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Use of Statistical Distance T^2 and ARL to Detect Cutting Tool Drift

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Abstract

This article exploits the statistical distance T^2 for the detection of cutting tool drift in the case of CNC machine tools. We present a control approach based on the T^2 chart and inertial control in a multivariate process. Inertial control is an approach for controlling CNC machine tools developed in recent years. Its principle is to link measurable parameters of the part to adjustable parameters on the machine (tool coordinators). In the case of machining mechanical parts by CNC machine tools, it involves linking tool offsets on the machine to deviations in dimensions measured on the part. This produces an incidence matrix representing the influence of machine parameters on product characteristics. When a tool offset affects several dimensions of the part, there is a correlation between these dimensions and the tool offset. The T^2 chart allows us to take this correlation into account to detect cutting tool misalignment, and inertial control will calculate the tool offset values to correct this misalignment. Furthermore, we use the ARL (Average Run Length) measurement to evaluate the performance of T^2 in comparison with the Shewhart control chart. ARL is commonly used to monitor processes and detect changes in their behavior. A comparative result based on ARLs of these charts is presented in the article. An example of a simple 1D part in which a tool offset is maximal in three dimensions is presented in this article to illustrate our point.

Keywords

Tool Offsets, Machine Setting, Machine Steering, Hotelling's T^2

1. Introduction

When a malfunction is observed in a manufacturing process, it must be corrected to maintain control over the process. It is in this context that Statistical Process

Control (SPC) (cf. Pillet [1]) is currently being implemented in several companies to best prevent incidents from occurring during production. Its main objective in industrial production is to maintain the quality characteristics of the manufactured product as close as possible to their target values defined in the design. To this end, the aim is to detect a malfunction as early as possible in order to trigger appropriate corrective action. There are several methods and tools in the literature for detecting out-of-control processes.

Walter A. Shewhart [2] did remarkable work by first proposing the one-dimensional control chart for monitoring production processes. He extended his idea to introduce the $\overline{X}$ control chart to monitor the means and standard deviation of a process. Later, this same Shewhart [3] showed that the statistical approach based on the detection of out-of-control situations is well-suited for solving manufacturing process problems.

E. Del Castillo [4] tested the $\overline{X}$ average control chart when the standard deviation is unknown, for small batches. He demonstrated that it is preferable to use the $\overline{X}$ chart with a standard deviation estimated from individual samples, rather than using it with an average standard deviation, which generates false alarms.

Other types of control charts, such as CUSUM (Cumulative Sum) or EWMA (Exponentially Weighted Moving Average), have been used by authors (Box [5], Yi-Dai [6], Huwang [7]) for monitoring manufacturing processes. Some authors have compared the effectiveness of different control charts, following the example of G. Brunschwig [8] or Boudaoud [9]. G. Brunschwig showed that the Shewhart control chart is more suitable for detecting sudden and significant drifts, while the CUSUM and EWMA charts are more effective in detecting slow drifts. N. Boudaoud and C. Zohra [9] proposed a drift detection approach using a multimodal control chart based on the statistical approach of Hotelling T^2 [10]. He made a comparative study of the CUSUM and EWMA charts and the multimodal T^2 chart. He showed that the ARL (Average Run Length) value of the T^2 chart is much higher than that of the first two charts, but that the CUCUM chart is better in performance in the case of detecting slow drifts.

The author Douglas C. Montgomery [11] demonstrated the joint use of SPC (Statistical Process Control) and EPC (Engineering Process Control) for process monitoring through the Shewhart control chart, CUSUM, and EWMA. He indicated that the role of SPC is to correct the process when it is deemed out of control, while the role of EPC is to continually adjust the process to avoid drift.

Frank E. Grubbs [12], in turn, proposed a statistical approach to detect outliers in individual samples based on a test criterion (Tn) which corresponds to the square root of the statistical distance. This criterion is associated with a risk of rejecting good observations. He shows that if the risk is between 1% and 5%, the test gives a more reliable detection result, but that beyond 5% of the risk, the result is not guaranteed.

Researchers (Lafaye [13], Epprecht [14]) have worked on multidimensional process monitoring using multivariate control charts to track multiple process parameters simultaneously.

Robert L. Mason and Young [15] have done excellent work on the statistical monitoring of multivariate processes. He used Hotelling's statistical distance (T^2) to design the T^2 chart to detect out-of-control situations using an elliptical representation of correlated variables. He shows that for a bivariate normal function, there is up to a 95% probability of finding all points with the same statistical distance within the elliptical contour. Points outside the elliptical contour correspond to out-of-control situations.

Del Castillo *et al.* [16] have developed a sequential process adjustment based on a stochastic approximation to approximate the average value of the target. They used the rule of Grubbs [12], which is to adjust the machine by a successive approach. Kibe *et al.* [17] proposed to adjust the initial position of tools using an in-situ measurement because the initial setting of the tools is very approximate. The authors (Bourdet [18], Anselmetti *et al.* [19]) showed that if we can act on tool offsets in the machine to reduce the deviation between the theoretical part and the machined part, it can also be reduced by acting on the optimum dimensions chosen intelligently. Goldschmidt [20] and Pairel (Pairel *et al.* [21], Pairel *et al.* [22]) have developed the methodology Copilot Pro®, which allows adjusting all the tools for roughing and finishing by measuring the manufacturing dimensions established by the Office of the methods. Works on an inertial approach for setting machines were conducted by Pillet *et al.* [23] in the case of inertial tolerancing.

The authors (Pillet [24], Del Castillo *et al.* [16]) worked on the statistical approach to process adjustment in the one-dimensional case. Pillet [25] showed that using the classic Shewhart map allows for early process adjustment because it generates false alarms. He then introduced a tuning rule that takes into account short-term dispersion and the measured deviation to calculate corrections. In this article, we exploit the statistical distance T^2 to detect cutting tool drift at an opportune time without generating false alarms.

This approach complements the work carried out by M. Pillet in the context of inertial control. It provides a significant improvement to inertial control for effectively adjusting processes in a multivariate control framework. The results of 10^6 simulations generated using data from the Mersenne Twister algorithm, performed on three types of maps, provide relevant results that validate the approach. A comparative result based on ARLs of these charts is presented in the article. We illustrate our remarks using an example of a simple part whose sole purpose is to show the influence of three correlated characteristics on the setting of the tool that produces them.

In the case of one-dimensional charting, in the literature, there are many methods and tools for the detection of a non-controlled process. Walter A. Shewhart [2] [3] accomplished a tremendous work with his one-dimensional control chart for process surveillance. Later, he extended his idea with the introduction of the $\overline{X}$ control chart for monitoring the mean values and the standard deviation of a process. E. Del Castillo [4] tested this type of chart when the standard deviation is unknown, in the case of small-sized batches. He demonstrated that it is better to use this chart with an estimated standard deviation from individual samples than with a mean standard deviation in terms of false alarm rate. Other types of control charts, such as CUSUM (Cumulative Sum) or EWMA (Exponentially Weighted Moving Average) charts, have been exploited by authors (Box [5], Dai [6], Huwang [7] for monitoring of the manufacturing process.

In the case of multidimensional charting, in practice, many industrial processes have their performances tied to a set of interrelated variables. Monitoring such processes is named Multivariate Statistical Process Control (MSPC). Multivariate process control is one of the subjects in the field of statistical process control in perpetual evolution. Many studies have been done about it. Mason and Young [15] and Bersimis [26] wrote a comprehensive review of the literature on multivariate process control chart techniques.

According to Zhang [27], multivariate process measurement benefits from the use of inherent multivariate methods rather than a collection of univariate charting methods applied to the individual components. The development of multivariate control charts originates from the work by Hotelling [10]. In his paper, the author develops a single control chart that regroups the EWMA method and the generalized likelihood ratio (GRL) test. Monitoring the process mean and variability through a mean vector and a covariance matrix.

N. Boudaoud and C. Zohra [9] proposed to detect drifts with a multimodal control chart based upon the Hoteling's T^2 statistic ([10]. He conducted a comparative study of CUSUM and EWMA charts and the multimodal T^2 chart. He has demonstrated that the ARL value (Average Run Length) for the T^2 chart was superior to the others. But concerning slow drifts, the CUSUM chart remains better.

In this paper, we will show how a specific use of the T^2 statistic in Inertial Steering can help deal with correlated dimensions by computing T^2 as a variable for correcting tools.

2. Limits of the Inertial Process Steering

The objective of any production is to produce parts that conform to the requirements of the geometry established by CAD (Computer Aided Design). This requirement is materialized by a digital target on which is added an acceptable level of variability (tolerances). The Inertial Process Steering approach is a tool that is able to reconcile the real workpiece to its digital model through the measured dimensions of the workpiece. Inertial tolerance is the quality indicator of the part in the Inertial Process Steering approach. The inertial tolerance of a surface is calculated by Equation (1) below.

2.1. Principe of Inertial Steering

For the manufacturing, we used the following materials:

In this section, the general implementation of the inertial steering approach will be exposed. The deviation between a measured dimension and its standard value is considered. The objective of the inertial steering approach is to have minimum deviation after correction, following the general paradigm in SPC.

Frequently, multiple deviations have to be treated during process control. So, the exercise consists of minimizing a set of deviations. If these deviations were to be independent, an n-dimensional problem could be interpreted as a one-dimensional problem. Otherwise, any attempt to reduce a multidimensional problem results in a loss of information. Less information is considered, then the results are less accurate. When many dimensions are related to the same surface, the tool correcting this surface will influence these dimensions at the same time. This is a typical case of multiple correlated dimensions, which often occurs in process control. The deviations cannot be controlled individually and must be viewed as a set. Mathematically, a set of variables can take the form of a vector. Here, it will be a deviation vector.

The inertial steering methodology can be summarized as the search for the nullification of deviation vectors whose terms are correlated values. Presently, the inertial steering manages to keep the multidimensional aspect of the situation until the computation of the corrections to steer the machine. But it is dropped when the question about filtering arises. The following sections in this paper will develop key components of Inertial to better understand how they can be used to improve filtering.

The aim of inertial steering (cf. Pillet *et al.* [25]) is to minimize the inertia, *i.e.*, the average of squared deviations between the target position of the theoretical surface and the actual position of the measured surface. The following Equation (1) gives the inertial tolerance of the surface.

$$I = \sqrt{\sigma^2 + (X - \tau)^2} \tag{1}$$

with

X: Measured dimension;

σ: Standard deviation of the measurements;

τ: Target dimension (Theoretical Exact Dimension);

I: Inertial tolerance (Inertial tolerances are defined by the French standard XP E04-008 (2009) in an experimental version).

2.1.1. Presentation of the Example for the Study of the Article

The objective of this example is to illustrate the implementation of the Inertial Steering approach for monitoring a multivariate process. **Figure 1** below gives the drawing of the mechanical part with target dimensions established in the design. The dimensions are represented by letters for ease of explanation. We develop our ideas from the following unidirectional (X axis, see **Figure 1**) example to facilitate understanding, but the principle remains the same for 2D or 3D parts with complex shapes. This paper will not detail what inertia stands for in manufacturing process control; for this part, one can refer to Pillet (2010).

The process deals with five manufacturing dimensions on a revolving part, illustrated by **Figure 1**. These dimensions are obtained using four tools (**Figure 2**).

We associated tool offsets with each cutting tool to correct its position. The position of the mechanical stop will be fixed to avoid all tool offset moves. The stop will therefore be a reference for adjusting the tools. However, if its position is changed over time due to random effects of the machine tool, other tools will be readjusted to compensate for the deviation. Figure 2 gives the manufacturing process of the workpiece.

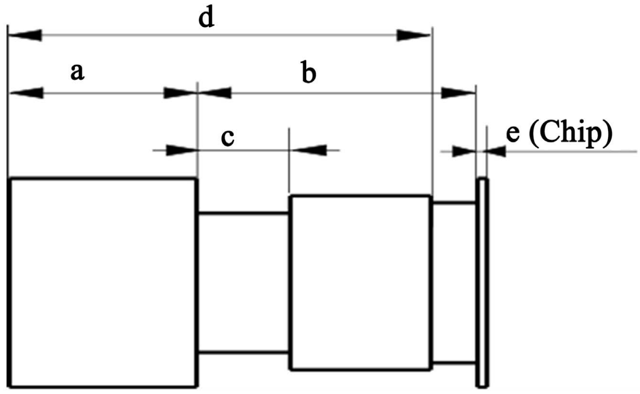

Figure 1. Definition drawing of the workpiece.

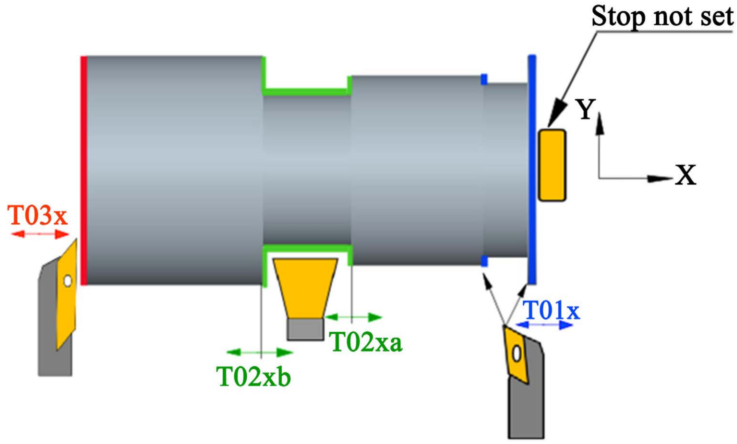

Figure 2. Manufacturing process of the workpiece.

2.1.2. Incidence Matrix

The principle of inertial steering is to connect the vector (C) of the settings parameters available on the machine (mainly tools offset) to the vector (ξ) of deviations of the measured dimension of the workpiece. We find a matrix relationship defined by Equation (2).

$$I = (\xi) = [a] \cdot (C) \tag{2}$$

$$(\xi) = \begin{pmatrix} D_a = \Delta a = X_a - \mu_a \\ D_b = \Delta b = X_b - \mu_b \\ D_c = \Delta c = X_c - \mu_c \\ D_d = \Delta d = X_d - \mu_d \\ D_e = \Delta e = X_e - \mu_e \end{pmatrix} \quad (C) = \begin{pmatrix} \text{TO1X} \\ \text{TO2Xa} \\ \text{TO2Xb} \\ \text{TO3X} \end{pmatrix}$$

With knowledge of one of the vectors, we can deduce the other from the incidence matrix.

Where [*a*] is the incidence matrix calculate from the following Equation (2) (Bourdet and Clement [28]), so, by hypothesis of small displacements in relation to the curvatures of the surfaces, makes it possible to linearize the deviation to the point P_j with respect to its target surface towards its displacement, according to the relation (Equation (3)):

$$a_{ij} = \left[\vec{T}_i \cdot \vec{n}_j + \left(\overrightarrow{P_0 P_j} \wedge \vec{R}_i \right) \vec{n}_j \right] \tag{3}$$

with:

a_{ij}: Incidence of tool offset *i* on dimension *j* (coefficients of the incidence matrix [*a*]);

T_i: Translation Tool offset *i*;

R_i: Rotation Tool offset *i*;

P_0: Origin applying rotation;

n_j: Components of the vector normal to the target surface.

In the case of this example $n_j = (1, 0, 0)$ or $n_j = (-1, 0, 0)$. This depends on the direction of displacement of the tool along the outer material. We do not make a rotation in the case of this example, where $R_i = 0$.

These dimensions are to be adjusted by acting only in translation on the tool offsets T01x, T02xa, T02xb, and T03x. The tool T02x will be adjusted by acting on two tool offsets (T02xa and T02xb) associated with each face that it machines on the workpiece.

$$a_{ij} = [\text{T01x} + \text{T02xa} + \text{T02xb} + \text{T03x}] \cdot nx$$

Table 1 gives the incidence matrix.

Table 1. Incidence matrix [*a*]

	TO1X	TO2Xa	TO2Xb	TO3X
a	0	0	1	−1
b	1	0	−1	0
c	0	1	−1	0
d	1	0	0	−1
e	−1	0	0	0

It is not the relation needed to solve a control problem, but it will serve as a stepping stone for the rest of the methodology, and it is easier to determine the arguments of this matrix than its effective counterpart.

2.1.3. Steering Matrix

The steering problem is to find (*D*), which is the possible best fit for the *X* Tools offsets. In the inertial steering approach, the best fit is obtained by minimizing the inertia. In Equation (1), the matrix[*a*] is generally not a square matrix then we used the well know Gauss pseudo inverse to calculate the best fit.

$$(X) = [a^*] \cdot (\xi) \tag{4}$$

where $[a^*] = ([a]^T \cdot [a])^{-1} \cdot [a]^T$.

It is the relation resulting in D knowing X that is to be used in the process control. This Steering Matrix is directly computed from the Incidence Matrix as its inverse or pseudo inverse $[a^*]$ for the non-square matrices (it will not work if the number of correctors is superior to the number of dimensions). Table 2 gives the steering matrix.

Table 2. Steering matrix.

	a	b	c	d	e (chip)
TO1X	0	0	0	0	−1
TO2Xa	0.33	−0.67	1	−0.33	−1
TO2Xb	0.33	−0.67	0	−0.33	−1
TO3X	−0.33	−0.33	0	−0.67	−1

2.1.4. Process Control

This Steering Matrix multiplied by the deviation vector (affiliated with measurements) results in the control vector (affiliated with corrections) (see Equation (5)). By applying the control vector to the commands, the resulting steering tends to minimize the deviation vector. So, the entire set of measurements is optimized while considering the correlations between the measurements.

$$\begin{pmatrix} \text{TO1X} \\ \text{TO2Xa} \\ \text{TO2Xb} \\ \text{TO3X} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0.33 & -0.67 & 1 & -0.33 & -1 \\ 0.33 & -0.67 & 0 & -0.33 & -1 \\ -0.33 & -0.33 & 0 & -0.67 & -1 \end{pmatrix} \begin{pmatrix} X_a - \mu_a \\ X_b - \mu_b \\ X_c - \mu_c \\ X_d - \mu_d \\ X_e - \mu_e \end{pmatrix} \tag{5}$$

That way, there is no risk of offsetting a dimension while adjusting for the deviation of another.

2.2. Out of Control Situations

Knowing how much correction leads to an optimal setting and knowing when such a correction is required are two different questions. The former has already been answered through the previous chapters. The latter is a call to the notion of filtering and is widely covered in unidimensional cases by the different charts used in Statistical Process Control (Brunschwig, 1992).

2.3. Filtering the Adjustments

When measuring a manufactured part, there will always be small deviations from the target values. They could be due to random dispersion phenomena. It is possible to adjust the machine for every shift in the value. Although this practice is very efficient for signaling problems with the tool's adjustment, its performance is lacking in the case of a small or absent drift. Furthermore, over-adjusting the tools when it is not necessary for the process quality can lead to an increase in dispersion by adding variation from tool settings to the process's natural variation. It is also time-consuming for the industry.

Filtering the data helps to run such an adjustment only when the presumption of a drifted or worn-out tool is strong. To keep a process under such control, the control charts are widely put to use according to their relative strengths and weaknesses.

2.4. Limit of the Mono-Dimensional Strategy of Filtering

There exist various strategies for filtering data. But presently, in Inertial Process Steering, they always lead to a one-dimensional point of view, even after the computation of the problem as multidimensional. And so, information is lost.

2.4.1. The Correlation Structures Are Not Considered

When looking at the practice of Inertial Process Steering, the correlations between dimensional values of the same set have yet to be exploited. Indeed, when a corrector is applied to different surfaces of the product, it is fair to assume that its wear or drift will appear as correlations between the variables. By not accounting for this information, the process may go on until the shift of a single value triggers an intervention, "as the charts say so". It is thought that a multivariate post-processing data analysis could ring an alarm sooner by revealing a cross-trend of the variables.

2.4.2. The Correlation Structures Revealed by the Incidence Matrix

The correlation structures between characteristics are highlighted by the Incidence Matrix given in Table 3.

Table 3. Incidence matrix and correlations (or no) between characteristics.

	TO1X	TO2Xa	TO2Xb	TO3X
a	0	0	1	−1
b	1	0	−1	0
c	0	1	−1	0
d	1	0	0	−1
e	−1	0	0	0
Correlation	Yes	No	Yes	Yes

In this table, a column with more than one non-null value displays a correlation structure because T^2 allows taking into account the correlations between characteristics that are non-null in the column. If we consider the first column, characteristics b, d, and e are corralled.

3. Multivariate Detection by Statistical Distance T^2

3.1. Computation of T^2

Mason and Young [15] used Hotelling's T^2 statistic to identify outlying values of correlated variables. With some modifications, it could be used to detect the correlations from a set of values.

The matrix form for the T^2 is as follows (Equations (6) and (7)):

$$T^2 = (\boldsymbol{X} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{X} - \boldsymbol{\mu}) \quad (6)$$

$$T^2 = \boldsymbol{D}^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \boldsymbol{D} \quad (7)$$

with $\boldsymbol{D}$: the deviation vector.

$\boldsymbol{\Sigma}^{-1}$: the inverse of the covariance matrix $\boldsymbol{\Sigma}$ for the variables, where:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & & & \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & & \\ \vdots & & \ddots & \\ \rho_{1p}\sigma_1\sigma_p & \cdots & \cdots & \sigma_p^2 \end{pmatrix}$$

with ρ_{ij} : the correlation factor between the variable i and j.

σ_i : the standard deviation of the variable i, if this value comes from an estimation S_i, then switch σ_i with $\frac{S_i}{\sqrt{m}}$ (m is the size of the batches).

Setting a limit is required to use T^2 as a multivariate filtering tool. According to Mason and Young [15], three statistics are selected according to the situation:

- if the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are known (Equation (8)):

$$limit = \chi^2_{(p)} \quad (8)$$

- if the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown and the estimators $\bar{\boldsymbol{X}}$ and S are used:

$$T^2 = (\boldsymbol{X} - \bar{\boldsymbol{X}})^{\mathrm{T}} \boldsymbol{S}^{-1} (\boldsymbol{X} - \bar{\boldsymbol{X}}) \quad (9)$$

  - if X is independent of $\bar{\boldsymbol{X}}$ and S:

$$limit = \left[\frac{p(n+1)(n-1)}{n(n-p)}\right] F_{(p,n-p)} \quad (10)$$

  - if X is included in the computation of $\bar{\boldsymbol{X}}$ and S:

$$limit = \left[\frac{(n-1)^2}{n}\right] B_{\left(\frac{p}{2},\frac{n-p-1}{2}\right)} \quad (11)$$

with:

- F: Law of F-Snedecor;
- B: Beta law;
- p: The number of dimensions considered;
- n: The number of observations'

The Incidence Matrix already charts the correlations between dimensions and sets them according to the involved tool. It is the statistical distance of a set of potentially correlated dimensions that is calculated with T^2. The correlation in one set would be the problem of one tool. The formula (1) has for parameters dimensions (or their deviations) and their standard deviations (included in the covariance matrix). But in the end, T^2 will reveal the status of the tool. One T^2 can be calculated for each tool (when a correlation is possible).

3.2. T^2 for Adjustment Detections in Inertial Process Steering

Mason and Young [15] proposed to use the T^2 statistic as a way to detect outlying values in a set of correlated variables. Here, the point is the detection of a correlation between monitored variables. If the tool is correctly adjusted, then only the random deviations will disturb the production. Over the monitored production, there should be no correlation appearing. Instead of computing T^2 as in (2), T^2 is calculated while "forcing" the correlation terms in the covariance matrix to be null. The formula is the same, but with the following covariance matrix:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & & & \\ 0 & \sigma_2^2 & & \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & \sigma_p^2 \end{pmatrix}$$

The data needed to calculate T^2, such as deviations and standard deviations, are known, and T^2 can be easily calculated. Compared to the standard T^2 calculation, applying Hotelling's T^2 in the context of inertial steering involves designing one T^2 chart per column of the incidence matrix, and therefore as many T^2 charts as there are possible characteristics. By setting the correlation terms in the covariance matrix to zero, only the interdependence of the characteristics is considered, acting solely on a tool offset.

3.3. Simulation

To evaluate the potential gain of a T^2 chart in Inertial Process Steering, in comparison with one-dimensional filtering strategies, a simulated production using the Mersenne Twister algorithm has been run following this protocol:

The studied part is the same as in Section 2.

- The deviation of 3 related measurements (a, b, and c linked by the tool TO2Xb, see **Table 3**), with a standard deviation of 1, will be registered over a 10^6-parts population. We suppose knowing σ^2 for each measurement, a, b, and c.
- The data are then computed into their T^2 corresponding value.
- The χ^2 limit has been set for 3 degrees of freedom. If $T^2 > \chi^2$, then the produced part counts as discarded; the ARL for a T^2 chart can be quantified.
- The procedure is repeated with the addition of a perturbation ranging from 0 to 3σ with a 0.2σ step.

The "Shewhart control chart" presented by Shewhart [2] is designed to judge the importance of variability. It is considered that the process is under control if it is subjected only to random variations (the sum of small variations, which, with the central limit theorem, leads to a distribution law of Gauss). In this case, it is not necessary to intervene in the process. The zone of random variation is determined by the "control limits". These control limits are traditionally set within $\pm 3\sigma$ of the target value, so we take a risk $\alpha = 2 \times 0.135\% = 0.27\%$ of disrupting a well-centered process. The control limits are calculated for $n = 1$, by Equation (12):

$$\begin{cases} \text{LCI} = X - u_\alpha \sigma \\ \text{LCS} = X + u_\alpha \sigma \end{cases} \tag{12}$$

with $u_\alpha = 3$, standard Gauss variable for the risk $\alpha = 0.27\%$.

LCI: Lower Control Limit of dimension;

LCS: Upper Control Limit of dimension.

The calculation of ARL depends on the specific type of control chart and the process being monitored. Here's a breakdown of common scenarios:

For Shewhart Charts (and other charts where a single point exceeding limits triggers an alarm):

The ARL is the reciprocal of the probability of a point exceeding the control limits:

ARL = $1/p$, where "p" is the probability of a point exceeding the control limits.

4. Average Run Length (ARL) Comparison

These results are to be compared with the corresponding ARL following the strategies:

- If 1 out of the 3 measurements is outside Shewhart's limits, the part is discarded.
- If the one measurement tagged as charted is outside Shewhart's limits, the part is discarded (The measurements have the same characteristics here, so the choice is irrelevant).

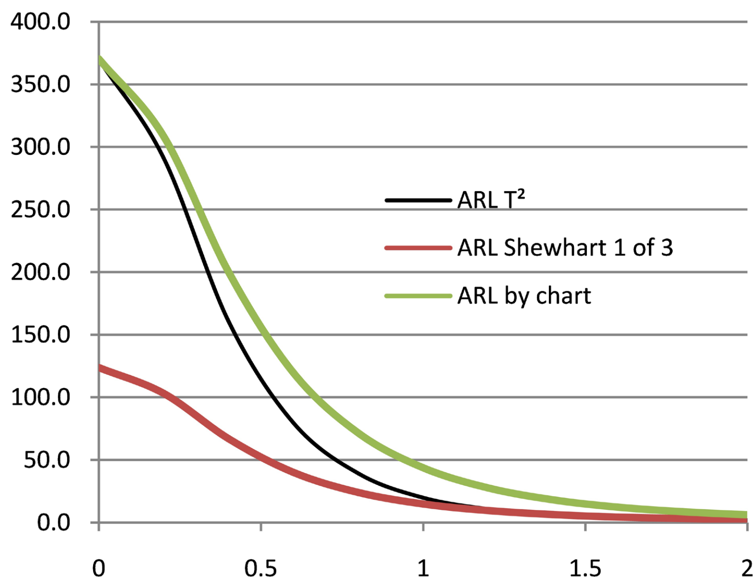

Figure 3. False alarm detection.

The ARL0 (no perturbation) is a false alarm indication. The higher the ARL0 is, the fewer the number of false alarms is. On the contrary, when there is a perturbation, a lower ARL signifies a better detection capability (**Figure 3**).

According to the graph realized with the simulation, both of the one-dimensional approaches have their advantages and disadvantages. The one-by-one strategy sets fewer false alarms on, whereas the "one chart" strategy has a better detection rate for perturbation.

The T^2 chart presents the advantages of both. Moreover, with a zoom (**Figure 4**), the T^2 detection rate is revealed as even better than the one-by-one chart when the perturbation exceeds about 1.5 σ.

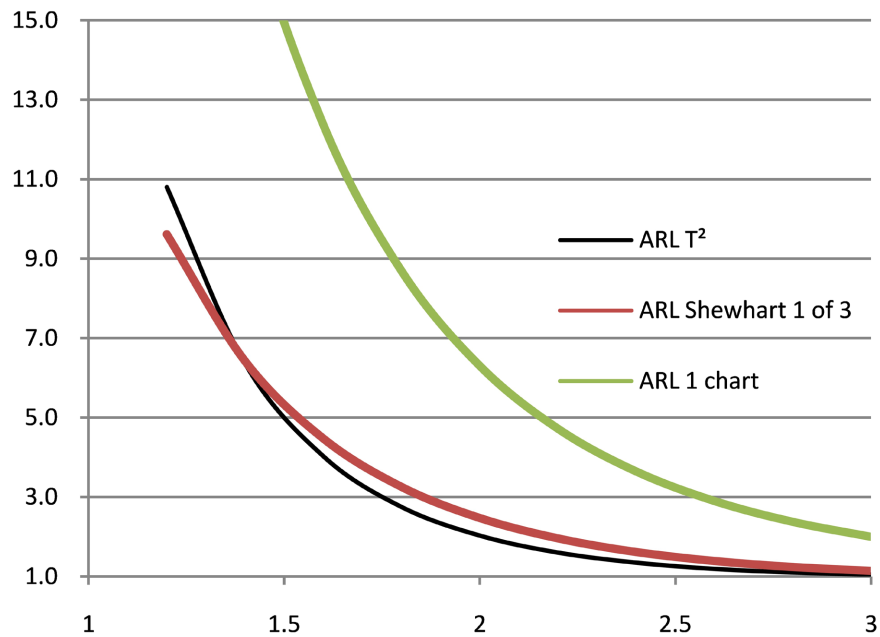

Figure 4. Tool drift detection performance profile using charts measured by ARL.

With a lower false alarm and higher perturbation detection rates, this approach appears as an improvement in the context of Inertial Process Steering filtering.

5. Conclusions

Today, the common practice in industrial manufacturing is to consider the product characteristics as independent. The Inertial Process Steering method is based on the establishment of a relation between a deviation vector and a correction vector. By nature, it is a multidimensional relation. The Steering Matrix computed from the Incidence matrix addresses both the multidimensional aspect of the problem and the plausible correlation in the data imputed by a problem of tool adjustment.

The use of Hotelling's T^2 for CNC machine tools allowed for reduced machine setting time, particularly in companies that manufacture mechanical parts. For example, in watchmaking companies where the watch plate contains no fewer than eighty (80) interdependent dimensions and whose setup takes approximately two (2) weeks, this will significantly reduce setup time, essentially reducing it to the time required to establish the incidence matrix. This approach also reduces scrap rates, improves tool life management, and simultaneously minimizes machine downtime.

This paper has shown, via a simulation, how Hoteling's T^2 statistic can help improve detection in Inertial Process Steering. First, by keeping a multidimensional point of view, the gain of information compared to one-dimensional Shewhart charting leads to a better efficiency at filtering the manufactured production. Second, when a process is out of control due to a problem with a tool adjustment, filtering the correction with a chart focusing on the tool instead of the dimensions should be more legitimate. Third, the computation of T^2 can be easily implemented in the already existing software. Indeed, all the data needed to compute T^2 are already input (the deviations and the standard deviations).

The performed simulation compared the efficiency between the standard Shewhart and T^2 charting. A narrower analysis could be realized by comparing this approach with CUSUM and EWMA charts.

This paper deals with the new perspective coming from the study of correlations in IPC, not with the optimal adjustment while running the method. This explains why the χ^2 statistic, for the purpose of convenience and common use, has been chosen.

One of the major difficulties in the case of vectorial systems is the complexity of the computation, which increases exponentially with the size of the vectors. Hence, through the T^2 statistic, reducing the multivariate situation to a scalar value while accounting for the values inputted in the deviation vector seems promising.

Using Hoteling's T^2 for CNC machine tool control reduces machine setup time, particularly in mechanical parts manufacturing plants. For example, in watchmaking plants where the watch plate contains no fewer than eighty (80) interdependent dimensions and whose setup takes approximately two (2) weeks, this will significantly reduce setup time, essentially reducing it to the time required to establish the incidence matrix. This approach also reduces scrap rates, improves tool life management, and simultaneously minimizes machine downtime.

The study was presented on a specific case of a 1D part, where the method considers the columnar information of the incidence matrix. However, it does not consider both the row and columnar information of the incidence matrix. The approach is limited in its ability to take into account both the correlations between tool offsets and the geometric characteristics of the part. This limits its generalization to more complex parts where several cutting tools act simultaneously on one or more geometric characteristics of the part.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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