

Algorithm to Assess Temporarily Reliability of a System Relying Upon the Intensity of the Failure and Recovery Flows of Three Autonomous Subsystems

Victor Kravets¹, Valerii Domanskyi², Illia Domanskyi³, Volodymyr Kravets⁴

¹Department of Automobiles and Automobile Sector, Dnipro University of Technology, Dnipro, Ukraine department

²Department of Electric Transport, O. M. Beketov National University of Urban Economy, Kharkiv, Ukraine

³Department of Power Engineering, Ukrainian State University of Science and Technologies, Dnipro, Ukraine

⁴Ivano-Frankivsk Professional College, Lviv National Environmental University, Ivano-Frankivsk, Ukraine

Email: ilya.domanskiy@gmail.com

How to cite this paper: Kravets, V., Domanskyi, V., Domanskyi, I. and Kravets, V. (2025) Algorithm to Assess Temporarily Reliability of a System Relying Upon the Intensity of the Failure and Recovery Flows of Three Autonomous Subsystems. *Open Journal of Applied Sciences*, 15, 1605-1624. <https://doi.org/10.4236/ojapps.2025.156110>

Received: April 27, 2025

Accepted: June 10, 2025

Published: June 13, 2025

Copyright © 2025 by author(s) and Scientific Research Publishing Inc. This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

An algorithm is being developed to conduct a computational experiment to study the dynamics of random processes in an asymmetric Markov chain with eight discrete states and continuous time. The algorithm is based on an exact analytical solution of the Kolmogorov differential equations, derived from the concept of harmonizing the mathematical description of models. The harmonization of the mathematical description is reflected in the asymmetric structure of the possible states of the system under study, which consists of three autonomous subsystems, the symmetric distribution of the roots of the Kolmogorov characteristic equation in the complex plane, the systematic representation of the used matrix tables and corresponding determinant tables. It is shown that presenting expanded formulas in the form of ordered tables allows for a compact description of a large volume of initial data, overcomes limitations related to the problem's dimensionality, and ensures the algorithm's adaptability to computer technologies, including the verification challenge. The algorithm is tested on a methodological example assessing the reliability of a military structure consisting of three separate autonomous branches of the armed forces. The probabilities of possible states of the military structure are determined depending on the intensity of loss and recovery flows in the branches. The abstract, dimensionless results of the state probability calculations are interpreted in terms of the physically significant factor—the time of combat readiness of the branches and the military structure as a whole. Verification of the calculation results, the algorithm, and the mathemati-

cal model is performed using a time-invariant condition that links the probabilities of the system's states. A general algorithm for studying asymmetric Markov chains and the corresponding Kolmogorov equations for complex systems is formulated.

Keywords

Reliability of Systems, Markovian Process, Kolmogorov Equation, State Probabilities

1. Introduction

Computational experiments are the key tool for studying multidimensional, non-linear, and controlled dynamic systems [1] [2], including random process dynamics [3] [4]. Theoretical foundation of a computational experiment involves the mathematical model-building techniques for dynamic systems [5]-[7], including Markovian models [8] [9], as well as methods of their analysis adapted to computer-aided technology. The analysis techniques are based upon the development of approximate numerical approaches to integrate systems of differential equations and formulation of accurate analytical procedures to solve systems of linear differential equations including Kolmogorov equation [10]-[12]. Problems admitting accurate analytical solutions cover analysis of random dynamic processes taking place in asymmetric Markovian chains with discrete states and continuous time for which Kolmogorov equations are a mathematical model [13] [14].

2. Literature Review

Compared with proximate numerical results, the advantages of accurate analytical techniques to solve various dynamic problems are well-known [15]. However, in view of objective reasons, only individual, partial problems admit analytical solutions [16] [17]. Consequently, the development of new analytical approaches to solve individual problems is extraordinary event being both actual and topical [18] [19]. Study of random Markovian processes with discrete states and continuous time amounts to consideration of mathematical models in the form of Kolmogorov equations [13]. In a number of problems, Kolmogorov equations are the systems of ordinary, linear, and homogeneous differential equations with constant coefficients. An analytical solution for such differential equations of the n th order is solving the relevant characteristic of the n th order equations [10]. To date, only the following individual analytical solutions of complete algebraic equations have been known: Cardano; Ferrari; Descartes-Euler; trigonometric; biquadratic and reciprocal equation; Moivre formula; etc. [11] [19].

3. The Study Objective and Tasks

In the context of asymmetric state graph of Markovian chain, the order of Kolmo-

gorov equations is defined as $n = 2^m$ where m is the number of autonomous subsystems in the system. In the case of a system with two or three autonomous subsystems, analytical solutions of quartic [17] and octic [20] Kolmogorov equations have been obtained. With increase of the problem order, it becomes problematic to describe systemically extensive information in the mathematical models and represent analytical solutions in the form being adapted to the current computer-aided technology and software [21]-[24]. The study is intended to solve the problem through the introduction of the specific ordered matrix tables as well as relevant determinants which order is identified by means of the problem dimension. The introduction of the ordered matrices and determinants helps provide compaction and visibility of the computational algorithm; the possibility of direct use of the standard software; and convenience while verifying both the algorithm and calculation results. The computational algorithm is illustrated on the systematic example of temporally reliability evaluation of a military structure depending upon the intensity of flows of losses and recoveries of three autonomous military structures. The computational algorithm is verified, and the calculation results of state probabilities of a military structure are interpreted temporally.

4. Methodology

A computational algorithm to evaluate the temporally reliability of a complex system relying upon the failure and recovery flows of three autonomous subsystems is developed based upon the obtained accurate analytical solution of octic Kolmogorov equations [20]. The Kolmogorov equations model dynamics of random processes in asymmetric Markovian chain with discrete states and continuous time.

4.1. Intensity Matrix

The ordered matrix of the intensity of the failure and recovery flows of three autonomous systems is introduced in following asymmetric form

$$R = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix}, \quad (1)$$

The ordered determinant is associated with the matrix

$$\Delta = \det[R]. \quad (2)$$

By reason of asymmetric structure of R matrix, it follows

$$\Delta = 0,$$

i.e., R matrix is of the specific nature. The property serves as a verification criterion for mathematical models of the considered problem class.

4.2. Characteristic Equation

Characteristic determinant is compiled for the ordered matrix R

$$\Delta(v) = \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 - v & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 - v & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 - v & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 - v & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 - v & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 - v & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 - v & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 - v \end{vmatrix}, \tag{3}$$

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgements and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, non-italic) in addition to the style provided by the drop-down menu to differentiate the head from the text.

The characteristic equation is solved

$$\Delta(v) = 0.$$

Real conjugated relative to a symmetry center characteristic equation roots are determined being expressed in the intensity of the failure and recovery flows of three autonomous subsystems

$$\begin{aligned} v_1 &= 0, & v_2 &= -\lambda_1 - \mu_1 - \lambda_2 - \mu_2 - \lambda_3 - \mu_3, \\ v_3 &= -\lambda_1 - \mu_1, & v_4 &= -\lambda_2 - \mu_2 - \lambda_3 - \mu_3, \\ v_5 &= -\lambda_2 - \mu_2, & v_6 &= -\lambda_1 - \mu_1 - \lambda_3 - \mu_3, \\ v_7 &= -\lambda_3 - \mu_3, & v_8 &= -\lambda_1 - \mu_1 - \lambda_2 + \mu_2. \end{aligned} \tag{4}$$

Each of the eight roots are verified using the normalized determinants

$$\Delta(v_k) = 0, \quad (k = 1, 2, 3, \dots, 8). \tag{5}$$

4.3. Resolving Octal Square Matrix

Column matrix of the specified initial conditions for probabilities of the eight asymmetric states is introduced

$$[P_i(0)], \quad (i = 1, 2, 3, \dots, 8). \tag{6}$$

In the eight determinants $\Delta(v_k), k = 1, 2, 3, \dots, 8$, i^{th} column is replaced by $[-P_i(0)]$ column; the normalized determinants of a following type are formed

$$\Delta_i(v_k), \quad (i, k = 1, 2, 3, \dots, 8), \tag{7}$$

being components of octic square matrix $[\Delta_i(v_k)]$.

4.4. Resolving Octal Column Matrix

Products of a following type are formed

$$\Pi_k = \prod_{\substack{s=2 \\ s \neq k}}^8 (v_k - v_s), \quad (8)$$

for each k^{th} root $k = 1, 2, 3, \dots, 8$; the column matrix is compiled

$$\begin{bmatrix} e^{v_k \cdot t} \\ \Pi_k \end{bmatrix}, \quad (9)$$

where t is Markovian process time.

4.5. Probabilities of States

Matrix probability formula of temporal eight states of the system is applied for the purpose

$$[P_i(t)] = [\Delta_i(v_k)] \cdot \begin{bmatrix} e^{v_k \cdot t} \\ \Pi_k \end{bmatrix}, \quad (i, k = 1, 2, 3, \dots, 8).$$

4.6. Verification of the Mathematical Model

The result is verified if $t = 0$

$$[P_i(0)] = [\Delta_i(v_k)] \cdot \begin{bmatrix} 1 \\ \Pi_k \end{bmatrix}, \quad (11)$$

where $[P_i(0)]$ is a column matrix of the specified initial conditions.

The stable condition is used to verify probabilities of states for arbitrary time moment t

$$\sum_{i=1}^8 P_i(t) = 1. \quad (12)$$

The stabilized (stationary) mode of random Markovian process is defined with the help of a limiting process $t \rightarrow \infty$ through a matrix formula in the form of the column matrix

$$[P_i(\infty)] = \begin{bmatrix} \Delta_i(v_1) \\ \Pi_1 \end{bmatrix}, \quad (13)$$

In this context, marginal probabilities of states $P_i(\infty)$ meet the condition

$$\sum_{i=1}^8 P_i(\infty) = 1. \quad (14)$$

Moreover, following algebraic approach helps identify the marginal probabilities. Kolmogorov differential equations

$$\frac{d}{dt} [P_i(t)] = R \cdot [P_i(t)] \quad (15)$$

are simplified in terms of the stabilized (stationary) mode of a continuous Mar-

kovian process owing to

$$\frac{d}{dt}[P_i(\infty)] = [0_i], \quad (16)$$

where $[0_i]$ is a zero octic column matrix, *i.e.*, Kolmogorov equations degenerate into the system of eight linear homogeneous algebraic equations

$$R \cdot [P_i(\infty)] = [0_i]. \quad (17)$$

where $[P_i(0)]$ is a column matrix of the specified initial conditions.

The stable condition is used to verify probabilities of states for arbitrary time moment t

$$\sum_{i=1}^8 P_i(\infty) = 1.$$

The derived eight variants of equivalent systems of linear homogeneous algebraic equations has the only identically equal solutions being verification criterion for the results and the mathematical model on the whole.

4.7. Interpretation of the State Probabilities

The intensity of the failure and recovery flows of three autonomous subsystems are defined using statistical approaches within $[0, T]$ time interval identified depending upon peculiarities of the applied problem being solved. Within the considered time interval, flow intensities are referred to as specified and constant. On the time interval T , continuous Markovian process has two different modes:

- transient (dynamic) within the time interval T_n and
- stabilized (stationary) within the time interval T_y . The modes are bound by the obvious condition

$$T_n + T_y = T.$$

Qualitatively, the nature of continuous Markovian process within time interval T is mainly defined by means of the specified initial conditions as well as the total of exponentials which indices are equal to characteristic equation roots. Transition process on the T time interval is defined through the total of the damped exponentials corresponding to negative roots of the characteristic equation. Stationary process within T_y time interval is identified asymptotically with the help of a zero root of the characteristic equation if $t \rightarrow \infty$. During the following time intervals, the intensity of the failure and recovery flows of three subsystems may vary discretely. Subsequently, dynamics of random Markovian process will be determined through new relevant roots of the characteristic equation; limit probabilities of states $P_i(\infty)$ at a previous time step are assumed as initial conditions.

To interpret probabilities of the system states temporally, time integration takes place within $[0, T]$ interval of an invariant condition

$$\int_0^T \sum_{i=1}^8 P_i(t) \cdot dt = \int_0^T 1 \cdot dt \quad (18)$$

whence it follows

$$\sum_{i=1}^8 \int_0^T P_i(t) dt = T \quad (19)$$

or

$$\sum_{i=1}^8 \int_0^{T_n} P_i(t) dt + \sum_{i=1}^8 \int_{T_n}^T P_i(\infty) dt = T, \quad (20)$$

where

$$\int_0^{T_n} P_i(t) dt = T_{in}, \quad \int_{T_n}^T P_i(\infty) dt = T_{iy} \quad (21)$$

T_{in} is the duration of stay at i^{th} state in the transitional mode; and

T_{iy} is the duration of stay at i^{th} state in the stationary mode,

i.e.,

$$T_{in} + T_{iy} = T_i \quad (22)$$

T_i is the system duration stay at i^{th} state.

In this context

$$T_{iy} = P_i(\infty) \int_{T_n}^T dt = P_i(\infty) \cdot (T - T_n) = P_i(\infty) \cdot T_y; \quad (23)$$

$$T_{in} = \sum_{k=1}^8 \frac{\Delta_i(v_k)}{\Pi_k} \int_0^{T_n} e^{v_k t} dt = \frac{\Delta_i(v_1)}{\Pi_1} T_n + \sum_{k=2}^8 \frac{\Delta_i(v_k)}{\Pi_k} \cdot \frac{1}{v_k} (e^{v_k T_n} - 1) \quad (24)$$

In such a way, verification condition takes the form

$$\sum_{i=1}^8 T_i = T. \quad (25)$$

5. Research Results

The computational algorithm for random continuous Markovian processes has been developed based upon analytical solutions. It is useful for a wide range of the applied problems [25]-[30]. The paper considers systematic example of a problem to evaluate the state probabilities of a military structure involving three autonomous substructures-branches of the armed forces: land arms, airpower, and marine troops. During the fighting, loss intensity of the military branches is assumed as statistically known. The intensity of recovery flows of the service arms is assumed as be specified and controlled depending upon the available reserves. Resulting from analytical modelling; restricting intensity of loss flows; and varying intensity of recovery flows it becomes possible to control efficiently random processes, and make informed as well as reasonable decisions if resources are limited.

5.1. Formulation of the Applied Problem to Be Used in a Military Field

Following indexing is introduced for the considered branches of the armed forces: land arms (1), air power (2), and marine troops (3). Probable states of every sub-

structure are defined as combat-effective \oplus or combat-ineffective \ominus . Hence, the military structure has eight probable states (Figure 1).

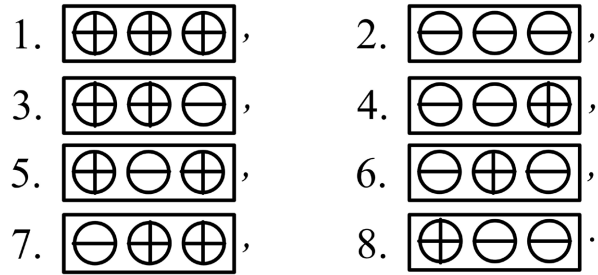


Figure 1. Probable states of the military structure.

Within the interval of military operation T (being a week, *i.e.*, seven days) specified by the problem, intensity of Poissonian flows of losses ($\lambda_1, \lambda_2, \lambda_3$) and recoveries (μ_1, μ_2, μ_3) of the combatant branches are assumed to be statistically defined on the average and constant

$$\lambda_1 = 2; \lambda_2 = 1; \lambda_3 = 0,5; \mu_1 = 1,5; \mu_2 = 0,5; \mu_3 = 2. \tag{26}$$

Initial state of the military structure ($t = 0$) is considered as the known; it corresponds to a probable state 1, *i.e.* the land arms, airpower, and marine troops are combat-effective. While using probabilities of the military structure states, we obtain

$$\begin{aligned} P_1(0) = 1; & P_3(0) = 0; P_5(0) = 0; P_7(0) = 0; \\ P_2(0) = 0; & P_4(0) = 0; P_6(0) = 0; P_8(0) = 0. \end{aligned} \tag{27}$$

It is required to identify probable states of the military structure at the current time

$$P_i(t) \quad (i = 1, 2, 3, 4, 5, 6, 7, 8) \tag{28}$$

and the anticipated future states ($t \rightarrow \infty$), *i.e.* limit probability of the states

$$P_i(\infty) \quad (i = 1, 2, 3, 4, 5, 6, 7, 8). \tag{29}$$

5.2. Computation and Verification of Characteristic Equation Roots

Relying upon the specified loss and recovery flows of the three combatant branches, analytical formulas are applied to calculate eight roots of the characteristic equation

$$\begin{aligned} \nu_1 = 0, & \quad \nu_2 = -7,5, \\ \nu_3 = -3,5, & \quad \nu_4 = -4, \\ \nu_5 = -1,5, & \quad \nu_6 = -6, \\ \nu_7 = -2,5, & \quad \nu_8 = -5. \end{aligned} \tag{30}$$

Each of the eight ν_k ($k = 1, 2, 3, \dots, 8$) roots is verified with the help of a characteristic determinant $\Delta(\nu)$ while k^{th} root substitution in the characteristic determinant, and its evaluation $\Delta(\nu_k)$ if

$$\Delta(v_k) = 0 \quad (k = 1, 2, 3, 4, 5, 6, 7, 8). \tag{31}$$

5.3. Illustration of the Expanded Calculation Formulas

Formulas of temporal probabilities of the structure states

$$P_1(t) = \sum_{k=1}^8 \frac{\Delta_1(v_k)}{\Pi_k} \cdot e^{v_k \cdot t}; \quad P_2(t) = \sum_{k=1}^8 \frac{\Delta_2(v_k)}{\Pi_k} \cdot e^{v_k \cdot t}; \quad \dots \tag{32}$$

where

$$\begin{aligned} \Pi_1 &= (v_1 - v_2)(v_1 - v_3)(v_1 - v_4)(v_1 - v_5)(v_1 - v_6)(v_1 - v_7)(v_1 - v_8) \\ \Pi_2 &= (v_2 - v_1)(v_2 - v_3)(v_2 - v_4)(v_2 - v_5)(v_2 - v_6)(v_2 - v_7)(v_2 - v_8), \\ \Pi_3 &= (v_3 - v_1)(v_3 - v_2)(v_3 - v_4)(v_3 - v_5)(v_3 - v_6)(v_3 - v_7)(v_3 - v_8), \\ &\vdots \\ \Pi_8 &= (v_8 - v_1)(v_8 - v_2)(v_8 - v_3)(v_8 - v_4)(v_8 - v_5)(v_8 - v_6)(v_8 - v_7). \end{aligned} \tag{33}$$

$$\begin{aligned} \Delta_1(v_k) &= \begin{vmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & -\mu_1 - \mu_2 - \mu_3 - v_k & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & -\mu_1 - \lambda_2 - \lambda_3 - v_k & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 - v_k & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 - v_k & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 - v_k & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 - v_k & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 - v_k \end{vmatrix}, \\ \Delta_2(v_k) &= \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 - v_k & -P_1(0) & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -P_2(0) & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & -P_3(0) & -\mu_1 - \lambda_2 - \lambda_3 - v_k & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & -P_4(0) & 0 & -\lambda_1 - \mu_2 - \mu_3 - v_k & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & -P_5(0) & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 - v_k & 0 & 0 & \mu_1 \\ 0 & -P_6(0) & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 - v_k & \lambda_1 & 0 \\ \lambda_3 & -P_7(0) & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 - v_k & 0 \\ 0 & -P_8(0) & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 - v_k \end{vmatrix}, \\ \Delta_3(v_k) &= \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 - v_k & 0 & -P_1(0) & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 - v_k & -P_2(0) & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -P_3(0) & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & -P_4(0) & -\lambda_1 - \mu_2 - \mu_3 - v_k & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & -P_5(0) & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 - v_k & 0 & 0 & \mu_1 \\ 0 & \mu_2 & -P_6(0) & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 - v_k & \lambda_1 & 0 \\ \lambda_3 & 0 & -P_7(0) & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 - v_k & 0 \\ 0 & \mu_3 & -P_8(0) & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 - v_k \end{vmatrix}, \\ \Delta_8(v_k) &= \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 - v_k & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & -P_1(0) \\ 0 & -\mu_1 - \mu_2 - \mu_3 - v_k & 0 & \lambda_1 & 0 & \lambda_2 & 0 & -P_2(0) \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 - v_k & 0 & 0 & \mu_3 & 0 & -P_3(0) \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 - v_k & \lambda_3 & 0 & \lambda_2 & -P_4(0) \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 - v_k & 0 & 0 & -P_5(0) \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 - v_k & \lambda_1 & -P_6(0) \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 - v_k & -P_7(0) \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -P_8(0) \end{vmatrix} \end{aligned}$$

or

$$\Delta_1(v_1) = \begin{pmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix},$$

$$\Pi_1 = (\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_2 + \mu_3)(\lambda_1 + \mu_1)(\lambda_2 + \lambda_3 + \mu_2 + \mu_3)(\lambda_2 + \mu_2)(\lambda_1 + \lambda_3 + \mu_1 + \mu_3)(\lambda_3 + \mu_3)(\lambda_1 + \lambda_2 + \mu_1 + \mu_2);$$

$$\Delta_1(v_2) = \begin{pmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & \lambda_1 + \lambda_2 + \lambda_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & \lambda_1 + \mu_2 + \mu_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & \mu_1 + \lambda_2 + \lambda_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & \mu_1 + \lambda_2 + \mu_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & \lambda_1 + \mu_2 + \lambda_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & \mu_1 + \mu_2 + \lambda_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & \lambda_1 + \lambda_2 + \mu_3 \end{pmatrix},$$

$$\Pi_2 = (-\lambda_1 - \lambda_2 - \lambda_3 - \mu_1 - \mu_2 - \mu_3)(-\lambda_1 - \mu_1)(-\lambda_2 - \lambda_3 - \mu_2 - \mu_3)(-\lambda_2 - \mu_2)(-\lambda_1 - \lambda_3 - \mu_1 - \mu_3)(-\lambda_3 - \mu_3)(-\lambda_1 - \lambda_2 - \mu_1 - \mu_2);$$

$$\Delta_1(v_3) = \begin{pmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & \lambda_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & \lambda_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & \mu_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & \mu_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & \lambda_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & \mu_1 - \lambda_2 - \mu_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & \lambda_1 - \mu_2 - \lambda_3 \end{pmatrix},$$

$$\Pi_3 = (-\lambda_1 + \lambda_2 + \lambda_3 - \mu_1 + \mu_2 + \mu_3)(-\lambda_1 - \mu_1)(\lambda_2 + \lambda_3 + \mu_2 + \mu_3)(\lambda_2 + \mu_2)(-\lambda_1 + \lambda_3 - \mu_1 + \mu_3)(\lambda_3 + \mu_3)(-\lambda_1 + \lambda_2 - \mu_1 + \mu_2);$$

5.4. Computational Results

Time interval in a day is considered

$$0 \leq t \leq 7.$$

The computational results are summarized in **Table 1**. **Figure 2** illustrates them.

Table 1. Computational results.

<i>t</i>	0	1	2	3	4	5	6
P1	1	0.175	0.126	0.117	0.115	0.114	0.114
P2	0	0.053	0.072	0.075	0.076	0.076	0.076
P3	0	0.218	0.168	0.156	0.153	0.153	0.152
P4	0	0.042	0.054	0.056	0.057	0.057	0.057
P5	0	0.189	0.218	0.226	0.228	0.228	0.229
P6	0	0.049	0.042	0.039	0.038	0.038	0.038
P7	0	0.039	0.031	0.029	0.029	0.029	0.029

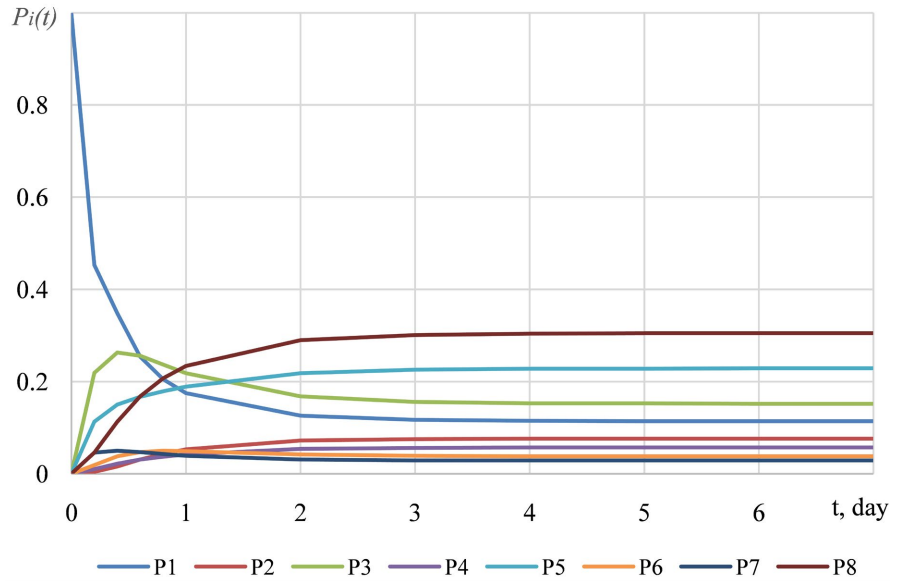


Figure 2. Temporal change in state probabilities of the military structure.

where $[0, 5]$ is time interval of a transient process; and $(5, 7]$ is time interval of stationary (stabilized) mode.

5.5. Verification of the Computational Results

At every time moment t , the computations are verified in terms of the invariant condition

$$\sum_{i=1}^8 P_i(t) = 1.$$

It is easy to show with the help of the Table that the total of each column elements is equal to 1. Specifically, it should be mentioned that under $t = 0$ and $t \rightarrow \infty$ the general formulas of state probabilities are simplified and take the form

$$\begin{aligned} P_1(0) &= \sum_{k=1}^8 \frac{\Delta_1(v_k)}{\Pi_k}; & P_2(0) &= \sum_{k=1}^8 \frac{\Delta_2(v_k)}{\Pi_k}; & \dots \\ P_1(\infty) &= \frac{\Delta_1(v_1)}{\Pi_1}; & P_2(\infty) &= \frac{\Delta_2(v_1)}{\Pi_1}; & \dots \end{aligned} \tag{34}$$

where the specified initial conditions $P_1(0), P_2(0), \dots$ as well as limit state probabilities $P_1(\infty), P_2(\infty), \dots$ comply with the equations

$$\sum_{i=1}^8 P_i(0) = 1; \quad \sum_{i=1}^8 P_i(\infty) = 1.$$

5.6. Algebraic Approach to Calculate Limit State Probabilities

Eight variants of equivalent systems of algebraic levels relative to the limit state probabilities are compiled in expanded form. Limit state probabilities are calculated using Cramer’s formulas.

Variant 1.

$$\begin{pmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\
 \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\
 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\
 \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\
 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\
 \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\
 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3
 \end{pmatrix}
 \begin{pmatrix}
 P_1(\infty) \\
 P_2(\infty) \\
 P_3(\infty) \\
 P_4(\infty) \\
 P_5(\infty) \\
 P_6(\infty) \\
 P_7(\infty) \\
 P_8(\infty)
 \end{pmatrix}
 =
 \begin{pmatrix}
 1 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}
 \quad (35)$$

Solution is

$$P_i(\infty) = \frac{\Delta_i}{\Delta}, \quad (i = 1, 2, 3, \dots, 8), \quad (36)$$

where

$$\Delta = \begin{vmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\
 \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\
 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\
 \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\
 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\
 \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\
 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3
 \end{vmatrix}, \quad (37)$$

$$\Delta_1 = \begin{vmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\
 0 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\
 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\
 0 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\
 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\
 0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\
 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3
 \end{vmatrix}, \quad (38)$$

$$\Delta_2 = \begin{vmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\
 \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\
 0 & 0 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\
 \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\
 0 & 0 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\
 \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\
 0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3
 \end{vmatrix}, \quad (39)$$

$$\Delta_3 = \begin{vmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\
 \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\
 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\
 \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\
 0 & \mu_2 & 0 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\
 \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\
 0 & \mu_3 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3
 \end{vmatrix}, \quad (40)$$

$$\Delta_8 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & 0 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & 0 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & 0 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0 \end{pmatrix}. \quad (41)$$

Variant 2.

$$\begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix} \cdot \begin{pmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (42)$$

Solution is

$$P_i(\infty) = \frac{\Delta_i}{\Delta}, \quad (i=1,2,3,\dots,8), \quad (43)$$

where

$$\Delta = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix}, \quad (44)$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ 0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix}, \quad (45)$$

$$\Delta_2 = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & 0 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & 0 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix}, \quad (46)$$

$$\Delta_3 = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & 0 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 - \mu_2 - \lambda_3 \end{pmatrix}, \quad (47)$$

$$\Delta_8 = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & 0 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0 \end{pmatrix}. \quad (48)$$

3, 4, 5, 6, and 7 variants are compiled and solved analogously.

Variant 8.

$$\begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (49)$$

solution is

$$P_i(\infty) = \frac{\Delta_i}{\Delta}, \quad (i = 1, 2, 3, \dots, 8), \quad (50)$$

where

$$\Delta = \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \quad (51)$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ 0 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ 0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \quad (52)$$

$$\Delta_2 = \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & 0 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & 0 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (53)$$

$$\Delta_3 = \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & 0 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (54)$$

$$\Delta_8 = \begin{vmatrix} -\lambda_1 - \lambda_2 - \lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1 - \mu_2 - \mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & 0 \\ \lambda_1 & 0 & -\mu_1 - \lambda_2 - \lambda_3 & 0 & 0 & \mu_3 & 0 & 0 \\ 0 & \mu_1 & 0 & -\lambda_1 - \mu_2 - \mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1 - \mu_2 - \lambda_3 & 0 & 0 & 0 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1 - \lambda_2 - \mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1 - \lambda_2 - \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}. \quad (55)$$

Solution equivalence of eight variants of the compiled systems of algebraic equations is the verification criterion for the calculation limit probabilities of states as well as initial mathematical model of asymmetric Markovian chain with eight states and continuous time.

5.7. Temporal Identification of State Probabilities of a Military Structure

Computational results of state probabilities of a military structure have helped understand that within the considered time interval ($T = 7$), $T_{\pi} = 5$ corresponds to a transitional process, and $T_{\gamma} = 2$ corresponds to the stabilized stationary mode respectively. Consequently, probable stay time of a military structure in the i^{th} state in a stationary mode is defined as follows

$$T_{iy} = P_i(\infty) \cdot 2 \quad (i = 1, 2, 3, 4, 5, 6, 7, 8), \quad (56)$$

where

$$P_1(\infty) = 0,1145; \quad P_2(\infty) = 0,076; \quad P_3(\infty) = 0,1525; \quad P_4(\infty) = 0,057; \\ P_5(\infty) = 0,2285; \quad P_6(\infty) = 0,038; \quad P_7(\infty) = 0,0285; \quad P_8(\infty) = 0,305.$$

i.e.

$$T_{1y} = 0,229; \quad T_{2y} = 0,152; \quad T_{3y} = 0,305; \quad T_{4y} = 0,114;$$

$$T_{5y} = 0,457; \quad T_{6y} = 0,076; \quad T_{7y} = 0,057; \quad T_{8y} = 0,610.$$

Following equation is the verification criterion

$$\sum_{i=1}^8 T_{iy} = 2. \tag{57}$$

Under the transitional process, probable stay time of a military structure in the i th state is identified using the formula

$$T_{in} = \frac{\Delta_i(v_1)}{\Pi_1} \cdot 5 + \sum_{k=2}^8 \frac{\Delta_i(v_k)}{\Pi_k} \cdot \frac{1}{v_k} (e^{v_k \cdot 5} - 1), \tag{58}$$

i.e., resulting from the calculations we determine that

$$T_{1n} = 0,87; \quad T_{2n} = 0,315; \quad T_{3n} = 0,878; \quad T_{4n} = 0,243;$$

$$T_{5n} = 1,028; \quad T_{6n} = 0,197; \quad T_{7n} = 0,165; \quad T_{8n} = 1,304.$$

The equality is the verification criterion

$$\sum_{i=1}^8 T_{in} = 5. \tag{59}$$

In such a way, probable stay time of a military structure in the i th during the analyzed $T = 7$ period is defined in the form

$$T_i = T_{in} + T_{iy} \quad (i = 1, 2, 3, 4, 5, 6, 7, 8), \tag{60}$$

i.e.,

$$T_1 = 1,099; \quad T_2 = 0,467; \quad T_3 = 1,183; \quad T_4 = 0,357;$$

$$T_5 = 1,485; \quad T_6 = 0,274; \quad T_7 = 0,222; \quad T_8 = 1,913.$$

Hence, the verification criterion looks like

$$\sum_{i=1}^8 T_i = 7. \tag{61}$$

The calculations show that under the specified initial data, the longest stay time of the military structure corresponds to state eight (*i.e.* $T_8 = 1.913$ days) where land arms are combat-effective, but air power and marine troops are combat-ineffective. The shortest stay time of the military structure corresponds to state seven (*i.e.* $T_7 = 0.222$) where land arms are combat-ineffective, but air power and marine troops are combat-effective. Stay time of each of the three service arms is introduced concerning combat-effective $T(+)$ or combat-ineffective $T(-)$ states for

land arms $T_1(+)$ or $T_1(-)$;

air power $T_2(+)$ or $T_2(-)$; and

marine troops $T_3(+)$ or $T_3(-)$

which are defined through following formulas

$$T_1(+)=T_1+T_3+T_5+T_8, \quad T_1(-)=T_2+T_4+T_6+T_7;$$

$$T_2(+)=T_1+T_3+T_6+T_7, \quad T_2(-)=T_2+T_4+T_5+T_8; \tag{62}$$

$$T_3(+)=T_1+T_4+T_5+T_7, \quad T_3(-)=T_2+T_3+T_6+T_8.$$

Within the analyzed one-week interval we obtain

$$\begin{aligned} T_1(+)&= 5,680, & T_1(-)&= 1,320; \\ T_2(+)&= 2,778, & T_2(-)&= 4,222; \\ T_3(+)&= 3,163, & T_3(-)&= 3,837. \end{aligned} \quad (63)$$

The equations are the conditions to verify the calculations

$$\begin{aligned} T_1(+)+T_1(-)&= 7; \\ T_2(+)+T_2(-)&= 7; \\ T_3(+)+T_3(-)&= 7. \end{aligned} \quad (64)$$

The interpreted probability of a military structure states helps forecast battle condition temporally and make the informed as well as reasonable decisions for the expedient control of random processes while varying intensity of recovery and loss flows of service arms.

6. Conclusions

Algorithm has been developed for computational experiments to assess the reliability of a multidimensional dynamic system temporally under the known initial state and the specified intensity of the failure and recovery flows of three autonomous subsystems within the analyzed time interval. The algorithm is based upon the results of the proposed analytical solution of octic Kolmogorov equations for asymmetric Markovian chain obtained relying upon a harmonization concept of mathematical description of models [31]-[33]. In this context, mathematical description harmonization is demonstrated in the form of the ordered octic matrix tables as well as relevant determinant tables. The applied table record of the expanded calculation formulas provides a systematization of the large initial data amount; the possibility of direct implementation of the algorithm on a computer; reducing the probability of programming errors owing to the order of the tables; and favours verification process of the algorithm.

The general algorithm has been used for the applied problem in a military field where among other things methods are used to identify period of combat-effective state of service arms, and evaluate overall state of a military structure under the conditions of intensive loss and recovery flows.

7. Implications

Based upon the obtained results, general algorithm is proposed defining probabilities of system states depending upon the intensity of the failure and recovery flows of a random number of autonomous subsystems. The algorithm includes following sequence of operations:

- 1) development of a set of asymmetric system states in the whole, being equal to $n = 2^m$ where m is the number of autonomous systems;
- 2) formation of square matrix of R intensities of n power according the specified and constant within the considered time interval $[0, T)$ failure λ_j and recovery

μ_j ($j = 1, 2, 3, \dots, m$) flows of autonomous subsystems;

3) formation of a characteristic determinant $\Delta(\nu)$ of intensity matrix R ;

4) determination of the total of ν_k ($k = 1, 2, 3, \dots, n$) roots of a characteristic equation $\Delta(\nu) = 0$ relying upon the intensity of the failure and recovery flows of the subsystems;

5) identification of various products of differences of roots of a characteristic equation Π_k ($k = 1, 2, 3, \dots, n$);

6) formation of a square matrix $[D_i(\nu_k)]$ ($i, k = 1, 2, 3, \dots, n$) on a characteristic determinant $\Delta(\nu)$ depending upon k^{th} roots and i^{th} states of the system while using a column matrix of the known initial conditions of the system state;

7) formation of exponential column matrix

$$\begin{bmatrix} e^{\nu_k \cdot t} \\ \Pi_k \end{bmatrix}$$

using the totality of roots ν_k and time t from the restricted interval $[0, T]$ where the intensities of the failure and recovery flows of the subsystems are assumed to be known and constant;

8) temporal determination of the system states $P_i(t)$ ($i = 1, 2, 3, \dots, n$) within the analyzed interval $[0, T]$ resulting from a product of the compiled matrices

$$[\Delta_i(\nu_k)] \cdot \begin{bmatrix} e^{\nu_k \cdot t} \\ \Pi_k \end{bmatrix};$$

9) solving the problem to define probabilities of the system states within the subsequent time intervals under the discrete changes in the failure and recovery flows is in the proposed algorithm cycle. Limit probabilities of states at the previous steps are assumed as the initial conditions at a subsequent step interval; and

10) verification and interpretation process of calculation results is based upon

$$\sum_{i=1}^n P_i(t) = 1,$$

equation, being invariable relative to time.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] Igdalov, I.M., Kuchma, L.D., Polyakov, N.V. and Sheptun, Y.D. (2010) *Dinamicheskoe proektirovanie raket. Zadachi dinamiki raket i ix kosmicheskix stupeney* [Dynamic Design of Missiles. The Tasks of the Dynamics of Rockets and Their Cosmic Stages]. Dnipro National University Publisher, 264.
- [2] Kravets, V.V., Bass, K.M., Kravets, T.V. and Tokar, L.V. (2015) Dynamic Design of Ground Transport with the Help of Computational Experiment. *Mechanics, Materials Science & Engineering Journal*, **1**, 105-111.
- [3] Hajek, B. (2015) *Random Processes for Engineers*. Cambridge University Press. <https://doi.org/10.1017/cbo9781316164600>
- [4] Domanskyi, V., Domanskyi, I., Zakurdai, S. and Liubarskyi, D. (2022) Development

- of Technologies for Selecting Energy-Efficient Power Supply Circuits of Railway Traction Networks. *Technology audit and production reserves*, **4**, 47-54.
<https://doi.org/10.15587/2706-5448.2022.263961>
- [5] Glushkov, V.M. (1980). Fundamental'nye issledovaniya i texnologiya programmiravaniya [Fundamental Research and Programming Technology]. *Programmirovaniye*, **2**, 3-13.
- [6] Andrews, J.G. and McLone, R.R. (1976) *Mathematical Modeling*. Butterworth-Heinemann, 260.
- [7] Van Tassel, D. (1978) *Design, Efficiency, Debugging, and Testing*. 2 Edition, Prentice Hall, 328.
- [8] Pender, J. (2014) Nonstationary Loss Queues via Cumulant Moment Approximations. *Probability in the Engineering and Informational Sciences*, **29**, 27-49.
<https://doi.org/10.1017/s0269964814000205>
- [9] Sadeghian, P., Han, M., Håkansson, J. and Zhao, M.X. (2024) Testing Feasibility of Using a Hidden Markov Model on Predicting Human Mobility Based on GPS Tracking Data. *Transportmetrica B: Transport Dynamics*, **12**, Article ID: 2336037.
<https://doi.org/10.1080/21680566.2024.2336037>
- [10] Ovchinnikov, P.P. (2000). *Vishha matematika [Higher Mathematics]*. Part 2. Tehnika Publisher, 797.
- [11] Korn, G. and Korn, T. (1984) *Spravochnik po matematike dlya nauchnyh rabotnikov i inzhenerov [Mathematical Handbook for Scientists and Engineers]*. Nauka Publisher, 832.
- [12] Seabrook, E. and Wiskott, L. (2023) A Tutorial on the Spectral Theory of Markov Chains. *Neural Computation*, **35**, 1713-1796. https://doi.org/10.1162/neco_a_01611
- [13] Ventcel', E.S. and Ovcharov, L.A. (1991) *Theory of Random Processes and Its Engineering Application*. Nauka Publisher, 384.
- [14] Kravets, V.V., Kapitsa, M.I., Domanskyi, I.V., Kravets, V.V., Hryshechkina, T.S., Zakurday, S.O., et al. (2024) Analytical Solution of Kolmogorov Equations for Asymmetric Markov Chains with Four and Eight States. *Mathematics and Computer Science: Contemporary Developments*, **10**, 140-162.
<https://doi.org/10.9734/bpi/mcscd/v10/3410>
- [15] Blexman, I.I., Myshkis, A.V. and Panovko, Y.G. (1983) *Mexanika i prikladnaya matematika: logika i osobennosti prilozhenij matematiki [Mechanics and Applied Mathematics: Logic and Especially Applications of Mathematics]*. Nauka Publisher, 328.
- [16] Kravets, V., Kravets, V. and Burov, O. (2021) Analytical Modeling of the Dynamic System of the Fourth Order. *Transactions on Machine Learning and Artificial Intelligence*, **9**, 14-24. <https://doi.org/10.14738/tmlai.93.9947>
- [17] Kravets, V.V., Bass, K.M., Kravets, V.V. and Tokar, L.A. (2014) Analytical Solution of Kolmogorov Equations for Four-Condition Homogenous, Symmetric and Ergodic System. *Open Journal of Applied Sciences*, **4**, 497-500.
<https://doi.org/10.4236/ojapps.2014.410048>
- [18] Alpatov, A., Kravets, V., Kravets, V. and Lapkhanov, E. (2021) Analytical Modeling of the Binary Dynamic Circuit Motion. *Transactions on Machine Learning and Artificial Intelligence*, **9**, 23-32. <https://doi.org/10.14738/tmlai.95.10922>
- [19] Kravets, V. and Chibushov, Y. (1994) Method of Finding the Analytical Solution of the Algebraic Particular Aspect Equation. *Poland, Rzeszow, Folia Scientiarum Universitatis Technicae Resoviensis, Math*, **16**, 104-117.
- [20] Kravets, V., Kapitsa, M., Domanskyi, I., Kravets, V., Hryshechkina, T. and Zakurday,

- S. (2024) Devising an Analytical Method for Solving the Eighth-Order Kolmogorov Equations for an Asymmetric Markov Chain. *Eastern-European Journal of Enterprise Technologies*, **5**, 33-41. <https://doi.org/10.15587/1729-4061.2024.312971>
- [21] Domanskyi, I.V. (2016) Osnovy enerhoefektyvnosti elektrychnykh system z tiahovymy navantazhenniamy. Kharkiv: TOV "Tsentri informatsiyi transportu Ukrainy", 224. http://library.kpi.kharkov.ua/files/new_postupleniya/oceesi.pdf
- [22] Bellman, R. (1997) Introduction to Matrix Analysis, Second Edition. Classics in Applied Mathematics, 403. <http://dx.doi.org/10.1137/1.9781611971170>
- [23] Sigorskij, V.P. (1977) Matematicheskij apparat inzhenera [Mathematical Engineer Unit]. Tehnika Publisher, 768.
- [24] Ayyub, B. and Mecuen, R. (1997) Probability, Statistics & Reliability for Engineers. CRC Press, 663.
- [25] Asmussen, S. (2008) Applied Probability and Queues. 2nd Edition, Springer, 438. https://books.google.co.uk/books?id=X1CacQAACAAJ&pg=PR1&hl=ru&source=gb_s_selected_pages&cad=1#v=onepage&q&f=false
- [26] Kravets, V.V., Kravets, V.V. and Burov, O.V. (2016) Reliability of Systems Part 2. Dynamics of Failures. Omni Scriptum GmbH & Co. KO., 2, 100.
- [27] Chen, X.Q., Li, L. and Shi, Q.X. (2015) Stochastic Evolutions of Dynamic Traffic Flow: Modeling and Applications. Springer. <https://doi.org/10.1007/978-3-662-44572-3>
- [28] Yun, M., Qin, W., Yang, X. and Liang, F. (2019) Estimation of Urban Route Travel Time Distribution Using Markov Chains and Pair-Copula Construction. *Transportmetrica B: Transport Dynamics*, **7**, 1521-1552. <https://doi.org/10.1080/21680566.2019.1637798>
- [29] Suliankatchi Abdulkader, R., Deneshkumar, V., Senthamarai Kannan, K., Koyilil, V., Paes, A.T. and Sebastian, T. (2021) An Application of Markov Chain Modeling and Semi-Parametric Regression for Recurrent Events in Health Data. *Communications in Statistics: Case Studies, Data Analysis and Applications*, **8**, 68-80. <https://doi.org/10.1080/23737484.2021.1973926>
- [30] Ray, S.N., Bose, S. and Chattopadhyay, S. (2020) A Markov Chain Approach to the Predictability of Surface Temperature over the Northeastern Part of India. *Theoretical and Applied Climatology*, **143**, 861-868. <https://doi.org/10.1007/s00704-020-03458-z>
- [31] Wigner, E.P. (1979) Symmetries and Reflections: Scientific Essays. Ox Bow Press, 280.
- [32] Elliot, J.P. and Dawber, P.G. (1985) Symmetry in Physics, Vol. 1: Principles and Simple Applications. Oxford University Press, 302.
- [33] Myamlin, S.V. and Kravets V.V. (2003) Simmetriya matematicheskoy modeli i dostovernost' vychistitelnogo eksperimenta [Symmetry of the Mathematical Model and Reliability of the Computational Experiment]. Vinnitsa AU Publisher, 15, 339-340.