Open Journal of Applied Sciences, 2026, 16(2), 718-740
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

CC-Integral on Interval-Valued Sugeno Probability Measure and Precisely to Solve Uncertain Multi-Criteria Decision Making Problem

Jiaxue Wei, Hong Yang*

College of Mathematics and Statistics, Northwest Normal University, Lanzhou, China
Email: *ysin888@163.com, weijiaxue0511@163.com

How to cite this paper: Wei, J.X. and Yang, H. (2026) CC-Integral on Interval-Valued Sugeno Probability Measure and Precisely to Solve Uncertain Multi-Criteria Decision Making Problem. *Open Journal of Applied Sciences*, **16**, 718-740.
https://doi.org/10.4236/ojapps.2026.162044

Received: January 8, 2026
Accepted: February 25, 2026
Published: February 28, 2026

Abstract

In the current era of big data and artificial intelligence, multi-criteria decision-making is facing numerous uncertain issues such as incomplete information and ambiguous evaluation. Nevertheless, the traditional precise mathematical models have significant limitations when it comes to expressing these uncertainties. In order to better address these kinds of uncertainty issues, the CC-integral on the interval-valued Sugeno probability measure has been proposed in this paper. By introducing the Choquet integral based on the Copula, a reasonable measure and integral are established to solve the complexity of the decision environment. Among them, due to different Copula functions, the application of the Choquet integral based on Copula is more extensive. So as to verify the effectiveness of the proposed method, it was applied to the end-of-life (EOL) strategy problem. Comparing with the decision results of the Choquet integral based on triangular norms, it is found that the method proposed in this paper is more accurate, simpler to calculate, and has a smaller computational cost.

Keywords

Fuzzy Analysis, Choquet Integral, CC-Integral, Uncertain Multi-Criteria Decision

1. Introduction

Multi-criteria decision making (MCDM) is regarded as a complex decision tool involving quantitative and qualitative factors, and it refers to the process of systematic evaluation, ranking or selection of limited alternatives under multiple conflicting objectives or criteria. The core challenge lies in the trade-off between decision criteria and uncertainty handling. The trade-off between criteria requires aggregation operators to be de-aggregated, such as weighting operators and geometric operators, but these operators are not applicable when decision criteria influence and interact with each other. The Choquet integral is an aggregate function that can be used to measure the level of uncertainty in a fuzzy set to connect the interactions between decision criteria through the determination of fuzzy measures.

In 1954, the French mathematician Choquet proposed the capacity theory for the first time and proposed the Choquet integral based on the capacity theory [1]. Since then, the study of the Choquet integral from the aspects of properties and measures began. A generalization of the Choquet integral started to surface in [2] in 1987. The Choquet integral was used by Barreneche *et al.* in 2013 as an aggregation function for the fuzzy rule-based classification system in [3]. A novel aggregation function enhanced by the Choquet integral was introduced in [4] in 2016. The aggregation function, the preaggregation function, and the ordered directional monotone function-referred to as the CC-integral [5], C_F-integral [6], and C_T-integral [7] replace the product operator in the Choquet integral in these generalizations. For the CC-integral, an aggregate class function is obtained by substituting a copula for the product operator in the Choquet integral.

In 1986, Schmeidler first applied Choquet integral to decision-making problems [8]. In 1991, Tanaka and Sugeno applied it to evaluate color printed images [9]. In 1995, Grabisch pointed out the advantages and characteristics of Choquet integral in decision-making. The decision techniques are also presented. So far, Choquet integral has been widely used in the field of decision making [10]-[13]. As an extension of Choquet integral, CC-integral has also been applied to solve MCDM problems [14] [15]. The EOL strategy and how to handle the product after use are the most prevalent MCDM issues. Marco *et al.* first presented the EOL method in 1994, and it has since undergone constant development and improvement [16] [17]. The MCDM problem of identifying the EOL strategy of refrigerator composition is a crucial study area to assess the elements of EOL strategy from a holistic perspective. EOL decision-making depends on numerous factors.

The evaluation value is typically fuzzy linguistic evaluation, and the criteria frequently interact with one another when solving the EOL strategy problem of refrigerator building. These language computations are typically transformed into triangle norms, which are difficult to compute. Instead, they are typically transformed into interval values or exact numbers. Consequently, the universal CC-integral must be extended to interval values. This study provides the CC-integral on interval-valued Sugeno probability measures. Before this, Chen proposed the Choquet integral on interval-valued Sugeno probability measures [18], Shang proposed the C_T-integral on interval-valued Sugeno probability measures in 2022 [19]. And taking Copula as the overlap function, it is applied to solve the MCDM problem in the context of EOL policy determination. In contrast to the former, the method presented in this study uses mathematical software to simplify the calculation process, and the findings are more commonly utilized and accurate.

The paper is organized as follows. In Section 2, we introduce some basic concepts, including the definition and theorem of aggregation functions and Choquet integral. In Section 3, we present the expression of the CC-integral on the interval-valued Sugeno probability measure, and provides the discrete expression when the Copula is taken as an overlap function. In Section 4, we apply the CC-integral on the interval-valued Sugeno probability measure to MCDM, and provides a detailed calculation process, demonstrating the effectiveness of the proposed method. Our conclusions are presented in Section 5.

2. Preliminaries

First of all, in this section we use the definition of the theorem for the necessary description and introduction. In the following, let $(X, \mathscr{R}_0, \mu)$ is a measure space, where X be the universal set, $\mathscr{R}_0$ be a $\sigma-$ ring of subsets of X, using a σ-algebra $\mathscr{T}$ as $\mathscr{R}_0$ is convenient for defining measures, μ be a measure on $\mathscr{R}_0$, $n > 0$.

Definition 2.1 [20] A function $\mu : 2^N \to [0,1]$ is said to be a fuzzy measure if, for all A, $B \subset N$, it satisfies the following properties:

(1) Increasing: if $A \subset B$, then $\mu(A) \subset \mu(B)$;

(2) Boundary conditions: $\mu(\varnothing) = 0$ and $\mu(N) = 1$.

Theorem 2.1 [20] Let $(X, \mathscr{R}_0, \mu)$ be a measure space. Then Measure μ has the following properties.

(1) Monotonicity: whenever $E \in \mathscr{R}_0, F \in \mathscr{R}_0$, and $E \subset F$ implies $\mu(E) \le \mu(F)$;

(2) Continuity from below: whenever $E_i \in \mathscr{R}_0, i = 1, 2, \cdots$, and $\{E_i\}$ is nondecreasing, implying that $\mu\left(\bigcup_{i=1}^{\infty}(E_i)\right) = \lim_{i\to\infty} \mu(E_i)$;

(3) Continuity from above: whenever $E_i \in \mathscr{R}_0, i = 1, 2, \cdots$, and $\{E_i\}$ is nonincreasing, and there exists i_0 such that $\mu\left(E_{i_0}\right) < \infty$ implying that $\mu\left(\bigcap_{i=1}^{\infty}(E_i)\right) = \lim_{i\to\infty} \mu(E_i)$;

Definition 2.2 [21] A bivariate function $C : [0,1]^2 \to [0,1]$ is called a Copula, iff

(1) $C(x, y) + C(x_1, y_1) \ge C(x, y_1) + C(x_1, y)$;

(2) $C(x, 0) = C(0, x) = 0$;

(3) $C(x, 1) = C(1, x) = x$,

For all x, x_1, y, $y_1 \in [0,1]$ with $x \le x_1$ and $y \le y_1$:

Copula are functions that link (two-dimensional) probability distribution functions to their one-dimensional margins, playing an significant role in the theory of probabilistic metric spaces and statistics [22]. **Table 1** presents some examples of Copulas, such as *t*-norms, overlap functions and other non-associative copulas. These functions play a crucial role in the subsequent research.

Definition 2.3 [18] Nonnegative set function μ is said to satisfy the $\sigma - \lambda$ rules iff:

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right)=\begin{cases}\frac{1}{\lambda}\left\{\prod_{i=1}^{\infty}\left[1+\lambda\mu(E_i)\right]-1\right\} & \text{if } \lambda=0,\\ \sum_{i=1}^{\infty}\mu(E_i) & \text{if } \lambda\neq 0,\end{cases}$$

where $\lambda\in\left(-\frac{1}{sup\mu},\infty\right)\bigcup 0$, $E_i\subset\mathcal{F}$, $E_i\bigcap E_j=\varnothing$ for all $i,j=1,2,\cdots$, and $i\neq j$.

Table 1. Examples of copulas.

(1) *t*-norms		
Definition	Name	Observations
$T_M(x,y)=\min\{x,y\}$	Minimum	Overlap function
$T_P(x,y)=xy$	Algebraic Pruduct	Overlap function
$T_L(x,y)=\max\{0,x+y-1\}$	Lukasiewicz	
$T_{HP}(x,y)=\begin{cases}0 & \text{if } x=y=0\\ \frac{xy}{x+y-xy} & \text{otherwise}\end{cases}$	Hamacher Product	Overlap function
(2) Non-associative overlap functions		
Definition	Reference	Observations
$O_B(x,y)=\min\{x\sqrt{y},y\sqrt{x}\}$	[[23], Theorem 8]	Cuadras-Augé family of copulas[24]
$O_{mM}(x,y)=\min\{x,y\}\max\{x^2,y^2\}$	[[25], Example 3.1]	
$O_\alpha(x,y)=xy(1+\alpha(1-x)(1-y))$ $\alpha\in[-1,0]\cup[0,1]$	[[22], Appendix A]	
(3) Other non-associative copulas		
Definition	Reference	Observations
$C_F(x,y)=xy+x^2y(1-x)(1-y)$	[[26], Example 9.5]	Non-commutative
$C_L(x,y)=\max\left\{\min\left\{x,\frac{y}{2}\right\},x+y-1\right\}$	[[22], Appendix A]	Non-commutative
$C_{Div}(x,y)=\frac{xy+\min\{x,y\}}{2}$	[[22], Appendix A]	

Definition 2.4 [18] A set function $\nu:\mathcal{A}\to R^+$, $\underline{\nu}:\mathcal{A}\to R^+$, $\overline{\nu}:\mathcal{A}\to R^+$, $\nu=[\underline{\nu},\overline{\nu}]$ is called an interval-valued fuzzy measure iff:

(1) $\underline{\nu}(\varnothing)=0$, $\overline{\nu}(\varnothing)=0$;

(2) if $E,F\subset X$, and $E\subset F$, then $\underline{\nu}(E)\leq\underline{\nu}(F)$, $\overline{\nu}(E)\leq\overline{\nu}(F)$;

(3) for every $E \subset X$, $\underline{\nu}(E) \le \overline{\nu}(E)$;

Definition 2.5 [18] If $\underline{\nu}$ and $\overline{\nu}$ follow the $\sigma-\lambda$ rules, and $\underline{\nu}(X)=1$, $\overline{\nu}(X)=1$, then $\nu=[\underline{\nu},\overline{\nu}]$ is referred an interval-valued Sugeno probability measure based on $\sigma-\lambda$ rules, denoted by $h_\lambda=\underline{h}_\lambda,\overline{h}_\lambda$.

Definition 2.6 [18] Let X be a finite set with power set 2^X, a set function $\nu:2^X \to [\underline{\nu},\overline{\nu}] \subset L(0,1)$ is called regular interval fuzzy measure iff:

(1) Boundary Conditions: $\underline{\nu}(\varnothing)=\overline{\nu}(\varnothing)=0$, $\underline{\nu}(X)=1$, $\overline{\nu}(X)=1$;

(2) Monotonicity: for any $M,N \in 2^X$ with $M \subset N$, then $\underline{\nu}(M) \le \underline{\nu}(N)$, $\overline{\nu}(M) \le \overline{\nu}(N)$.

Definition 2.7 [18] Suppose X be a finite set with power set 2^X, a set function $\nu:2^X \to [\underline{\nu},\overline{\nu}] \subset L(0,1)$ is called regular λ-interval fuzzy measure iff:

(1) $\underline{\nu}(\varnothing)=\overline{\nu}(\varnothing)=0$, $\underline{\nu}(X)=1$, $\overline{\nu}(X)=1$;

(2) if $E \subset X$, $F \subset X$, $E \cap F=\varnothing$, then

$\underline{\nu}(E \cup F)=\underline{\nu}(E)+\underline{\nu}(F)+\lambda\underline{\nu}(E)\underline{\nu}(F)$,

and,

$\overline{\nu}(E \cup F)=\overline{\nu}(E)+\overline{\nu}(F)+\lambda\overline{\nu}(E)\overline{\nu}(F)$, $\lambda \in (-1,\infty)$,

where $L(0,1)$ is the set of all closed subintervals of the unit interval.

Theorem 2.2 [20] [27] The following formulas determine the parameters $\lambda=(\lambda_1,\lambda_2)$ of the regular interval Sugeno probability measure:

$$\prod_{i=1}^{n}\left(1+\lambda_1\underline{h}_{\lambda_i}\right)=1+\lambda_1,$$

$$\prod_{i=1}^{n}\left(1+\lambda_2\underline{h}_{\lambda_i}\right)=1+\lambda_2.$$

Proof. Because $\lambda=(\lambda_1,\lambda_2)$ and take into account that $X=\{x_1,x_2,\cdots,x_n\}$ is a finitee set, $g_{\lambda_i}=g_\lambda(x_i)(i=1,2,\cdots,n)$ is said to be a measure of density. Then,

$$\frac{1}{\lambda}\left\{\prod_{i=1}^{n}\left[1+\lambda_1 g_{\lambda_i}\left(x_{(i)}\right)\right]-1\right\}=1,$$

$$\prod_{i=1}^{n}\left[1+\lambda_1\underline{g}_{\lambda_i}\left(x_{(i)}\right)\right]-1=\lambda_1,$$

$$\prod_{i=1}^{n}\left[1+\lambda_1\underline{g}_{\lambda_i}\left(x_{(i)}\right)\right]=1+\lambda_1,$$

3. The CC-Integral on Interval-Valued Sugeno Probability Measure

The definition of the classical Lebesgue integral takes into account additive measures, while the Choquet integral is a generalization of the Lebesgue integral and it considers fuzzy measures. In this section, by introducing the definition of discrete Choquet integral, the generalization of Choquet integral, namely CC-integral, is studied, and the expression of CC-integral on Sugeno probability measure is proposed.

Think of $N=\{1,\cdots,n\}$ as a finite set.

Definition 3.1 [28] Suppose $\nu:2^N \to [0,1]$ be a fuzzy measure. The discrete Choquet integral is the function $\mathfrak{C}_{\mathrm{m}}:[0,1]^n \to [0,1]$, defined, for all of

$\vec{x} = \left(f\left(x_1\right), \cdots, f\left(x_n\right)\right) \in [0,1]^n$, by $C_m\left(\vec{x}\right) = \sum_{i=1}^{n}\left(f\left(x_{(i)}\right) - f\left(x_{(i-1)}\right)\right) \cdot \nu\left(A_{(i)}\right)$,

observe that the equation can be also written as,

$$C_m\left(\vec{x}\right) = \sum_{i=1}^{n}\left(f\left(x_{(i)}\right) \cdot \nu\left(A_{(i)}\right) - f\left(x_{(i-1)}\right) \cdot \nu\left(A_{(i)}\right)\right),$$

which we call the Choquet integral in its expanded form.

The discrete Choquet integral in its enlarged form can be combined with copulas to create a family of aggregation functions by simply replacing the product operation with a copula, according to Reference [5]. We refer to these functions as CC-integral.

Definition 3.2 [5] Suppose $\nu : 2^N \to [0,1]$ be a fuzzy measure and $C : [0,1]^2 \to [0,1]$ be a bivariate copula. The Choquet-like Copula-based integral with respect to ν is defined as a function $\mathfrak{C}_{\mathrm{m}} : [0,1]^n \to [0,1]$, given, for all $x \in [0,1]^n$, by,

$$C_m^C\left(\vec{x}\right) = \sum_{i=1}^{n} C\left(f\left(x_{(i)}\right), \nu\left(A_{(i)}\right)\right) - C\left(f\left(x_{(i-1)}\right), \nu\left(A_{(i)}\right)\right),$$

where

(1) $\left(f\left(x_{(1)}\right), \cdots, f\left(x_{(n)}\right)\right)$ is an increasing permutation on the input $f(x)$, that is, $0 \le f\left(x_{(1)}\right) \le \cdots \le f\left(x_{(n)}\right)$;

(2) $f\left(x_{(0)}\right) = 0$;

(3) $A_{(i)} = \left\{(i), \cdots, (n)\right\}$ is the subset of indices corresponding to the $n-i+1$ largest components of $\vec{x}$.

Definition 3.3 For a non-negative measurable interval-valued function $F : \Omega \to I\left(R^+\right)$, and any set $A \in \mathcal{A}$, the Choquet integral of F is characterized by its measurable selections:

$$(c)\int_A F \mathrm{d}\nu = \left\{(c)\int_A f \mathrm{d}\nu : f \in S_F\right\},$$

where the set S_F consists of all measurable selections $f : \Omega \to R^+$.

Definition 3.4 [29] If there is a C-integral function $h : \Omega \to P_0\left(R^+\right)$, for any measurable selection $f \in S_F$, $A \in \mathcal{A}$ has,

$$(c)\int_A f \mathrm{d}\nu \le (c)\int_A h \mathrm{d}\nu,$$

then F is said to be C-integrally bounded.

Theorem 3.1 [18] If C-be integrally bounded, F be a non-negative measurable, $A \in \mathcal{A}$; then, X is C-integrable on A and,

$$(c)\int_A X \mathrm{d}\nu = \left[(c)\int_A \underline{X} \mathrm{d}\nu, (c)\int_A \overline{X} \mathrm{d}\nu\right].$$

Remark If $(c)\int_A \underline{f} \mathrm{d}\mu$ and $(c)\int_A \overline{f} \mathrm{d}\mu$ exists and is bounded, then the interval-valued function f is C-integrable on A.

Theorem 3.2 Suppose f is an interval-valued function on $X = \left\{x_1, x_2, \cdots, x_n\right\}$ and C is a Copula. Then, the CC-integral of f as regards the interval-valued Sugeno probability measure g_λ on X is proposed by,

$$(c)\int_X f\,\mathrm{d}g_\lambda=\sum_{i=1}^{n}\left[C\left(g_\lambda\left(X_i'\right),f\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),f\left(X_i'\right)\right)\right],$$

where $x_1',x_2',\cdots,x_n'$ is a permutation of $x_1,x_2,\cdots,x_n$ such that $f\left(x_0'\right)\le f\left(x_1'\right)\le\cdots\le f\left(x_n'\right)$, $f\left(x_0'\right)=[0,0]$, $X_i'=\left\{x_i',x_{i+1}',\cdots,x_n'\right\}$, $i=1,2,\cdots,n$ and $X_{n+1}'=\varnothing$.

Proof. Due to f is an interval-valued function on X, according to Theorem 3.1, we have,

$$(c)\int_X f\,\mathrm{d}g_\lambda=\left[(c)\int_X \underline{f}\,\mathrm{d}g_\lambda,(c)\int_X \overline{f}\,\mathrm{d}g_\lambda\right],$$

Because the CC-integral is monotonic and continuous, let us assume that $\underline{f}$ and $\overline{f}$ are real-valued functions on x, respectively. Meanwhile, noted then nonnegativity and the monotonicity of the fuzzy measure, we can obtain

$$\begin{aligned}(c)\int_X f\,\mathrm{d}g_\lambda&=\left\{\sum_{i=1}^{n}\left[C\left(g_\lambda\left(X_i'\right),\underline{f}\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),\underline{f}\left(X_i'\right)\right)\right],\right.\\&\qquad\left.\sum_{i=1}^{n}\left[C\left(g_\lambda\left(X_i'\right),\overline{f}\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),\overline{f}\left(X_i'\right)\right)\right]\right\}\\&=\sum_{i=1}^{n}\left\{\left[C\left(g_\lambda\left(X_i'\right),\underline{f}\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),\underline{f}\left(X_i'\right)\right)\right],\right.\\&\qquad\left.\left[C\left(g_\lambda\left(X_i'\right),\overline{f}\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),\overline{f}\left(X_i'\right)\right)\right]\right\}\\&=\sum_{i=1}^{n}\left\{C\left(g_\lambda\left(X_i'\right),\left[\underline{f}\left(X_i'\right),\overline{f}\left(X_i'\right)\right]\right)\right.\\&\qquad\left.-C\left(g_\lambda\left(X_{i+1}'\right),\left[\underline{f}\left(X_i'\right),\overline{f}\left(X_i'\right)\right]\right)\right\}\\&=\sum_{i=1}^{n}\left\{C\left(g_\lambda\left(X_i'\right),f\left(X_i'\right)\right)-C\left(g_\lambda\left(X_{i+1}'\right),f\left(X_i'\right)\right)\right\}.\end{aligned}$$

where C can be $T_M,T_P,T_L,T_{HP},O_B,O_{mM},O_\alpha,C_F,C_L,C_{Div}$, when C is T_L, $T_L(x,y)=\max\{0,x+y-1\}$, we have an expression of the C_{TL}-integral on interval-valued Sugeno probability measure:

$$\begin{aligned}(c)\int_X f\,\mathrm{d}g_\lambda&=\sum_{i=1}^{n}\left\{\max\left(0,g_\lambda\left(X_i'\right)+f\left(X_i'\right)-1\right)-\max\left(0,g_\lambda\left(X_{i+1}'\right)+f\left(X_i'\right)-1\right)\right\}\\&=\sum_{i=1}^{n}\left\{\max\left(0,g_\lambda\left(X_i'\right)+\left[\underline{f}\left(X_i'\right),\overline{f}\left(X_i'\right)\right]-1\right)\right.\\&\qquad\left.-\max\left(0,g_\lambda\left(X_{i+1}'\right)+\left[\underline{f}\left(X_i'\right),\overline{f}\left(X_i'\right)\right]-1\right)\right\}\\&=\sum_{i=1}^{n}\left\{\left[\max\left(0,g_\lambda\left(X_i'\right)+\underline{f}\left(X_i'\right)-1\right)-\max\left(0,g_\lambda\left(X_{i+1}'\right)+\underline{f}\left(X_i'\right)-1\right)\right],\right.\\&\qquad\left.\left[\max\left(0,g_\lambda\left(X_i'\right)+\overline{f}\left(X_i'\right)-1\right)-\max\left(0,g_\lambda\left(X_{i+1}'\right)+\overline{f}\left(X_i'\right)-1\right)\right]\right\}\\&=\left\{\sum_{i=1}^{n}\left[\max\left(0,g_\lambda\left(X_i'\right)+\underline{f}\left(X_i'\right)-1\right)-\max\left(0,g_\lambda\left(X_{i+1}'\right)+\underline{f}\left(X_i'\right)-1\right)\right],\right.\\&\qquad\left.\sum_{i=1}^{n}\left[\max\left(0,g_\lambda\left(X_i'\right)+\overline{f}\left(X_i'\right)-1\right)-\max\left(0,g_\lambda\left(X_{i+1}'\right)+\overline{f}\left(X_i'\right)-1\right)\right]\right\}.\end{aligned}$$

It is worth noting that when C is O_{mM}, $O_{mM}(x,y)=\min\{x,y\}\max\{x^2,y^2\}$, Copula is an overlap function, we have an expression of the $C_{O_{mM}}$-integral on interval-valued Sugeno probability measure:

$$
\begin{aligned}
(c)\int_X f\,\mathrm{d}g_\lambda =& \sum_{i=1}^{n}\Big[\min\big(g_\lambda(X_i'),f(X_i')\big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big(f(X_i')\big)^2\Big)\\
&-\min\big(g_\lambda(X_{i+1}'),f(X_i')\big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big(f(X'_i)\big)^2\Big)\Big]\\
=&\sum_{i=1}^{n}\Big\{\min\Big(g_\lambda(X_i'),\big[\underline{f}(X_i'),\overline{f}(X_i')\big]\Big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big[\underline{f}(X_i'),\overline{f}(X_i')\big]^2\Big)\\
&-\min\Big(g_\lambda(X_{i+1}'),\big[\underline{f}(X_i'),\overline{f}(X_i')\big]\Big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big[\underline{f}(X_i'),\overline{f}(X_i')\big]^2\Big)\Big\}\\
=&\sum_{i=1}^{n}\Big\{\Big[\min\big(g_\lambda(X_i'),\underline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big(\underline{f}(X_i')\big)^2\Big)\Big]\\
&-\Big[\min\big(g_\lambda(X_{i+1}'),\underline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big(\underline{f}(X_i')\big)^2\Big)\Big],\\
&\Big[\min\big(g_\lambda(X_i'),\overline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big(\overline{f}(X_i')\big)^2\Big)\Big]\\
&-\Big[\min\big(g_\lambda(X_{i+1}'),\overline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big(\overline{f}(X_i')\big)^2\Big)\Big]\Big\}\\
=&\sum_{i=1}^{n}\Big\{\Big[\min\big(g_\lambda(X_i'),\underline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big(\underline{f}(X_i')\big)^2\Big)\Big]\\
&\Big[\min\big(g_\lambda(X_i'),\overline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_i')\big)^2,\big(\overline{f}(X_i')\big)^2\Big)\Big]\Big\}\\
&-\sum_{i=1}^{n}\Big\{\Big[\min\big(g_\lambda(X_{i+1}'),\underline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big(\underline{f}(X_i')\big)^2\Big)\Big]\\
&\Big[\min\big(g_\lambda(X_{i+1}'),\overline{f}(X_i')\big)\max\Big(\big(g_\lambda(X_{i+1}')\big)^2,\big(\overline{f}(X_i')\big)^2\Big)\Big]\Big\}.
\end{aligned}
$$

4. Application in Multi-Criteria Decision-Making Problems

MCDM is a decision-making process in which a decision maker needs to evaluate, compare, and eventually select (or rank, classify) one or more alternatives from a limited set of alternatives under multiple, often conflicting criteria. It can systematically deal with complex decision scenarios, especially when the decision involves multiple conflicting objectives, limited alternatives and subjective preferences, MCDM provides a scientific, transparent and defensible framework to transform multi-dimensional conflicts into computable optimization problems.

4.1. Context of Decision-Making

In this Decision-making problems, We demonstrate the applicability of CC-integral on interval-valued Sugeno probability measure using a refrigerator component EOL strategy determination MCDM problem as a case study. This MCDM problem includes fourteen sub-criteria and four primary criteria, as shown in Figure 1. The primary parts and a few minor connectors of the refrigerator are the only parts that communicate with each other. The case data sets come from [17] [30].

Component EOL Evaluation Characteristics	Component Quality (x_1)	Component Durability (x_1^1)
		Component EOL Condition (x_1^2)
	Function/Module Complexity (x_2)	Difficulty with Component's Disassembly (x_2^1)
		Level of Integration (x_2^2)
		Quantity of Parts (x_2^3)
		Difficulty of Dismantling Part Attachments (x_2^4)
		Component Weight (x_2^5)
	Material (x_3)	Quantity of High Value Material (x_3^1)
		Material Calorific Capacity (x_3^2)
		Amount of Hazardous Materials (x_3^3)
		Amount of Different Materials (x_3^4)
	External (x_4)	Regulation Support (x_4^1)
		Customer Preference (x_4^2)
		Succedaneum Price (x_4^3)

Figure 1. Structure of the decision attributes.

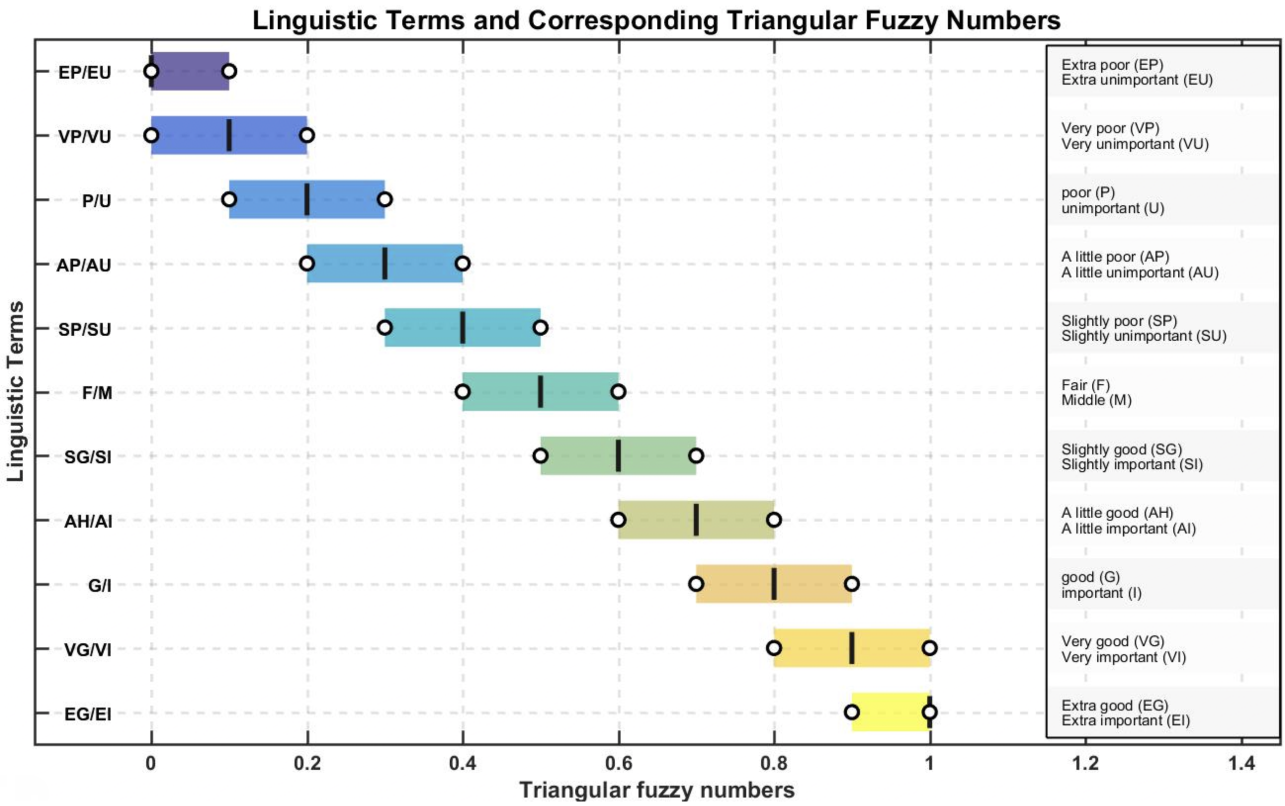

Figure 2. Linguistic terms and the corresponding triangular fuzzy number.

In [10], linguistic evaluation and each attribute are represented by triangular fuzzy numbers. Since the participation of decision makers is required in the process of language evaluation, the method adopted is to use natural language to progressively evaluate, and then transform natural language into a triangular fuzzy number. The language evaluation and the corresponding triangular fuzzy number are shown in **Figure 2**. Taking the cabinet framework as an example, the linguistic evaluations of the main criteria and sub-criteria are shown in **Table 2**, and the corresponding triangular fuzzy numbers are shown in **Table 3**.

Table 2. The importance of criteria and linguistic assessment of EOL in relation to cabinet frame.

Criteria	Weights	Linguistic evaluation of EOL options f_{it}^{j}					
		A_1	A_2	A_3	A_4	A_5	A_6
x_1	VG						
x_1^1	VG	G	G	F	VG	VG	VP
x_1^2	F	G	G	VG	G	F	VP
x_2	G						
x_2^1	G	G	G	F	F	VP	VP
x_2^2	F	VG	G	G	G	G	P
x_2^3	G	F	VG	VG	F	VG	G
x_2^4	VG	F	G	G	G	F	F
x_2^5	F	G	VG	F	G	P	VP
x_3	VG						
x_3^1	VG	G	P	VG	F	VG	VG
x_3^2	F	P	VP	VG	G	P	G
x_3^3	G	VG	F	G	G	VP	G
x_3^4	P	F	F	G	F	F	G
x_4	F						
x_4^1	F	G	G	F	P	F	F
x_4^2	F	VG	G	VG	P	VG	F
x_4^3	F	VG	VG	VG	VP	VG	VG

Table 3. The evaluation of a triangle fuzzy number in relation to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$(0.8,0.9,1)$						
x_1^1	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0,0.1,0.2)$
x_1^2	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0,0.1,0.2)$
x_2	$(0.7,0.8,0.9)$						
x_2^1	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.4,0.5,0.6)$	$(0,0.1,0.2)$	$(0,0.1,0.2)$

Continued

x_2^2	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.1,0.2,0.3)$
x_2^3	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$
x_2^4	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.4,0.5,0.6)$
x_2^5	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.1,0.2,0.3)$	$(0,0.1,0.2)$
x_3	$(0.8,0.9,1)$						
x_3^1	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.1,0.2,0.3)$	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$
x_3^2	$(0.4,0.5,0.6)$	$(0.1,0.2,0.3)$	$(0,0.1,0.2)$	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.1,0.2,0.3)$	$(0.7,0.8,0.9)$
x_3^3	$(0.7,0.8,0.9)$	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0,0.1,0.2)$	$(0.7,0.8,0.9)$
x_3^4	$(0.1,0.2,0.3)$	$(0.4,0.5,0.6)$	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$
x_4	$(0.4,0.5,0.6)$						
x_4^1	$(0.4,0.5,0.6)$	$(0.7,0.8,0.9)$	$(0.7,0.8,0.9)$	$(0.4,0.5,0.6)$	$(0.1,0.2,0.3)$	$(0.4,0.5,0.6)$	$(0.4,0.5,0.6)$
x_4^2	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.7,0.8,0.9)$	$(0.8,0.9,1)$	$(0.1,0.2,0.3)$	$(0.8,0.9,1)$	$(0.4,0.5,0.6)$
x_4^3	$(0.4,0.5,0.6)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0,0.1,0.2)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$

4.2. Decision-Making Studies and Solutions

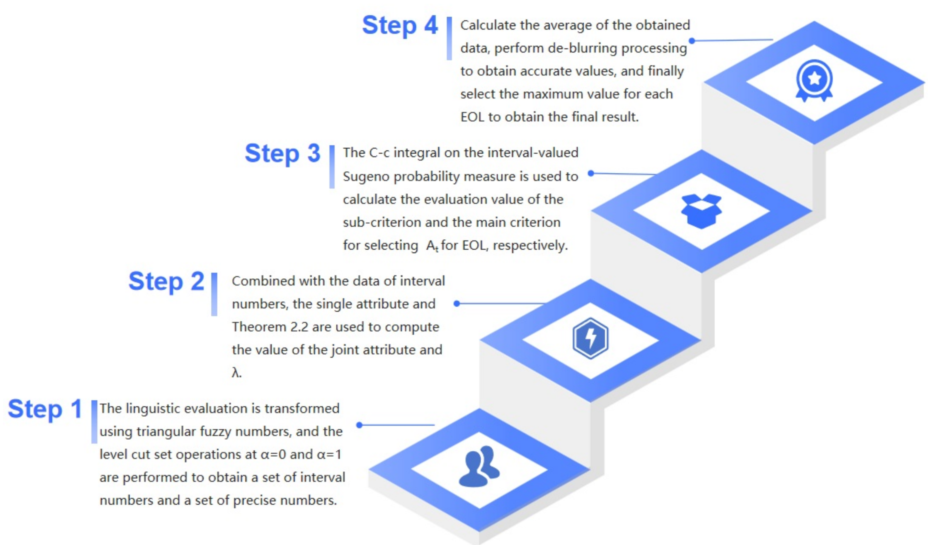

Figure 3. Calculation process.

The MCDM problem for determining the EOL policy of refrigerator components is divided into four steps, as shown in Figure 3. Figure 1 depicts the four main criteria included in this case represented by $x_i\ (i=1,2,3,4)$, and the 14 sub-criteria represented by x_i^j, that is if $i=1$, then $j=1,2$; if $i=2$, then $j=1,2,3,4,5$; if $i=3$, then $j=1,2,3,4$; if $i=4$, then $j=1,2,3$. Finally, the six EOL choices (Reuse, Rermanufacture, Primary recycle, Secondary, Incineration and Landfill) are denoted by $A_1, A_2, A_3, A_4, A_5, A_6$ respectively.

First step: The evaluated linguistic information is transformed into triangular fuzzy numbers, and $\alpha = 0$ and $\alpha = 1$ level cut set operations are applied to the triangular fuzzy numbers. The results are shown in Table 4 and Table 5.

Table 4. The α-level cut set value for $\alpha = 0$.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.8,1]$						
x_1^1	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$
x_1^2	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.8,1]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0,0.2]$
x_2	$[0.7,0.9]$						
x_2^1	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0,0.2]$	$[0,0.2]$
x_2^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.1,0.3]$
x_2^3	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$
x_2^4	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$
x_2^5	$[0.4,0.6]$	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0,0.2]$
x_3	$[0.8,1]$						
x_3^1	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$
x_3^2	$[0.4,0.6]$	$[0.1,0.3]$	$[0,0.2]$	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.7,0.9]$
x_3^3	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0,0.2]$	$[0.7,0.9]$
x_3^4	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$
x_4	$[0.4,0.6]$						
x_4^1	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$
x_4^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.8,1]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$
x_4^3	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$	$[0.8,1]$	$[0.8,1]$

Table 5. The α-level cut set value for $\alpha = 1$.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.9]$						
x_1^1	$[0.9]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.9]$	$[0.9]$	$[0.1]$
x_1^2	$[0.5]$	$[0.8]$	$[0.8]$	$[0.9]$	$[0.8]$	$[0.5]$	$[0.1]$
x_2	$[0.8]$						

Continued

x_2^1	$[0.8]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.5]$	$[0.1]$	$[0.1]$
x_2^2	$[0.5]$	$[0.9]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.2]$
x_2^3	$[0.8]$	$[0.5]$	$[0.9]$	$[0.9]$	$[0.5]$	$[0.9]$	$[0.8]$
x_2^4	$[0.9]$	$[0.5]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.5]$
x_2^5	$[0.5]$	$[0.8]$	$[0.9]$	$[0.5]$	$[0.8]$	$[0.2]$	$[0.1]$
x_3	$[0.9]$						
x_3^1	$[0.9]$	$[0.8]$	$[0.2]$	$[0.9]$	$[0.5]$	$[0.9]$	$[0.9]$
x_3^2	$[0.5]$	$[0.2]$	$[0.1]$	$[0.9]$	$[0.8]$	$[0.2]$	$[0.8]$
x_3^3		$[0.9]$	$[0.5]$	$[0.8]$	$[0.8]$	$[0.1]$	$[0.8]$
x_3^4	$[0.2]$	$[0.5]$	$[0.5]$	$[0.8]$	$[0.5]$	$[0.5]$	$[0.8]$
x_4	$[0.5]$						
x_4^1	$[0.5]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.2]$	$[0.5]$	$[0.5]$
x_4^2	$[0.5]$	$[0.9]$	$[0.8]$	$[0.9]$	$[0.2]$	$[0.9]$	$[0.5]$
x_4^3	$[0.5]$	$[0.9]$	$[0.9]$	$[0.9]$	$[0.1]$	$[0.9]$	$[0.9]$.

Second step: Using $\alpha = 0$ as an example, the joint attribute and the value of λ are computed using a single attribute based on the data in Table 4, Theorem 2.2, and Definition 2.3. Table 6 displays the computation results.

Third step: The CC-integral on the interval-valued Sugeno probability measure is used to calculate the evaluation value of the sub-criterion and the main criterion for selecting A_t for EOL, respectively, where $\left(f_{i,t}^j\right)_\alpha$ represents the α-level cut set of the function $f_{i,t}^j$, $\left(f_{i,t}^j\right)_0$ denotes the 0-level cut set of the function $f_{i,t}^j$.

For EOL options A_2 with respect to criteria x_2, $\alpha = 0$:

(1) It can be seen from Table 4: $\left(f_{2,2}^1\right)_0 = [0.7, 0.9]$, $\left(f_{2,2}^2\right)_0 = [0.7, 0.9]$, $\left(f_{2,2}^3\right)_0 = [0.8, 1]$, $\left(f_{2,2}^4\right)_0 = [0.7, 0.9]$, $\left(f_{2,2}^5\right)_0 = [0.8, 1]$.

(2) It also can be know from Table 4: $(g_\lambda)_2^1 = [0.7, 0.9]$, $(g_\lambda)_2^2 = [0.4, 0.6]$, $(g_\lambda)_2^3 = [0.7, 0.9]$, $(g_\lambda)_2^4 = [0.8, 1]$, $(g_\lambda)_2^5 = [0.4, 0.6]$.

(3) According to Definition 2.5, $\underline{f}_{i,t}^j$ and $\overline{f}_{i,t}^j$ represent the left and right endpoints of the α-level cut set of the function $f_{i,t}^j$, respectively, and $\underline{f}_{2,1}^1 = \underline{f}_{2,1}^2 = \underline{f}_{2,1}^4 \le \underline{f}_{2,1}^3 = \underline{f}_{2,1}^5$, then $x_2^{1'}$, $x_2^{2'}$, $x_2^{3'}$, $x_2^{4'}$, $x_2^{5'}$ is a permutation of x_2^1, x_2^2, x_2^3, x_2^4, x_2^5, where $x_2^{1'} = x_2^1$, $x_2^{2'} = x_2^2$, $x_2^{3'} = x_2^4$, $x_2^{4'} = x_2^3$, $x_2^{5'} = x_2^5$.

The values of all measurements and the required parameter λ obtained from step three are shown in Table 6.

Table 6. The fuzzy measure of sub-criteria value for $\alpha = 0$.

A_1	A_2	A_3
$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$
$\underline{g}_\lambda(2) = 0.4\overline{g}_\lambda(2) = 0.6$	$\underline{g}_\lambda(2) = 0.4\overline{g}_\lambda(2) = 0.6$	$\underline{g}_\lambda(2) = 0.4\overline{g}_\lambda(2) = 0.6$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_\lambda(5) = 0.4\overline{g}_\lambda(5) = 0.6$	$\underline{g}_\lambda(5) = 0.4\overline{g}_\lambda(5) = 0.6$	$\underline{g}_\lambda(5) = 0.7\overline{g}_\lambda(5) = 0.9$
$\underline{g}_\lambda(4) = 0.641\overline{g}_\lambda(4) = 0.84$	$\underline{g}_\lambda(4) = 0.822\overline{g}_\lambda(4) = 0.96$	$.\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1.$
$\underline{g}_\lambda(3) = 0.896\overline{g}_\lambda(3) = 0.984$	$\underline{g}_\lambda(3) = 0.967\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$
$\underline{g}_\lambda(4) = 0.7\overline{g}_\lambda(4) = 0.9$	$\underline{g}_\lambda(4) = 0.1\overline{g}_\lambda(4) = 0.3$	$\underline{g}_\lambda(4) = 0.4\overline{g}_\lambda(4) = 0.6$
$\underline{g}_\lambda(3) = 0.964\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.733\overline{g}_\lambda(3) = 0.93$	$\underline{g}_\lambda(3) = 0.888\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.972\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.972\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.901\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$
$\underline{g}_\lambda(3) = 0.4\overline{g}_\lambda(3) = 0.6$	$\underline{g}_\lambda(3) = 0.4\overline{g}_\lambda(3) = 0.6$	$\underline{g}_\lambda(3) = 0.4\overline{g}_\lambda(3) = 0.6$
$\underline{g}_\lambda(2) = 0.729\overline{g}_\lambda(2) = 0.875$	$\underline{g}_\lambda(2) = 0.729\overline{g}_\lambda(2) = 0.875$	$\underline{g}_\lambda(2) = 0.729\overline{g}_\lambda(2) = 0.875$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
A_4	A_5	A_6
$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$
$\underline{g}_\lambda(2) = 0.8\overline{g}_\lambda(2) = 0.1$	$\underline{g}_\lambda(2) = 0.8\overline{g}_\lambda(2) = 0.1$	$\underline{g}_\lambda(2) = 0.4\overline{g}_\lambda(2) = 0.6$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_\lambda(5) = 0.7\overline{g}_\lambda(5) = 0.9$	$\underline{g}_\lambda(5) = 0.7\overline{g}_\lambda(5) = 0.9$	$\underline{g}_\lambda(5) = 0.7\overline{g}_\lambda(5) = 0.9$
$\underline{g}_\lambda(4) = 0.822\overline{g}_\lambda(4) = 0.96$	$\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1$	$\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1$
$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$

Continued

$\underline{g}_{\lambda}(2)=0.984\overline{g}_{\lambda}(2)=1$	$\underline{g}_{\lambda}(2)=0.984\overline{g}_{\lambda}(2)=1$	$\underline{g}_{\lambda}(2)=0.984\overline{g}_{\lambda}(2)=1$
$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$
$\lambda_1=-0.975\lambda_2=-1$	$\lambda_1=-0.975\lambda_2=-1$	$\lambda_1=-0.975\lambda_2=-1$
$\underline{g}_{\lambda}(4)=0.7\overline{g}_{\lambda}(4)=0.9$	$\underline{g}_{\lambda}(4)=0.8\overline{g}_{\lambda}(4)=1$	$\underline{g}_{\lambda}(4)=0.8\overline{g}_{\lambda}(4)=1$
$\underline{g}_{\lambda}(3)=0.827\overline{g}_{\lambda}(3)=1$	$\underline{g}_{\lambda}(3)=0.823\overline{g}_{\lambda}(3)=1$	$\underline{g}_{\lambda}(3)=0.823\overline{g}_{\lambda}(3)=1$
$\underline{g}_{\lambda}(2)=0.846\overline{g}_{\lambda}(2)=0.927$	$\underline{g}_{\lambda}(2)=0.908\overline{g}_{\lambda}(2)=1$	$\underline{g}_{\lambda}(2)=0.927\overline{g}_{\lambda}(2)=1$
$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$
$\lambda_1=-0.443\lambda_2=-0.904$	$\lambda_1=-0.443\lambda_2=-0.904$	$\lambda_1=-0.443\lambda_2=-0.904$
$\underline{g}_{\lambda}(3)=0.4\overline{g}_{\lambda}(3)=0.6$	$\underline{g}_{\lambda}(3)=0.4\overline{g}_{\lambda}(3)=0.6$	$\underline{g}_{\lambda}(3)=0.4\overline{g}_{\lambda}(3)=0.6$
$\underline{g}_{\lambda}(2)=0.729\overline{g}_{\lambda}(2)=0.875$	$\underline{g}_{\lambda}(2)=0.729\overline{g}_{\lambda}(2)=0.875$	$\underline{g}_{\lambda}(2)=0.729\overline{g}_{\lambda}(2)=0.875$
$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$	$\underline{g}_{\lambda}(1)=1\overline{g}_{\lambda}(1)=1$

(4) The value of the subcriterion $(x_i, i=1,2,3,4)$ with respect to the EOL option $A_t(t=1,2,3,4,5)$ is computed by CC-integral as follows:

$$
\begin{aligned}
(c)\int \underline{f}_{2,2}^{j}\mathrm{d}\underline{g}_{\lambda} =\ & \min\left\{\underline{f}_{2,2}^{1},\underline{g}_{\lambda}(1)\right\}*\max\left\{\left(\underline{f}_{2,2}^{1}\right)^2,\left(\underline{g}_{\lambda}(1)\right)^2\right\} \\
& +\left\{\min\left\{\underline{f}_{2,2}^{2},\underline{g}_{\lambda}(2)\right\}*\max\{(\left\{\left(\underline{f}_{2,2}^{2}\right)^2,\left(\underline{g}_{\lambda}(2)\right)^2\right\}\right. \\
& \left.-\min\left\{\underline{f}_{2,2}^{1},\underline{g}_{\lambda}(2)\right\}*\max\left\{\left(\underline{f}_{2,2}^{1}\right)^2,\left(\underline{g}_{\lambda}(2)\right)^2\right\}\right\} \\
& +\left\{\min\left\{\underline{f}_{2,2}^{4},\underline{g}_{\lambda}(3)\right\}*\max\left\{\left(\underline{f}_{2,2}^{4}\right)^2,\left(\underline{g}_{\lambda}(3)\right)^2\right\}\right. \\
& \left.-\min\left\{\underline{f}_{2,2}^{2},\underline{g}_{\lambda}(3)\right\}*\max\left\{\left(\underline{f}_{2,2}^{2}\right)^2,\left(\underline{g}_{\lambda}(3)\right)^2\right\}\right\} \\
& +\left\{\min\left\{\underline{f}_{2,2}^{3},\underline{g}_{\lambda}(4)\right\}*\max\left\{\left(\underline{f}_{2,2}^{3}\right)^2,\left(\underline{g}_{\lambda}(4)\right)^2\right\}\right. \\
& \left.-\min\left\{\underline{f}_{2,2}^{4},\underline{g}_{\lambda}(4)\right\}*\max\left\{\left(\underline{f}_{2,2}^{4}\right)^2,\left(\underline{g}_{\lambda}(4)\right)^2\right\}\right\} \\
& +\left\{\min\left\{\underline{f}_{2,2}^{5},\underline{g}_{\lambda}(5)\right\}*\max\left\{\left(\underline{f}_{2,2}^{5}\right)^2,\left(\underline{g}_{\lambda}(5)\right)^2\right\}\right. \\
& \left.-\min\left\{\underline{f}_{2,2}^{3},\underline{g}_{\lambda}(5)\right\}*\max\left\{\left(\underline{f}_{2,2}^{3}\right)^2,\left(\underline{g}_{\lambda}(5)\right)^2\right\}\right\} \\
=\ & \min(0.7000,1.000)*\max(0.4900,1.0000) \\
& +\left(\min(0.7000,0.984)*\max(0.4900,0.9683)\right. \\
& \left.-\min(0.7000,0.984)*\max(0.4900,0.9683)\right) \\
& +\left(\min(0.7000,0.967)*\max(0.4900,0.9351)\right. \\
& \left.-\min(0.7000,0.967)*\max(0.4900,0.9351)\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\min(0.8000,0.822)*\max(0.6400,0.6757)\right.\\
&\left.-\min(0.7000,0.822)*\max(0.4900,0.6757)\right)\\
&+\left(\min(0.8000,0.400)*\max(0.6400,0.1600)\right.\\
&\left.-\min(0.8000,0.400)*\max(0.6400,0.1600)\right)\\
&=0.7*1+(0.7*0.9683-0.7*0.9683)\\
&\quad+(0.7*0.9351-0.7*0.9351)\\
&\quad+(0.8*0.6757-0.7*0.6757)\\
&\quad+(0.4*0.64-0.4*0.64)\\
&=0.7+0+0+0.0676+0\\
&=0.7676.
\end{aligned}
$$

Similarly, $(c)\int \overline{f}_{2,2}^{j}\mathrm{d}\overline{g}_{\lambda}=1$. Therefore,

$$
(c)\int f_{2,2}^{j}\mathrm{d}g_{\lambda}=\left[(c)\int \underline{f}_{2,2}^{j}\mathrm{d}\underline{g}_{\lambda},(c)\int \overline{f}_{2,2}^{j}\mathrm{d}\overline{g}_{\lambda}\right]=[0.7676,1].
$$

As seen in **Table 7**, we may compute the evaluation values for the remaining sub-criteria in a similar manner. In a similar manner, $\alpha=1$ can also be derived in **Table 8**.

According to the data in **Table 7**, the evaluation value of the main criterion is calculated using the same method, For EOL option A_2, $a=0$, that is, $(f_{1,2})_0=[0.7,0.9]$, $(f_{2,2})_0=[0.7676,1]$, $(f_{3,2})_0=[0.2557,0.5595]$, $(f_{4,2})_0=[0.7600,1]$, the corresponding value of λ is shown in **Table 9**. The specific calculation process is as follows:

$$
\begin{aligned}
(c)\int \underline{f}_{i,2}\mathrm{d}\underline{g}_{\lambda}&=\min\left\{\underline{f}_{3,2},\underline{g}_{\lambda}(1)\right\}*\max\left\{\left(\underline{f}_{3,2}\right)^2,\left(\underline{g}_{\lambda}(1)\right)^2\right\}\\
&\quad+\left\{\min\left\{\underline{f}_{1,2},\underline{g}_{\lambda}(2)\right\}*\max\left\{\left(\underline{f}_{1,2}\right)^2,\left(\underline{g}_{\lambda}(2)\right)^2\right\}\right.\\
&\quad\left.-\min\left\{\underline{f}_{3,2},\underline{g}_{\lambda}(2)\right\}*\max\left\{\left(\underline{f}_{3,2}\right)^2,\left(\underline{g}_{\lambda}(2)\right)^2\right\}\right\}\\
&\quad+\left\{\min\left\{\underline{f}_{4,2},\underline{g}_{\lambda}(3)\right\}*\max\left\{\left(\underline{f}_{4,2}\right)^2,\left(\underline{g}_{\lambda}(3)\right)^2\right\}\right.\\
&\quad\left.-\min\left\{\underline{f}_{1,2},\underline{g}_{\lambda}(3)\right\}*\max\left\{\left(\underline{f}_{1,2}\right)^2,\left(\underline{g}_{\lambda}(3)\right)^2\right\}\right\}\\
&\quad+\left\{\min\left\{\underline{f}_{2,2},\underline{g}_{\lambda}(4)\right\}*\max\left\{\left(\underline{f}_{2,2}\right)^2,\left(\underline{g}_{\lambda}(4)\right)^2\right\}\right.\\
&\quad\left.-\min\left\{\underline{f}_{4,2},\underline{g}_{\lambda}(4)\right\}*\max\left\{\left(\underline{f}_{4,2}\right)^2,\left(\underline{g}_{\lambda}(4)\right)^2\right\}\right\}\\
&=\min(0.2557,1.000)*\max(0.0654,1.0000)\\
&\quad+\min(0.7,0.970)*\max(0.4900,0.9403)\\
&\quad-\min(0.2557,0.970)*\max(0.0654,0.9403)\\
&\quad+\min(0.7600,0.822)*\max(0.5116,0.6757)\\
&\quad-\min(0.7000,0.822)*\max(0.4900,0.6757)\\
&\quad+\min(0.7676,0.700)*\max(0.5892,0.4900)\\
&\quad-\min(0.76,0.7)*\max(0.5776,0.4900)
\end{aligned}
$$

$$
\begin{aligned}
&= 0.2557 * 1 + (0.7 * 0.9403 - 0.2557 * 0.9403) \\
&\quad + (0.76 * 0.6757 - 0.7 * 0.6757) \\
&\quad + (0.7 * 0.5892 - 0.7 * 0.5776) \\
&= 0.2557 + 0.4178 + 0.0405 + 0.0081 \\
&= 0.7222.
\end{aligned}
$$

In the same manner, $(c)\int \overline{f}_{i,2} \mathrm{d}\overline{g}_{\lambda} = 1$, then,

$$
\begin{aligned}
(c)\int f_{2,2} \mathrm{d}g_{\lambda} &= \left[(c)\int \underline{f}_{2,2} \mathrm{d}\underline{g}_{\lambda}, (c)\int \overline{f}_{2,2} \mathrm{d}\overline{g}_{\lambda}\right] \\
&= [0.7222, 1].
\end{aligned}
$$

Similarly, the evaluation values for the other main criteria can also be calculated, and the results are shown in **Table 10**, the above calculation process and results were all accomplished using the Matlab software.

Fourth step: Since triangular fuzzy numbers cannot be directly applied, the mean method was used to perform defuzzification on the triangular fuzzy numbers obtained in **Table 10**. This process transformed the fuzzy numbers into precise figures, thereby determining the optimal EOL selection.

Table 7 and **Table 10** present the optimal EOL choices derived from the calculation method of this paper. **Table 11** and **Table 12** show the data we used in [19]. By comparing the two approaches, it is discovered that while the acquired data varies, the ultimate EOL strategy are the same. In contrast to the C_T-integral on the interval-valued Sugeno probability measure, the C_T-integral on the interval-valued Sugeno probability measure not only makes the calculation process easier and more efficient by using the mathematical program Matlab, but more careful calculations can yield more remarkable and distinct results. Meanwhile, since the Copula function encompasses a series of commonly functions such as triangular norms and overlap functions, different Copula-based Choquet integrals can be derived, which broadens the application scope and significantly expands the application space in the field of integration.

Table 7. The evaluation value of the major criteria for $\alpha = 0$ in relation to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.592,0.984]$	$[0.7640,0.919]$	$[0.0.656,0.664]$	$[0,0.2]$
x_1^1	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$
x_1^2	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.8,1]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0,0.2]$
x_2	$[0.7,0.9]$	$[0.7008,1]$	$[0.7676,1]$	$[0.7867,1]$	$[0.6817,0.9]$	$[0.7509,1]$	$[0.5082,0.843]$
x_2^1	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0,0.2]$	$[0,0.2]$
x_2^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.1,0.3]$
x_2^3	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$

Continued

x_2^4	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$
x_2^5	$[0.4,0.6]$	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0,0.2]$
x_3	$[0.8,1]$	$[0.7672,1]$	$[0.2557,0.5595]$	$[0.7789,1]$	$[0.6052,0.9]$	$[0.5416,1]$	$[0.7640,1]$
x_3^1	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$
x_3^2	$[0.4,0.6]$	$[0.1,0.3]$	$[0,0.2]$	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.7,0.9]$
x_3^3	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0,0.2]$	$[0.7,0.9]$
x_3^4	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$
x_4	$[0.4,0.6]$	$[0.7946,1]$	$[0.7600,1]$	$[0.6540,1]$	$[0.0531,0.2766]$	$[0.654,1]$	$[0.592,0.984]$
x_4^1	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$
x_4^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.8,1]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$
x_4^3	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$	$[0.8,1]$	$[0.8,1]$

Table 8. The evaluation value of the major criteria for $\alpha = 1$ in relation to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.9]$	$[0.8]$	$[0.8]$	$[0.78]$	$[0.8810]$	$[0.824]$	$[0.1]$
x_1^1	$[0.9]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.9]$	$[0.9]$	$[0.1]$
x_1^2	$[0.5]$	$[0.8]$	$[0.8]$	$[0.9]$	$[0.8]$	$[0.5]$	$[0.1]$
x_2	$[0.8]$	$[0.8796]$	$[0.8810]$	$[0.9306]$	$[0.7858]$	$[0.8728]$	$[0.6789]$
x_2^1	$[0.8]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.5]$	$[0.1]$	$[0.1]$
x_2^2	$[0.5]$	$[0.9]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.2]$
x_2^3	$[0.8]$	$[0.5]$	$[0.9]$	$[0.9]$	$[0.5]$	$[0.9]$	$[0.8]$
x_2^4	$[0.9]$	$[0.5]$	$[0.8]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.5]$
x_2^5	$[0.5]$	$[0.8]$	$[0.9]$	$[0.5]$	$[0.8]$	$[0.2]$	$[0.1]$
x_3	$[0.9]$	$[0.9235]$	$[0.4111]$	$[0.8912]$	$[0.7452]$	$[0.7723]$	$[0.881]$
x_3^1	$[0.9]$	$[0.8]$	$[0.2]$	$[0.9]$	$[0.5]$	$[0.9]$	$[0.9]$
x_3^2	$[0.5]$	$[0.2]$	$[0.1]$	$[0.9]$	$[0.8]$	$[0.2]$	$[0.8]$
x_3^3	$[0.8]$	$[0.9]$	$[0.5]$	$[0.8]$	$[0.8]$	$[0.1]$	$[0.8]$
x_3^4	$[0.2]$	$[0.5]$	$[0.5]$	$[0.8]$	$[0.5]$	$[0.5]$	$[0.8]$
x_4	$[0.5]$	$[0.9317]$	$[0.8850]$	$[0.828]$	$[0.1654]$	$[0.828]$	$[0.78]$
x_4^1	$[0.5]$	$[0.8]$	$[0.8]$	$[0.5]$	$[0.2]$	$[0.5]$	$[0.5]$
x_4^2	$[0.5]$	$[0.9]$	$[0.8]$	$[0.9]$	$[0.2]$	$[0.9]$	$[0.5]$
x_4^3	$[0.5]$	$[0.9]$	$[0.9]$	$[0.9]$	$[0.1]$	$[0.9]$	$[0.9]$

Table 9. The fuzzy measure value on the main criteria for $\alpha = 0$ in relation to the cabinet frame.

A_1	A_2	A_3
$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.992\lambda_2 = -1$	$\lambda_1 = -0.992\lambda_2 = -1$	$\lambda_1 = -0.992\lambda_2 = -1$
$\underline{g}_\lambda(4) = 0.4\overline{g}_\lambda(4) = 1$	$\underline{g}_\lambda(4) = 0.7\overline{g}_\lambda(4) = 0.9$	$\underline{g}_\lambda(4) = 0.8\overline{g}_\lambda(4) = 1$
$\underline{g}_\lambda(3) = 0.882\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.822\overline{g}_\lambda(3) = 0.96$	$\underline{g}_\lambda(3) = 0.944\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.9697\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.9697\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.9697\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
A_4	A_5	A_6
$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	$\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.992\lambda_2 = -1$	$\lambda_1 = -0.992\lambda_2 = -1$	$\lambda_1 = -0.992\lambda_2 = -1$
$\underline{g}_\lambda(4) = 0.8\overline{g}_\lambda(4) = 1$	$\underline{g}_\lambda(4) = 0.7\overline{g}_\lambda(4) = 1$	$\underline{g}_\lambda(4) = 0.8\overline{g}_\lambda(4) = 1$
$\underline{g}_\lambda(3) = 0.944\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.944\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.944\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.995\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.9697\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.9697\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$

Table 10. The thorough assessment of EOL possibilities in relation to the cabinet frame.

Main criteria	$(c)\int f\,\mathrm{d}g_\lambda$	Crisp number
	A_1 A_2 A_3	A_1 A_2 A_3
Overall EOL	$(0.770, 0.926, 1.0)$ $(0.722, 0.865, 1.0)$ $(0.767, 0.946, 1.0)$	0.899 0.862 0.904*
	options value	
x_1	$(0.7, 0.8, 0.9)$ $(0.7, 0.8, 0.9)$ $(0.592, 0.78, 0.984)$	0.8 0.8 0.785
x_2	$(0.7008, 0.8796, 1)$ $(0.7676, 0.8810, 1)$ $(0.7867, 0.9306, 1)$	0.860 0.883 0.906
x_3	$(0.7672, 0.9235, 1)$ $(0.2557, 0.4111, 0.5595)$ $(0.7789, 0.8912, 1)$	0.897 0.409 0.890
x_4	$(0.7946, 0.9317, 1)$ $(0.7600, 0.8850, 1)$ $(0.6540, 0.828, 1)$	0.909 0.882 0.827

Continued

Main criteria	$(c)\int f\,\mathrm{d}g_{\lambda}$ A_4 A_5 A_6	Crisp number A_4 A_5 A_6
Overall EOL	$(0.720, 0.860, 0.920)$ $(0.722, 0.864, 1.0)$ $(0.663, 0.849, 1)$	0.833 0.862 0.837
	options value	
x_1	$(0.7640, 0.8810, 0.919)$ $(0.656, 0.824, 0.664)$ $(0, 0.1, 0.2)$	0.855 0.715 0.1
x_2	$(0.6817, 0.7858, 0.9)$ $(0.7509, 0.8728, 1)$ $(0.5082, 0.6789, 0.843)$	0.789 0.875 0.677
x_3	$(0.6052, 0.7452, 0.9)$ $(0.5416, 0.7723, 1)$ $(0.7640, 0.881, 1)$	0.750 0.771 0.882
x_4	$(0.0531, 0.1654, 0.2766)$ $(0.654, 0.828, 1)$ $(0.592, 0.78, 0.984)$	0.165 0.827 0.785

Table 11. The evaluation value of the major criteria for $\alpha = 0$ in relation to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.8,1]$	$[0.8,1]$	$[0.7,0.8]$	$[0,0.2]$
x_1^1	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$
x_1^2	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.8,1]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0,0.2]$
x_2	$[0.7,0.9]$	$[0.8,1]$	$[0.8,1]$	$[0.8,1]$	$[0.7,0.9]$	$[0.8,1]$	$[0.8,1]$
x_2^1	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0,0.2]$	$[0,0.2]$
x_2^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.1,0.3]$
x_2^3	$[0.7,0.9]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$
x_2^4	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$
x_2^5	$[0.4,0.6]$	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0,0.2]$
x_3	$[0.8,1]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.8,1]$	$[0.8,1]$
x_3^1	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$
x_3^2	$[0.4,0.6]$	$[0.1,0.3]$	$[0,0.2]$	$[0.8,1]$	$[0.7,0.9]$	$[0.1,0.3]$	$[0.7,0.9]$
x_3^3	$[0.7,0.9]$	$[0.8,1]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0,0.2]$	$[0.7,0.9]$
x_3^4	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.4,0.6]$	$[0.7,0.9]$

Continued

x_4	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.8,1]$	$[0.1,0.3]$	$[0.8,1]$	$[0.8,1]$
x_4^1	$[0.4,0.6]$	$[0.7,0.9]$	$[0.7,0.9]$	$[0.4,0.6]$	$[0.1,0.3]$	$[0.4,0.6]$	$[0.4,0.6]$
x_4^2	$[0.4,0.6]$	$[0.8,1]$	$[0.7,0.9]$	$[0.8,1]$	$[0.1,0.3]$	$[0.8,1]$	$[0.4,0.6]$
x_4^3	$[0.4,0.6]$	$[0.8,1]$	$[0.8,1]$	$[0.8,1]$	$[0,0.2]$	$[0.8,1]$	$[0.8,1]$

Table 12. The thorough assessment of EOL possibilities in relation to the cabinet frame.

Main criteria	$(c)\int f\,\mathrm{d}g_\lambda$			Crisp number
	A_1	A_2	A_3	A_1 A_2 A_3
Overall EOL	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0.8,0.99,1)$	0.9 0.9 0.933*
Main criteria	$(c)\int f\,\mathrm{d}g_\lambda$			Crisp number
	A_4	A_5	A_6	A_4 A_5 A_6
Overall EOL	$(0.8,0.9,1)$	$(0.8,0.9,1)$	$(0.8,0.9,1)$	0.9 0.9 0.9

5. Conclusion and Prospects

In this article, we expanded on the Choquet integral based on the interval-valued Sugeno probability measure, obtaining the CC-integral on the interval-valued Sugeno probability measure. We then applied it to MCDM. By comparing with the decision results of the Choquet integral based on triangular norms, it was found that the proposed method is superior to the general method. Due to the wide application of the Copula function, when the Copula function is $C(x,y)=xy$, the CC-integral is the general Choquet integral. The main application of this article not only lies in simplifying the calculation process by using the overlapping function as the Copula function, making it more straightforward and efficient, but also more remarkable and distinct results can be obtained through more meticulous calculations thereby demonstrating the practicality of the proposed method. On the other hand, since the Choquet integral based on the Copula function is a generalization of the standard Choquet integral, and the Choquet integral is a generalization of the classical Lebesgue integral, compared with the general integral, the integral based on the Copula function has a wider application range and establishes the connection between the Copula function and the Choquet integral.

Furthermore, this paper only presents the discrete representation and applications of the CC-integral on the interval-valued Sugeno probability measure. However, its specific properties, potential applications, and the CC-integral formed by different types of Copula functions still need to be studied in order to better understand the CC-integral.

Acknowledgements

This work is supported by National Natural Science Foundation of China (NO. 12161082).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Choquet, G. (1954) Theory of Capacities. *Annales de l'Institut Fourier*, 5, 131-295. https://doi.org/10.5802/aif.53

[2] Sugeno, M. and Murofushi, T. (1987) Pseudo-Additive Measures and Integrals. *Journal of Mathematical Analysis and Applications*, **122**, 197-222. https://doi.org/10.1016/0022-247x(87)90354-4

[3] Barrenechea, E., Bustince, H., Fernandez, J., Paternain, D. and Sanz, J. (2013) Using the Choquet Integral in the Fuzzy Reasoning Method of Fuzzy Rule-Based Classification Systems. *Axioms*, **2**, 208-223. https://doi.org/10.3390/axioms2020208

[4] Lucca, G., Sanz, J.A., Dimuro, G.P., Bedregal, B., Mesiar, R., Kolesarova, A., *et al.* (2016) Preaggregation Functions: Construction and an Application. *IEEE Transactions on Fuzzy Systems*, **24**, 260-272. https://doi.org/10.1109/tfuzz.2015.2453020

[5] Lucca, G., Sanz, J.A., Dimuro, G.P., Bedregal, B., Asiain, M.J., Elkano, M., *et al.* (2017) CC-Integrals: Choquet-Like Copula-Based Aggregation Functions and Its Application in Fuzzy Rule-Based Classification Systems. *Knowledge-Based Systems*, **119**, 32-43. https://doi.org/10.1016/j.knosys.2016.12.004

[6] Lucca, G., Antonio Sanz, J., Dimuro, G.P., Bedregal, B., Bustince, H. and Mesiar, R. (2018) CF-Integrals: A New Family of Pre-Aggregation Functions with Application to Fuzzy Rule-Based Classification Systems. *Information Sciences*, **435**, 94-110. https://doi.org/10.1016/j.ins.2017.12.029

[7] Dimuro, G.P., Fernández, J., Bedregal, B., Mesiar, R., Sanz, J.A., Lucca, G., *et al.* (2020) The State-of-Art of the Generalizations of the Choquet Integral: From Aggregation and Pre-Aggregation to Ordered Directionally Monotone Functions. *Information Fusion*, **57**, 27-43. https://doi.org/10.1016/j.inffus.2019.10.005

[8] Schmeidler, D. (1986) Integral Representation without Additivity. *Proceedings of the American Mathematical Society*, **97**, 255-261. https://doi.org/10.1090/s0002-9939-1986-0835875-8

[9] Tanaka, K. and Sugeno, M. (1991) A Study on Subjective Evaluations of Printed Color Images. *International Journal of Approximate Reasoning*, **5**, 213-222. https://doi.org/10.1016/0888-613x(91)90009-b

[10] Chen, S. and Hwang, C.L. (1992) Fuzzy Multiple Attribute Decision-Making: Methods and Application. Springer.

[11] Garg, H., Krishankumar, R. and Ravichandran, K.S. (2022) Decision Framework with Integrated Methods for Group Decision-Making under Probabilistic Hesitant Fuzzy Context and Unknown Weights. *Expert Systems with Applications*, **200**, Article 117082. https://doi.org/10.1016/j.eswa.2022.117082

[12] Garg, H. (2018) Some Methods for Strategic Decision-Making Problems with Immediate Probabilities in Pythagorean Fuzzy Environment. *International Journal of Intelligent Systems*, **33**, 687-712. https://doi.org/10.1002/int.21949

[13] Rani, P., Govindan, K., Mishra, A.R., Mardani, A., Alrasheedi, M. and Hooda, D.S. (2020) Unified Fuzzy Divergence Measures with Multi-Criteria Decision Making Problems for Sustainable Planning of an E-Waste Recycling Job Selection. *Symmetry*, **12**, Article 90. https://doi.org/10.3390/sym12010090

[14] Wieczynski, J.C., Lucca, G. and Borges, E. (2021) CC-Separation Measure Applied in Business Group Decision Making. *Proceedings of the* 23*rd International Conference*

on Enterprise Information Systems, Online, 26-28 April 2021, 26-28.

[15] Wieczynski, J.C., Fumanal-Idocin, J. and Lucca, G. (2022) d-XC Integrals: On the Generalization of the Expanded Form of the Choquet Integral by Restricted Dissimilarity Functions and Their Applications. *IEEE Transactions on Fuzzy Systems*, **30**, 5376-5389. https://doi.org/10.1109/tfuzz.2022.3176916

[16] Di Marco, P., Eubanks, C.F. and Ishii, K. (1994) Compatibility Analysis of Product Design for Recyclability and Reuse. *ASME* 1994 *International Computers in Engineering Conference and Exhibition*, Minneapolis, 11-14 September 1994, 11-14. https://doi.org/10.1115/cie1994-0392

[17] Ma, J., Kremer, G.E.O. and Ray, C.D. (2018) A Comprehensive End-of-Life Strategy Decision Making Approach to Handle Uncertainty in the Product Design Stage. *Research in Engineering Design*, **29**, 469-487. https://doi.org/10.1007/s00163-017-0277-0

[18] Chen, L., Duan, G., Wang, S. and Ma, J. (2020) A Choquet Integral Based Fuzzy Logic Approach to Solve Uncertain Multi-Criteria Decision Making Problem. *Expert Systems with Applications*, **149**, Article 113303. https://doi.org/10.1016/j.eswa.2020.113303

[19] Yang, H., Shang, L.Y. and Gong, Z.T. (2022) CC-Integral on Interval-Valued Sugeno Probability Measure and Its Application in Multi-Criteria Decision-Making Problems. *Axioms*, **11**, Article 317.

[20] Sugeno, M. (1974) Theory of Fuzzy Integral and Its Application. Doctorial Dissertation, Tokyo Institute of Technology.

[21] Mayor, G. and Trillas, E. (1986) On the Representation of Some Aggregation Functions. *Proceedings of IEEE International Symposium on Multiple- Valued Logic*, Los Alamitos, 27-29 May 1986, 110-114.

[22] Alsina, C., Frank, M.J. and Schweizer, B. (2006). Associative Functions: Triangular Norms and Copulas. World Scientific Publishing. https://doi.org/10.1142/9789812774200

[23] Bustince, H., Fernandez, J., Mesiar, R., Montero, J. and Orduna, R. (2010) Overlap Functions. *Nonlinear Analysis: Theory, Methods & Applications*, **72**, 1488-1499. https://doi.org/10.1016/j.na.2009.08.033

[24] Nelsen, R.B. (1999) An Introduction to Copulas. Springer.

[25] Dimuro, G.P. and Bedregal, B. (2014) Archimedean Overlap Functions: The Ordinal Sum and the Cancellation, Idempotency and Limiting Properties. *Fuzzy Sets and Systems*, **252**, 39-54. https://doi.org/10.1016/j.fss.2014.04.008

[26] Klement, E.P., Mesiar, R. and Pap, E. (2000) Triangular Norms. Kluwer Academic Publisher.

[27] Wang, Z. and Zklir, G.J. (1992) Fuzzy Measure Theory. Plenum Press.

[28] Beliakov, G., Pradera, A. and Calvo, T. (2007) Aggregation Functions: A Guide for Practitioners. *Studies in Fuzziness and Soft Computing*, Springer. https://doi.org/10.1007/978-3-540-73721-6

[29] Guo, Y.W. (2021) Interval-Valued Choquet Integrals and Their Properties. Journal of Shanxi Normal University (Natural Science Edition), **35**, 16-21.

[30] Chung, W., Okudan, G.E. and Wysk, R.A. (2011) A Modular Design Approach to Improve the Life Cycle Performance Derived from Optimized Closed-Loop Suply Chain. *Proceedings of the ASME* 2011 *International Design Engineering Technical Conference Computers and Information in Engineering Conference*, Washington, 28-31 August 2011, 28-31.