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Global Stability of Fractional-Order Fuzzy Memristive Neural Networks with Time Delay and Impulses

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Abstract

In this paper, the author studies the global stability of fractional-order fuzzy memristor neural networks with time delay and impulses. By applying the contraction mapping principle, the author proves the existence and uniqueness of the equilibrium point in this paper, thereby obtaining the prerequisite conditions for the stability of the system. Then, by establishing an appropriate Lyapunov function and using the relevant knowledge of fractional calculus, the author provides the relevant criteria for the stability of the system. Finally, the feasibility of the theoretical results is verified through numerical simulation experiments.

Keywords

Fuzzy Memristor Neural Network, Global Stability, Fractional Order, Time Delay, Impulse

1. Introduction

Neural networks are a powerful tool capable of effectively solving many complex problems in the real world. Their development originated from the neuron theory established by Spanish anatomist Cajal at the end of the 19th century, which has made significant contributions to subsequent research on artificial neural networks. An artificial neural network is a dynamic system with a directed graph as its topological structure, which processes information by responding to continuous or discontinuous inputs in a state manner. It is an information processing system designed to imitate the structure and function of the human brain [1].

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With the rapid development of neural networks in practical applications, the theoretical research requirements for neural networks are also increasing simultaneously. This demands researchers to develop neural network models and their dynamic behaviors that can more accurately describe real-world situations. In real life, time delays are widespread. For instance, in the field of biology, infectious diseases often have incubation periods. In network communication, signal transmission can also encounter delays. Similarly, in elasticity mechanics, the response of materials may be delayed, and so on. In neural networks, due to the limited switching rate of neuron amplifiers and equipment aging, the signal transmission between neurons cannot be completed instantaneously. This makes the current dynamic behavior of the system not only dependent on the current state but also related to the past state. As a result, time delay in neural networks is inevitable and often leads to system instability, even chaotic phenomena and other undesirable effects. Therefore, analyzing the dynamic behavior of time-delay neural networks has significant research significance [2]-[4]. Due to the important characteristics of time-delay neural networks compared to general neural networks, they can be widely applied in multiple fields [5]-[7]. In the practical application of mathematical modeling with neural networks, not only time delays are encountered, but also factors such as uncertainty, approximation, and fuzziness. This has led to the increasing importance of fuzzy theory as an appropriate approach. In 1996, Yang *et al.* [8] [9] first introduced fuzzy logic (fuzzy AND and fuzzy OR) into neural networks, establishing the model of fuzzy cellular neural networks. Unlike general cellular neural networks, the modules of fuzzy cellular neural networks, in addition to the product-sum operation with inputs and outputs, also possess fuzzy logic operations. This gives it a wider range of applications [10]-[12].

In real life, various systems may deviate from their original trajectories at certain moments due to sudden external disturbances, forming impulses. The short-term interference of impulses will inevitably have certain impacts on the systems. Therefore, when analyzing the dynamic characteristics of nonlinear power grids, it is necessary to consider the influence of impulses. Yang [13] obtained some new results on global synchronization by combining the interaction of impulses through multiple recurrent neural networks with time delays. Ramal and Smith [14] studied the influence of the correlation between the input and output degrees of randomly directed networks on the performance of the same impulse-coupled oscillator synchronization system.

In 1971, Tsai Shao-Tang [15] derived and predicted through symmetry theory that there might exist a fourth fundamental circuit element in addition to the three known basic components of capacitance, resistance, and inductance, which could relate to magnetic flux and charge. He named it the memristor, but its physical entity had not yet emerged at that time. It was not until 2008 that the Hewlett-Packard Laboratory [16] first constructed a physical model of the memristor in real life, thereby confirming its existence. Due to its memory-like behavior, similar functions to biological synapses, and advantages such as scalability, small size, and low power consumption, the memristor has been widely applied in various fields [17]-[19]. Traditional neural networks suffer from inherent lack of parallelism, complex circuits, excessive circuit area, inflexible synaptic regulation, and inability of sensor arrays to learn in real-time and dynamically. However, the memristor, with its unique characteristics, has been introduced into neural network models and serves as an important tool for mimicking the neural processes of the human brain [20].

Fractional calculus emerged over 300 years ago and was first mentioned in a correspondence between Hospital and Leibnitz in 1695 [21] [22]. However, it has only been intensively studied for over 20 years. In fact, fractional calculus has advantages over integer-order calculus. For instance, it can describe certain phenomena more accurately with fractional derivatives and integrals, offers greater freedom in constructing system models, and can effectively describe hereditary and memory characteristics. As a result, some researchers have introduced it into neural networks to study the related characteristics of neural networks more accurately, thus giving rise to fractional-order neural networks.

Stability is one of the important dynamic behaviors in dynamics and holds significant importance in research areas such as medical science [23], secure communication [24], intelligent control and image processing (such as cryptography [25]). Meanwhile, the study of stability in neural networks is also abundant. In reference [26], the author established delay-dependent asymptotic stability conditions in the form of linear matrix inequalities (LMIs) by applying the Razumikhin theorem and LMIs and discussing the delay-dependent stability and stability of a class of fractional-order memristor neural networks with time-varying delays. In reference [27], the authors, based on Jensen's integral inequality, established new delay-dependent and order-dependent stability conditions for fractional-order memristor neural networks with time-varying delays. In reference [28], the authors analyzed the unified criteria for global dissipation and stability of delay fractional-order systems of multi-valued fractional-order memristor neural networks (FSMVMNNs). However, a few papers have discussed the global stability problem of fractional-order fuzzy memristor neural networks with time delay and impulses, which is one of the characteristics of this paper.

Inspired by the above discussion, this paper studies the global stability problem of fractional-order fuzzy meristem neural networks with time delay and impulses. The main contributions of this paper can be summarized as follows: 1) This paper utilizes the relevant knowledge of the contraction mapping principle to prove the existence of equilibrium points. 2) Compared with literature [26], this paper considers the stability of the system under the influence of impulses.

The structure of the remaining part of this paper is as follows. The model description and preliminary knowledge are presented in Section 2. Section 3 provides sufficient conditions for the global stability of fractional-order fuzzy memristive neural networks with time delay and impulses. Section 4 presents two numerical examples to demonstrate the effectiveness of our method. Finally, the conclusion is given in Section 5.

2. Preliminaries

Then some assumptions, definitions and lemmas will be introduced in this section.

$$\begin{cases} D^{\alpha}u_i(t) = \sum_{j=1}^{n} b_{ij}(u_i(t)) f_j(u_j(t)) + \sum_{j=1}^{n} c_{ij}(u_i(t)) g_j(u_j(t-\tau(t))) \\ \qquad + \wedge_{j=1}^{n} \alpha_{ij} g_j(u_j(t-\tau(t))) + \wedge_{j=1}^{n} T_{ij} v_j + \vee_{j=1}^{n} S_{ij} v_j \\ \qquad - a_i(u_i(t)) u_i(t) + \vee_{j=1}^{n} \beta_{ij} g_j(u_j(t-\tau(t))) + \sum_{j=1}^{m} d_{ij} v_j + I_i, \\ \Delta u_i(t_k) = \lambda_{ik} u_i(t_k^-), t = t_k, \\ u_i(t) = \psi_i(t), t \in [-\tau, 0], \end{cases} \tag{1}$$

where $u_i(t_k^+) = \lim_{t\to t_k^+} u_i(t)$, $u_i(t_k^-) = \lim_{t\to t_k} u_i(t)^-$ represents the left and right limits of the impulse moment $t = t_k$. Suppose that the solution of system (1) is left-continuous at t_k. Let $t_0 = 0$, $u_i(t_k^-) = u_i(t_k)$, $\Delta u_i(t_k) = u_i(t_k^+) - u_i(t_k^-)$. Then consider the following time-delay fractional-order fuzzy memristive neural network model:

$$\begin{cases} D^{\alpha}u_i(t) = \sum_{j=1}^{n} b_{ij}(u_i(t)) f_j(u_j(t)) + \sum_{j=1}^{n} c_{ij}(u_i(t)) g_j(u_j(t-\tau(t))) \\ \qquad + \wedge_{j=1}^{n} \alpha_{ij} g_j(u_j(t-\tau(t))) + \wedge_{j=1}^{n} T_{ij} v_j + \vee_{j=1}^{n} S_{ij} v_j \\ \qquad - a_i(u_i(t)) u_i(t) + \vee_{j=1}^{n} \beta_{ij} g_j(u_j(t-\tau(t))) + \sum_{j=1}^{m} d_{ij} v_j + I_i, \\ u_i(t) = \psi_i(t), t \in [-\tau, 0], \end{cases} \tag{2}$$

Among them, the initial state $u(s) = \psi(s) = (\psi_1(s), \psi_2(s), \psi_3(s), \cdots, \psi_n(s))^{\mathrm{T}}$, $\psi(s) \in C([-\tau, 0], R^n)$. $u_i(t)$ is the state variable of the i-th neuron. The self-feedback coefficient of a neuron is denoted by the symbol $a_i(u_i(t))$, the connection weights of the non-delayed and delay-related memristors are respectively denoted as $b_{ij}(u_i(t))$ and $c_{ij}(u_i(t))$. The transmission delay of the j-th neuron is represented by the following symbol: $\tau(t)$. The delay-free feedback function and the delay feedback function are set as f_j, g_j ($R \to R$). $\tau(t)$ is a time-varying delay and continuously differentiable function. It satisfies $0 \le \tau(t) \le \tau$, $0 \le \tau'(t) < 1$. v_i represents the bias of the i-th neuron. The elements of the MIN and MAX fuzzy feedback templates, as well as the MIN and MAX fuzzy feedforward templates are respectively represented as $\alpha_{ij}, \beta_{ij}, T_{ij}, S_{ij}$. $\wedge$ and $\vee$ are fuzzy AND and fuzzy OR. I_i is recorded as external input. The connection weights based on memristors are $a_{ij}(u_i(t)), b_{ij}(u_i(t))$ and $c_{ij}(u_i(t))$. They satisfy:

$$a_i(u_i(t)) = \begin{cases} \acute{a}_i, |u_i(t)| \le T_i, \\ \grave{a}_i, |u_i(t)| > T_i, \end{cases}$$

$$b_{ij}(u_i(t)) = \begin{cases} \acute{b}_{ij}, |u_i(t)| \le T_j, \\ \grave{b}_{ij}, |u_i(t)| > T_j, \end{cases}$$

$$c_{ij}(u_i(t)) = \begin{cases} \acute{c}_{ij}, |u_i(t)| \le T_j, \\ \grave{c}_{ij}, |u_i(t)| > T_j, \end{cases}$$

here, T_i is a jump instruction. $\acute{a}_i, \grave{a}_i, \acute{b}_{ij}, \grave{b}_{ij}, \acute{c}_{ij}, \grave{c}_{ij}$ $(i, j = 1, 2, \cdots, n)$, $(k = 1, 2, \cdots, K)$ These constants are related to memristors. According to the Filippov solution and the differential inclusion theorem, we obtain:

$$\begin{cases} D^{\alpha}u_i(t) \in \sum_{j=1}^{n}\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]f_j\left(u_j(t)\right)+\sum_{j=1}^{n}\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]g_j\left(u_j\left(t-\tau(t)\right)\right) \\ \qquad +\sum_{j=1}^{m}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+\wedge_{j=1}^{n}T_{ij}v_j+\vee_{j=1}^{n}S_{ij}v_j \\ \qquad -\overline{co}\left[\hat{a}_i,\breve{a}_i\right]u_i(t)+\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+I_i+N_i(t), \\ u_i(t)=\psi_i(t), t\in[-\tau,0], \end{cases} \tag{3}$$

where $\hat{a}_i=\min\{\acute{a}_\iota,\grave{a}_\iota\}$, $\breve{a}_i=\max\{\acute{a}_\iota,\grave{a}_\iota\}$, $\hat{b}_{ij}=\min\{\acute{b}_{\iota J},\grave{b}_{\iota J}\}$, $\breve{b}_{ij}=\max\{\acute{b}_{\iota J},\grave{b}_{\iota J}\}$, $\hat{c}_{ij}=\min\{\acute{c}_{\iota J},\grave{c}_{\iota J}\}$, $\breve{c}_{ij}=\max\{\acute{c}_{\iota J},\grave{c}_{\iota J}\}$; and there exist measurable functions $\widetilde{a_i^1}\in\overline{co}\left[\hat{a}_i,\breve{a}_i\right]$, $\widetilde{b_{ij}^1}\in\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]$, $\widetilde{c_{ij}^1}\in\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]$, where, $i,j\in N$, So

$$\begin{cases} D^{\alpha}u_i(t)=-\widetilde{a_i^1}u_i(t)+\sum_{j=1}^{n}\widetilde{b_{ij}^1}f_j\left(u_j(t)\right)+\sum_{j=1}^{n}\widetilde{c_{ij}^1}g_j\left(u_j\left(t-\tau(t)\right)\right) \\ \qquad +\sum_{j=1}^{m}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+\wedge_{j=1}^{n}T_{ij}v_j \\ \qquad +\vee_{j=1}^{n}S_{ij}v_j+\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+I_i+N_i(t), \\ u_i(t)=\psi_i(t), t\in[-\tau,0], \end{cases} \tag{4}$$

where, $u(t)^*=\left(u_1^*,u_2^*,\cdots,u_n^*\right)^{\mathrm{T}}$ is called an equilibrium point in the sense of Filippov, if and only if

$$\begin{aligned} 0\in &-\overline{co}\left[\hat{a}_i,\breve{a}_i\right]u_i^*+\sum_{j=1}^{n}\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]f_j\left(u_j^*\right)+\sum_{j=1}^{n}\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]g_j\left(u_j^*\right) \\ &+\sum_{j=1}^{m}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j^*\right)+\wedge_{j=1}^{n}T_{ij}v_j \\ &+\vee_{j=1}^{n}S_{ij}v_j+\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j^*\right)+I_i. \end{aligned} \tag{5}$$

Then there exist $\widetilde{a_i^*}\in\overline{co}\left[\hat{a}_i,\breve{a}_i\right],\widetilde{b_{ij}^*}\in\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right],\widetilde{c_{ij}}\in\overline{co}\left[\hat{c}_{ij}^k,\breve{c}_{ij}\right]$; Meanwhile,

$$\begin{aligned} 0=&-\widetilde{a_i^*}u_i^*+\sum_{j=1}^{n}\widetilde{b_{ij}^*}f_j\left(u_j^*\right)+\sum_{j=1}^{n}\widetilde{c_{ij}^*}g_j\left(u_j^*\right) \\ &+\sum_{j=1}^{n}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j^*\right)+\vee_{j=1}^{n}S_{ij}v_j \\ &+\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j^*\right)+\wedge_{j=1}^{n}T_{ij}v_j+I_i, \end{aligned} \tag{6}$$

where $i,j\in N$.

Remark 1 This paper appropriately introduces the impulse effect into the system, enabling the system to reach a stable state under the premise of having an equilibrium point. After the introduction of pulse effects, the original system usually becomes unstable. To ensure the stability of the system, the relevant conditions of Theorem 2 below need to be satisfied.

The following assumptions are made for this chapter:

(Al) If the activation functions $f_j(\cdot)$ and $g_j(\cdot)$ satisfy the Lipschitz condition, and for any $u,v\in R$ and $u\neq v$, then there exist constants L_j and K_j such that:

$$\left|f_j(u)-f_j(v)\right|\le L_j\left|u-v\right|,\left|g_j(u)-g_j(v)\right|\le K_j\left|u-v\right|,$$

where $j=1,2,3,\cdots,n$.

Lemma 1 [29] Suppose u_j and v_j are two states of a neural network, then it can be said that:

$$\left|\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j\right)-\wedge_{j=1}^{n}\alpha_{ij}g_j\left(v_j\right)\right|\le\sum_{j=1}^{n}\left|\alpha_{ij}\right|\left|g_j\left(u_j\right)-g_j\left(v_j\right)\right|,$$

$$\left|\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j\right)-\vee_{j=1}^{n}\beta_{ij}g_j\left(v_j\right)\right|\le\sum_{j=1}^{n}\left|\beta_{ij}\right|\left|g_j\left(u_j\right)-g_j\left(v_j\right)\right|.$$

Lemma 2 [30] Under assumption (Al), if $f_j\left(\pm T_j\right)=g_j\left(\pm T_j\right)=0$, then it follows that:

$$\left|-\overline{co}\left[\hat{a}_i,\breve{a}_i\right]u_i(t)+\overline{co}\left[\hat{a}_i,\breve{a}_i\right]v_i(t)\right|\le m_i^*\left|u_i(t)-v_i(t)\right|,$$

$$\left|\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]f_j\left(u_j(t)\right)-\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]f_j\left(v_j(t)\right)\right|\le f_{ij}^*F_j\left|u_j(t)-v_j(t)\right|,$$

$$\left|\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]g_j\left(u_j(t)\right)-\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]g_j\left(v_j(t)\right)\right|\le g_{ij}^*G_j\left|u_j(t)-v_j(t)\right|,$$

for any $\widetilde{a_i^*},\overline{a_i^*}\in\overline{co}\left[\hat{a}_i,\breve{a}_i\right]$, $\widetilde{b_{ij}^*},\overline{b_{ij}^*}\in\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]$, $\widetilde{c_{ij}^*},\overline{c_{ij}^*}\in\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]$, we can obtain:

$$\left|-\widetilde{a_i^*}u_i(t)+\overline{a_i^*}v_i(t)\right|\le m_i^*\left|u_i(t)-v_i(t)\right|$$

$$\left|\widetilde{b_{ij}^*}f_j\left(u_j(t)\right)-\overline{b_{ij}^*}f_j\left(v_j(t)\right)\right|\le f_{ij}^*F_j\left|u_j(t)-v_j(t)\right|,$$

$$\left|\widetilde{c_{ij}^*}g_j\left(u_j(t)\right)-\overline{c_{ij}^*}g_j\left(v_j(t)\right)\right|\le g_{ij}^*G_j\left|u_j(t)-v_j(t)\right|.$$

where $m_i^*=\max\{|\acute{a}_\iota|,|\grave{a}_\iota|\}$, $f_{ij}^*=\max\{|\acute{b}_{\iota J}|,|\grave{b}_{\iota J}|\}$, $g_{ij}^*=\max\{|\acute{c}_{\iota J}|,|\grave{c}_{\iota J}|\}$, $i,j\in N$.

Lemma 3 [31] let $g(t)$ be a continuously differentiable function on $t\in[0,+\infty)$, then for $0<q<1$, one has:

$$D^q\left|g(t)\right|\le\operatorname{sign}\left(g(t)\right)D^q g(t).$$

Lemma 4 [32] if $0<\alpha<1$, $\left|\arg(s)\right|<\frac{\pi}{2}$, make $N>1$, $s\ne0$, M-L function expansion:

$$E_{\alpha,\beta}(s)=\frac{1}{\alpha}s^{\frac{1-\beta}{\alpha}}\mathrm{e}^{s^{\frac{1}{\alpha}}}-\sum_{k=1}^{N}\frac{s^{-k}}{\Gamma(\beta-\alpha r)}+O\left(\frac{1}{S^{N+1}}\right).$$

Definition 1 [33] Let the impulse sequence be $\{t_k\}$, $o(t_0,t)$ denote the number of impulses in the sequence $\{t_k\}$ within the time interval (t_0,t). If there exist T and T_0 such that:

$$\frac{t-t_0}{T}-T_0\le o(t_0,t)\le\frac{t-t_0}{T}+T_0.$$

Then T is called the average impulse interval of the impulse sequence $\{t_k\}$.

Definition 2 [34] The Caputo fractional derivative of order $q\in(0,1)$ for a function $h(t)\in R^1\left([0,+\infty),R\right)$ is defined as:

$$D_t^q=\frac{1}{\Gamma(1-q)}\int_{t_0}^{t}(t-s)^{-q}h'(s)\mathrm{d}s.$$

Definition 3 [35] A set-valued map F with nonempty values is said to be upper-semi continuous at $y_0\in\epsilon\subset R^n$, if for any open set M containing $F(y_0)$, there exists a neighborhood O of y_0 such that $F(O)\subset M$. $F(y)$ is said to have a closed (convex, compact) image if for each $y\in\epsilon$, $F(y)$ is closed.

Definition 4 [36] Consider the system $\mathrm{d}y/\mathrm{d}t = f(y)$, $y \in R^n$, with discontinuous right-hand sides, a set-valued map is defined as:

$$\varphi(y) = \bigcap_{\delta>0} \bigcap_{\mu(M)=0} \overline{co}\left[f\left(B(y,\delta)\right)\setminus M\right],$$

where $\overline{co}$ is the closure of the convex hull of set *F*, $B(y,\delta) = \{z \| z - y \| \le \delta\}$ and $\mu(M)$ is the Lebesgue measure of the set *M*. A solution in Filippovs sense of the Cauchy problem for this system with initial condition $y(0) = y_0$ is a continuous function $y(t), t \in [0,T]$, which satisfies $y(0) = y_0$ and differential inclusion:

$$\frac{\mathrm{d}y}{\mathrm{d}t} \in \varphi(y),$$

for $t \in [0,T]$.

Definition 5 [37] Define M-L function:

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{+\infty} \frac{t^k}{\Gamma(\alpha k + \beta)}.$$

where $\alpha > 0, \beta > 0, E_\alpha(t) = E_{\alpha,1}(t)$.

Definition 6 [37] If $\alpha \in (0,1)$ and $V(t) \in [t_0,\infty)$ is a continuous function, and for the constant ρ

$$D_t^\alpha V(t) \le \rho V(t),$$

Then

$$V(t) \le V(t_0) E_\alpha\left(\rho (t - t_0)^\alpha\right).$$

Definition 7 [38] Consider pulse sequences t_k, $o(t_0,t)$ represents the number of impulses. If exist T and T_0 satisfy:

$$\frac{t - t_0}{T} - T_0 \le o(t_0,t) \le \frac{t - t_0}{T} + T_0.$$

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3. Main Results

In this chapter, the existence of equilibrium point conditions is effectively proved by using the contraction mapping principle, and the stability of fractional-order fuzzy memristor neural networks with time delay and impulses is discussed. Moreover, the stability criteria of the system are obtained by establishing an appropriate Lyapunov function.

Theorem 1 If assumption (Al) holds and

(A) $w=\sum_{i=1}^{n}\max_{1\le j\le n}\left(\dfrac{f_{ij}^{*}F_j+G_j\left(g_{ij}^{*}+\left|\alpha_{ij}\right|+\left|\beta_{ij}\right|\right)}{\widetilde{a_j^{*}}}\right)<1$.

Then system (2) has an equilibrium point.

Proof. Let $v(t)=\left(v_1(t),v_2(t),\cdots,v_n(t)\right)^{\mathrm{T}}=\left(\widetilde{a_1^{*}}u_1(t),\widetilde{a_2^{*}}u_2(t),\cdots,\widetilde{a_n^{*}}u_n(t)\right)^{\mathrm{T}}\in R^n$. Consider the mapping $\zeta:X\to X$, $\zeta(u)=\left(\zeta_1(u),\zeta_2(u),\cdots,\zeta_n(u)\right)^{\mathrm{T}}$, then:

$$
\begin{aligned}
\zeta_i(v)&=\sum_{j=1}^{n}\widetilde{b_{ij}^{*}}\left(\frac{v_j(t)}{\widetilde{a_j^{*}}}\right)+\sum_{j=1}^{n}\widetilde{c_{ij}^{*}}g_j\left(\frac{v_j(t)}{\widetilde{a_j^{*}}}\right)\\
&\quad+\sum_{j=1}^{m}d_{ij}v_j+\bigwedge_{j=1}^{n}\alpha_{ij}g_j\left(\frac{v_j(t)}{\widetilde{a_j^{*}}}\right)+\bigvee_{j=1}^{n}S_{ij}v_j \qquad (7)\\
&\quad+\bigvee_{j=1}^{n}\beta_{ij}g_j\left(\frac{v_j(t)}{\widetilde{a_j^{*}}}\right)+\bigwedge_{j=1}^{n}T_{ij}v_j+I_i.
\end{aligned}
$$

where $\widetilde{a_i^{*}}\in\overline{co}\left[\hat{a}_i,\breve{a}_i\right]$, $\widetilde{b_{ij}^{*}}\in\overline{co}\left[\hat{b}_{ij},\breve{b}_{ij}\right]$, $\widetilde{c_{ij}^{*}}\in\overline{co}\left[\hat{c}_{ij},\breve{c}_{ij}\right]$. For any two vectors $x=\left(x_1,x_2,\cdots,x_n\right)^{\mathrm{T}}$, $y=\left(y_1,y_2,\cdots,y_n\right)^{\mathrm{T}}$, then:

$$
\begin{aligned}
\left|\zeta_i(x)-\zeta_i(y)\right|&\le\left|\sum_{j=1}^{n}\widetilde{b_{ij}^{1}}f_j\left(\frac{x_j}{\widetilde{a_j^{*}}}\right)-\sum_{j=1}^{n}\widetilde{b_{ij}^{*}}f_j\left(\frac{y_j}{\widetilde{a_j^{*}}}\right)\right|\\
&\quad+\left|\sum_{j=1}^{n}\widetilde{c_{ij}^{1}}g_j\left(\frac{x_j}{\widetilde{a_j^{*}}}\right)-\sum_{j=1}^{n}\widetilde{c_{ij}^{*}}g_j\left(\frac{y_j}{\widetilde{a_j^{*}}}\right)\right|+\bigwedge_{j=1}^{n}\alpha_{ij}g_j\left(\frac{x_j}{\widetilde{a_j^{*}}}\right)\\
&\quad-\bigwedge_{j=1}^{n}\alpha_{ij}g_j\left(\frac{y_j}{\widetilde{a_j^{*}}}\right)+\bigvee_{j=1}^{n}\beta_{ij}g_j\left(\frac{x_j}{\widetilde{a_j^{*}}}\right)-\bigvee_{j=1}^{n}\beta_{ij}g_j\left(\frac{y_j}{\widetilde{a_j^{*}}}\right) \qquad (8)\\
&\le\sum_{j=1}^{n}\left(\frac{f_{ij}^{*}F_j}{\widetilde{a_j^{*}}}\left|x_j-y_j\right|\right)+\sum_{j=1}^{n}\left(\frac{g_{ij}^{*}G_j}{\widetilde{a_j^{*}}}\left|x_j-y_j\right|\right)\\
&\quad+\sum_{j=1}^{n}\frac{G_j\left|\alpha_{ij}\right|}{\widetilde{a_j^{*}}}\left|x_j-y_j\right|+\sum_{j=1}^{n}\frac{G_j\left|\beta_{ij}\right|}{\widetilde{a_j^{*}}}\left|x_j-y_j\right|\\
&=\sum_{j=1}^{n}\left(\frac{f_{ij}^{*}F_j+G_j\left(g_{ij}^{*}+\left|\alpha_{ij}\right|+\left|\beta_{ij}\right|\right)}{\widetilde{a_j^{*}}}\right)\left|x_j-y_j\right|,
\end{aligned}
$$

Then:

$$
\begin{aligned}
\left\|\zeta(x)-\zeta(y)\right\|&=\sum_{i=1}^{n}\left|\zeta_i(x)-\zeta_i(y)\right|\\
&\le\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{f_{ij}^{*}F_j+G_j\left(g_{ij}^{*}+\left|\alpha_{ij}\right|+\left|\beta_{ij}\right|\right)}{\widetilde{a_j^{*}}}\right)\left|x_j-y_j\right|\\
&\le\sum_{i=1}^{n}\max_{1\le j\le n}\left(\frac{f_{ij}^{*}F_j+G_j\left(g_{ij}^{*}+\left|\alpha_{ij}\right|+\left|\beta_{ij}\right|\right)}{\widetilde{a_j^{*}}}\right)\sum_{j=1}^{n}\left|x_j-y_j\right| \qquad (9)\\
&=\sum_{i=1}^{n}\max_{1\le j\le n}\left(\frac{f_{ij}^{*}F_j+G_j\left(g_{ij}^{*}+\left|\alpha_{ij}\right|+\left|\beta_{ij}\right|\right)}{\widetilde{a_j^{*}}}\right)\left|x-y\right|.
\end{aligned}
$$

where $\left\|\zeta(x)-\zeta(y)\right\|<q|x-y|,\ \ x\neq y$.

Therefore, there exists a fixed point $x^*\in R^n$ such that $\zeta(x^*)=x^*$ and

$$x^*=\sum\nolimits_{j=1}^{n}\widetilde{b_{ij}^*}f_j\left(\frac{x^*}{\widetilde{a_j^*}}\right)+\sum\nolimits_{j=1}^{n}\widetilde{c_{ij}^*}g_j\left(\frac{x^*}{\widetilde{a_j^*}}\right)+\sum\nolimits_{j=1}^{m}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(\frac{x^*}{\widetilde{a_j^*}}\right)$$
$$+\vee_{j=1}^{n}S_{ij}v_j+\vee_{j=1}^{n}\beta_{ij}g_j\left(\frac{x^*}{\widetilde{a_j^*}}\right)+\wedge_{j=1}^{n}T_{ij}v_j+I_i. \tag{10}$$

Let $u_i^*=\dfrac{x_i^*}{a_i}$, then:

$$0=-a_iu_i^*+\sum\nolimits_{j=1}^{n}\widetilde{b_{ij}^*}f_j\left(u_j^*\right)+\sum\nolimits_{j=1}^{n}\widetilde{c_{ij}^*}g_j\left(u_j^*\right)$$
$$+\sum\nolimits_{j=1}^{m}d_{ij}v_j+\wedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j^*\right)+\wedge_{j=1}^{n}T_{ij}v_j \tag{11}$$
$$+\vee_{j=1}^{n}S_{ij}v_j+\vee_{j=1}^{n}\beta_{ij}g_j\left(u_j^*\right)+I_i.$$

where $i=1,2,\cdots,n$. We have derived that the system has a unique equilibrium point u^*. It is proved.

Theorem 2 If assumption (Al) holds and

(B) $w=\sum\nolimits_{i=1}^{n}\max\nolimits_{1\le j\le n}\left(\dfrac{f_{ij}^*F_j+G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)}{\widetilde{a_j^*}}\right)<1$,

(C) $z=\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n}G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)|\phi_i(s)|<0$,

(D) $o=\dfrac{\ln\left(\dfrac{\lambda_i}{\alpha}\right)}{T}+p^{\frac{1}{\alpha}}<0$.

where $p=\left(m_i^*+\sum\nolimits_{j=1}^{n}f_{ij}^*F_j+\sum\nolimits_{j=1}^{n}G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)\right)$. Then system (1) is globally exponentially stable.

Proof. Let the equilibrium point of system (1) be $u(t)^*=\left(u_1(t)^*,u_2(t)^*,\cdots,u_n(t)^*\right)^{\mathrm{T}}$ and the initial conditions are κ^*, If system (1) has a solution, then any solution is $u(t)=\left(u_1(t),u_2(t),\cdots,u_n(t)\right)^{\mathrm{T}}$ and the initial conditions are $u(s)=\psi(s)$, where $\psi(s)\in C\left([-\tau,0),R^n\right)$. Let $d_i(t)=u_i(t)-u_i^*$. We consider the following impulsive system:

$$\begin{cases} D^\alpha d_i(t)=\left(-\widetilde{a_i^1}u_i(t)+\widetilde{a_i^*}u_i^*\right)+\sum\limits_{j=1}^{n}\left(\widetilde{b_{ij}^1}f_j\left(u_j(t)\right)-\widetilde{b_{ij}^*}f_j\left(u_j^*\right)\right) \\ \qquad +\sum\limits_{j=1}^{n}\left(\widetilde{c_{ij}^1}g_j\left(u_j\left(t-\tau(t)\right)\right)-\widetilde{c_{ij}^*}g_j\left(u_j^*\right)\right) \\ \qquad +\bigvee\limits_{j=1}^{n}\left(\beta_{j=1}g_j\left(u_j\left(t-\tau(t)\right)\right)-\beta_{ij}g_j\left(u_j^*\right)\right) \\ \qquad +\left(\alpha_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)-\alpha_{ij}g_j\left(u_j^*\right)\right) \\ \qquad +\bigwedge\limits_{j=1}^{n}\left(\alpha_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)-\alpha_{ij}g_j\left(u_j^*\right)\right), \\ \Delta d_i(t_k)=\lambda_{ik}d_i\left(t_k^-\right),t=t_k, \\ d_i(t)=\phi_i(t),t\in[-\tau,0], \end{cases} \tag{12}$$

where $d_i(t)=u_i(t)-u_i^*$ and $d(s)=\phi(s)$, $\phi(s)\in C([-\tau,0),R^n)$. $d_i(t_k^+)=\lim_{t\to t_k^+} d_i(t)$, $d_i(t_k^-)=\lim_{t\to t_k} d_i(t)^-$ representative impulse moment $t=t_k$ left and right limits. Suppose the solution of system 2 is left-continuous at t_k. Let $t_0=0$, $d_i(t_k^-)=d_i(t_k)$, $\Delta d_i(t_k)=d_i(t_k^+)-d_i(t_k^-)$. We consider the following Lyapunov function:

$$V(t)=\sum_{i=1}^n |d_i(t)|.$$

Taking the Caputo derivative of $V(t)$ along system (11), Then when $t\in(t_{k-1},t_k]$:

$$D^\alpha V(d(t))\le \sum_{i=1}^n \operatorname{sgn}(d_i(t))D^\alpha d_i(t). \tag{13}$$

Based on Lemma 1 and Lemma 2, the following conclusion can be drawn:

$$\begin{aligned}
D^\alpha V(d(t))\le& \sum_{i=1}^n \operatorname{sgn}(d_i(t))\left(-\widetilde{a_i^1}u_i(t)+\widetilde{a_i^*}u_i^*\right)\\
&+\operatorname{sgn}(d_i(t))\left(\sum_{i=1}^n\left(\widetilde{b_{ij}^1}f_j(u_j(t))-\widetilde{b_{ij}^*}f_j(u_j^*)\right)\right)\\
&+\operatorname{sgn}(d_i(t))\left(\sum_{i=1}^n\left(\widetilde{c_{ij}^1}g_j(u_j(t-\tau(t)))-\widetilde{c_{ij}^*}g_j(u_j^*)\right)\right)\\
&+\operatorname{sgn}(d_i(t))\left(\left(\alpha_{ij}g_j(u_j(t-\tau(t)))-\alpha_{ij}g_j(u_j^*)\right)\right)\\
&+\operatorname{sgn}(d_i(t))\left(\bigvee_{j=1}^n\left(\beta_{ij}g_j(u_j(t-\tau(t))-\alpha_{ij}g_j(u_j^*))\right)\right)\\
&+\operatorname{sgn}(d_i(t))\left(\bigwedge_{j=1}^n\left(\alpha_{ij}g_j(u_j(t-\tau(t)))-\alpha_{ij}g_j(u_j^*)\right)\right)\\
\le& \sum_{i=1}^n m_i^*|d_i(t)|+\sum_{i=1}^n\sum_{j=1}^n f_{ij}^*F_j|d_j(t)|\\
&+\sum_{i=1}^n\sum_{j=1}^n G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)|d_j(t-\tau(t))|.
\end{aligned} \tag{14}$$

And

$$\begin{aligned}
d_i(t-\tau(t))&\le \sup_{-\tau\le s\le t}|d_i(s)|\le \sup_{-\tau\le s\le t}|d_i(s)|+\sup_{0\le s\le t}|d_i(s)|\\
&=|\phi_i(s)|+|d_i(t)|.
\end{aligned}$$

This leads to:

$$\begin{aligned}
D^\alpha V(d(t))\le& \sum_{i=1}^n m_i^*|d_i(t)|+\sum_{i=1}^n\sum_{j=1}^n f_{ij}^*F_j|d_j(t)|\\
&+\sum_{i=1}^n\sum_{j=1}^n G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)\left(|\phi_i(s)|+|d_i(t)|\right)\\
=& \sum_{i=1}^n\left(m_i^*+\sum_{j=1}^n f_{ij}^*F_j+\sum_{j=1}^n G_j\left(g_{ij}^*+|\alpha_{ij}|+|\beta_{ij}|\right)\right)|d_j(t)|\\
\le& pV(t).
\end{aligned} \tag{15}$$

when $t=t_k$,

$$V(t_k^+)=\sum_{i=1}^n|d_i(t_k^+)|=\sum_{i=1}^n|1+\lambda_{ik}|d_i(t_k^-)=\lambda_i V(t_k^-). \tag{16}$$

where $\lambda_i = |1+\lambda_{ik}|$.

When $t\in(0,t_1]$, $V(t)\le E_\alpha\left(p(t-t_0)^\alpha\right)V(t_0^+)$, then $V(t_1)\le E_\alpha\left(p(t_1-t_0)^\alpha\right)V(t_0^+)$, at the same time, $V(t_1^+)\le \lambda_i E_\alpha\left(p(t_1-t_0)^\alpha\right)V(t_0^+)$. When $t\in(t_1,t_2]$, $V(t_2^+)\le \lambda_i^2 E_\alpha\left(p(t_2-t_1)^\alpha\right)V(t_1^+)E_\alpha\left(p(t_1-t_0)^\alpha\right)V(t_0^+)$. When $t\in(t_k,t_{k+1}]$, $V(t)\le \prod_{i=1}^{K}E_\alpha\left(p(t_i-t_{i-1})^\alpha\right)\lambda_i^K E_\alpha\left(p(t-t_K)^\alpha\right)V(t_0^+)$.

Set the impulse count to $\sigma(t_0,t)$, Then, according to Definition 1:

$$\begin{aligned}V(t)&\le \prod_{i=1}^{K}E_\alpha\left(p(t_i-t_{i-1})^\alpha\right)\lambda_i^K E_\alpha\left(p(t-t_K)^\alpha\right)V(t_0^+)\\&\le \frac{1}{\alpha^{\sigma(t_0,t)}}e^{\sum_{i=1}^{\sigma(t_0,t)}p^{\frac{1}{\alpha}}(t_i-t_{i-1})}\lambda_i^{\sigma(t_0,t)}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}\left(t-t_{\sigma(t_0,t)}\right)}\\&\le\left(\frac{\lambda_i}{\alpha}\right)^{\sigma(t_0,t)}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}(t-t_0)}.\end{aligned}\tag{17}$$

When $0<\frac{\lambda_i}{\alpha}<1$, then:

$$\begin{aligned}V(t)&\le\left(\frac{\lambda_i}{\alpha}\right)^{\frac{t-t_0}{T}-T_0}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}(t-t_0)}\\&=\left(\frac{\lambda_i}{\alpha}\right)^{-T_0}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}(t-t_0)}e^{\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}(t-t_0)}\\&=\left(\frac{\lambda_i}{\alpha}\right)^{-T_0}\frac{1}{\alpha}e^{\left(\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}+p^{\frac{1}{\alpha}}\right)(t-t_0)}.\end{aligned}\tag{18}$$

Let $\xi=\left(\frac{\lambda_i}{\alpha}\right)^{-T_0}\frac{1}{\alpha}$, $\epsilon=\left(\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}+p^{\frac{1}{\alpha}}\right)<0$, then $V(t)\le\xi e^{\epsilon(t-t_0)}V(t_0^+)$.

When $\frac{\lambda_i}{\alpha}>1$,

$$\begin{aligned}V(t)&\le\left(\frac{\lambda_i}{\alpha}\right)^{\frac{t-t_0}{T}+T_0}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}(t-t_0)}\\&=\left(\frac{\lambda_i}{\alpha}\right)^{T_0}\frac{1}{\alpha}e^{p^{\frac{1}{\alpha}}(t-t_0)}e^{\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}(t-t_0)}\\&=\left(\frac{\lambda_i}{\alpha}\right)^{T_0}\frac{1}{\alpha}e^{\left(\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}+p^{\frac{1}{\alpha}}\right)(t-t_0)}.\end{aligned}\tag{19}$$

Let $\xi_1=\left(\frac{\lambda_i}{\alpha}\right)^{T_0}\frac{1}{\alpha}$, $\epsilon_1=\left(\frac{\ln\left(\frac{\lambda_i}{\alpha}\right)}{T}+p^{\frac{1}{\alpha}}\right)<0$, then $V(t)\le\xi_1 e^{\epsilon_1(t-t_0)}V(t_0^+)$. In conclusion, when $t \to \infty$, $V(t) \to 0$. This means that the system can achieve global exponential stability.

Remark 2 This paper creatively links the fractional-order neural network system with the influence of impulses, and because the system in this paper is more general, it is more applicable.

4. Numerical Simulation

Next, let's consider the following example to verify the validity of the conclusion.

4.1. Example 1

Consider the following model:

$$\begin{cases} D^{\alpha}u_i(t) = \sum_{j=1}^{n} b_{ij}\left(u_i(t)\right) f_j\left(u_j(t)\right) + \sum_{j=1}^{n} c_{ij}\left(u_i(t)\right) g_j\left(u_j\left(t-\tau(t)\right)\right) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} g_j\left(u_j\left(t-\tau(t)\right)\right) + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} S_{ij} v_j \\ \qquad - a_i\left(u_i(t)\right) u_i(t) + \bigvee_{j=1}^{n} \beta_{ij} g_j\left(u_j\left(t-\tau(t)\right)\right) + \sum_{j=1}^{m} d_{ij} v_j + I_i, \\ \Delta u_i(t_k) = \lambda_{ik} u_i\left(t_k^-\right), t = t_k, \\ u_i(t) = \psi_i(t), t \in [-\tau, 0], \end{cases} \tag{20}$$

Let $\alpha = 0.5$, $f_1(u(t)) = f_2(u(t)) = g_1(u(t)) = g_2(u(t)) = \frac{1}{2}\left(|u+1| - |u-1|\right)$. $\tau(t) = \frac{e^t}{1+e^t}$. $I_1 = 0.2$, $I_2 = 0.4$, $v_1 = v_2 = 1$, $(\alpha_{ij})_{2\times2} = \begin{pmatrix} 1.2 & -1.1 \\ -1.5 & 1.1 \end{pmatrix}$, $(\beta_{ij})_{2\times2} = \begin{pmatrix} 1.4 & -1.3 \\ -0.3 & 1.4 \end{pmatrix}$, $(\lambda_{ik})_{2\times2} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$, $(d_{ij})_{2\times2} = (S_{pl})_{2\times2} = (H_{pl})_{2\times2} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$, Initial value $(\psi_1(0), \psi_2(0)) = (-0.1, 0.2)^{\mathrm{T}}$. Among them, the impulse moment t_i satisfies: $t_{8q+1} = \frac{\pi}{4} + 2\pi l$, $t_{8q+2} = \frac{\pi}{2} + 2\pi l$, $t_{8q+3} = \frac{3\pi}{4} + 2\pi l$, $t_{8q+4} = \pi + 2\pi l$, $t_{8q+5} = \frac{5\pi}{4} + 2\pi l$, $t_{8q+6} = \frac{3\pi}{2} + 2\pi l$, $t_{8q+7} = \frac{7\pi}{4} + 2\pi l$, $t_{8q+8} = 2\pi + 2\pi l$, $t_{8q+9} = \frac{9\pi}{4} + 2\pi l$, $q = 0, 1, 2, \cdots$

$$a_1\left(u_1(t)\right) = \begin{cases} 0.8, & |u_1(t)| < 1, \\ 1.4, & |u_1(t)| > 1, \end{cases} \quad a_2\left(u_2(t)\right) = \begin{cases} 1.4, & |u_2(t)| < 1, \\ 0.8, & |u_2(t)| > 1, \end{cases}$$

$$b_{11}\left(u_1(t)\right) = \begin{cases} \frac{1}{8}, & |u_1(t)| < 1, \\ -\frac{1}{8}, & |u_1(t)| > 1, \end{cases} \quad b_{12}\left(u_1(t)\right) = \begin{cases} \frac{1}{7}, & |u_1(t)| < 1, \\ -\frac{1}{7}, & |u_1(t)| > 1, \end{cases}$$

$$b_{21}\left(u_2(t)\right) = \begin{cases} \frac{1}{6}, & |u_2(t)| < 1, \\ -\frac{1}{6}, & |u_2(t)| > 1, \end{cases} \quad b_{22}\left(u_2(t)\right) = \begin{cases} \frac{1}{4}, & |u_2(t)| < 1, \\ -\frac{1}{4}, & |u_2(t)| > 1, \end{cases}$$

$$c_{11}(u_1(t)) = \begin{cases} \frac{1}{9}, & |u_1(t)| < 1, \\ -\frac{1}{9}, & |u_1(t)| > 1, \end{cases} \quad c_{12}(u_1(t)) = \begin{cases} \frac{1}{5}, & |u_1(t)| < 1, \\ -\frac{1}{5}, & |u_1(t)| > 1, \end{cases}$$

$$c_{21}(u_2(t)) = \begin{cases} \frac{1}{3}, & |u_2(t)| < 1, \\ -\frac{1}{3}, & |u_2(t)| > 1, \end{cases} \quad c_{22}(u_2(t)) = \begin{cases} \frac{1}{7}, & |u_2(t)| < 1, \\ -\frac{1}{7}, & |u_2(t)| > 1, \end{cases}$$

For the system without impulse interference as shown in Figure 1, it can be obtained through calculation that $w = 0.44 < 1$, $p = 0.76 > 0$, $z = -0.61 < 0$, $o = -0.33 < 0$. According to Theorem 2, it can be concluded that system (1) is globally exponentially stable. The experimental results are shown in Figure 2.

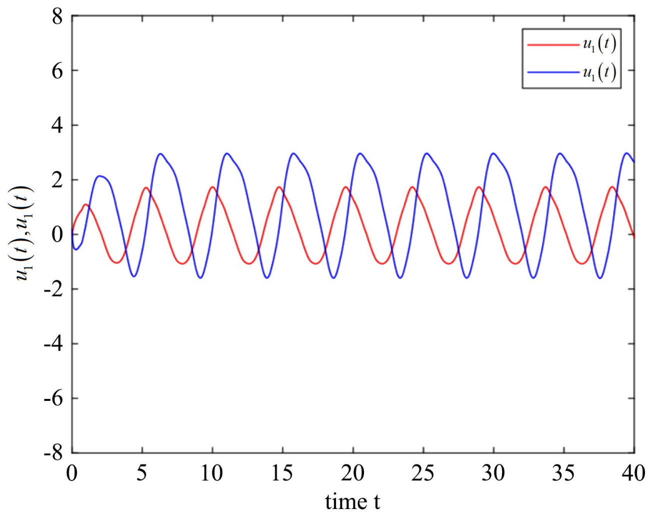

Figure 1. The orbits of $u_1(t)$ and $u_2(t)$ without impulse interference.

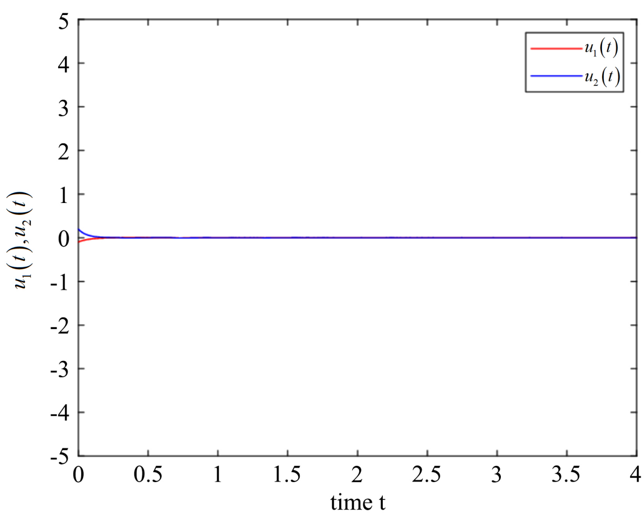

Figure 2. Orbits of $u_1(t)$ and $u_2(t)$ with impulse interference.

4.2. Example 2

Consider the following model:

$$\begin{cases} D^{\alpha}u_i(t)=\sum_{j=1}^{n}b_{ij}\left(u_i(t)\right)f_j\left(u_j(t)\right)+\sum_{j=1}^{n}c_{ij}\left(u_i(t)\right)g_j\left(u_j\left(t-\tau(t)\right)\right) \\ \qquad +\bigwedge_{j=1}^{n}\alpha_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+\bigwedge_{j=1}^{n}T_{ij}v_j+\bigvee_{j=1}^{n}S_{ij}v_j \\ \qquad -a_i\left(u_i(t)\right)u_i(t)+\bigvee_{j=1}^{n}\beta_{ij}g_j\left(u_j\left(t-\tau(t)\right)\right)+\sum_{j=1}^{m}d_{ij}v_j+I_i, \\ \Delta u_i(t_k)=\lambda_{ik}u_i\left(t_k^-\right), t=t_k, \\ u_i(t)=\psi_i(t), t\in[-\tau,0], \end{cases} \tag{21}$$

Let $\alpha=0.8$, $f_1(u(t))=f_2(u(t))=g_1(u(t))=g_2(u(t))=\sin(x+1)$. $\tau(t)=\frac{e^t}{1+e^t}$. $(\alpha_{ij})_{2\times2}=\begin{pmatrix}1.5 & -1.2\\ -1.5 & 1.6\end{pmatrix}$, $(\beta_{ij})_{2\times2}=\begin{pmatrix}1.7 & -1.6\\ -0.5 & 1.6\end{pmatrix}$, $(\lambda_{ik})_{2\times2}=\begin{pmatrix}0.3 & 0\\ 0 & 0.3\end{pmatrix}$, $(d_{ij})_{2\times2}=(S_{pl})_{2\times2}=(H_{pl})_{2\times2}=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix}$, Initial value $(\psi_1(0),\psi_2(0))=(0.2,-0.3)^{\mathrm{T}}$. where the impulse time t_i satisfies:

$t_{8q+1}=\frac{\pi}{4}+2\pi l$, $t_{8q+2}=\frac{\pi}{2}+2\pi l$, $t_{8q+3}=\frac{3\pi}{4}+2\pi l$, $t_{8q+4}=\pi+2\pi l$, $t_{8q+5}=\frac{5\pi}{4}+2\pi l$, $t_{8q+6}=\frac{3\pi}{2}+2\pi l$, $t_{8q+7}=\frac{7\pi}{4}+2\pi l$, $t_{8q+8}=2\pi+2\pi l$, $t_{8q+9}=\frac{9\pi}{4}+2\pi l$, $q=0,1,2,\cdots$.

$$a_1(u_1(t))=\begin{cases}0.9, & |u_1(t)|<1,\\ 1.3, & |u_1(t)|>1,\end{cases}\quad a_2(u_2(t))=\begin{cases}1.3, & |u_2(t)|<1,\\ 0.9, & |u_2(t)|>1,\end{cases}$$

$$b_{11}(u_1(t))=\begin{cases}\frac{1}{7}, & |u_1(t)|<1,\\ -\frac{1}{7}, & |u_1(t)|>1,\end{cases}\quad b_{12}(u_1(t))=\begin{cases}\frac{1}{8}, & |u_1(t)|<1,\\ -\frac{1}{8}, & |u_1(t)|>1,\end{cases}$$

$$b_{21}(u_2(t))=\begin{cases}\frac{1}{5}, & |u_2(t)|<1,\\ -\frac{1}{5}, & |u_2(t)|>1,\end{cases}\quad b_{22}(u_2(t))=\begin{cases}\frac{1}{4}, & |u_2(t)|<1,\\ -\frac{1}{4}, & |u_2(t)|>1,\end{cases}$$

$$c_{11}(u_1(t))=\begin{cases}\frac{1}{7}, & |u_1(t)|<1,\\ -\frac{1}{7}, & |u_1(t)|>1,\end{cases}\quad c_{12}(u_1(t))=\begin{cases}\frac{1}{6}, & |u_1(t)|<1,\\ -\frac{1}{6}, & |u_1(t)|>1,\end{cases}$$

$$c_{21}(u_2(t))=\begin{cases}\frac{1}{4}, & |u_2(t)|<1,\\ -\frac{1}{4}, & |u_2(t)|>1,\end{cases}\quad c_{22}(u_2(t))=\begin{cases}\frac{1}{7}, & |u_2(t)|<1,\\ -\frac{1}{7}, & |u_2(t)|>1,\end{cases}$$

In a system without impulse interference as shown in **Figure 3**, it can be calculated that $w=0.58<1$, $p=0.74>0$, $z=-0.51<0$, $o=-0.37<0$. According to Theorem 2, it can be concluded that system 1 is globally exponentially stable. The experimental results are shown in **Figure 4**.

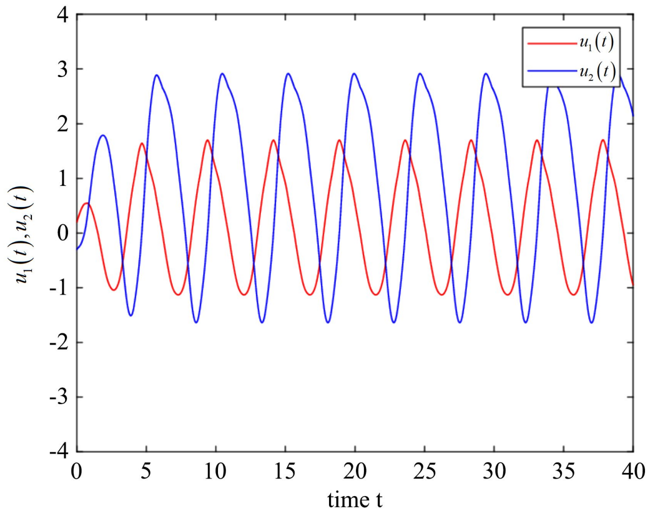

Figure 3. The orbits of $u_1(t)$ and $u_2(t)$ without impulse interference.

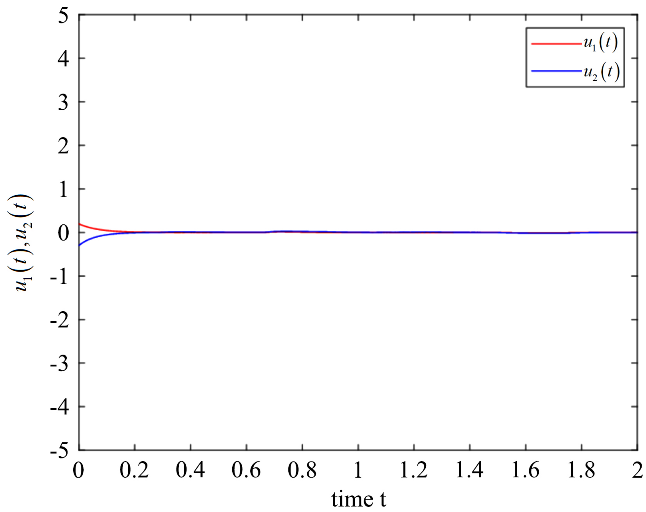

Figure 4. Orbits of $u_1(t)$ and $u_2(t)$ with impulse interference.

5. Conclusion

This paper studies the global stability problem of fractional-order fuzzy memristive neural networks with time delay and impulses. The existence of the equilibrium point is proved by using the knowledge of the contraction mapping principle. Then, the stability criteria of the system are given by establishing an appropriate Lyapunov function. Finally, two simulation results are provided to verify the theoretical results. In the future, research on neural networks, we will continue to study the stability problem of non-autonomous neural networks with diffusion and impulses.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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