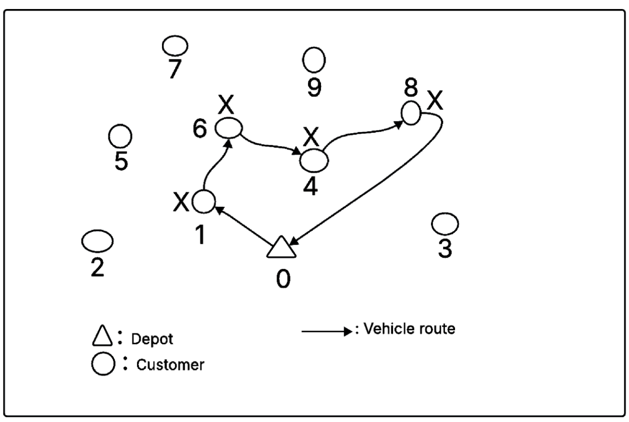

Figure 3. Example of a vehicle's route.

2) Phase 2: **Improvement of the solution**

a) inter-route swap

A local improvement is applied to the initial solution to explore better solutions:

- Two distinct routes are selected at random from among the existing routes.
- Two customers are selected at random from each of the two routes.
- Customers are exchanged simultaneously between the two routes if and only if the new routes obtained satisfy the following conditions:
  - vehicle capacities.
  - time windows.
  - the total cost of the solution decreases significantly.

If an improvement is found, the best local solution is updated; otherwise, the exchange of customers between different routes is cancelled.

b) intra-route swap

After the inter-route swap, an intra-route swap improvement is applied for each route individually:

- All pairs of customers on the same route are exchanged as and when each new route obtained verifies:
  - vehicle capacities.
  - time windows.
  - the total cost of the solution decreases significantly.

c) Criterion for stopping the main programme (global search)

There is an evaluation criterion defined in the main programme. This criterion minimises transport costs. If an improvement is achieved after the inter-route swap and intra-route swap, the best overall solution is updated; otherwise, it is retained.

Phases 1 and 2 are repeated several times after a set maximum number of iterations.

Let us examine Figure 4, Figure 5, and Figure 6 to illustrate the improvement of an initial solution obtained.

In our example shown in Figure 4, we have 10 customers, 2 mobility levels, and 2 vehicles that served the customers. In the initial solution, vehicle 1 follows the route: 0-1-5-12-3-7-0, and vehicle 2 follows the route: 0-6-9-4-8-10-0.

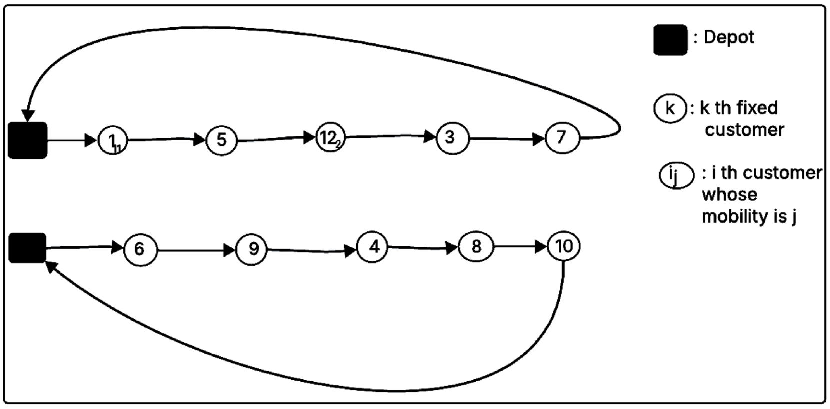

Figure 4. Example of an initial solution to a problem with 10 customers, 2 mobile customers and 2 vehicles.

In our example shown in Figure 5, we have an illustration of the application of inter-route swapping on the two routes of the initial solution. Customer 12 on route 1 is swapped with customer 9 on route 2, and customer 7 on route 1 is swapped with customer 8 on route 2. Both swaps are performed simultaneously, and we check whether the constraints of the problem are verified and whether there is an improvement in the solution; if not, we cancel these swaps.

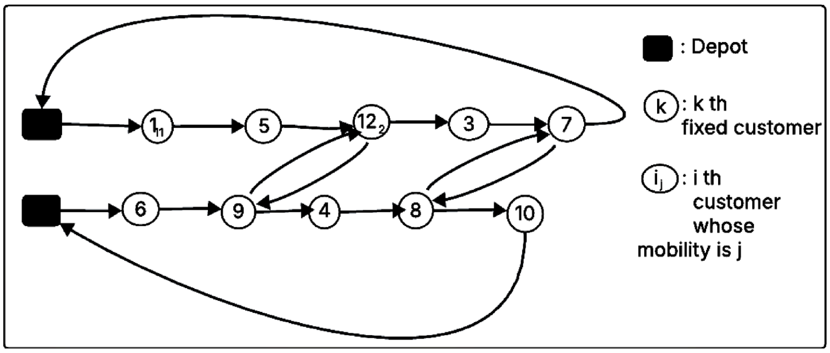

Figure 5. Example of inter-route swap application on the initial solution in Figure 4.

In our example shown in Figure 6, we have an illustration of the application of intra-route swapping on a route obtained at the end of inter-route swapping. This is the route: 0-1-5-9-3-8-0. Here, we tried to swap customers 1 and 5 on the same route if possible and see if this improves the solution; if not, we leave the swap as it is. We obtained the new route 0-5-1-9-3-8-0.

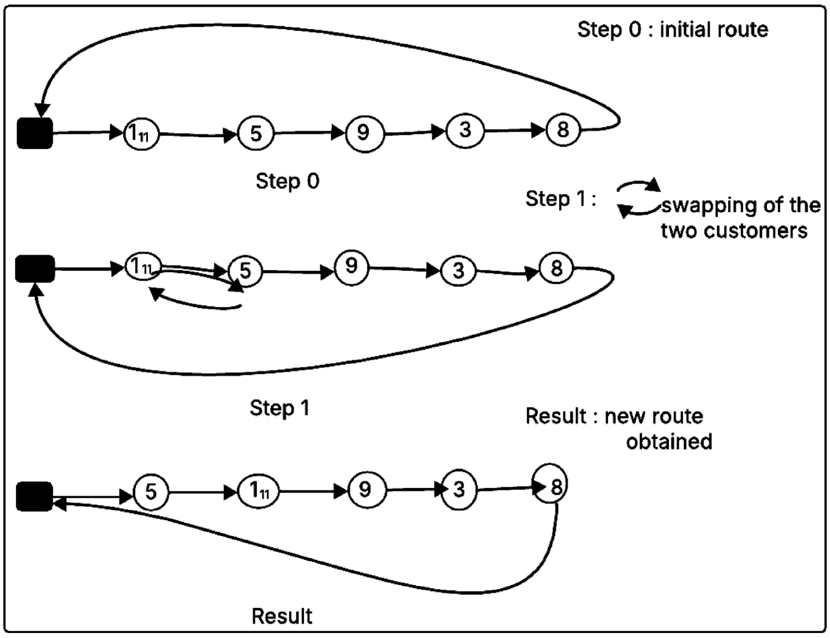

Figure 6. Example illustrating the application of intra-route swapping.

Figure 7 shows us the organisational chart for a better understanding of this method.

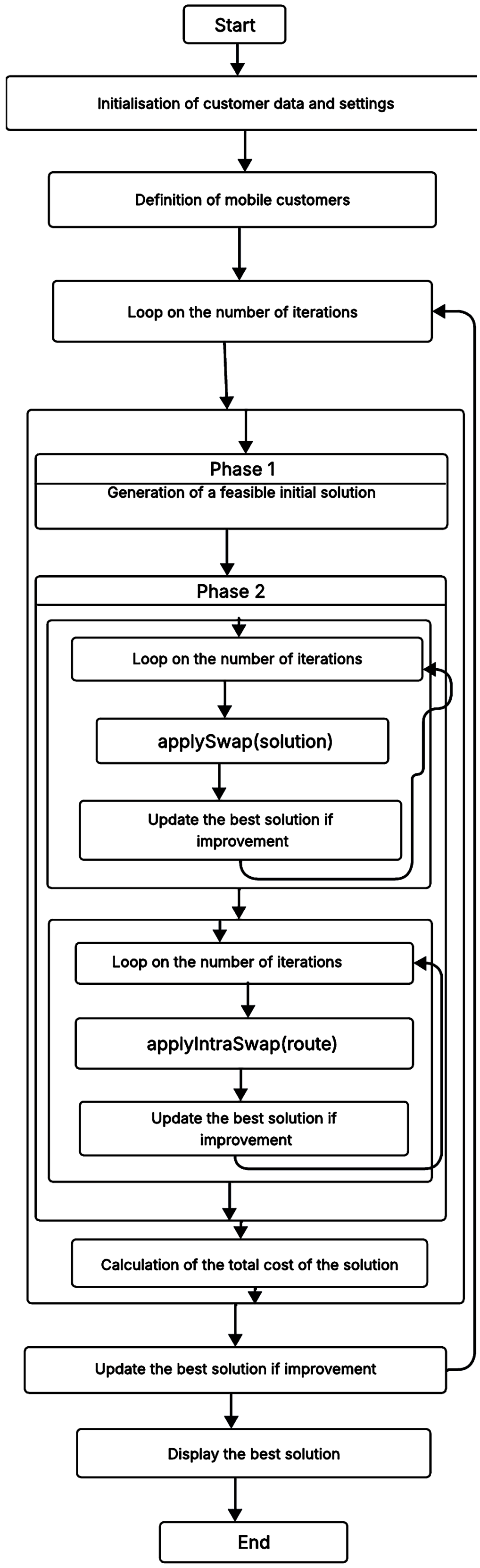

Figure 7. Organisational chart of the two-phase decomposition heuristic method for mobile customers (TPDH-FMC).

6. Experimentation

We used Solomon's benchmark [14] with instances of 8 customers, 12 customers, 16 customers and 20 customers of the VRPTW. It should be noted that Solomon's benchmark instances are grouped into three classes. C1 instances where customers are grouped into clusters and time windows are relatively short, C2 instances where customers are grouped into clusters and time windows are relatively long, R1 instances where customers are randomly distributed and time windows are relatively short, R2 instances where customers are randomly distributed and time windows are relatively long, RC1 instances where customers are distributed in a mixture of clusters and random distribution and time windows are relatively short, RC2 instances where customers are distributed in a mixture of clusters and random distribution and time windows are relatively long. The capacity of each vehicle is 200.

We therefore used instances C1, C2, R1 and R2 to test our model. The instances are denoted as follows: (Example C101-20-10_5, where the first digit 1 before C indicates that we have a C1 type instance, the two digits after C1 indicate the C1 instance number, which is 01, 20 indicates the number of customers, 10 indicates the number of available vehicles, and 5 indicates the number of mobility options). For each instance, we varied the number of mobile customers by 25%, 50%, 75% and 100% of the number of customers in the instance. That is, for an instance with 8 customers, we considered 25% of 8 (2 mobile customers), 50% of 8 (4 mobile customers), 75% of 8 (6 mobile customers) and 100% of 8 (8 mobile customers) in order to see the influence of the number of mobile customers on the results. We took 3 instances C1, 3 instances C2, 3 instances R1 and 3 instances R2, giving a total of 12 instances, for which we varied the number of customers and, for each fixed number of customers, we varied the number of mobile customers. We used a maximum number of vehicles set at half the number of customers to perform our tests. So for 8 customers, we had a total of 4 vehicles available. The results are shown in the table below. We solved the exact method of our model with IBM's Cplex using Microsoft Visual C++. We used an HP computer with an Intel(R) Core(TM) i7-10610U CPU @1.80 GHz (2.30 GHz) processor and 32 GB of RAM.

7. Results and Analysis

As mentioned above, we used instances C1, C2, R1, and R2 from Solomon's benchmark, containing 8, 12, 16, and 20 customers. We created comparative tables of the results and performed an analysis. The comparative tables comprise seven (07) columns. The first column (INSTANCES) is the column for the instances used. The notation for these instances was described in the experimentation section. We also have the B&C column, which shows the value obtained using the exact method, while the TPDH-FMC column shows the optimal value obtained using TPDH-FMC. The GAP column shows the relative difference between the solution obtained using TPDH-FMC and the lower bound or the known optimum. It indicates how far the current approximate solution is from the optimal solution. In our table, it is expressed as a percentage, which allows us to evaluate the quality and efficiency of our heuristic. The COSTS column is the cost column and the CPU column is the column that gives the CPU (Central Processing Unit) time, which represents the computing time consumed by the processor to execute our method. The CPU time corresponds to the time during which the CPU worked to solve the problem, which allows us to evaluate the efficiency and speed of the algorithm. In our case, it is expressed in seconds (s). We also have the AVERAGES lines, which provide the averages of the observed columns. In addition, the AVERAGE DEVIATIONS lines provide the average deviations in the costs of the instances using the two methods, which shows the variations in costs around their averages by adjusting the number of mobile customers in these instances. To properly analyse our results, we grouped them into twelve (12) different tables. In these tables, we compared the results of the exact method (B&C) and the heuristic method (TPDH-FMC). We set the compilation of (B&C) to a maximum time of 03 hours (10,800 s). In TPDH-FMC, we performed the inter-route swap three hundred (300) times, then we performed the intra-route method n! (n factorial) times, where n represents the number of customers on the route to which the intra-route swap is applied. Finally, we applied the number of iterations five hundred (500) times to retain the best solution. The mathematical formula for the GAP in cost expressed as a percentage is GAP $= \frac{Val_{(TPDH\text{-}FMC)} - Val_{(B\&C)}}{val_{(B\&C)}} \times 100$ where $Val_{(TPDH\text{-}FMC)}$ is the approximate value obtained by TPDH-FMC and $Val_{(B\&C)}$ is the value obtained by B&C. Similarly, the mathematical formula for the CPU GAP expressed as a percentage is GAP CPU $= \frac{Val_{T_{(TPDH\text{-}FMC)}} - Val_{T_{(B\&C)}}}{val_{T_{(B\&C)}}} \times 100$ where $Val_{T_{(TPDH\text{-}FMC)}}$ is the time taken to solve the problem by TPDH-FMC and $Val_{T_{(B\&C)}}$ is the time taken to solve the problem by B&C. The 12 tables contain instances C101, C102, C103, C201, C202, C203, R101, R102, R103, R201, R202 and R203. For each instance, we used 8, 12, 16 and 20 customers to perform our tests. In each case of customers considered, we used 25%, 50%, 75% and 100% as mobile.

In Table 1, which contains instance C101, we see that regardless of the number of customers and mobile customers, B&C provides a better cost than TDPH-FMC, as indicated by the positive values in the GAP columns under COSTS. Furthermore, B&C was faster in solving the problem than TPDH-FMC, as shown by the positive values in the GAP columns in CPU.

For 8 customers, we find that regardless of the number of mobile customers, the costs obtained by both the B&C and TPDH-FMC methods remain unchanged. For B&C, the cost is 50 and for TPDH-FMC, the cost is 83. This explains their average differences with zero values. Also, the cost GAP with 8 customers is 66%, which shows that the cost obtained by TPDH-FMC is a little too far from B&C. On average, the problem is solved in 0.32 seconds for B&C and 2.32 seconds for TPDH-FMC.

Table 1. The results of instance C101.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C101-8-4_2	50	83	66	0.23	1.64	591.59
C101-8-4_4	50	83	66	0.26	2	649.06
C101-8-4_6	50	83	66	0.36	2.49	579.56
C101-8-4_8	50	83	66	0.41	3.14	665.69
AVERAGES	**50**	**83**	**66**	**0.32**	**2.32**	**621.47**
AVERAGE DEVIATIONS	**0**	**0**				
C101-12-6_3	129	162	25.58	0.36	2.13	482.83
C101-12-6_6	129	166	28.68	0.76	2.84	274.73
C101-12-6_9	129	160	24.03	0.81	3.62	344.96
C101-12-6_12	84	128	52.38	0.75	4.47	489.85
AVERAGES	**117.75**	**154**	**32.66**	**0.67**	**3.27**	**398.09**
AVERAGE DEVIATIONS	**16.87**	**13**				
C101-16-8_4	144	221	53.47	0.57	2.24	293.15
C101-16-8_8	144	249	72.91	0.61	3.26	431.54
C101-16-8_12	136	216	58.82	0.72	4.02	454.95
C101-16-8_16	136	191	40.44	2.70	5.99	121.52
AVERAGES	**140**	**219.25**	**56.41**	**1.15**	**3.88**	**325.29**
AVERAGE DEVIATIONS	**4**	**15.75**				
C101-20-10_5	174	264	51.72	1.15	2.69	133.13
C101-20-10_10	174	256	47.12	2.38	3.65	53.26
C101-20-10_15	174	270	55.17	3.06	5.19	69.51
C101-20-10_20	148	205	38.51	3.31	7.38	122.64
AVERAGES	**167.5**	**248.75**	**48.13**	**2.48**	**4.73**	**94.63**
AVERAGE DEVIATIONS	**9.75**	**21.87**				

Furthermore, for 12 customers, the average cost for B&C is 117.75, and we also obtain an average cost variation of 16.87 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 154 and on average, we observe a variation in cost of 13 around the average for the different cases of mobile customers. The problem is solved on average in 0.67 seconds for B&C and 3.27 seconds for TPDH-FMC.

For 16 customers, the average cost for B&C is 140, and on average there is a slight variation in cost around the average for B&C, as this variation is 4. However, the average cost of TPDH-FMC is 219.25, and on average the variation in costs for the different mobile customer cases around this average is 15.75. On average, the problem is solved in 1.15 seconds for B&C and 3.88 seconds for TPDH-FMC.

Also, for 20 customers, the average cost for B&C is 167.5, and on average we observe a cost variation of 9.75 around the average for the different mobile customer cases. For TPDH-FMC, the average cost is 248.75, and on average, the cost variation for the different mobile customer cases is 21.87 around the average. On average, the problem is solved in 2.48 seconds for the B&C and 4.73 seconds for the TPDH-FMC.

In **Table 2**, where we have instance C102, we see that regardless of the number of customers and mobile customers, B&C gives a better cost than TPDH-FMC except when we have 20 customers and all of them are mobile. However, this time, TPDH-FMC was faster at solving the problem than B&C. This is why we see negative values in the CPU GAP columns.

Table 2. The results of instance C102.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C102-8-4_2	49	80	63.26	2129.27	1.79	−99.91
C102-8-4_4	49	80	63.26	8638.06	1.83	−99.97
C102-8-4_6	49	80	63.26	2039.7	1.90	−99.90
C102-8-4_8	49	80	63.26	501.72	2.24	−99.55
AVERAGES	**49**	**80**	**63.26**	**3327.18**	**1.94**	**−99.83**
AVERAGE DEVIATIONS	**0**	**0**				
C102-12-6_3	101	128	26.73	828.87	2.75	−99.66
C102-12-6_6	101	128	26.73	2404.29	3.50	−99.85
C102-12-6_9	101	116	14.85	4552.98	5.20	−99.88
C102-12-6_12	78	109	39.74	6670.48	5.39	−99.91
AVERAGES	**95.25**	**120.25**	**27.01**	**3614.15**	**4.21**	**−99.83**
AVERAGE DEVIATIONS	**8.62**	**7.75**				
C102-16-8_4	143	214	49.65	2881.32	2.63	−99.90
C102-16-8_8	143	216	51.04	1241.61	3.66	−99.70
C102-16-8_12	135	218	61.48	5308.03	4.70	−99.91
C102-16-8_16	134	182	35.82	10.800	7.02	−99.93
AVERAGES	**138.75**	**207.5**	**49.50**	**5057.74**	**4.50**	**−99.86**
AVERAGE DEVIATIONS	**4.25**	**12.75**				
C102-20-10_5	167	250	49.70	289.29	3.04	−98.94
C102-20-10_10	167	256	53.29	2937.85	3.93	−99.86
C102-20-10_15	202	241	19.30	2433.72	6	−99.75
C102-20-10_20	258	216	−16.27	371.44	8.23	−97.78
AVERAGES	**198.5**	**240.75**	**26.50**	**1508.07**	**5.30**	**−99.08**
AVERAGE DEVIATIONS	**31.5**	**12.37**				

Similarly, for 8 customers, we find that regardless of the number of mobile customers, the costs obtained by both the B&C and TPDH-FMC methods remain unchanged. For B&C, the cost is 49, and for TPDH-FMC, the cost is 80. This explains their average deviations with zero values.

Also, the AVERAGE GAP in costs with 8 clients is 63%, which also shows that the cost obtained by the TPDH-FMC is a little too far from the B&C. On average, the problem is solved in 3327.18 seconds (55 minutes and 27 seconds) for the B&C and 1.94 seconds for the TPDH-FMC, which means that the TPDH-FMC is much faster than the B&C.

Furthermore, for 12 customers, the average cost for B&C is 95.25, and we also obtain an average a cost variation of 8.62 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 120.25 and on average, we observe a cost variation of 7.75 around the average for the different mobile customer cases. The problem is solved on average in 0.67 seconds for B&C and 3.27 seconds for TPDH-FMC. The best cost for B&C and TPDH-FMC is obtained when all 12 customers are mobile. For B&C, the value is 78 and for TPDH-FMC, the value is 109.

For 16 customers, the average cost for B&C is 138.75, and on average there is a slight variation in cost around the average for B&C, as this variation is 4.25. However, the average cost for TPDH-FMC is 240.75, and on average, the variation in costs for the different mobile customer cases around this average is 12.75. On average, the problem is solved in 5057.74 seconds (1 hour 24 minutes 17 seconds) for B&C and 4.5 seconds for TPDH-FMC. Hence, TPDH- is very fast compared to B&C. In addition, the AVERAGE GAP in cost is 49.50%.

Also, for 20 customers, we observe a negative cost gap when all 20 customers are mobile, which means that TPDH-FMC achieved a better cost than B&C. Here, the cost achieved by TPDH-FMC is 216, while that of B&C is 258. TPDH-FMC was able to achieve the best cost because the value obtained by B&C was not optimal. The average cost for B&C is 198.5, and we observe an average cost variation of 31.5 around the average for the different cases of mobile customers. For TPDH-FMC, the average cost is 240.75, and on average, the variation in cost for the different cases of mobile customers is 12.37 around the average. On average, the problem is solved in 1508.05 seconds (25 minutes 8 seconds) for B&C and 5.30 seconds for TPDH-FMC. The average cost gap is 26.50%.

In **Table 3**, we have instance C103. We also see that regardless of the number of customers and mobile customers, B&C gives a better cost than TPDH-FMC except when we have 20 customers and all of these customers are mobile. But once again, TPDH-FMC was faster at solving the problem than B&C.

Similarly, for 8 customers, we find that regardless of the number of mobile customers, the cost obtained by B&C does not change; this cost is 49. Also, the AVERAGE GAP in costs with 8 customers is 64%, which also shows that the cost obtained by TPDH-FMC is a little too far from that of B&C. On average, the problem is solved in 6100.05 seconds (1 hour 41 minutes and 40 seconds) for B&C and 1.97 seconds for TPDH-FMC, which means that TPDH-FMC is much faster than B&C.

Furthermore, for 12 customers, the average cost for B&C is 95, and we also obtain an average cost variation of 9 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 114.5, and on average, we observe a cost variation of 14.25 around the average for the different mobile customer cases. The problem is solved on average in 5590.82 seconds (1 hour 33 minutes 10 seconds) for B&C and 5.25 seconds for TPDH-FMC. The best cost for B&C and TPDH-FMC is obtained when all 12 customers are mobile. For B&C, the value is 77, and for TPDH-FMC, the value is 86.

Table 3. The results of instance C103.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C103-8-4_2	49	80	63.26	2089.84	1.79	−99.91
C103-8-4_4	49	80	63.26	9267.56	1.84	−99.98
C103-8-4_6	49	81	65.30	5065.65	21.84	−99.96
C103-8-4_8	49	81	65.30	7977.16	2.41	−99.96
AVERAGES	**49**	**80.5**	**64.28**	**6100.05**	**1.97**	**−99.95**
AVERAGE DEVIATIONS	**0**	**0.5**				
C103-12-6_3	101	128	26.73	1404.95	2.84	−99.79
C103-12-6_6	101	128	26.73	2391.42	3.40	−99.85
C103-12-6_9	101	116	14.85	10.800	5.82	−99.94
C103-12-6_12	77	86	11.68	7766.91	8.92	−99.88
AVERAGES	**95**	**114.5**	**20**	**5590.82**	**5.25**	**−99.87**
AVERAGE DEVIATIONS	**9**	**14.25**				
C103-16-8_4	143	176	23.07	10800	2.85	−99.97
C103-16-8_8	143	166	16.08	10800	3.95	−99.96
C103-16-8_12	136	141	3.67	10800	5.61	−99.94
C103-16-8_16	144	146	1.38	9047.21	9.74	−99.89
AVERAGES	**141.5**	**157.25**	**11.05**	**10361.80**	**5.54**	**−99.94**
AVERAGE DEVIATIONS	**2.75**	**13.75**				
C103-20-10_5	162	195	20.37	10800	3.60	−99.96
C103-20-10_10	162	192	18.51	10800	5.36	−99.95
C103-20-10_15	186	190	2.15	7665.38	7.45	−99.90
C103-20-10_20	224	184	−17.85	7185.17	10.20	−99.85
AVERAGES	**183.5**	**190.25**	**5.79**	**9112.63**	**6.65**	**−99.91**
AVERAGE DEVIATIONS	**21.5**	**3.25**				

For 16 customers, the average cost for B&C is 141.5, and on average there is a slight variation in cost around the average for B&C, as this variation is 2.75. However, the average cost for TPDH-FMC is 157.25 and on average the variation in costs for the different mobile customer cases around this average is 13.75. On average, the problem is solved in 10,361.80 seconds (2 hours 52 minutes 41 seconds) for B&C and 5.54 seconds for TPDH-FMC. Hence the speed of TPDH-FMC compared to B&C. In addition, the AVERAGE GAP in cost is 11.50%.

Also, for 20 customers, when all 20 customers are mobile, TPDH-FMC achieved a better cost than B&C. Here, the cost achieved by TPDH-FMC is 184, while that of B&C is 224. Obviously, TPDH-FMC was able to achieve the best cost in this case as well because the value obtained by B&C was not optimal. The average cost for B&C is 183.5, and we observe an average cost variation of 21.5 around the average for the different cases of mobile customers. For TPDH-FMC, the average cost is 190.25, and on average, the variation in cost for different mobile customer cases is 3.25 around the average. On average, the problem is solved in 9,112.63 seconds (2 hours 31 minutes 52 seconds) for B&C and 6.65 seconds for TPDH-FMC. The average cost gap is 5.79%.

In **Table 4**, we have instance C201. We can see that in some cases, B&C and TPDH-FMC give the same cost. This explains the zero cost gap. Furthermore, B&C was faster at solving the problem than TPDH-FMC in all cases.

Table 4. The results of instance C201.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C201-8-4_-2	134	134	0	0.16	1.47	801.21
C201-8-4_-4	134	134	0	0.18	1.82	903.29
C201-8-4_-6	115	134	16.52	0.20	2.75	1243.41
C201-8-4_-8	90	90	0	0.24	3.31	1231.72
AVERAGES	**118.25**	**123**	**4.13**	**0.2**	**2.34**	**1044.91**
AVERAGE DEVIATIONS	**15.75**	**16.5**				
C201-12-6_3	176	200	13.63	0.21	2.01	826.14
C201-12-6_6	138	198	43.47	0.26	3.03	1048.48
C201-12-6_9	104	183	75.96	0.32	4.22	1211.49
C201-12-6_12	104	168	61.53	0.69	5.30	664.69
AVERAGES	**130.5**	**187.25**	**48.65**	**0.37**	**3.64**	**937.70**
AVERAGE DEVIATIONS	**26.5**	**11.75**				
C201-16-8_4	187	215	14.97	0.29	2.64	790.57
C201-16-8_8	117	193	64.95	0.40	4.08	901.71
C201-16-8_12	117	189	61.53	0.48	5.2	983.33
C201-16-8_16	117	193	64.95	0.63	7.13	1025.23
AVERAGES	**134.5**	**197.5**	**51.60**	**0.45**	**4.76**	**925.21**
AVERAGE DEVIATIONS	**26.25**	**8.75**				
C201-20-10_5	194	312	60.82	0.41	3.38	714.69

Continued

C201-20-10_10	137	277	102.18	0.92	4.99	438.03
C201-20-10_15	137	260	89.78	0.72	6.68	828.19
C201-20-10_20	135	207	53.33	1.87	9.15	387.79
AVERAGES	**150.75**	**264**	**76.53**	**0.98**	**6.05**	**592.18**
AVERAGE DEVIATIONS	**21.62**	**30.5**				

For 8 customers, we find that for 2, 4 and 8 mobile customers, B&C and TPDH-FMC give us the same costs (134) for 2 and 4 mobile customers and (90) for 8 mobile customers. In the only case where we did not have the same costs with both methods, the cost GAP is 16.52%. Also, the AVERAGE cost GAP with 8 customers is 4.13%. The average cost for B&C is 118.25, and we also obtain an average cost variation of 15.75 around the average for the different cases of mobile customers, while for TPDH-FMC, the average cost is 123, and on average, we observe a cost variation of 16.5 around the average for the different cases of mobile customers. On average, the problem is solved in 0.2 seconds for B&C and 2.34 seconds for TPDH-FMC.

Furthermore, for 12 customers, the average cost for B&C is 130.5, and we also obtain an average a cost variation of 26.5 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 187.25 and on average, we observe a cost variation of 11.75 around the average for the different mobile customer cases. The problem is solved on average in 0.37 seconds for B&C and 3.64 seconds for TPDH-FMC. The AVERAGE GAP in cost is 48.65%.

For 16 customers, the average cost for B&C is 134.5 and the average variation in cost around the average is 26.25 for B&C, whereas the average cost for TPDH-FMC is 197.5 and the average variation in costs for the different mobile customer cases around this average is 8.75. On average, the problem is solved in 0.45 seconds for B&C and 4.76 seconds for TPDH-FMC. In addition, the AVERAGE GAP in cost is 51.60%.

Also, for 20 customers, the average cost for B&C is 150.75, and we observe an average cost variation of 21.62 around the average for the different mobile customer cases. For TPDH-FMC, the average cost is 264, and on average, the cost variation for the different mobile customer cases is 30.5 around the average. On average, the problem is solved in 0.98 seconds for B&C and 6.05 seconds for TPDH-FMC. The average cost gap is 76.53%.

In **Table 5**, we have instance C202. TPDH-FMC was faster in solving the problem than B&C in all cases.

For 8 customers, we find that when all customers are mobile, B&C and TPDH-FMC give us the same cost (87). Therefore, the AVERAGE GAP in costs with 8 customers is 3.96%. This means that the costs obtained by TPDH-FMC are very close to those obtained by B&C. The average cost for B&C is 113.25, and we also obtain an average cost variation of 13.75 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 117.75 and, on average, we observe a cost variation of 15.37 around the average for the different mobile customer cases. On average, the problem is solved in 401.75 seconds (6 minutes 41 seconds) for B&C and 1.66 seconds for TPDH-FMC.

Table 5. The results of instance C202.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C202-8-4_2	127	128	0.78	3.77	1.43	−62.20
C202-8-4_4	127	128	0.78	9.07	1.43	−84.16
C202-8-4_6	112	128	14.28	1587.6	1.73	−99.89
C202-8-4_8	87	87	0	6.57	2.04	−68.87
AVERAGES	**113.25**	**117.75**	**3.96**	**401.75**	**1.66**	**−78.74**
AVERAGE DEVIATIONS	**13.75**	**15.37**				
C202-12-6_3	171	195	14.03	936.32	2.77	−99.70
C202-12-6_6	129	195	51.16	827.28	4.85	−99.41
C202-12-6_9	104	182	75	6.87	5.98	−12.92
C202-12-6_12	104	169	62.5	501.93	7.47	−98.51
AVERAGES	**127**	**185.25**	**50.67**	**568.10**	**5.27**	**−77.63**
AVERAGE DEVIATIONS	**23**	**9.75**				
C202-16-8_4	169	225	33.13	88.01	3.13	−96.44
C202-16-8_8	114	199	74.56	37.71	4.62	−87.73
C202-16-8_12	114	188	64.91	146.19	5.91	−95.95
C202-16-8_16	114	196	71.92	51.67	8.66	−83.23
AVERAGES	**127.75**	**202**	**61.13**	**80.90**	**5.58**	**−90.84**
AVERAGE DEVIATIONS	**20.62**	**11.5**				
C202-20-10_5	177	311	75.70	751.72	3.76	−99.49
C202-20-10_10	137	279	103.64	37.65	5.05	−86.58
C202-20-10_15	137	244	78.10	65.74	6.92	−89.47
C202-20-10_20	153	221	44.44	2004.35	10.69	−99.46
AVERAGES	**151**	**263.75**	**75.47**	**714.86**	**6.60**	**−93.75**
AVERAGE DEVIATIONS	**14**	**31.25**				

Furthermore, for 12 customers, the average cost for B&C is 127, and we also obtain an average a cost variation of 13 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 185.25 and on average, we observe a cost variation of 9.75 around the average for the different mobile customer cases. The problem is solved on average in 568.10 seconds (9 minutes 28 seconds) for B&C and 5.27 seconds for TPDH-FMC. The AVERAGE GAP in cost is 50.67%.

For 16 customers, the average cost for B&C is 127.75 and the average variation in cost around the average is 20.62 for B&C, whereas the average cost of TPDH-FMC is 202 and the average variation in costs for the different mobile customer cases around this average is 11.5. On average, the problem is solved in 80.90 seconds (1 minute 20 seconds) for B&C and 5.58 seconds for TPDH-FMC. In addition, the AVERAGE GAP in cost is 61.3%.

Also, for 20 customers, the average cost for B&C is 151, and we observe an average variation in cost of 14 around the average for the different mobile customer cases. For TPDH-FMC, the average cost is 263.75, and on average, the variation in cost for the different mobile customer cases is 31.25 around the average. On average, the problem is solved in 714.86 seconds (11 minutes 54 seconds) for B&C and 6.60 seconds for TPDH-FMC. The average cost gap is 75.47%.

In **Table 6**, we have instance C203. TPDH-FMC was faster in solving the problem than B&C in all cases.

Table 6. The results of instance C203.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
C203-8-4_2	127	128	0.78	3.77	1.28	−65.98
C203-8-4_4	127	128	0.78	9.32	1.37	−85.27
C203-8-4_6	112	125	11.60	10800	2.21	−99.97
C203-8-4_8	87	88	1.14	3687.22	2.10	−99.94
AVERAGES	**113.25**	**117.25**	**3.58**	**3625.08**	**1.74**	**−87.79**
AVERAGE DEVIATIONS	**13.75**	**14.62**				
C203-12-6_3	167	195	16.76	192.80	2.59	−98.65
C203-12-6_6	123	167	35.77	734.95	5.69	−99.22
C203-12-6_9	105	136	29.52	2588.16	6.75	−99.73
C203-12-6_12	109	136	24.77	615.87	7.38	−99.80
AVERAGES	**126**	**158.5**	**26.70**	**1032.94**	**5.60**	**−99.10**
AVERAGE DEVIATIONS	**20.5**	**22.5**				
C203-16-8_4	169	209	23.66	10.800	3.17	−99.97
C203-16-8_8	168	177	5.35	236.90	5.44	−97.70
C203-16-8_12	117	177	51.28	7302.12	6.15	−99.91
C203-16-8_16	129	177	37.20	3996.18	10.04	−99.74
AVERAGES	**145.75**	**185**	**29.37**	**5583.80**	**6.20**	**−99.33**

Continued

AVERAGE DEVIATIONS	**22.75**	**12**				
C203-20-10_5	240	268	11.66	2967.92	4.28	−99.85
C203-20-10_10	191	240	25.65	7353.71	6.42	−99.91
C203-20-10_15	208	240	15.38	6779.53	9.55	−99.85
C203-20-10_20	201	207	2.98	544.99	13.23	−97.57
AVERAGES	**210**	**238.75**	**13.92**	**4411.53**	**8.37**	**−99.29**
AVERAGE DEVIATIONS	**15**	**15.87**				

For 8 customers, the AVERAGE GAP in costs is 3.58%. This means that the costs obtained by the TPDH-FMC are very close to those obtained by the B&C. The average cost for the B&C is 113.25 and we also obtain an average cost variation of 13.75 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 117.25 and, on average, we observe a cost variation of 14.62 around the average for the different mobile customer cases. On average, the problem is solved in 3625.08 seconds (1 hour 0 minutes 25 seconds) for B&C and 1.74 seconds for TPDH-FMC.

Furthermore, for 12 customers, the average cost for B&C is 126, and we also obtain an average a cost variation of 20.5 around the average for the different mobile customer cases, whereas for TPDH-FMC, the average cost is 158.5 and on average, we observe a cost variation of 22.5 around the average for the different mobile client cases. The problem is solved on average in 1032.94 seconds (17 minutes 12 seconds) for B&C and 5.60 seconds for TPDH-FMC. The AVERAGE GAP in cost is 26.70%.

For 16 customers, the average cost for B&C is 145.75 and the average variation in cost around the average is 22.75 for B&C, whereas the average cost for TPDH-FMC is 185 and the average variation in costs for the different mobile customer cases around this average is 12. On average, the problem is solved in 5583.80 seconds (1 hour 33 minutes 3 seconds) for B&C and 6.20 seconds for TPDH-FMC. In addition, the AVERAGE GAP in cost is 29.37%.

Also, for 20 customers, the average cost for B&C is 210, and we observe an average variation in cost of 15 around the average for the different mobile customer cases. For TPDH-FMC, the average cost is 238.75, and on average, the variation in cost for the different mobile customer cases is 15.85 around the average. On average, the problem is solved in 4,411.53 seconds (1 hour 13 minutes 31 seconds) for B&C and 8.37 seconds for TPDH-FMC. The average cost gap is 13.92%.

In Table 7, we have instance R101. B&C was faster in solving the problem than TPDH-FMC in all cases.

For 8 customers, the AVERAGE GAP in costs is 2.46%. This means that the costs obtained by HDDP-PCM are very close to those obtained by B&C. The average cost for B&C is 182.75 and the average cost for TPDH-FMC is 187.5. B&C and TPDH-FMC gave the same cost of 167 when 6 customers are mobile and when all 8 customers are mobile. We observe an average cost variation of 16.62 around the average for B&C. While for TPDH-FMC, we observe a cost variation of 20.5 around the average for the different cases of mobile customers. On average, the problem is solved in 0.34 seconds for B&C and 1.65 seconds for TPDH-FMC. Furthermore, for 12 customers, the average cost for B&C is 289.5 and we also obtain an average cost variation of 14.5 around the average for the different mobile customer cases, while for TPDH-FMC, the average cost is 315.5 and on average we observe a cost variation of 17 around the average for the different mobile customer cases. The average problem resolution time is 0.56 seconds for B&C and 1.87 seconds for TPDH-FMC. The AVERAGE GAP in cost is 9.02%.

Table 7. The results of instance R101.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R101-8-4_2	216	223	3.24	0.29	1.22	318.02
R101-8-4_4	181	193	6.62	0.33	1.41	325.52
R101-8-4_6	167	167	0	0.34	1.74	401.44
R101-8-4_8	167	167	0	0.39	2.21	460.10
AVERAGES	**182.75**	**187.5**	**2.46**	**0.34**	**1.65**	**376.27**
AVERAGE DEVIATIONS	**16.62**	**20.5**				
R101-12-6_3	306	341	11.43	0.34	1.13	225.78
R101-12-6_6	302	324	7.28	0.43	1.50	250.46
R102-12-6_9	281	286	1.77	0.58	2.05	251.79
R102-12-6_12	269	311	15.61	0.90	2.78	207.86
AVERAGES	**289.5**	**315.5**	**9.02**	**0.56**	**1.87**	**233.97**
AVERAGE DEVIATIONS	**14.5**	**17**				
R101-16-8_4	394	438	11.16	0.46	1.45	213.36
R101-16-8_8	394	440	11.67	0.61	1.86	202.59
R101-16-8_12	341	396	16.12	0.96	2.72	182.76
R101-16-8_16	291	352	20.96	1.48	3.82	158.16
AVERAGES	**355**	**406.5**	**14.98**	**0.88**	**2.46**	**189.22**
AVERAGE DEVIATIONS	**39**	**32.5**				
R101-20-10_5	483	527	9.10	0.64	1.92	198.14
R101-20-10_10	425	470	10.58	0.82	2.89	251.64
R101-20-10_15	414	516	24.63	1.59	3.98	150.12
R101-20-10_20	368	455	23.64	2.21	5.09	130.09
AVERAGES	**422.5**	**492**	**16.99**	**1.31**	**3.47**	**182.50**
AVERAGE DEVIATIONS	**31.5**	**29.5**				

For 16 customers, the average cost for B&C is 355 and the average variation in cost around the average is 39 for B&C, whereas the average cost of TPDH-FMC is 406.5 and the average variation in costs for the different mobile customer cases around this average is 32.5. On average, the problem is solved in 0.88 seconds for B&C and 2.46 seconds for TPDH-FMC. In addition, the AVERAGE GAP in cost is 14.98%.

Also, for 20 customers, the average cost for B&C is 422.5, and we observe an average cost variation of 31.5 around the average for the different mobile customer cases. For TPDH-FMC, the average cost is 492, and on average, the cost variation of the different mobile customer cases is 29.5 around the average. On average, the problem is solved in 1.31 seconds for B&C and 3.47 seconds for TPDH-FMC. The average cost gap is 16.99%.

In Table 8, we have instance R102. TPDH-FMC was faster than B&C in solving our problem in all cases.

Table 8. The results of instance R102.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R102-8-4_2	177	183	3.38	2	1.78	−11.17
R102-8-4_4	152	159	4.60	3.11	2.27	−26.86
R102-8-4_6	145	145	0	5.53	3.01	−45.50
R102-8-4_8	145	145	0	5.45	3.96	−27.24
AVERAGES	**154.75**	**158**	**1.99**	**4.02**	**2.76**	**−27.69**
AVERAGE DEVIATIONS	**11.12**	**13**				
R102-12-6_3	264	277	4.92	56.16	1.66	−97.03
R102-12-6_6	266	282	6.01	3602.72	2.08	−99.94
R102-12-6_9	246	248	0.81	9179.85	3.10	−99.96
R102-12-6_12	249	233	-6.42	6037.53	3.84	−99.93
AVERAGES	**256.25**	**260**	**1.33**	**4719.06**	**2.67**	**−99.22**
AVERAGE DEVIATIONS	**8.75**	**19.5**				
R102-16-8_4	356	367	3.08	10.800	1.91	−99.98
R102-16-8_8	328	380	15.85	6436	2.57	−99.95
R102-16-8_12	287	325	13.24	10.800	3.94	−99.96
R102-16-8_16	236	363	53.81	5265.79	5.64	−99.89
AVERAGES	**301.75**	**358.75**	**21.49**	**8325.44**	**3.52**	**−99.94**
AVERAGE DEVIATIONS	**40.25**	**16.87**				
R102-20-10_5	433	473	9.23	3526.32	2.25	−99.93
R102-20-10_10	381	462	21.25	10.800	3.58	−99.96
R102-20-10_15	455	440	−3.29	858.59	5.37	−99.37
R102-20-10_20	328	403	22.86	10.800	7.13	−99.93
AVERAGES	**399.25**	**444.5**	**12.51**	**6496.22**	**4.58**	**−99.80**
AVERAGE DEVIATIONS	**44.75**	**23**				

For 8 customers, we find that TPDH-FMC and B&C gave the same cost of 145 when we have 6 mobile customers and 8 mobile customers. Also, the AVERAGE GAP in costs with 8 customers is 1.99%, which also shows that the average cost obtained by TPDH-FMC is close to that obtained by B&C. On average, the problem is solved in 4.02 seconds for B&C and 2.76 seconds for TPDH-FMC.

For 12 customers, TPDH-FMC gave a better cost of 233 than that of B&C, which is 249. The AVERAGE GAP in costs for 12 customers is 1.33%. The average time taken to solve the problem is 4719.06 seconds (1 hour 18 minutes 39 seconds) for B&C and 2.67 seconds for TPDH-FMC.

Furthermore, for 16 customers, the AVERAGE GAP in costs is 21.49%. The problem is solved on average in 8325.44 seconds (2 hours 18 minutes 45 seconds) for B&C and 3.52 seconds for TPDH-FMC.

Also, for 20 customers, we find that TPDH-FMC gave a better cost of 440 than that of B&C, which is 455 when we have 15 mobile customers. The AVERAGE GAP in cost is 12.51%. The average problem resolution time is 6496.22 seconds (1 hour 48 minutes 16 seconds) for B&C and 4.58 seconds for TPDH-FMC.

In Table 9, we have instance R103.

Table 9. The results of instance R103.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R103-8-4_2	177	183	3.38	1.57	1.77	12.52
R103-8-4_4	152	159	4.60	2.06	2.32	12.70
R103-8-4_6	145	145	0	3.25	3.73	14.74
R103-8-4_8	144	145	0.69	1865.97	5.76	−99.69
AVERAGES	**154.5**	**158**	**2.17**	**468.21**	**3.39**	**−14.92**
AVERAGE DEVIATIONS	**11.25**	**13**				
R103-12-6_3	264	277	4.92	10.800	1.69	−99.98
R103-12-6_6	230	250	8.69	7002.9	2.57	−99.96
R103-12-6_9	215	208	−3.25	5259.96	4.49	−99.91
R103-12-6_12	199	208	4.52	2009.1	5.97	−99.70
AVERAGES	**227**	**235.75**	**3.72**	**6267.99**	**3.68**	**−99.89**
AVERAGE DEVIATIONS	**20**	**27.75**				
R103-16-8_4	326	342	4.90	5503.82	2.13	−99.96
R103-16-8_8	292	318	8.90	311.19	3.27	−98.94
R103-16-8_12	273	298	9.15	608.23	5.12	−99.15
R103-16-8_16	244	238	−2.45	8844.79	6.88	−99.92
AVERAGES	**283.75**	**299**	**5.12**	**3817**	**4.35**	**−99.49**
AVERAGE DEVIATIONS	**25.25**	**31**				
R103-20-10_5	372	399	7.25	5080.76	3.03	−99.94

Continued

R103-20-10_10	298	315	5.70	10.800	5.58	−99.94
R103-20-10_15	306	318	3.92	10.800	7.87	−99.92
R103-20-10_20	344	295	−14.24	3056.99	9.57	−99.68
AVERAGES	**330**	**331.75**	**0.66**	**7434.43**	**6.51**	**−99.87**
AVERAGE DEVIATIONS	**28**	**33.62**				

For 8 customers, TPDH-FMC and B&C gave the same cost of 145 when we have 6 mobile customers. Also, the AVERAGE GAP in costs is 2.17%, which also shows that the average cost obtained by TPDH-FMC is close to that obtained by B&C. On average, the problem is solved in 468.21 seconds (7 minutes 48 seconds) for B&C and 3.39 seconds for TPDH-FMC.

For 12 customers, TPDH-FMC gave a better cost of 208 than B&C, which is 215. The AVERAGE GAP in costs for 12 customers is 3.72%. The problem is solved on average in 6267.99 seconds (1 hour 44 minutes 27 seconds) for B&C and 3.68 seconds for TPDH-FMC.

Furthermore, for 16 customers, TPDH-FMC gave a better cost of 238 than B&C, which was 244 when all 16 customers were mobile. The AVERAGE GAP in costs is 5.12%. The problem is solved on average in 3817 seconds (1 hour 3 minutes 37 seconds) for B&C and 4.35 seconds for HDDP-PCM.

Also, for 20 customers, we observe that TPDH-FMC gave a better cost of 295 than that of B&C, which is 344 when all 20 customers are mobile. The AVERAGE GAP in cost is 0.66%. The average problem resolution time is 7434.43 seconds (2 hours 3 minutes 54 seconds) for B&C and 6.51 seconds for TPDH-FMC.

In **Table 10**, we have instance R201. For 8 customers, the AVERAGE GAP in costs is 11.92%. On average, the problem is solved in 0.72 seconds for B&C and 3.71 seconds for TPDH-FMC.

Table 10. The results of instance R201.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R201-8-4_2	200	204	2	1	2.31	130.60
R201-8-4_4	162	180	11.11	0.36	2.86	677.98
R201-8-4_6	153	186	21.56	0.39	4.14	939.09
R201-8-4_8	146	165	13.01	1.14	5.51	384.03
AVERAGES	**165.25**	**183.75**	**11.92**	**0.72**	**3.71**	**532.93**
AVERAGE DEVIATIONS	**17.37**	**11.25**				
R201-12-6_3	286	315	10.13	2.12	3.02	42.59
R201-12-6_6	272	315	15.80	10.65	3.79	−64.35
R201-12-6_9	224	263	17.41	6.44	5.50	−14.50
R201-12-6_12	224	273	21.87	14.50	7.49	−48.33

Continued

AVERAGES	**251.5**	**291.5**	**16.30**	**8.43**	**4.95**	**−21.15**
AVERAGE DEVIATIONS	**27.5**	**23.5**				
R201-16-8_4	301	320	6.31	25.54	4.13	−83.82
R201-16-8_8	294	341	15.98	129.34	5.49	−95.75
R201-16-8_12	262	306	16.79	215.56	8.09	−96.24
R201-16-8_16	236	295	25	111.42	12.75	−88.55
AVERAGES	**273.25**	**315.5**	**16.02**	**120.47**	**7.61**	**−91.09**
AVERAGE DEVIATIONS	**24.25**	**15**				
R201-20-10_5	379	446	17.67	399.69	5.53	−98.61
R201-20-10_10	341	408	19.64	32.79	7.28	−77.80
R201-20-10_15	341	436	27.85	69.07	9.59	−86.1
R201-20-10_20	320	432	35	343.32	12.36	−96.39
AVERAGES	**345.25**	**430.5**	**25.04**	**211.22**	**8.69**	**−89.72**
AVERAGE DEVIATIONS	**16.87**	**11.25**				

For 12 customers, the AVERAGE GAP in costs is 16.30%. On average, the problem is solved in 8.43 seconds for B&C and 54.95 seconds for TPDH-FMC.

Furthermore, for 16 customers, the AVERAGE GAP in costs is 16.02%. The problem is solved on average in 120.47 seconds (2 minutes) for B&C and 7.61 seconds for TPDH-FMC.

Also, for 20 customers, the AVERAGE GAP in costs is 25.04%. On average, the problem is solved in 211.22 seconds (3 minutes 31 seconds) for B&C and 8.69 seconds for TPDH-FMC.

In **Table 11**, we have instance R202.

For 8 customers, TPDH-FMC and B&C gave the same cost of 142 for 2 mobile customers and the same cost of 118 for 4 mobile customers. The AVERAGE GAP in costs is 10.59%. On average, the problem is solved in 1.93 seconds for B&C and 4.02 seconds for TPDH-FMC.

For 12 customers, the AVERAGE GAP in costs is 9.68%. On average, the problem is solved in 2467.60 seconds (41 minutes 7 seconds) for B&C and 4.85 seconds for TPDH-FMC.

Furthermore, for 16 customers, the AVERAGE GAP in costs is 13.02%. The problem is solved on average in 5983.49 seconds (1 hour 39 minutes 43 seconds) for B&C and 7.05 seconds for TPDH-FMC.

Also, for 20 customers, TPDH-FMC gave a better cost of 369 than B&C, which is 410 when we have 15 mobile customers. The AVERAGE GAP in costs is 8.58%. On average, the problem is solved in 3690.01 seconds (1 hour 1 minute 30 seconds) for B&C and 9.16 seconds for TPDH-FMC.

In **Table 12**, we have instance R203.

Table 11. The results of instance R202.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R202-8-4_2	142	142	0	0.75	2.86	278.59
R202-8-4_4	118	118	0	0.94	2.86	201.58
R202-8-4_6	118	143	21.18	2.93	4.81	64.35
R202-8-4_8	118	143	21.18	3.10	5.53	78.08
AVERAGES	**124**	**136.5**	**10.59**	**1.93**	**4.02**	**155.65**
AVERAGE DEVIATIONS	**9**	**9.25**				
R202-12-6_3	236	252	6.77	401.5	3.15	−99.22
R202-12-6_6	232	260	12.06	888.57	4.39	−99.50
R202-12-6_9	194	216	11 .34	1290.14	4.97	−99.61
R202-12-6_12	199	216	8.54	7286.62	6.91	−99.90
AVERAGES	**215.25**	**236**	**9.68**	**2467.60**	**4.85**	**−99.56**
AVERAGE DEVIATIONS	**18.75**	**20**				
R202-16-8_4	276	340	23.18	7511.5	4.36	−99.94
R202-16-8_8	276	315	14.13	10416.9	5.69	−99.94
R202-16-8_12	223	247	10.76	1196.22	7.68	−99.35
R202-16-8_16	224	233	4.01	4809.34	10.45	−99.78
AVERAGES	**249.75**	**283.75**	**13.02**	**5983.49**	**7.05**	**−99.75**
AVERAGE DEVIATIONS	**26.25**	**43.75**				
R202-20-10_5	335	406	21.19	3156.16	5.16	−99.83
R202-20-10_10	302	363	20.19	10800	7.71	−99.92
R202-20-10_15	410	369	−10	494.67	9.88	−98
R202-20-10_20	339	349	2.94	309.21	13.87	−95.51
AVERAGES	**346.5**	**371.75**	**8.58**	**3690.01**	**9.16**	**−98.31**
AVERAGE DEVIATIONS	**31.75**	**17.12**				

Table 12. The results of instance R203.

	COSTS			CPU		
INSTANCES	B&C	TPDH-FMC	GAP	B&C	TPDH-FMC	GAP
R203-8-4_2	142	142	0	0.78	2.16	174.90
R203-8-4_4	118	118	0	0.99	2.41	142.38
R203-8-4_6	118	148	25.42	2.58	0.40	−84.53
R203-8-4_8	118	142	20.33	3.32	0.50	−84.71
AVERAGES	**124**	**135.5**	**11.44**	**1.92**	**1.37**	**37.01**
AVERAGE DEVIATIONS	**9**	**9.75**				
R203-12-6_3	236	260	10.16	14760.3	2.86	−99.98

Continued

R203-12-6_6	224	232	3.57	10.800	3.93	−99.96
R203-12-6_9	180	208	15.55	5952.8	3.95	−99.93
R203-12-6_12	173	206	19.07	1418.1	4.67	−99.67
AVERAGES	**203.25**	**226.5**	**12.09**	**8232.80**	**3.85**	**−99.88**
AVERAGE DEVIATIONS	**26.75**	**19.5**				
R203-16-8_4	265	336	26.79	10.800	3.35	−99.96
R203-16-8_8	322	343	6.52	196.72	5.11	−97.39
R203-16-8_12	216	254	17.59	4900.49	8.09	−99.83
R203-16-8_16	235	235	0	999.65	11.98	−98.80
AVERAGES	**259.5**	**292**	**12.72**	**4224.21**	**7.13**	**−99.00**
AVERAGE DEVIATIONS	**34**	**47.5**				
R203-20-10_5	367	389	5.99	3533.17	4.28	−99.87
R203-20-10_10	313	313	0	7238.17	7.02	−99.90
R203-20-10_15	297	280	−5.72	4219.02	12.52	−99.70
R203-20-10_20	314	275	−12.42	595.06	16.05	−97.30
AVERAGES	**322.75**	**314.25**	**−3.03**	**3896.35**	**9.97**	**−99.19**
AVERAGE DEVIATIONS	**22.12**	**37.37**				

For 8 customers, TPDH-FMC and B&C gave the same cost of 142 for 2 mobile customers and the same cost of 118 for 4 mobile customers. The AVERAGE GAP in costs is 11.44%. On average, the problem is solved in 1.92 seconds for B&C and 1.37 seconds for TPDH-FMC.

For 12 customers, the AVERAGE GAP in costs is 12.09%. On average, the problem is solved in 8,232.80 seconds (2 hours 17 minutes 12 seconds) for B&C and 3.85 seconds for TPDH-FMC.

Furthermore, for 16 customers, TPDH-FMC and B&C gave the same cost of 235 when all 16 customers are mobile. The AVERAGE GAP in costs is 12.72%. The problem is solved on average in 4224.21 seconds (1 hour 10 minutes 24 seconds) for B&C and 7.13 seconds for TPDH-FMC.

Also, for 20 customers, TPDH-FMC gave a better cost of 280 than B&C, which is 297 when we have 15 mobile customers. Similarly, TPDH-FMC gave a better cost of 275 than B&C, which is 314 when all 20 customers are mobile. The AVERAGE GAP in costs is -3.03%. Thus, TPDH-FMC performed better than B&C in solving the problem. On average, the problem is solved in 3896.35 seconds (1 hour 4 minutes 56 seconds) for B&C and 9.97 seconds for TPDH-FMC.

Table 13 lists the 10 different instances for which TPDH-FMC provided the best feasible solution found within the allotted time limit of 3 hours (10,800 seconds). No best solution found by TPDH-FMC was better than an optimum proven by B&C. That is, the cost gap between B&C is zero. This table contains the names of the instances in the INSTANCES column, the values obtained by the TPDH-

FMC in the TPDH-FMC column and the values obtained by the B&C in the B&C column. We have added a BEST BOUND column which gives the lower bounds obtained by the B&C and a GAP (B&C) column which gives the GAP as a percentage obtained by the B&C.

Table 13. The 10 instances in which TPDH-FMC performed better than B&C.

	COSTS			
INSTANCES	TPDH-FMC	B&C	BEST BOUND	GAP (B&C)
C102-20-10_20	216	258	84.3	67.31
C103-20-10_20	184	224	54	75.89
R102-12-6_12	233	249	137	44.97
R102-20-10_15	440	455	237.4	47.83
R103-12-6_9	208	215	142	33.95
R103-16-8_16	238	244	170.5	30.11
R103-20-10_20	295	344	179.8	47.73
R202-20-10_15	369	410	202.9	50.51
R203-20-10_15	280	297	184	38.04
R203-20-10_20	275	314	168	46.49

We observe that in all cases where TPDH-FMC gave a better result, the solution obtained by B&C is far from optimal. Indeed, the GAPs (B&C) vary from 30% to 76%.

8. Conclusions

We addressed the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) with mobility of certain customers, a new problem that can be seen as a variant of the VRPMTW. We also saw that the exact method for solving this problem is difficult for certain instances, which is why we used a two-phase decomposition heuristic method for mobile customers (TPDH-FMC). We used 12 Solomon instances. For each instance, we used 8 customers, 12 customers, 16 customers, and 20 customers. For each fixed number of customers, we performed our tests by taking 25%, 50%, 75%, and 100% of these customers as mobile. This gave us a total of 192 results. We saw that of the 192 results obtained, in 7.81% (15 results) of cases, B&C and TPDH-FMC gave the same cost. In 5.20% (10 results) of cases, TPDH-FMC gave a better result in terms of cost compared to B&C. In addition, we saw that the number of mobile customers has an influence on the cost values.

Given that the quality of our heuristic solution depends on obtaining a good initial solution, the contribution of machine learning to the search for an initial solution would be advantageous.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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