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Modeling the Lubrication of Hydrodynamic Bearings Using Ferrofluids

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Abstract

This study investigates the lubrication performance of hydrodynamic journal bearings using ferrofluids-colloidal suspensions of magnetic nanoparticles as smart lubricants. By leveraging the Shliomis model**,** which accounts for rotational viscosity and thermal effects**,** the work integrates magnetic field dynamics into a modified Reynolds framework.

Keywords

Lubrication, Hydrodynamique, Ferrofluids, Viscosity, Bearing

1. Introduction

Hydrodynamic bearings play a crucial role in rotary mechanical systems by supporting radial loads [1]. The use of ferrofluids for lubrication represents an innovative approach, especially in extreme environments or at high speeds. Due to their responsiveness to magnetic fields, ferrofluids enable active control of the lubricating film [2].

This work aims to:

- Model the ferrofluid flow in bearings with finite dimensions.
- Study the influence of physical parameters (ϕ, θ) on tribological behavior.

2. Theoretical Models

In general, three major models are considered:

2.1. Neuringer-Rosensweig Model

This model introduces the volumetric magnetic force $\boldsymbol{f}_m = \mu_0 (\boldsymbol{M} \cdot \nabla) \boldsymbol{H}$, where

$\boldsymbol{M}$ is the magnetization and $\boldsymbol{H}$ is the magnetic field [3]. It does not account for magnetic moment or rotation.

2.2. Jenkins Model

A static isothermal model based on angular momentum [4]. The equation of motion incorporates:

- Magnetic force coefficient λ_3;
- Jenkins viscosity $\lambda_4 = \eta_j$;
- Direct influence of the magnetic field on ferromagnetic particles.

2.3. Shliomis Model

This model accounts for rotational viscosity and magnetic moment [5]. The fluid equations include:

- $\eta_s = \dfrac{\mu_0 M_0^2 \tau}{4}$
- Volumetric torque $\boldsymbol{T}_v = \boldsymbol{M} \times \boldsymbol{H}$
- Thermal and relaxation effects: τ_s, λ_1, λ_2

This study will focus particularly on the Shliomis model. The Shliomis model takes into account the rotational viscosity and the magnetization parameter, which shows that this model is realistic as it fully considers the effects of the magnetic field without excessive assumptions.

3. Fundamental Equations

The governing equations for ferrofluid flow are:

Navier-Stokes:

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla p + \eta \nabla^2 \boldsymbol{v} + \boldsymbol{f}_m \tag{1}$$

Maxwell's Equations:

$$\nabla \cdot \boldsymbol{B} = 0, \boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right) \tag{2}$$

Continuity Equation:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{3}$$

Modified Reynolds Equation:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\eta U \frac{\partial h}{\partial x} \tag{4}$$

Modeling of the Lubricating Film Thickness

We have:

$$h = O_c M - O_c M'$$

$$R_c - O_c M' = R_a + C - O_c M'$$

By applying the law of sines to triangle $O_a M' O_c$ (**Figure 1**), we get:

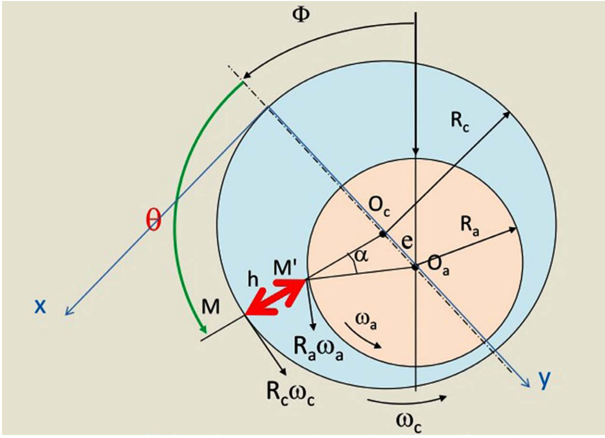

Figure 1. Diagram of a lubricated bearing [1].

$$\frac{O_aM'}{\sin(O_aO_cM')} = \frac{e}{\sin\alpha} = \frac{O_cM'}{\sin(O_cO_aM')} = \frac{R_a}{\sin(\pi-\theta)} = \frac{R_a}{\sin\theta}$$

Thus,

$$\begin{cases} \sin\alpha = \dfrac{e}{R_a}\sin\theta \\ O_cM' = \dfrac{R_a}{\sin\theta}\cdot\sin(O_cO_aM') \end{cases}$$

Since $O_cO_aM' = \theta - \alpha$, we have:

$$O_cM' = \frac{R_a}{\sin\theta}\cdot\sin\left[\theta - \arcsin\left(\frac{e}{R_a}\sin\theta\right)\right]$$

After development and simplification, we obtain:

$$O_cM' = R_a\sqrt{1-\left(\frac{e}{R_a}\sin\theta\right)^2} - e + \cos\theta$$

Since the ratio $\frac{e}{R_a} < \frac{c}{R}$ is very small (on the order of 10^{-3}), we can neglect the term $\left(\frac{e}{R_a}\sin\theta\right)^2$.

The film thickness can then be expressed as:

$$h(\theta) = c(1+\varepsilon\cos\theta)$$

4. Numerical Methods

Hydrodynamic lubrication is applied to a bearing of finite dimensions. The boundary conditions used in this study are those of Gumbel or Half-Sommerfeld (**Figure 2**), which impose the cancellation of negative pressure terms between $\theta = \pi$ and $\theta = 2\pi$. It should be noted that these conditions result in a discontinuity in the flow at $\theta = \pi$.

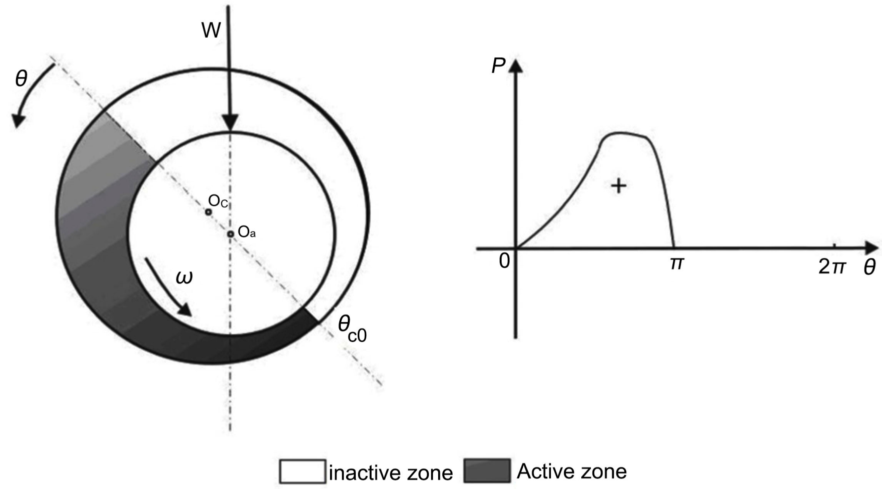

Figure 2. Boundary conditions related to the lubricant flow (Gümbel conditions) [2].

$$\begin{cases} \tilde{P}\left(\theta=0,\tilde{Z}\right)=0 \\ \tilde{P}\left(\theta=\pi,\tilde{Z}\right)=0 \\ \tilde{P}\left(\theta,\tilde{Z}\right)=0 \ \text{ si } \pi<\theta<2\pi \end{cases} \tag{5}$$

These conditions do not ensure the conservation of flow rate between the active and inactive zones of the bearing. The Gümbel conditions are frequently used for short bearings, as they make it possible to obtain results close to experimental findings.

4.1. Discretization

Finite difference methods are used to discretize the modified Reynolds equation:

$$\begin{aligned} &\frac{\partial f\left(\theta,\tilde{z}\right)}{\partial\theta}\frac{\partial\tilde{p}}{\partial\theta}+f\left(\theta,\tilde{z}\right)\frac{\partial^2\tilde{p}}{\partial\theta^2}+\gamma_1\frac{\partial g\left(\theta,\tilde{z}\right)}{\partial\tilde{z}}\frac{\partial\tilde{p}}{\partial\tilde{z}}+\gamma_1 g\left(\theta,\tilde{z}\right)\frac{\partial^2\tilde{p}}{\partial\tilde{z}^2} \\ &=6\frac{\partial\tilde{h}}{\partial\theta}+\gamma_2\frac{\partial F\left(\theta,\tilde{z}\right)}{\partial\theta}\frac{\partial\tilde{H}_x}{\partial\theta}+\gamma_2 F\left(\theta,\tilde{z}\right)\frac{\partial^2\tilde{H}_x}{\partial\theta^2} \end{aligned} \tag{6}$$

where:

$$f\left(\theta,\tilde{z}\right)=\frac{\tilde{h}^3}{1+\frac{3}{2}\varphi\frac{\xi-\tanh\xi}{\xi+\tanh\xi}},\ \text{with}\ \tilde{h}\left(\theta\right)=1+\varepsilon\cos\theta$$

$$\xi=\lambda_1\tilde{H},\ \lambda_1=\frac{\mu_0\mu}{K_B T}H_0,\ \gamma_1=\frac{R^2}{L^2}$$

$$g\left(\theta,\tilde{z}\right)=\frac{\tilde{h}^3}{1+\frac{3}{2}\varphi\frac{\xi-\tanh\xi}{\xi+\tanh\xi}\left(\frac{\tilde{H}_y}{\tilde{H}}\right)^2},\ \gamma_2=\lambda_2\left(\frac{\varphi}{1+2.5\varphi}\right)$$

$$\lambda_2=\frac{C^2\mu_0 H_0\mu}{R^2\eta_0\omega V},\ F\left(\theta,z\right)=\left(\coth\xi-\frac{1}{\xi}\right)\frac{\tilde{H}_x}{\tilde{H}}$$

4.2. Solution Method

After discretizing the modified Reynolds equation, the Gauss-Seidel method with relaxation is used:

$$\tilde{p}_{i,j}^{N_{\text{iter}}+1} = (1-\sigma)\,\tilde{p}_{i,j}^{N_{\text{iter}}} + \sigma\left(a_{ij}\,\tilde{p}_{i+1,j}^{N_{\text{iter}}} + b_{ij}\,\tilde{p}_{i-1,j}^{N_{\text{iter}}+1} + c_{ij}\,\tilde{p}_{i,j+1}^{N_{\text{iter}}} + d_{ij}\,\tilde{p}_{i,j-1}^{N_{\text{iter}}+1} + e_{ij}\right) \quad (7)$$

where σ is the relaxation coefficient, and N_{iter} is the iteration number.

To ensure convergence, in the MATLAB solution we set the condition: if $N_{\text{iter}_{\max}} = N_{\text{iter}}$, then no convergence; otherwise, convergence.

5. Results and Discussion

MATLAB simulations show the following:

Dimensionless Pressure

The curves of dimensionless pressure as a function of different values of ferrofluid concentration ϕ and skew angle θ are presented below:

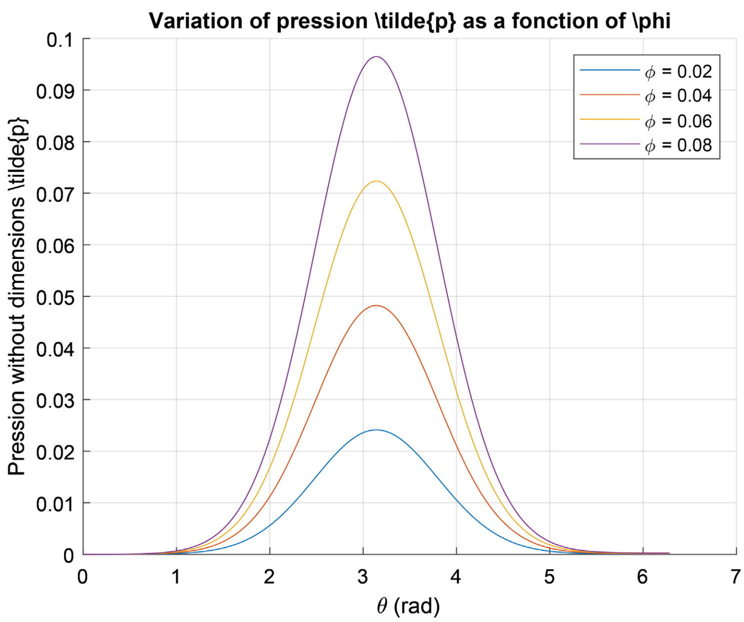

Figure 3. Pressure curve as a function of ferromagnetic particle concentration.

From **Figure 3**, we observe that the pressure varies with the concentration ϕ. The higher the value of ϕ, the higher the dimensionless pressure. Similarly, the angle θ also influences the pressure. The curve shows a rise in dimensionless pressure, reaching a peak around $\theta = 3$, followed by a decrease. Overall, the greater the concentration, the higher the dimensionless pressure. This observation was also noted in Study [1]. The variation of the dimensionless pressure field is entirely attributed to the presence of ferrofluid particles and the magnetic field. Without the magnetic field, the result is no longer the same. Similarly, if the ferrofluid particles are removed, the situation reverts to lubrication with conventional fluids, and the dimensionless pressure is then no longer as significant.

Dimensionless Load Capacity

The dimensionless load capacity directly depends on the dimensionless pressure. The curve of the dimensionless load, considering variations in ϕ (ferrofluid concentration) and angle θ, is shown below:

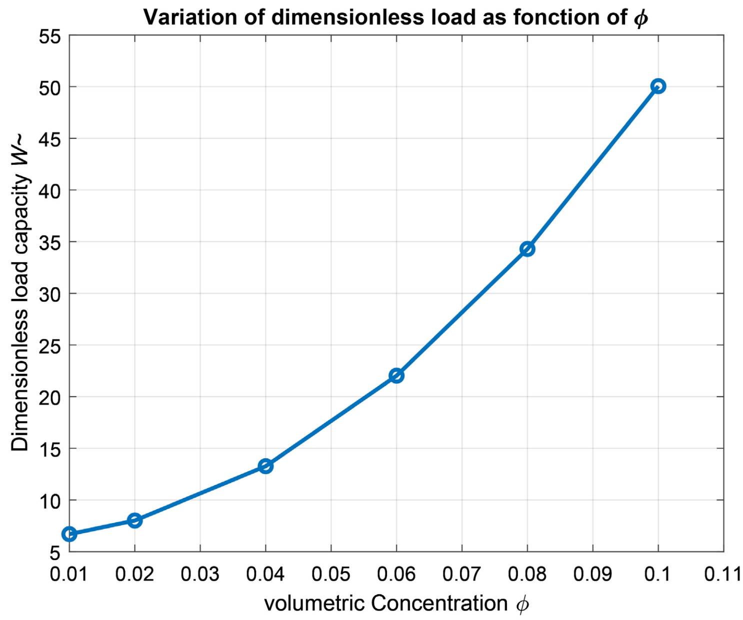

Figure 4. Load capacity curve as a function of ferromagnetic particle concentration.

From **Figure 4**, it can be seen that the load capacity depends on the volumetric concentration of the ferrofluid. For low concentrations, the load is small. The dimensionless load increases with higher concentrations of ferromagnetic particles. This variation depends on the concentration of ferrofluid particles while keeping the magnetic field constant.

As demonstrated by Rajest *et al.* [2] in their study on the film of magnetic fluid compressed between two curved, porous, rotating disks, this study confirms that the properties of the fluid are considerably enhanced in the presence of a magnetic field. They found that the pressure field becomes more significant, as well as the load-carrying capacity, which is consistent with our own findings.

6. Conclusions

This study demonstrates that ferrofluids are effective lubricants for hydrodynamic bearings operating in extreme environments. The Shliomis model allows for a more realistic representation of tribological behavior.

- Simulations confirm the influence of the magnetic field through ferrofluid concentration ϕ and geometry;
- An increase in both dimensionless pressure and load capacity is observed with higher concentrations of ferromagnetic particles;
- Future directions: dynamic lubrication, magnetic feedback control, integration with renewable energy systems (e.g., in the Sahel region). In the Sahel region, the climate is harsh, which causes fluids to lose their viscosity and provide little

support to the bearings. We need to control the quality of the fluid as effectively as possible.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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