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Enhancement of Polarized Optomechanical Entanglement in Lossy Cavity

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Abstract

We investigate quantum bipartite entanglement between mechanical and polarized optical modes in a lossy optomechanical cavity driven by a polarized electromagnetic field. A key feature of our scheme is that the polarization of the driving field permits phase-dependent modulation of the components of the optomechanical coupling strength. We demonstrate that this modulation offers a precise control mechanism, enabling the generation of optomechanical entanglement primarily oriented along a desired direction (vertical or horizontal). The intracavity loss acts as a resource, leading to an enhancement of the generated polarized optomechanical entanglements. Moreover, this loss is shown to be crucial for significantly increasing the robustness and stability of the generated entanglement against thermal noise. This work paves the way for advanced quantum state engineering and robust quantum information processing protocols within optomechanical systems.

Keywords

Bipartite Entanglement, Loss, Polarization, Optomechanics

1. Introduction

Bipartite entanglement is the non-classical correlation between two separate quantum systems, causing them to act as a single system. It is an essential resource that unlocks the power of all modern quantum technologies and is absolutely vital for major domains including quantum computation [1] [2], quantum communication [3]-[5], quantum metrology [6]-[9], and quantum information processing [10]-[12]. Generating and controlling entangled states demands versatile and highly convenient platforms that possess the capability to coherently manipulate light with high precision. Optomechanical systems [13] precisely address this fundamental requirement by offering a unique hybrid interface. They are systems that take advantage of the radiation pressure force to manipulate the interaction between the light field and the mechanical resonator. This makes optomechanics an ideal setting for quantum state engineering and the study of the bipartite entanglement.

Early works on bipartite entanglement in optomechanics primarily focused on the single-cavity architecture [14] [15], relying on maximizing the inherent radiation pressure interaction between the optical and mechanical modes. Shifting from basic setups, the field quickly moved to more complex methods to generate and enhance bipartite entanglement. This included exploiting inherent system nonlinearities [16]-[19], utilizing parametric interactions [20]-[22] for stronger effects, and applying various forms of modulation [23]-[25]. Although these works successfully demonstrated the possibility of enhancing optomechanical entanglement, they were inherently limited by the constraints of the single-cavity model. Consequently, the double optomechanical cavities and other models became a dominant focus, offering a richer environment for studying bipartite entanglement. New approaches have also emerged. Among these approaches, we have approaches based on nonlinear effects [26] [27], dark mode engineering [28], synthetic gauge [29] [30], and molecular systems [31] [32].

A recent focus has been on lossy cavities, based on the surprising idea that system loss can act as a quantum resource. The engineering of loss mechanisms can be physically realized via external interferometric techniques or through reservoir engineering, thereby enabling precise control over the cavity dissipation. This controlled dissipation is a crucial tool because it helps create and stabilize nonclassical correlations [33] [34]. By using loss strategically, the system naturally finds a steady, protected quantum state that is both stronger against environmental noise and easier to achieve experimentally. Combining the loss-engineering approach with another one, such as a polarized driven field [35], could offer a powerful synergy to achieve enhanced and highly robust bipartite entanglement.

In this paper, we aim to enhance optomechanical entanglement. Our benchmark system is a lossy cavity consisting of a mechanical resonator driven by a polarized electromagnetic field, allowing the modulation and control of the polarized optomechanical coupling strengths via the phase ϕ. The introduction of the loss in the cavity induces a significant enhancement of the polarized optomechanical entanglements, while the phase is crucial to modulate these entanglements in one direction or another. The presence of the loss actively makes the generated polarized optomechanical entanglements substantially more robust against thermal noise compared to the loss-free scenario. Our work provides a way to enhance polarized optomechanical entanglements with a simple modulated optomechanical system.

The rest of the paper is organized as follows, Section 2 presents the model and derives the dynamical equations. The enhancement of the polarized optomechanical entanglements is presented in Section 3. Our work is concluded in Section 4.

2. Model and Dynamical Equations

Our benchmark optomechanical system consists of a lossy cavity driven by a linearly polarized electromagnetic field. This setup is represented in **Figure 1**.

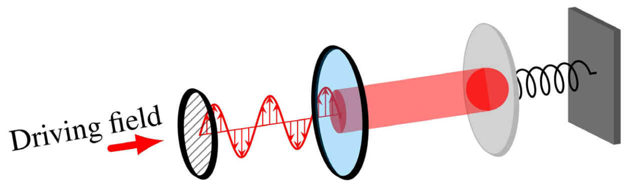

Figure 1. Sketch of our benchmark system.

In the frame rotating with respect to the frequency $\Omega_l\left(a_{\updownarrow}^{\dagger}a_{\updownarrow}+a_{\leftrightarrow}^{\dagger}a_{\leftrightarrow}\right)$, our system is described by the following Hamiltonian ($\hbar=1$),

$$H=H_0+H_{OM}+H_{loss}+H_{Drive}, \tag{1}$$

where

$$H_0=-\sum_{p=\updownarrow,\leftrightarrow}\Delta_p a_p^{\dagger}a_p+\omega_m b^{\dagger}b, \tag{2}$$

$$H_{\text{OM}}=-\sum_{p=\updownarrow,\leftrightarrow}g_p a_p^{\dagger}a_p\left(b+b^{\dagger}\right), \tag{3}$$

$$H_{\text{loss}}=-\sum_{p=\updownarrow,\leftrightarrow}if_s a_p^{\dagger}a_p, \tag{4}$$

$$H_{\text{drive}}=\sum_{p=\updownarrow,\leftrightarrow}iE_p\left(a_p^{\dagger}+a_p\right). \tag{5}$$

In the above Hamiltonian, H_0, H_{OM}, H_{loss}, and H_{drive} represent respectively the free Hamiltonian, the optomechanical interaction, the loss introduced in the cavity and the polarized driving fields. The terms a_p ($a_p^{\dagger}$) and b ($b^{\dagger}$) are the annihilation (creation) operators of the p-polarized optical mode (with frequency ω_p) and the mechanical mode (with frequency ω_m). The optomechanical couplings between the p-polarized driving field and the mechanical resonator are captured by g_p. The loss induced in the cavity is denoted by the term f_s, which is the intracavity loss rate. The amplitude of the driving field is represented by E defined as $|E|^2=|E_{\updownarrow}|^2+|E_{\leftrightarrow}|^2$, where $E_{\updownarrow}=E\cos\phi$ and $E_{\leftrightarrow}=E\sin\phi$ are the projections of the driven field E onto the vertical and horizontal modes, respectively. The polarization enables the manipulation of the light through the phase ϕ representing the angle between the two involved directions. The frequency detuning is defined by $\Delta_p=\omega_l-\omega_{c_p}$.

Using the Heisenberg equation applied to the previous Hamiltonian, we obtain the full set of dynamical equations,

$$\begin{cases}\dot{a}_p=\left(i\left(\Delta_p+g\left(b+b^{\dagger}\right)\right)-\kappa-f_s\right)a_p+E_p+\sqrt{2\kappa}a_p^{in}\\ \dot{b}=-\left(\dfrac{\gamma_m}{2}+i\omega_m\right)b+i\sum_{p=\updownarrow,\leftrightarrow}ga_p^{\dagger}a_p+\sqrt{2\gamma_m}b^{in}\end{cases}, \tag{6}$$

where κ and γ_m are the dissipations related to each mode of our system. We assume that the two polarized modes are coupled to the mechanical resonator at the same rate ($g = g_p$). The quantities a_p^{in} and b_m^{in} stand for the thermal quantum noise operators. These operators are characterized by zero mean value, and the following correlation functions,

$$\left\langle a_p^{in}(t) a_p^{in\dagger}(t') \right\rangle = \delta(t-t'), \quad \left\langle a_p^{in\dagger}(t) a_p^{in}(t') \right\rangle = 0, \tag{7}$$

$$\left\langle b^{in}(t) b^{in^\dagger}(t') \right\rangle = (n_{th}+1)\delta(t-t'), \tag{8}$$

$$\left\langle b^{in^\dagger}(t) b^{in}(t') \right\rangle = n_{th}\delta(t-t'), \tag{9}$$

where n_{th} is the thermal phonon occupation number of the mechanical resonator b that is defined as $n_{th} = \left[\exp\left(\frac{\hbar\omega_m}{k_b T}\right) - 1\right]^{-1}$, with k_b the Boltzmann constant and T the bath temperature.

To investigate the quantum bipartite entanglement in our system, we need to capture quantum fluctuations around the mean values of the operators involved in our Quantum Langevin Equations (QLEs) displayed in Equation (6). To implement this, we use the standard linearization procedure, which involves splitting each quantum operator into its dominant classical mean value and a smaller quantum fluctuation. By doing this, we obtain a separate set of equations for the mean values,

$$\begin{cases} \dot{\alpha}_p = \left(i\tilde{\Delta}_p - \kappa - f_s\right)\alpha_p + E_p \\ \dot{\beta} = -\left(\frac{\gamma_m}{2} + i\omega_m\right)\beta + ig\sum_{p=\updownarrow,\leftrightarrow}\left|\alpha_p\right|^2 \end{cases}, \tag{10}$$

and a set of the fluctuation dynamical equations,

$$\begin{cases} \delta\dot{a}_p = \left(i\tilde{\Delta}_p - \kappa - f_s\right)\delta a_p + iG_p\left(\delta b + \delta b^\dagger\right) + \sqrt{2\kappa}a_p^{in} \\ \delta\dot{b} = -\left(\frac{\gamma_m}{2} + i\omega_m\right)\delta b + i\sum_{p=\updownarrow,\leftrightarrow}\left(G_p^*\delta a_p + G_p\delta a_p^\dagger\right) + \sqrt{2\gamma_m}b^{in} \end{cases}, \tag{11}$$

where $\tilde{\Delta}_p = \Delta_p + g\left(\beta + \beta^\dagger\right)$ is the effective detuning and $G_p = g\alpha_p$ represents the p-polarized optomechanical coupling strength corresponding to the steady-state value of the p-polarized optical mode,

$$\alpha_p = \frac{-E_p}{i\tilde{\Delta}_p - \kappa - f_s}. \tag{12}$$

This p-polarized optomechanical coupling strength is a component of the total optomechanical coupling strength G ($G_\updownarrow = G\cos\phi$ or $G_\leftrightarrow = G\sin\phi$), which is precisely modulated through the phase ϕ, enabling us to selectively steer and align the entire quantum interaction toward either the horizontal or vertical polarization direction.

Based on the above fluctuation equations (Equation (11)), we define the following position and momentum quadrature operators, $\delta X_{\mathcal{O}} = \frac{\delta\mathcal{O}^\dagger + \delta\mathcal{O}}{\sqrt{2}}$,

$\delta Y_{\mathcal{O}} = i\frac{\delta\mathcal{O}^{\dagger} - \delta\mathcal{O}}{\sqrt{2}}$, where $\mathcal{O} \equiv a_p, b$, together with their related noise quadratures $\delta X_{\mathcal{O}}^{in} = \frac{\mathcal{O}^{\dagger in} + \mathcal{O}^{in}}{\sqrt{2}}$, $\delta Y_{\mathcal{O}}^{in} = i\frac{\mathcal{O}^{\dagger in} - \mathcal{O}^{in}}{\sqrt{2}}$. This procedure allows us to rewrite the fluctuation dynamical equations in a compact form,

$$\dot{u} = \mathrm{M}u + z^{in}, \tag{13}$$

where $u = \left(\delta X_{a_{\updownarrow}}, \delta Y_{a_{\updownarrow}}, \delta X_{a_{\leftrightarrow}}, \delta Y_{a_{\leftrightarrow}}, \delta X_b, \delta Y_b\right)^{\top}$,

$z^{in} = \left(\sqrt{2\kappa}X_{a_{\updownarrow}}^{in}, \sqrt{2\kappa}Y_{a_{\updownarrow}}^{in}, \sqrt{2\kappa}X_{a_{\leftrightarrow}}^{in}, \sqrt{2\kappa}Y_{a_{\leftrightarrow}}^{in}, \sqrt{2\gamma_m}X_b^{in}, \sqrt{2\gamma_m}Y_b^{in}\right)^{\top}$ and the matrix M is given by,

$$\begin{pmatrix} -(\kappa + f_s) & -\tilde{\Delta}_{\updownarrow} & 0 & 0 & 0 & 0 \\ \tilde{\Delta}_{\updownarrow} & -(\kappa + f_s) & 0 & 0 & 2G_{\updownarrow} & 0 \\ 0 & 0 & -(\kappa + f_s) & -\tilde{\Delta}_{\leftrightarrow} & 0 & 0 \\ 0 & 0 & \tilde{\Delta}_{\leftrightarrow} & -(\kappa + f_s) & 2G_{\leftrightarrow} & 0 \\ 0 & 0 & 0 & 0 & -\gamma_m & \omega_m \\ 2G_{\updownarrow} & 0 & 2G_{\leftrightarrow} & 0 & -\omega_m & -\gamma_m \end{pmatrix}. \tag{14}$$

In these expressions, the polarized optomechanical coupling strengths G_p have been assumed to be real for simplicity.

3. Polarized Optomechanical Entanglements

To quantify the optomechanical entanglement in our system, we need to evaluate the covariance matrix whose elements are defined as $V_{ij} = \frac{\langle u_i u_j + u_j u_i \rangle}{2}$, which also satisfy the motional equation,

$$\dot{V} = \mathrm{M}V + V\mathrm{M}^{\mathrm{T}} + D, \tag{15}$$

where D is the diagonal diffusion matrix expressed as
$D = \mathrm{Diag}\left[\kappa, \kappa, \kappa, \kappa, \gamma_m(2n_{th}+1), \gamma_m(2n_{th}+1)\right]$. To carry out the entanglement analysis, the matrix M must fulfills the Routh-Hurwitz stability criterion, *i.e.*, all its eigenvalues should have negative real parts [36]. Violating these criteria results in an unstable system, irrelevant to the investigation of steady-state entanglement. We have checked that our used parameters satisfy this stability condition. To capture the steady-state entanglement, we enforce the condition that all dynamical variables in Equation (15) are time-independent. This requirement simplifies the full differential equation into a Lyapunov equation,

$$\mathrm{M}V + V\mathrm{M}^{\top} = -D. \tag{16}$$

The elements V_{ij} of the covariance matrix can be numerically solved from the previous equation, which allows us to determine the complete, general form of the steady-state covariance matrix V,

$$V = \begin{pmatrix} V_{\updownarrow} & V_{\updownarrow,\leftrightarrow} & V_{\updownarrow,m} \\ V_{\updownarrow,\leftrightarrow}^{\top} & V_{\leftrightarrow} & V_{\leftrightarrow,m} \\ V_{\updownarrow,m}^{\top} & V_{\leftrightarrow,m}^{\top} & V_m \end{pmatrix}, \tag{17}$$

where V_i and V_{ij} are blocs of 2×2 matrices (with $i, j \equiv \updownarrow, \leftrightarrow, m$). The diagonal blocks V_i correspond to the polarized optical mode ($i = p = \updownarrow, \leftrightarrow$) and to the mechanical mode ($i = m$), respectively. The off-diagonal blocks capture the correlations between the different subsystems. For instance, $V_{\updownarrow,\leftrightarrow}$ stands for the correlations between the two polarized driving fields, $V_{\updownarrow,m}$ represents the correlations between the vertically polarized driving field and the mechanical resonator, while $V_{\leftrightarrow,m}$ describes the correlations between the horizontal polarized driving field and the mechanical resonator. In our investigation, we will focus on the entanglement between the p-polarized driving field and the mechanical resonator. To evaluate such bipartite entanglement, we use the logarithmic negativity (E_N), which is computed by tracing out the non-necessary third mode. This logarithmic negativity E_N is defined as,

$$E_N = \max\left[0, -\ln\left(2\nu^-\right)\right], \tag{18}$$

where $\nu^- = 2^{-1/2}\left[\Delta_\chi - \sqrt{\Delta_\chi^2 - 4I_4}\right]^{1/2}$. The reduced covariance matrix χ, capturing the dynamics of our targeted optomechanical subsystem is defined as,

$$\chi = \begin{pmatrix} V_p & V_{p,m} \\ V_{p,m}^\top & V_m \end{pmatrix}, \tag{19}$$

in such a way that $\Delta_\chi = I_1 + I_2 - 2I_3$, with the following symplectic invariants $I_1 = \det V_p$, $I_2 = \det V_m$, $I_3 = \det V_{p,m}$, and $I_4 = \det\chi$. In what follows, we assume that our system operates in the red-sideband regime ($\tilde{\Delta} = -\omega_m$), which is essential for high-fidelity quantum engineering, as it ensures that the dominant heating channels are effectively suppressed. This operating regime is crucial because it favors the beam-splitter interaction while mitigating the parametric amplification interaction. This facilitates a continuous and sustained transfer of energy from the mechanical resonator to the optical cavity and is consequently more suitable for the investigation of quantum correlations such as entanglement.

We begin our investigation by analyzing the optomechanical entanglement in terms of the optomechanical coupling strength ($G^2 = \left|G_\updownarrow\right|^2 + \left|G_\leftrightarrow\right|^2$) and the loss in **Figure 2**. By setting a zero angle between the polarized components, we established a key alignment where our results clearly demonstrate that the intracavity loss induces a strong enhancement in the steady-state optomechanical entanglements. Moreover, the entanglement generated between the vertically polarized optical mode and the mechanical mode (**Figure 2(a)**) is more pronounced than the horizontal counterpart. This outcome is a direct consequence of setting the phase to $\phi = 0$, which is precisely the condition required to annihilate the horizontal component of the optomechanical coupling strength. Consequently, the entire available interaction strength is concentrated and efficiently channeled into driving the entanglement between the vertically polarized optical mode and the mechanical mode.

To better visualize the influence of the loss on the entanglement, we extract the key dependencies from **Figure 2** and present them in **Figure 3**. **Figure 3** clearly demonstrates that the intracavity loss induces an enhancement of the steady-state

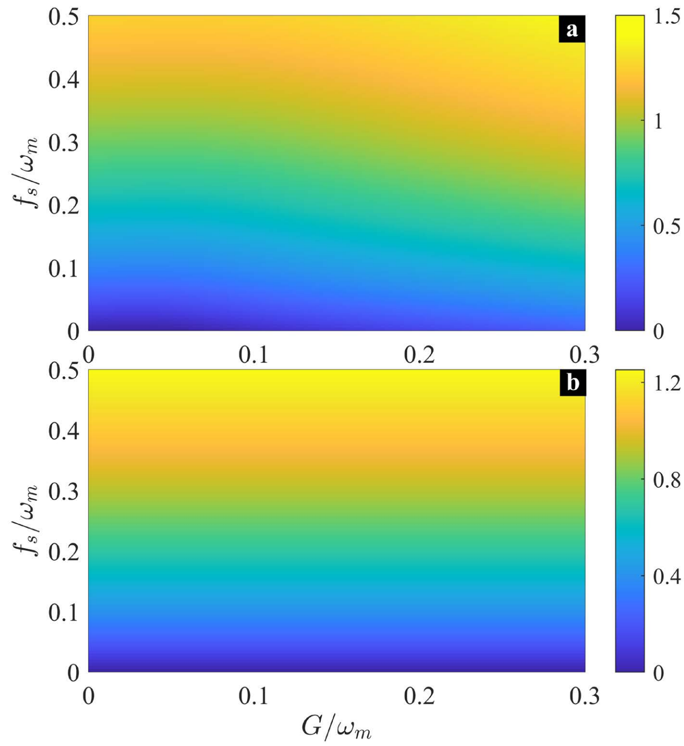

Figure 2. Bipartite entanglement between (a) the vertically polarized optical mode and the mechanical resonator, and (b) the horizontally polarized optical mode and the mechanical resonator versus the optomechanical coupling and the loss. Parameters used are $\Delta_p = -\omega_m$, $\kappa = 2\times10^{-1}\omega_m$, $\gamma_m = 10^{-5}\omega_m$, $\phi = 0$ and $n_{th} = 100$.

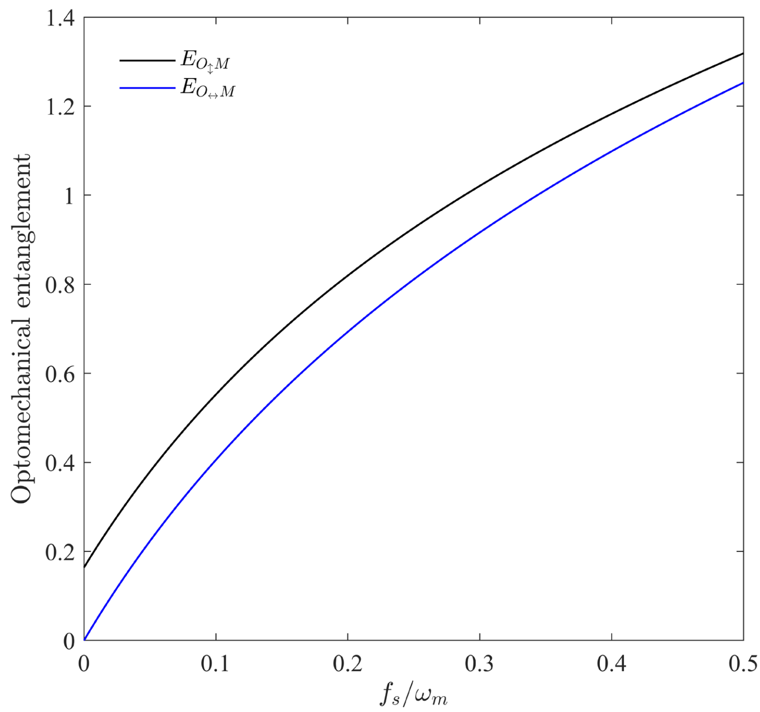

Figure 3. Optomechanical entanglements versus the loss for $G = 0.2\omega_m$. The other parameters are the same as those in **Figure 2**.

optomechanical entanglements. Regarding the vertically polarized optomechanical entanglement, it is possible to generate it even in the absence of loss, as this entanglement relies on the inherent polarized optomechanical coupling strength between these two modes. In this case, the introduction of loss acts purely as an enhancement factor. Conversely, for the entanglement between the horizontal polarized optical mode and the mechanical mode, the entanglement is induced solely by the cavity loss, since the chosen phase ($\phi = 0$) completely suppresses the direct coupling mechanism between these two specific modes.

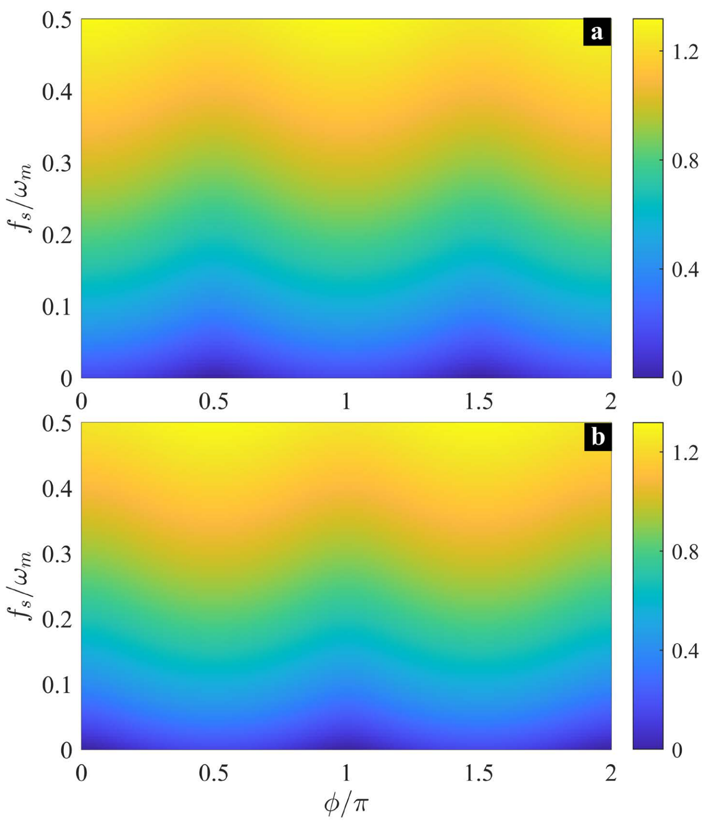

Figure 4. Bipartite entanglement between (a) the vertically polarized optical mode and the mechanical resonator, and (b) the horizontally polarized optical mode and the mechanical resonator versus the phase and the loss for $G = 0.2\omega_m$. The other parameters are the same as those in **Figure 2**.

As mentioned previously, the phase ϕ plays a crucial role in our system, as it modulates the p -polarized optomechanical coupling strength. This direct influence is clearly depicted in **Figure 4**. We observe that for a given optomechanical coupling strength ($G = 0.2\omega_m$), the entanglements are enhanced by the presence of loss, exhibiting pronounced peaks at specific phase values. Specifically, the peaks for the vertically polarized optomechanical entanglement (**Figure 4(a)**) are concentrated around $\phi = n\pi$, which corresponds precisely to the condition for maximal vertical optomechanical coupling strength. In contrast, the horizontal optomechanical entanglement (**Figure 4(b)**) experiences its maximum peaks around

$\phi = \frac{\pi}{2}(2n+1)$, aligning with the condition for maximal horizontal optomechanical coupling strength. These strong entanglements directly demonstrate the essential role of the phase in modulating the optomechanical coupling strength components, which in turn dictates the magnitude of the generated bipartite entanglements.

A more precise view of the phase influence is provided in **Figure 5**, which clearly depicts this modulatory effect. We can see that when the entire optomechanical coupling strength is concentrated on the vertical (horizontal) component, the corresponding vertical (horizontal) polarized entanglement reaches its peak, while the entanglement in the perpendicular direction is simultaneously driven to its dip. On the other hand, when the total optomechanical coupling strength is split across both components, both vertical and horizontal optomechanical entanglements are present, each with an amplitude directly proportional to its respective polarized optomechanical coupling strength. This crucial ability to manipulate the distribution of the polarized optomechanical coupling is the key factor enabling us to modulate and steer the bipartite entanglement toward our desired polarization direction.

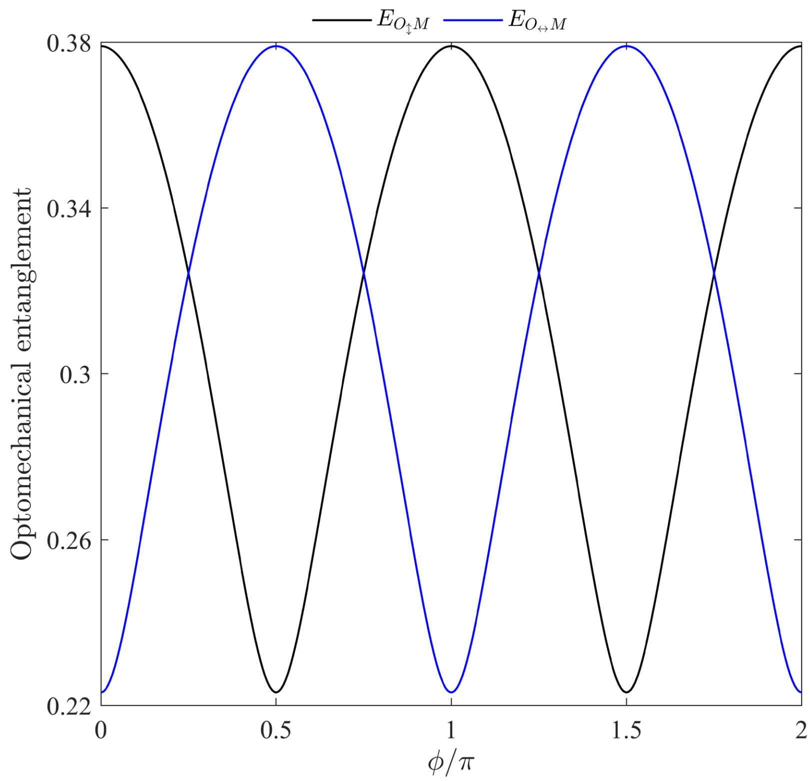

Figure 5. Optomechanical entanglements versus the phase for $G = 0.2\omega_m$ and $f_s = 5\times10^{-2}\omega_m$. The other parameters are the same as those in **Figure 2**.

Figure 6 illustrates the optomechanical entanglements versus the thermal noise. In the absence of the noise and a maximal vertically polarized optomechanical coupling strength (solid curves), the vertically polarized optomechanical entanglement shows a marginal resilience against the thermal noise, while the horizontally polarized optomechanical entanglement is nonexistent. When moving the total optomechanical coupling to the horizontal component (dashed dot curves), the previous scenario changes; the horizontally polarized optomechanical entanglement is limited and resilient, and the vertically polarized optomechanical entanglement is nonexistent. In the presence of the loss and for optimal polarized optomechanical coupling strength (dashed curves) in one case or another ($\phi = 0$ for vertical and $\phi = \pi/2$ for horizontal), we observe a gap compared to the two previous scenarios. There is not only an enhancement of entanglements, but the generated entanglements are more robust, paving the way for the stable implementation of these entangled states even at higher temperatures, such as ambient room temperatures.

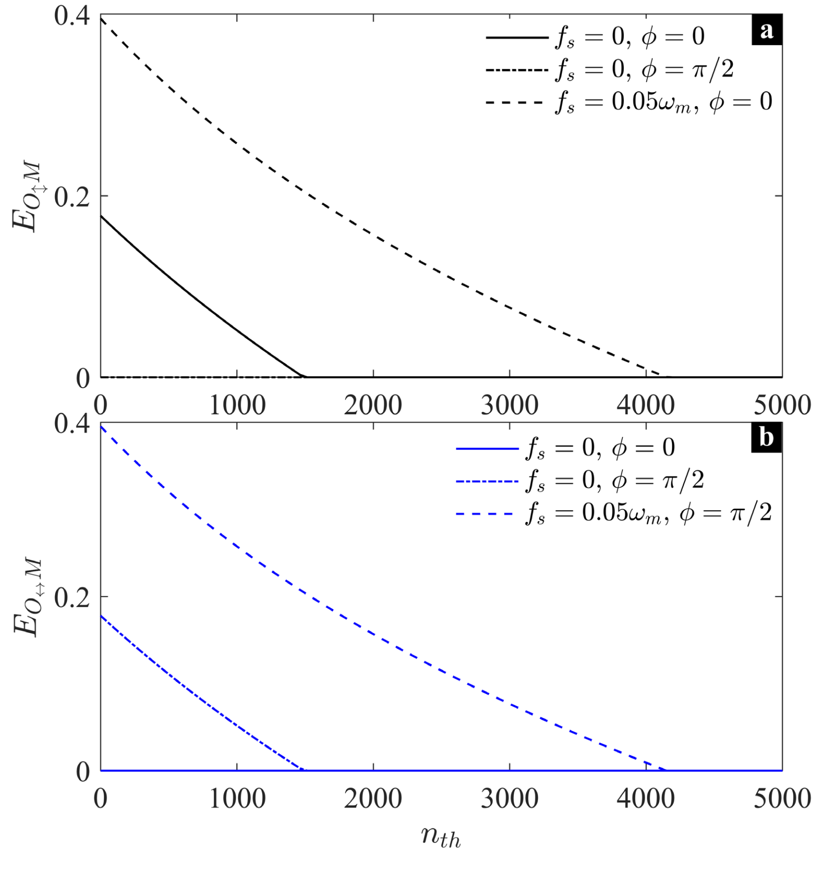

Figure 6. Bipartite entanglement between (a) the vertically polarized optical mode and the mechanical resonator, and (b) the horizontally polarized optical mode and the mechanical resonator versus the thermal noise for $G = 0.2\omega_m$. The other parameters are the same as those in **Figure 2**.

4. Conclusion

This work investigated the bipartite entanglement in an optomechanical cavity. Our benchmark system is made of a mechanical resonator driven by a polarized field. The cavity is subject to loss, which is an interesting factor for enhancing the bipartite entanglement. The polarization in our system enables the modulation of the components of the optomechanical coupling strength. The combination of the intracavity loss and the modulation of the phase results in an enhancement of polarized optomechanical entanglements, robust against thermal noise. This robustness is a critical finding because it facilitates the engineering of entangled quantum states under ambient conditions. This approach represents a more flexible scheme to generate and enhance the optomechanical entanglements. This work paves the way for the design of robust, phase-tunable quantum state engineering protocols and the practical implementation of optomechanical quantum devices at ambient temperatures.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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