

Combinations of Different Friction Models: Visualisation of the Curves

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Abstract

The study compares six composite dry-friction models (AV, CS, CVS, DV, SD, SV) built by combining classical static, Coulomb, viscous and Dahl terms plus Stribeck corrections. Using a numerical example of two mild-steel plates (1 kg load, 15 N driving force), the authors plot force-velocity or force-displacement curves and evaluate which combinations reproduce key phenomena such as stiction, hysteresis, Stribeck effect, and stick-slip. The qualitative inspection suggests that the static + viscous + Dahl (SV) model offers the most complete behaviour representation.

Keywords

Phenomenon, Friction, Adhesion, Stribeck Effect, Model

1. Introduction

Friction is present in most mechanical systems, and in most of them, this phenomenon causes performance losses such as trajectory tracking errors, stick-slip effects, or the occurrence of limit cycles. To mitigate these effects, the engineer must model these phenomena. The choice of a friction model among all those existing in the literature depends on a compromise managed by the user:

- Mainly, it is between the [1] [2] physical characteristics of real friction and those of the model. In other words, is the model capable of representing (or identifying) the observable and relevant physical phenomena?
- Then comes the model's complexity, *i.e.*, its implementation and simulation time. Thus, the most complete model is not necessarily the best.

For problems where friction modelling can be done roughly, simple models are sufficient. Here too, the user must specify his or her needs. For example, when inertia is small compared to the friction torque in rotational motion, more detailed modelling must be carried out. In most cases, authors use the computation time of open-loop simulations for two mechanical systems subjected to the stick-slip phenomenon as a performance criterion for [3] the different friction models, and the relevance of predicting limit cycles in a closed-loop experiment.

In this work, we will attempt to combine several models and then make a comparison to determine the optimal model subjected to stick-slip, Stribeck effect, and other physical friction phenomena defined above.

2. Limitations of Classical Friction Models

Classical friction models (such as Coulomb, viscous, or simple combinations) are widely used because of their simplicity, but they suffer from several major limitations:

- **No velocity dependence**

Coulomb friction assumes a constant friction force, independent of velocity, which is unrealistic in many real systems.

- **No representation of the Stribeck effect**

The decrease of friction force near zero velocity (Stribeck curve) is not captured by most simple models.

- **Absence of pre-sliding behavior**

The micro-displacements that occur before gross sliding are not represented, leading to poor accuracy in stick-slip or precision motion systems.

- **Lack of hysteresis modeling**

Classical models cannot reproduce the memory effect of friction (hysteresis loop), which is important in dynamic conditions.

- **No viscous contribution at higher velocities**

Pure Coulomb or Stribeck models do not account for the velocity-proportional viscous term that becomes dominant at higher speeds.

- **Over-simplification**

While easy to implement, they fail to reproduce complex experimental observations, especially in transition regimes (static-dynamic).

3. Combination of Some Friction Models

Some new friction models are proposed to identify friction behavior and to compensate as best as possible for its effects at the pre-sliding stage, or precise micro-displacements. For this purpose, a simulation will be performed to better characterize these models.

Let us take an experiment of sliding between two flat surfaces, where an external force of 15 N is applied to move the solid S_B of 1 kg (See **Figure 1**).

The parameters are represented by the Coulomb, static, [4] and viscous coefficients. These friction coefficients in **Table 1** correspond to typical values found

within a defined range for mild steel.

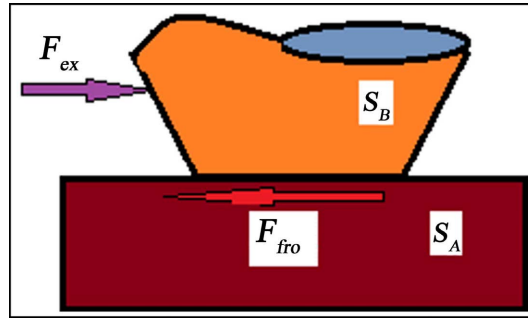


Figure 1. Contact between two solids with friction.

Table 1. Constants.

| F_{ex} | F_N | μ_c | μ_s | μ | \dot{X}_s | σ |
|----------|-------|---------|---------|-------|-------------|----------|
| 15 | 10 | 6 | 0.75 | 0.5 | 0.1 | 5 |

σ : The stiffness coefficient of asperities, behaving like a spring [4].

3.1. Combination of Adhesion and Viscous Friction (AV)

In this model, adhesion friction and viscous friction are considered, as these two terms. In this model, adhesion friction and viscous friction are considered, since these two terms represent the most used friction model. This simple model is not always sufficient for describing and compensating for friction. The hysteresis effect is considered a characteristic of the pre-sliding regime. The Stribeck velocity limit represents the start-up phase of the mechanism. The terms that make up this new set are:

The static friction denoted by par F_s , the viscous friction denoted by F_v , and the Stribeck effect defined by the terms C_1 and C_2 with \dot{X}_s the Stribeck velocity limit. The expression of the friction force F_{fro} is given by:

$$F_{fro} = F_s + F_v + C_1 + C_2 . \tag{1}$$

hence:

$$F_{fro} = \mu_s F_N + \mu_v \dot{X} + C_1 + C_2 . \tag{2}$$

With C_1 and C_2

$$C_1 = \begin{cases} C_1 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{si } \dot{X}_t > 0 \\ 0 & \text{si } \dot{X}_t \leq 0 \end{cases} \tag{3}$$

$$C_2 = \begin{cases} 0 & \text{si } \dot{X}_t \geq 0 \\ -C_2 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{si } \dot{X}_t < 0 \end{cases} \tag{4}$$

With X being the virtual displacement defined as

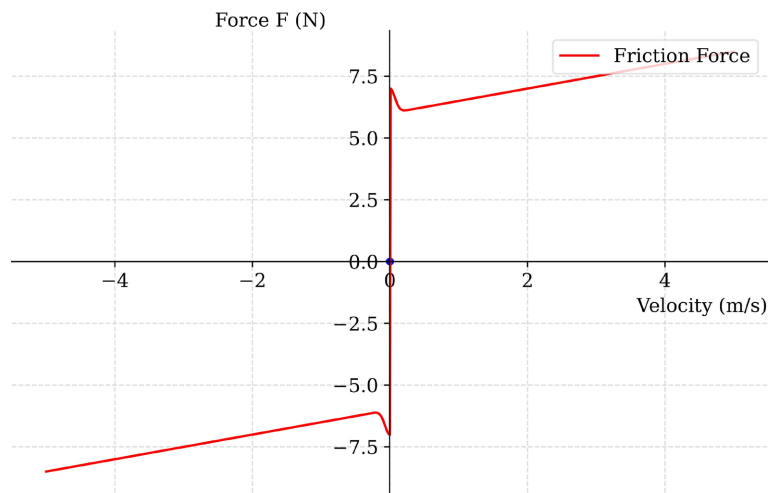


Figure 2. Adhesion and viscous friction curve.

Figure 2 shows the variation of the friction force as a function of velocity. A sharp drop in friction is observed near zero velocity, representing the Stribeck effect, followed by stabilization and a linear growth at higher velocities, which reflects the combination of Coulomb friction with a viscous contribution.

The simulation was performed using a 4th-order Runge-Kutta time integration method, which provides a good balance between accuracy and numerical efficiency. A time step of $\Delta t = 10^{-3}$ s was adopted to properly capture the rapid transitions around zero velocity while ensuring numerical stability. The temporal discretization was therefore chosen fine enough to reproduce the critical transient phenomena. In addition, a relative tolerance of 10^{-6} was imposed in the solver to control numerical errors.

Convergence was verified by comparing results obtained with different time steps and tolerance levels. Since the differences between the curves were negligible, it was concluded that the solution is robust and independent of the numerical parameters.

3.2. Combination of Coulomb and Static Friction (CS)

In this model, Coulomb friction and static friction are considered, since these two terms represent the most commonly used friction model. This simple model is not always sufficient for describing and compensating for friction. The stick-slip effect is accounted for by the static friction model characteristic. The start-up phase of the mechanism is represented by the Stribeck velocity limit (**Figure 3**). The hysteresis effect is expressed by introducing velocity into the terms C_1 and C_2 .

$$F_{fro} = F_C + F_S + C_1 + C_2. \tag{5}$$

$$F_{fro} = \begin{cases} F_{ex} & \text{if } \dot{X} = 0 \text{ and } |F_{ex}| < F_s \\ \mu_c \text{ sign}(\dot{X}) + C_1 + C_2 & \text{if } \dot{X} \neq 0 \text{ and } |F_{ex}| \geq F_s \end{cases} \tag{6}$$

And

$$C_1 = \begin{cases} C_1 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{if } \dot{X}_t > 0 \\ 0 & \text{if } \dot{X}_t \leq 0 \end{cases}$$

$$C_2 = \begin{cases} 0 & \text{if } \dot{X}_t \geq 0 \\ -C_2 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{if } \dot{X}_t < 0 \end{cases}$$

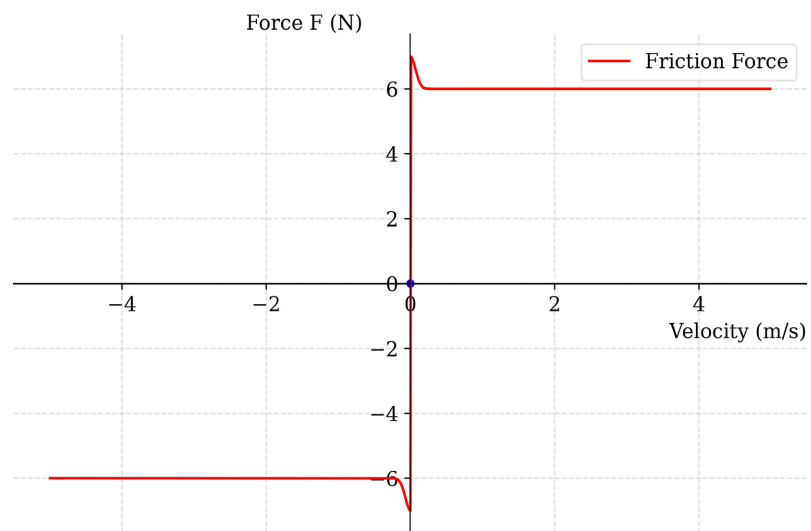


Figure 3. Friction between Coulomb and static.

The figure illustrates the evolution of the friction force as a function of velocity. A rapid decrease of the force is observed near zero velocity, representing the Stribeck effect, followed by stabilisation toward a constant value in the dynamic regime (Coulomb friction). The simulation was carried out using a 4th-order Runge-Kutta integration method with a time step $\Delta t = 10^{-3}$ s, ensuring both numerical stability and accuracy. The temporal discretisation was chosen sufficiently fine to accurately capture the critical zone around zero velocity. A relative tolerance of 10^{-6} was imposed in the solver to control numerical error. Convergence was verified by comparing results obtained with different time steps and tolerances: as the variations between curves were negligible, the robustness and independence of the solution with respect to the numerical parameters were confirmed.

3.3. Combination of Coulomb, Viscous, and Static Friction (CVS)

We then consider the three most well-known and commonly used friction models: Coulomb friction, viscous friction, and static friction. These simple models are not always sufficient to describe phenomena related to friction. In the terms C_1 and C_2 , the Stribeck effect appears through the introduction of velocity, as the hysteresis effect does not occur during the velocity decay phase (Figure 4). The expression for the friction force is given by:

$$F_{fro} = \begin{cases} \text{sign}(F_{ex}) \min(|F_{ex}|, F_s) & \text{if } \dot{X} = 0 \\ \mu_c \text{sign}(\dot{X}) + \mu_v \dot{X} + C_1 + C_2 & \text{if } \dot{X} \neq 0 \end{cases} \quad (7)$$

$$\text{And } C_1 = \begin{cases} C_1 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{if } \dot{X}_t > 0 \\ 0 & \text{if } \dot{X}_t \leq 0 \end{cases}$$

$$C_2 = \begin{cases} 0 & \text{if } \dot{X}_t \geq 0 \\ -C_2 e^{-\left(\frac{\dot{X}}{\dot{X}_s}\right)^2} & \text{if } \dot{X}_t < 0 \end{cases}$$

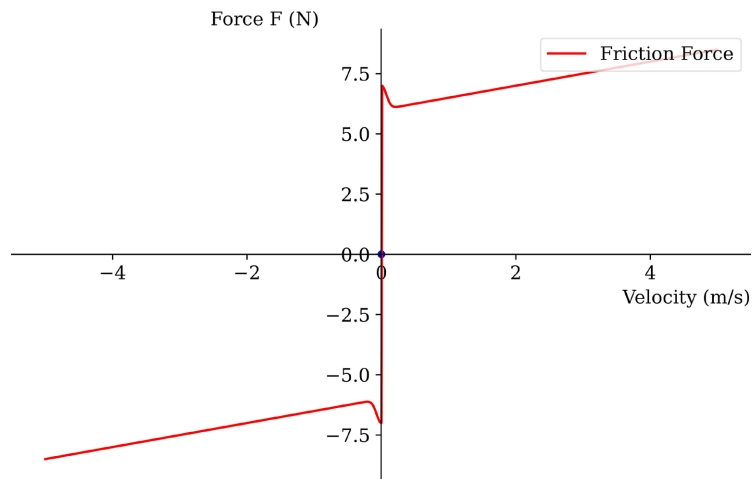


Figure 4. Friction combining Coulomb, viscous, and static components.

The red curve shows the friction law as a function of velocity:

- A sharp discontinuity appears near zero velocity (static - kinetic transition).
- For positive velocities, the friction force tends toward a positive increasing value.
- For negative velocities, it tends toward a negative decreasing value.
- The “kinks” around $v = 0$ suggest a regularized friction model (e.g., Coulomb + viscous component).

Such a law $F(v)$ is typically used in nonlinear dynamics with dry friction.

- The problem is stiff due to the discontinuity at $v = 0$.
- Therefore, implicit stable methods are often employed (Runge-Kutta).
- Explicit methods would require extremely small time steps to capture the sharp transition properly.

The time step must be small enough to resolve the rapid change around zero velocity.

- Typically, Δt is chosen so that the velocity change per step is much smaller than the transition region.
- If Δt is too large \rightarrow numerical oscillations or divergence may occur.
- Since the curve is smooth here, an adaptive time-stepping strategy was most likely used (automatically reducing Δt near $v \approx 0$).

3.4. Combination of Dahl and Viscous Friction (DV)

In this model, we consider the P. Dahl friction model along with the viscous model. Since the P. Dahl model does not account for the viscous behavior of friction, the viscous friction model is added to capture this important [4] physical phenomenon when the contact is lubricated. This model is therefore expressed as:

$$F_{fro} = \mu_v \dot{X} + F_c \left(1 - \exp\left(\frac{-\sigma |X|}{F_c}\right) \right) \text{sign}(\dot{X}) \quad (8)$$

$$\text{sign}(\dot{X}) = \begin{cases} 1 & \text{pour } \dot{X} > 0 \\ 0 & \text{pour } \dot{X} = 0 \\ -1 & \text{pour } \dot{X} < 0 \end{cases} \quad (9)$$

The equations are an exception to the prescribed specifications of this template. Thus, at a velocity different from zero, the model would be linearly dependent on the relative velocity. For this combination, friction is a function of two variables: velocity and displacement, both of which are time dependent. Therefore, for the simulation of this model, the displacement is defined as a function of time, and then the duration over which the movement between the two surfaces occurs is specified. For this work, we chose the velocity $\dot{X}(t) = t(t+1)$ over the time interval from zero to two seconds: $t = [0, 3]$ seconds, giving a velocity $\dot{X} = 26 \text{ m/s}$. Since $\dot{X} = \frac{dX}{dt}$, the [5] [6], displacement is: $X(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2$

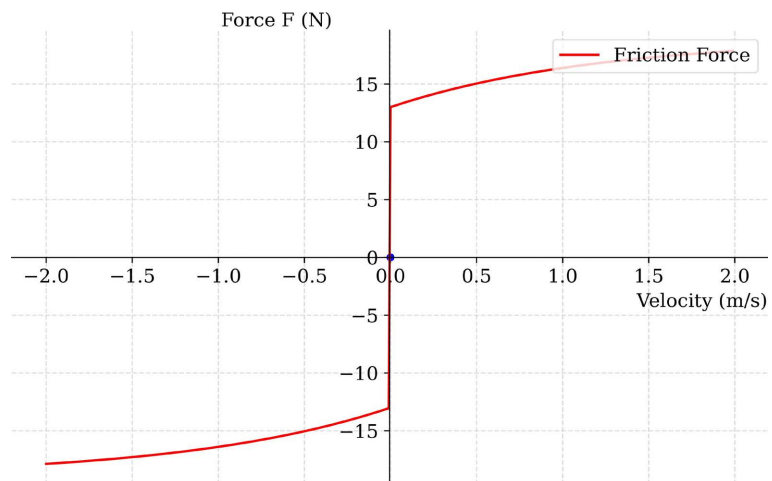


Figure 5. Friction combining Dahl and viscous components.

The red curve shows how friction evolves with displacement (See Figure 5):

- At zero displacement, there's again a jump discontinuity.
- For positive shifts, the friction force saturates toward a finite positive value.
- For negative shifts, it saturates toward a finite negative value.
- This is typical of a hysteresis or regularized friction law (e.g., Bouc-Wen or Dahl models).
- Since this is a force–displacement relationship with discontinuities, the system

is nonlinear and stiff.

- Common integration approaches:
 Implicit time integration (Backward Euler, Newmark, BDF).
 Newton-Raphson iterations at each step to solve the nonlinear equilibrium equations.
- The time (or load) increment must be small enough to resolve the steep slope near zero shift.
- In nonlinear static/dynamic problems, this corresponds to:
- Adaptive stepping: the solver automatically reduces the increment size when convergence slows (near discontinuities).
- If increments are too large, the solution will jump across the discontinuity and miss critical behavior.

3.5. Combination of Static and Dahl Friction (SD)

In this case, we consider the static friction model together with the Dahl model. Since the P. Dahl model does not account for the static friction phenomenon, this combined model consists of the Dahl model and the static [7] [8] friction model. The expression for this model is as follows:

$$F_{fro} = \mu_s F_N + F_{fc} \left(1 - \exp\left(\frac{-\sigma |X|}{F_{fc}}\right) \right) \text{sign}(\dot{X}) \tag{10}$$

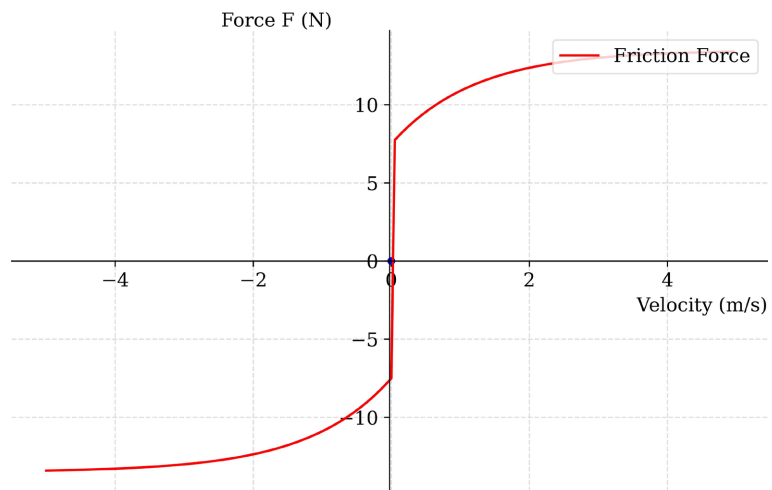


Figure 6. Friction combining Static and Dahl components.

The red curve represents a nonlinear friction law as a function of displacement (See **Figure 6**):

- For negative shifts, the friction force is strongly negative and tends asymptotically to a limiting value.
- For positive shifts, the force is strongly positive and approaches a higher limiting value.
- At Shift = 0, there is a sharp discontinuity, where the force jumps from about

–10 to about +8.

This reflects a nonlinear friction law with a threshold, characteristic of the stick–slip transition (adhesion → sliding).

Because the problem is highly nonlinear with a discontinuity, an implicit incremental integration scheme is most suitable.

Typically:

- Newton–Raphson iterations are used to solve the nonlinear equilibrium equations at each increment.
- Possible schemes: implicit Euler, implicit Newmark (for dynamic problems), or a quasi-static incremental approach.

Explicit schemes would require extremely small step sizes and would be unstable near the discontinuity.

The time step/displacement increment must be very small near Shift = 0, where the force changes abruptly.

An adaptive stepping strategy is generally applied:

- larger increments when the response is smooth,
- automatic reduction near discontinuities.

3.6. Combination of Static and Viscous Friction (SV)

In this final proposed model, we consider the combination of the static and viscous models, associated with the Dahl model, to address the two phenomena not accounted for by the Dahl model: the viscous and static behavior of friction. However, the Stribeck effect remains unsatisfied [9]-[11].

$$F_{fro} = \mu_s F_N + \mu_v \dot{X} + F_{fc} \left(1 - \exp\left(\frac{-\sigma |X|}{F_{fc}}\right) \right) \text{sign}(\dot{X}) \quad (11)$$

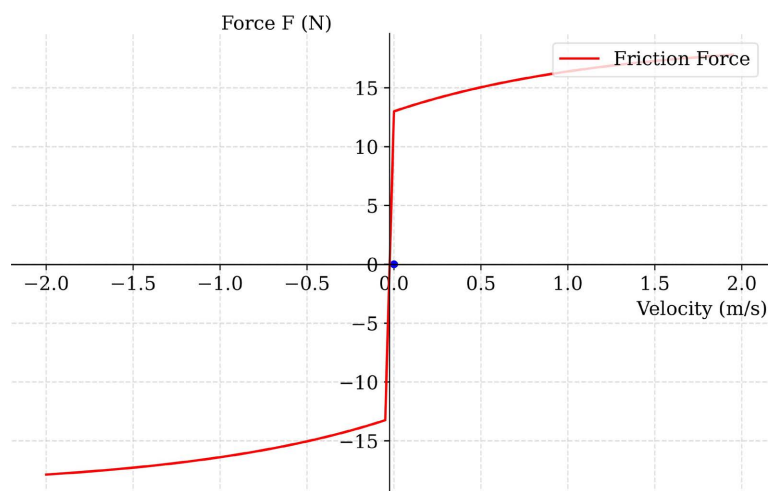


Figure 7. Friction combining Static and Viscous components [12] [13].

The red curve shows the nonlinear friction law as a function of displacement (See **Figure 7**):

- A sharp discontinuity occurs at Shift = 0, where the force jumps from a large negative value to a large positive value.
- For positive shifts, the force gradually approaches a limiting value (~17).
- For negative shifts, it tends toward (~-17).

This behavior is typical of a regularized dry-friction model, with a clear stick-slip transition at the origin.

The blue point indicates the initial or equilibrium state.

Since this is a highly nonlinear problem with discontinuities, an implicit incremental integration scheme is generally required:

- Newton-Raphson iterations to solve the nonlinear equilibrium equations at each step.
- Possible methods: Backward Euler, implicit Newmark (for dynamics), or incremental quasi-static schemes (for statics).

Explicit methods would be unstable here or require extremely small step sizes.

The time step/displacement increment must be very small near the discontinuity to capture the sudden transition in force.

Typically, an adaptive stepping strategy is used:

- larger increments when the response is smooth,
- automatic reduction near the discontinuity.

If increments are too large, the solution will jump across discontinuity and lose accuracy.

4. Summary of Friction Models

We provide a review of the friction models found in the literature according to their properties and the physical phenomena involved during contact between two surfaces. **Table 2** summarizes the properties of each of the proposed friction models

Table 2. Summary of the proposed friction models.

| Friction Model | AV | CS | CVS | DV | SD | SV |
|------------------------|----|----|-----|----|----|----|
| Numbers of Parameters | 4 | 4 | 4 | 3 | 3 | 4 |
| Stiction | 1 | 1 | 1 | 1 | 1 | 1 |
| pre-sliding | 1 | 1 | 1 | 1 | 1 | 1 |
| Stribeck effect | 1 | 1 | 1 | 0 | 0 | 0 |
| Viscous friction | 1 | 0 | 1 | 1 | 0 | 1 |
| Hysteresis | 1 | 1 | 1 | 1 | 1 | 1 |
| stick-slip | 1 | 1 | 0 | 0 | 1 | 1 |
| Rising static friction | 0 | 0 | 0 | 0 | 0 | 0 |

The following interpretations can help us better understand this table:

- **Stiction:** All other combinations respect the stiction phenomenon. At zero velocity, even if there is a discontinuity, they effectively define the friction force.

- **Pre-sliding:** This phenomenon represents friction on a scale that mainly depends on displacement rather than sliding velocity. It is only accounted for by models that use a spring to represent the elastic connection between the two contacting surfaces. Therefore, all dynamic model have this property.
- **Stribeck effect:** It is represented by most dynamic friction models except the Dahl model. No static friction model accounts for the Stribeck effect. Among the newly proposed models, the first three exhibit this Stribeck effect.
- **Viscous friction:** This is represented by all the proposed models, except for the Dahl-plus-static combination and the Coulomb-plus-static combination with the Stribeck effect.
- **Hysteresis:** All the newly proposed models exhibit the [12] [13] hysteresis effect.
- **Stick-slip:** Except for the Dahl-plus-adhesion model and the Coulomb-plus-static-plus-viscous combination with the Stribeck effect, the rest exhibit the stick-slip phenomenon.
- **Rising static friction:** None of the newly proposed models has this property.

5. Conclusions

Given the inadequacy of existing static friction models in the literature to fully capture the behavior of the friction phenomenon, we have proposed here necessary combinations of these models to better represent friction mechanisms. Six combinations were developed, with the choice of each depending on the model's ability to reproduce various behaviors (static, Stribeck, dynamic regime, stick-slip, etc.) as well as simulation feasibility. These models can be used to:

- Detect the degradation of adhesion (total or partial) or path deviation.
- Compensate for resistive forces while avoiding wear or improving force application.
- Maintain adhesion to optimise transmitted force in braking scenarios.
- Ensure path-tracking criteria at both low and high speeds.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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