Scientific Research Publishing

Open Journal of Applied Sciences, 2026, 16(3), 993-1005
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Principles of a Novel Method in Geometrical Optics and Its Application to the Determination of the Conjugation Formulae

Siaka Touré

Félix Houphouet-Boigny University of Cocody, Abidjan, Côte d'Ivoire
Email: siakaahtoure@yahoo.fr

How to cite this paper: Touré, S. (2026) Principles of a Novel Method in Geometrical Optics and Its Application to the Determination of the Conjugation Formulae. *Open Journal of Applied Sciences*, **16**, 993-1005.
https://doi.org/10.4236/ojapps.2026.163058

Received: February 7, 2026
Accepted: March 28, 2026
Published: March 31, 2026

Abstract

The present study introduces a novel axiomatic method for the study of optical elements and systems. This novel method is based on four principles. Their application allowed the determination of the conjugation formulae of the optical elements, such as spherical refractive surface, plane refractive surface, spherical mirror, and plane mirror, without any geometrical construction. They were also applied to find the conjugation formulae in the case of two successive refractive spherical surfaces. That allowed to find the conjugation formula, without any geometrical construction, in the case of a thin lens, then in the case of a parallel-sided refractive glass. The case of a thick lens was also investigated. Hence, the accuracy of the four novel principles is proved. Therefore, the new method can be used to study the conjugation formulae regarding other successive optical elements, without any geometrical construction, and be a very good tool of modelling the optical elements and the optical systems.

Keywords

Specific Refractive Index, Image of a Luminous Object, Refractive Medium Boundary, Conjugation Formulae, Axiomatic Method

1. Introduction

Since Newton, Snell, Fermat, Descartes, Gauss and several other researchers, the formulae linking a luminous object and its conjugated point were studied. Geometrical constructions are very often used to determine those conjugation formulae. Those formulae are widely used in the study of plane and curved mirrors, plane and spherical refractive surfaces, lenses, and optical instruments [1]-[5].

Geometrical optics is based on some fundamental principles:

- It is admitted that light travels in a medium following straight lines that carry the energy.
- According to the principle of Fermat, if n is the refractive index and d the distance, the path followed by light is the one for which the optical path $L = n.d$ is constant.

The objective of the present paper is to present an axiomatic method for which no geometrical construction is needed to find the conjugation formulae. The main principles of our methods are given.

Finally, from those principles, some conjugation formulae are derived, showing the relevance of the new method.

2. Method

Light is an electromagnetic radiation. Maxwell equations show the link between the electric and magnetic fields.

In vacuum, light, which is a form of energy, propagates with a velocity c, which is deemed to be constant. In a dense material, according to the wave theory, the velocity of light is v, which is linked to the refractive index by the following relationship

$$n = \frac{c}{v} \tag{1}$$

In the following study, some assumptions will be stated that are summarised hereafter:

- The refractive media are homogenous and isotropic;
- The luminous rays entering in an optical element or an optical system are paraxial, so that rays near the principal axis are considered;
- In sign convention, it is assumed that light travels from left to right, so that directed distances are positive to the right and negative to the left;
- Heights above the principal axis are positive;
- Angles that have positive slope relative to the principal axis are positive, negative otherwise;
- For reflective surfaces, the refractive index changes sign.

Let a boundary be considered between a dense medium 1, characterised by a reflective index n_1 and a dense medium 2 whose reflective index is n_2.

We consider a luminous object A in medium 1 and a point I at the boundary. A' is the conjugated point of A in medium 2. Hence a ray AI in medium 1 is followed by a ray IA' in medium 2, as shown in **Figure 1**. Another position of the point I can be S, a vertex of the curved boundary.

As a result, the refraction process is linked to a variation $\Delta n = n_2 - n_1$ of the refractive index that indicates a change of the velocity of the light ray and therefore, a change of energy, during the travel of light from A towards its image A'.

The whole optical path between the luminous object A and its image A' which is $L = n_1 \cdot \overline{AI} + n_2 \cdot \overline{IA'}$, is important because of the principle of Fermat.

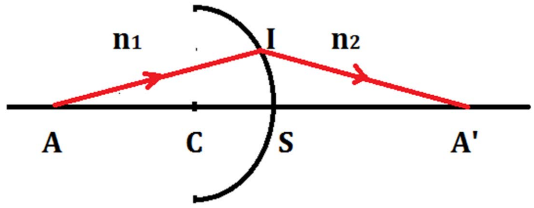

Figure 1. The path between two conjugated points A and A'.

In the case of a spherical refractive surface, the quantity C, which is the vergence, is defined as

$$C = \frac{\Delta n}{\overline{SC}} = \frac{\Delta n}{R} \tag{2}$$

The vergence C indicates the ability of a refractive medium to veer off course a luminous ray.

Our idea is to define a parameter $I = \frac{n}{R}$, linked to the velocity of a ray over a distance R. We call it the specific refractive index (index per unit length).

The dimension of the specific refractive index is m^{-1}. It has the same dimension as the vergence which is also expressed in dioptre (D).

The refractive index physically is an optical power that expresses the curvature of the wave front that hits the refractive surface. That curvature depends on the refractive index.

Hence, referring to **Figure 1**, it is defined in medium 1 and medium 2, respectively

$$I_A = \frac{n_1}{\overline{AI}} \tag{3}$$

$$I_{A'} = \frac{n_2}{\overline{IA'}} \tag{4}$$

Our method uses four principles as a tool to find the conjugation formulae.

2.1. Principle 1

It is the principle of additivity of the specific refractive indexes during the travel of a luminous ray. As seen previously, I_A expresses a curvature of the wave front in medium 1, and $I_{A'}$ a second curvature in medium 2. Hence, the overall curvature is the sum of the two curvatures. That is the basis of the principle of additivity.

Hence, from A to A', a combined specific refractive index is defined as $I_{AA'} = I_A + I_{A'}$. Hence we have

$$I_{AA'} = \frac{n_1}{\overline{AI}} + \frac{n_2}{\overline{IA'}} \tag{5}$$

The point I may be everywhere on the boundary surface. Hence, it may be in S, a vertex of the curved boundary, or very close to it. Then we have

$$I_{AA'} = \frac{n_1}{\overline{AS}} + \frac{n_2}{\overline{SA'}} \tag{6}$$

Equation (6) may also be written as

$$I_{AA'} = -\frac{n_1}{SA} + \frac{n_2}{SA'} = -\left(\frac{n_1}{SA} - \frac{n_2}{SA'}\right) \tag{7}$$

2.2. Principle 2

Let us call C the centre of curvature of the boundary. For each travel path, from A to A', there is an equivalent path of ray which is: movement of the ray from C to S in medium 1, followed by a movement from S to C in medium 2. This path is equivalent to the path from A to A' because it has the same vergence contribution.

This principle is obvious in the case where the curved surface is a spherical mirror.

In the case of refraction, this equivalence path via the centre of curvature is also valid. Of course, let the conjugation formula of a spherical refractive surface be considered. It will be seen later that its conjugation formula, expressed by Equation (23), is

$$\frac{n_1}{SA} - \frac{n_2}{SA'} = \frac{n_1 - n_2}{SC}$$

This equation can also be written as

$$\frac{n_1}{AS} + \frac{n_2}{SA'} = \frac{n_1}{CS} + \frac{n_2}{SC}$$

It can be seen that Equation (6)

$$I_{AA'} = \frac{n_1}{AS} + \frac{n_2}{SA'}$$

Hence, we have

$$I_{AA'} = I_{CS} + I_{SC}$$

Of course, the first movement corresponds to I_{CS} and, for the second movement, we have I_{SC}. Those specific refractive indices are defined as

$$I_{CS} = \frac{n_1}{CS} = -\frac{n_1}{SC} \tag{8}$$

$$I_{SC} = \frac{n_2}{SC} \tag{9}$$

Hence, the validity of principle 2 is proved.

According to principle 1, it is defined as a combined specific index

$$I_{CSC} = I_{CS} + I_{SC} = -\frac{n_1}{SC} + \frac{n_2}{SC} = -\frac{n_1 - n_2}{SC} \tag{10}$$

2.3. Principle 3

The combined specific reflective index does not depend on the location of the luminous object. This principle is stated because a conjugation formula is not linked to a particular position of the luminous object.

The important thing is the medium in which is the object, and the medium in which is the image. A movement from A to A', via S, induces an overall curvature of the wave front due to $\Delta n = n_2 - n_1$. Another movement from C to C, via S, for which the first movement C to S is in medium 1 and the second movement from S to C is in medium 2, induces the same overall curvature of the wave front, due to the same $\Delta n = n_2 - n_1$.

Hence, combining principle 1 and principle 2, we can write

$$I_{AA'} = I_{CSC} \tag{11}$$

As a matter of fact, this statement has also been demonstrated in subsection 2.2. The whole representation of the refractive surface is given in **Figure 2**.

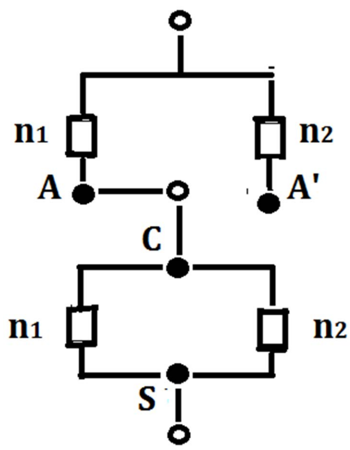

Figure 2. General representative diagram for a spherical refractive surface.

2.4. Principle 4

When two or more successive optical elements are considered, the global specific reflective index takes into account the principle of additivity (principle 1) and principle 2. Hence, for a path from A to A', the global specific index $I_{AA'}$ is the sum of the partial specific indices of the path.

The case of two successive curved surfaces is shown in **Figure 3**.

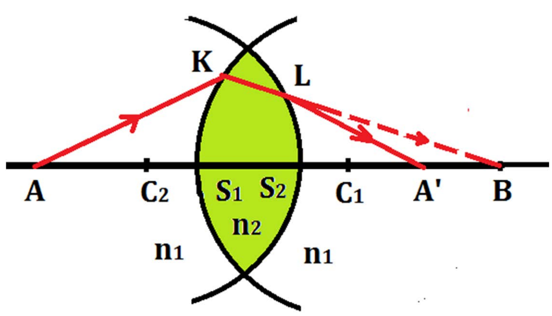

Figure 3. Case of two successive refractive surfaces.

The luminous object A is in medium 1, characterised by the reflective index n_1. The first refractive surface is characterized by its centre C_1 and its vertex S_1. B is the conjugated point of A in the dense medium 2 whose reflective index is n_2. Therefore, B is in medium 2.

Hence, the luminous incident ray AK is refracted and gives the ray KB, as B is the image of A.

The second refractive surface is characterized by its centre C_2 and its vertex S_2. A' is the conjugated point of B in the medium 1 whose reflective index is n_1.

Therefore, the ray KB intercepts the second surface in L, resulting in the ray LA' which is in medium 1.

According to principle 1, and because paraxial rays are considered, we define I_{AB} and $I_{BA'}$ as

$$I_{AB} = \frac{n_1}{AK} + \frac{n_2}{KB} \approx -\frac{n_1}{S_1A} + \frac{n_2}{S_1B} \tag{12}$$

$$I_{BA'} = \frac{n_2}{BL} + \frac{n_1}{LA'} \approx -\frac{n_2}{S_2B} + \frac{n_1}{S_2A'} \tag{13}$$

According to principle 4, we write

$$I_{AA'} = I_{AB} + I_{BA'} \tag{14}$$

Then, from Equation (12) and Equation (13), we have

$$I_{AA'} = -\frac{n_1}{S_1A} + \frac{n_2}{S_1B} - \frac{n_2}{S_2B} + \frac{n_1}{S_2A'} \tag{15}$$

$$I_{AA'} = n_1\left(-\frac{1}{S_1A} + \frac{1}{S_2A'}\right) + n_2\left(\frac{1}{S_1B} - \frac{1}{S_2B}\right) \tag{16}$$

Now, according to principle 2, we define the paths $C_1S_1C_1$ and $C_2S_2C_2$.

According to principle 1, we can write

$$I_{C_1S_1C_1} = \frac{n_1}{C_1S_1} + \frac{n_2}{S_1C_1} = \frac{n_2 - n_1}{S_1C_1} \tag{17}$$

$$I_{C_2S_2C_2} = \frac{n_2}{C_2S_2} + \frac{n_1}{S_2C_2} = \frac{n_1 - n_2}{S_2C_2} \tag{18}$$

Then, according to principle 4, a specific refractive index from C_1 to C_2 is defined as

$$I_{C_1C_2} = I_{C_1S_1C_1} + I_{C_2S_2C_2} \tag{19}$$

$$I_{C_1C_2} = \left(n_2 - n_1\right)\left(\frac{1}{S_1C_1} - \frac{1}{S_2C_2}\right) \tag{20}$$

According to principle 2, we can write

$$I_{AA'} = I_{C_1C_2} \tag{21}$$

The representative diagram of the two successive refractive surfaces is given in Figure 4.

3. Results and Discussions

3.1. Case of a Spherical Refractive Surface

As a consequence of principles 1 to 4, Equation (7), Equation (10) and Equation

(11) lead to the following equation for two conjugated points A and A':

$$-\left(\frac{n_1}{\overline{SA}}-\frac{n_2}{\overline{SA'}}\right)=-\frac{n_1-n_2}{\overline{SC}} \tag{22}$$

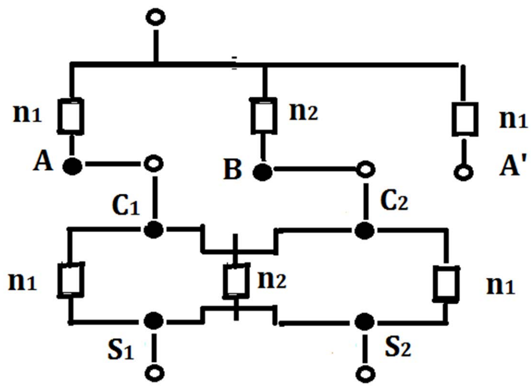

Figure 4. Representative diagram for two successive refractive surfaces.

As a result, we have the following equation

$$\frac{n_1}{\overline{SA}}-\frac{n_2}{\overline{SA'}}=\frac{n_1-n_2}{\overline{SC}} \tag{23}$$

This Equation (23) is the conjugation relation between a luminous object A and its image A', in the case where the incident ray is close to the principal axis.

In literature, this formula is found in the study of refractive spherical surfaces by geometrical constructions. In our study, it is obtained only by applying our four fundamental principles, without geometrical construction.

The representative diagram of the imaging system of the spherical refractive surface is shown in Figure 2.

3.2. Case of a Plane Refractive Surface

The case of a plane refractive surface is a particular case of the spherical surface. It is of course obtained for a radius of curvature $R=\overline{SC}\to\infty$.

As a consequence, the representative diagram of the imaging system of the plane refractive surface is shown in Figure 5.

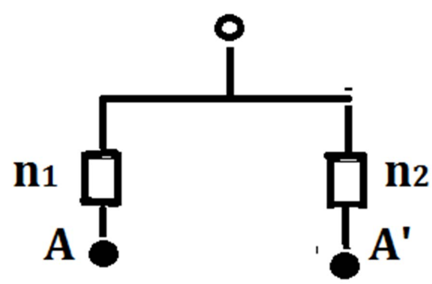

Figure 5. Representative diagram for a plane refractive surface.

As a consequence, Equation (23) becomes

$$\frac{n_1}{\overline{SA}} = \frac{n_2}{\overline{SA'}} \tag{24}$$

Equation (24) maybe written as

$$\frac{n_1}{\overline{AS}} + \frac{n_2}{\overline{SA'}} = I_{AA'} = 0 \tag{25}$$

Equation (25) means that the combined specific refractive index $I_{AA'} = 0$ when a luminous ray travels from an object A, to its conjugated image A', in the case of a plane refractive surface.

Equation (24) may also be written as

$$\frac{\overline{SA}}{n_1} = \frac{\overline{SA'}}{n_2} \tag{26}$$

Equation (24) and Equation (26) are gotten in literation by geometrical constructions, which are not needed in our study. They are the conjugation relation for the plane refractive surface for rays close to the normal to the surface.

3.3. Case of a Spherical Mirror

The spherical mirror is another curved surface. All the four principles are applied as in the spherical refractive surface. But here, we have $n_2 = -n_1$.

Then Equation (23) becomes

$$\frac{n_1}{\overline{SA}} + \frac{n_1}{\overline{SA'}} = \frac{2n_1}{\overline{SC}} \tag{27}$$

Finally, one gets

$$\frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}} \tag{28}$$

Equation (28) is the conjugation relation of the spherical mirror, for rays close to the principal axis. The representative diagram of the spherical mirror is shown in **Figure 6**. Here we set that $n_1 = n$ and $n_2 = -n$.

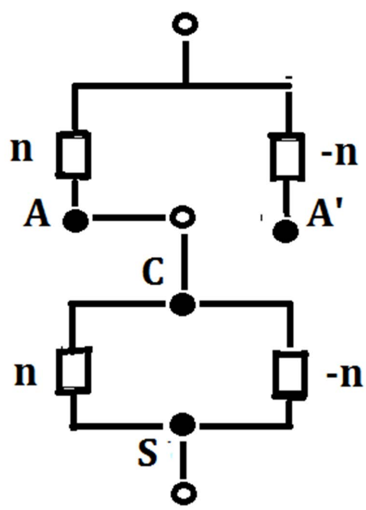

Figure 6. Representative diagram for a spherical mirror.

3.4. Case of a Plane Mirror

The plane mirror is a particular case of the spherical mirror. It is obtained for a radius of curvature $R = \overline{SC} \to \infty$.

As a consequence, from Equation (28), one gets

$$\overline{SA'} = -\overline{SA}$$

Here again, we have $n_2 = -n_1$ and the combined specific refractive index $I_{AA'}$ is given by

$$I_{AA'} = \frac{n_1}{\overline{AS}} + \frac{n_2}{\overline{SA'}} = 0$$

The optical path L from A to A' is given by

$$L = n_1 \cdot \overline{AS} + n_2 \overline{SA'} = n_1 \cdot \overline{AS} + (-n_1)(-\overline{SA}) = 0$$

Since the optical path is constant, the object A and its image A' are rigorously conjugated. The representative diagram of the plane mirror is shown in **Figure 7**.

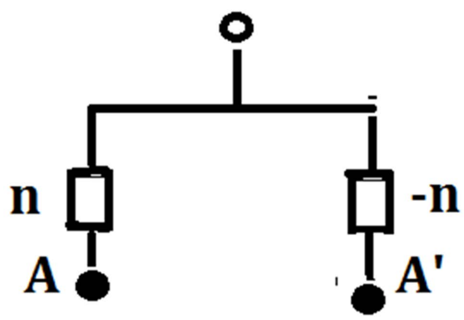

Figure 7. Representative diagram for a plane mirror.

Those first results show that, when the optical element is plane (a plane refractive surface or a plane mirror), the combined specific refractive index $I_{AA'}$ equals nought.

3.5. Case of Two Successive Refractive Curved Surfaces

Equation (16), Equation (20) and Equation (21) were derived from principles 1, 2, 3 and 4. Hence, we have

$$I_{AA'} = n_1\left(-\frac{1}{\overline{S_1A}} + \frac{1}{\overline{S_2A'}}\right) + n_2\left(\frac{1}{\overline{S_1B}} - \frac{1}{\overline{S_2B}}\right)$$

$$I_{C_1C_2} = (n_2 - n_1)\left(\frac{1}{\overline{S_1C_1}} - \frac{1}{\overline{S_2C_2}}\right)$$

$$I_{AA'} = I_{C_1C_2}$$

As a result, we can write

$$n_1\left(-\frac{1}{\overline{S_1A}} + \frac{1}{\overline{S_2A'}}\right) + n_2\left(\frac{1}{\overline{S_1B}} - \frac{1}{\overline{S_2B}}\right) = (n_2 - n_1)\left(\frac{1}{\overline{S_1C_1}} - \frac{1}{\overline{S_2C_2}}\right) \quad (29)$$

Equation (29) can be written as

$$n_1\left(\frac{1}{\overline{S_1A}}-\frac{1}{\overline{S_2A'}}\right)-n_2\left(\frac{1}{\overline{S_1B}}-\frac{1}{\overline{S_2B}}\right)=\left(n_2-n_1\right)\left(\frac{1}{\overline{S_2C_2}}-\frac{1}{\overline{S_1C_1}}\right) \tag{30}$$

An example of two successive refractive spherical surfaces is the lens. For a thin lens, we have $S_1 \approx S_2 \approx O$. If we write $R_1=\overline{S_1C_1}$ and $R_2=\overline{S_2C_2}$, one gets the conjugation formula of a thin lens, where O is the centre of the lens:

$$n_1\left(\frac{1}{\overline{OA}}-\frac{1}{\overline{OA'}}\right)+n_2\left(\frac{1}{\overline{OB}}-\frac{1}{\overline{OB}}\right)=\left(n_2-n_1\right)\left(\frac{1}{R_2}-\frac{1}{R_1}\right) \tag{31}$$

Finally one gets

$$n_1\left(\frac{1}{\overline{OA}}-\frac{1}{\overline{OA'}}\right)=\left(n_2-n_1\right)\left(\frac{1}{R_2}-\frac{1}{R_1}\right) \tag{32}$$

Equation (32) is the conjugation formula found in literature for a thin lens; formula found here only by applying our four principles.

3.6. Case of Two Successive Refractive Plane Surfaces

It is a particular case of the two successive refractive curved surfaces, in which we consider the radius of curvature infinite. Hence, $\overline{S_1C_1}=\overline{S_2C_2}=\infty$.

A glass whose refractive index is n_2 has two parallel sides. Each side is limited by a refractive medium whose refractive index is n_1.

The representative diagram of a parallel-sided refractive glass is shown in Figure 8.

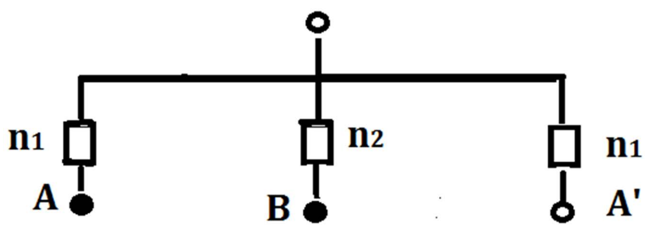

Figure 8. Representative diagram for a parallel sided refractive glass.

For that case, Equation (30) becomes

$$n_1\left(\frac{1}{\overline{S_1A}}-\frac{1}{\overline{S_2A'}}\right)-n_2\left(\frac{1}{\overline{S_1B}}-\frac{1}{\overline{S_2B}}\right)=0 \tag{33}$$

Equation (33) can be written as

$$n_1\left(\frac{1}{\overline{S_2A'}}-\frac{1}{\overline{S_1A}}\right)=n_2\left(\frac{1}{\overline{S_2B}}-\frac{1}{\overline{S_1B}}\right) \tag{34}$$

Equation (34) is the conjugation formula of a parallel-sided refractive glass.

The imaging system of this system is shown in Figure 9.

Equation (34) is also easily found from the conjugation formula of a plane refractive surface.

Using Equation (24), we can of course write:

- for surface 1

$$\frac{n_1}{\overline{S_1A}} = \frac{n_2}{\overline{S_1B}} \tag{35}$$

- for surface 2

$$\frac{n_1}{\overline{S_2A'}} = \frac{n_2}{\overline{S_2B}} \tag{36}$$

Equation (35) and Equation (36) lead to Equation (34).

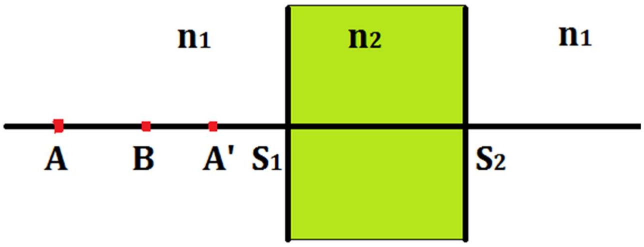

Figure 9. Imaging system of an object A given by a parallel-sided refractive glass.

Here, the combined refractive index $I_{AA'}$ is given by

$$I_{AA'} = n_1\left(-\frac{1}{\overline{S_1A}} + \frac{1}{\overline{S_2A'}}\right) + n_2\left(\frac{1}{\overline{S_1B}} - \frac{1}{\overline{S_2B}}\right) = 0 \tag{37}$$

The combined specific refractive index $I_{AA'}$ equals nought as in the case of a single refractive plane.

3.7. Case of Thick Lens

The aim of this subsection is to show the influence of the thickness on the conjugation formula of a lens. The thick lens, which is another case of two successive curved refractive surfaces is shown in **Figure 10**.

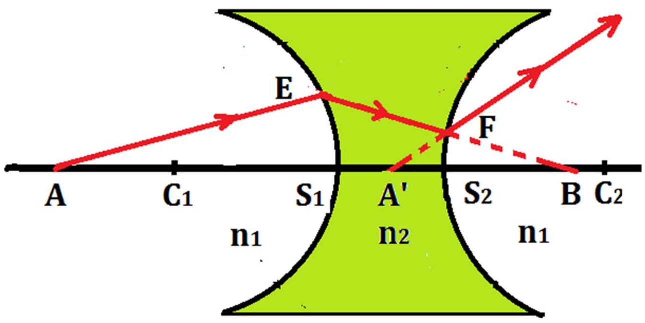

Figure 10. Case of a thick lens.

The lens is thick so that its thickness $e = \overline{S_1S_2}$ is not neglected. The intermediate image B is taken into account. Here, we have for I_{AB} and $I_{BA'}$

$$I_{AB} = \frac{n_1}{\overline{AE}} + \frac{n_2}{\overline{EB}} \approx -\frac{n_1}{\overline{S_1A}} + \frac{n_2}{\overline{S_1B}} \tag{38}$$

$$I_{BA'} = \frac{n_2}{\overline{BF}} + \frac{n_1}{\overline{FA'}} \approx -\frac{n_2}{\overline{S_2B}} + \frac{n_1}{\overline{S_2A'}} \quad (39)$$

According to principle 4, we write

$$I_{AA'} = I_{AB} + I_{BA'} \quad (40)$$

Then, from Equation (38) and Equation (39), we have

$$I_{AA'} = -\frac{n_1}{\overline{S_1A}} + \frac{n_2}{\overline{S_1B}} - \frac{n_2}{\overline{S_2B}} + \frac{n_1}{\overline{S_2A'}} \quad (41)$$

According to principles 1 and 2, we have for the paths $C_1S_1C_1$ and $C_2S_2C_2$

$$I_{C_1S_1C_1} = \frac{n_1}{\overline{C_1S_1}} + \frac{n_2}{\overline{S_1C_1}} = \frac{n_2 - n_1}{\overline{S_1C_1}} \quad (42)$$

$$I_{C_2S_2C_2} = \frac{n_2}{\overline{C_2S_2}} + \frac{n_1}{\overline{S_2C_2}} = \frac{n_1 - n_2}{\overline{S_2C_2}} \quad (43)$$

The specific refractive index from C_1 to C_2 is

$$I_{C_1C_2} = I_{C_1S_1C_1} + I_{C_2S_2C_2} \quad (44)$$

$$I_{C_1C_2} = (n_2 - n_1)\left(\frac{1}{\overline{S_1C_1}} - \frac{1}{\overline{S_2C_2}}\right) \quad (45)$$

We have, according to principle 2

$$I_{AA'} = I_{C_1C_2}$$

Finally, one gets the conjugation formula

$$n_1\left(\frac{1}{\overline{S_1A}} - \frac{1}{\overline{S_2A'}}\right) - n_2\left(\frac{1}{\overline{S_1B}} - \frac{1}{\overline{S_2B}}\right) = (n_2 - n_1)\left(\frac{1}{\overline{S_2C_2}} - \frac{1}{\overline{S_1C_1}}\right) \quad (46)$$

It is similar to Equation (30), but here, we take into account the thickness and the intermediate image B, so that we obtain

$$n_1\left(\frac{1}{\overline{S_1A}} - \frac{1}{\overline{S_2A'}}\right) + \frac{e \cdot n_2}{\overline{S_1B}\left(-e + \overline{S_1B}\right)} = (n_2 - n_1)\left(\frac{1}{\overline{S_2C_2}} - \frac{1}{\overline{S_1C_1}}\right) \quad (47)$$

The location of the intermediate image B, $\overline{S_1B}$, which is in Equation (47), is given by the following equation:

$$\frac{n_2}{\overline{S_1B}} = \frac{n_1}{\overline{S_1A}} + \frac{n_2 - n_1}{\overline{S_1C_1}} \quad (48)$$

Hence, for a given location of the object A, the location of the image A' is obtained from Equation (47) and Equation (48).

4. Conclusion

In this paper, an axiomatic method has been developed. It is based on four novel principles we stated. The conjugation formulae of the optical elements, such as spherical refractive surface, plane refractive surface, and construction. Those principles were also applied to find the conjugation formulae in the case of two successive refractive spherical surfaces. That allowed to find the conjugation formula, in the case of a thin lens, then in the case of a parallel-sided refractive glass. The case of a thick lens was also investigated. Here again, the conjugation formulae were found without any geometrical construction. Hence, the accuracy of the four novel principles is proved. Therefore, the new method can be used to study the conjugation formulae regarding other successive optical elements, without any geometrical construction. Finally, it is a very good tool for modelling the optical elements and the optical systems.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Nelkon, M. and Parker, P. (1977) Advanced Level Physics. 4th Ed., Heinemann Educational Books.

[2] Roussel, J. (2011) Cours de Physique. Optique Géométrique, femto-physique.fr/Optique.

[3] Bakkali, A. (2023) Cours d'optique géométrique. Université Moulay Ismaï, Maroc.

[4] Beldjilali, S.A. (2015) Optique géométrique. Université des Sciences et de la Technologie d'Oran, Algérie.

[5] Yazid, H. (2021) Chapitre III. Optique Géométrique Partie iii. University of Batna 2, Algérie. https://staff.univ-batna2.dz/sites/default/files/hedjar_yazid/files/Optique_geomrtrique_partie_iii.pdf