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Initial Model for the Impact of Social Distancing on COVID-19 Spread

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Abstract

The initial stages of the CoVID-19 coronavirus pandemic all around the world exhibited a nearly exponential rise in the number of infections with time. Planners, governments, and agencies scrambled to figure out "*How much*? *How bad*?" and how to effectively treat the potentially large numbers of simultaneously sick people. Modeling the CoVID-19 pandemic using an exponential rise implicitly assumes a nearly unlimited population of uninfected persons ("*dilute pandemic*"). Once a significant fraction of the population is infected ("*saturated pandemic*"), an exponential growth no longer applies. A new model is developed here, which modifies the standard exponential growth function to account for factors such as *Social Distancing*. A *Social Mitigation Parameter* [*SMP*] α_S is introduced to account for these types of society-wide changes, which can modify the standard exponential growth function, as follows:

$$N(t) = N_o \exp\left[+K_o t/(1+\alpha_S t)\right].$$

The *doubling-time* $t_{dbl} = (\ln 2)/K_o$ can also be used to substitute for K_o, giving a $\{t_{dbl}, \alpha_S\}$ parameter pair for comparing to actual CoVID-19 data. This model shows how the number of CoVID-19 infections can self-limit before reaching a *saturated pandemic* level. It also provides estimates for: 1) the timing of the *pandemic peak*, 2) the maximum number of new daily cases that would be expected, and 3) the expected total number of CoVID-19 cases. This model shows fairly good agreement with the presently available CoVID-19 pandemic data for several individual States, and for the USA as a whole (6 *Figures*), as well as for various countries around the World (9 *Figures*). An augmented model with two *Mitigation Parameters*, α_S and β_S, is also developed, which can give better agreement with the daily new CoVID-19 data. Data-to-model comparisons also indicate that using α_S by itself likely provides a worst-case estimate, while using both α_S and β_S likely provides a best-case estimate for the CoVID-19 spread.

Keywords

COVID-19, Pandemic Modeling, Social Distancing, Social Mitigation

1. Introduction

The Coronavirus 2019 disease (CoVID-19), caused by the Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) pathogen, has become a worldwide pandemic. In many localities, the number of cases $N(t)$ was found to have an initial period of nearly exponential growth:

$$N(t) = N_o \exp[+K_D t], \tag{1}$$

$$K_D \equiv (\ln 2)/t_D, \tag{2}$$

aside of the first few cases, which may be untraceable. In Equations (1)-(2), N_o is the initial number of infections at the $t = 0$ start of data tracking, K_D is an exponential growth factor, and t_D is the *doubling-time*. Each locality can have its own $\{N_o, t_D, t = 0\}$ values, with both K_D and t_D being nearly constant during this initial period of CoVID-19 spread.

Standard epidemiology identifies the number of people N_G a known infected individual had recent contact with. Contacts of that N_G group are tracked next, followed by additional tracking stages. This process sets the K_D value.

Society-wide *Mitigation Measures* such as: 1) *Social Distancing*, 2) wearing face masks in public, 3) prohibiting large gatherings, 4) implementing large-scale population testing, 5) disinfecting high-touch surfaces in public areas, 6) enhanced cleaning of items brought into homes, and 7) minimizing human contact with likely virus-containing materials and matter; all can help reduce $N(t)$ growth. These *Mitigation Measures* can modify the Equation (1) epidemiology model by causing the local t_D values to lengthen.

In order to model these *Mitigation Measures*, t_D and K_D become explicit functions of time, $t_D(t)$ and $K_D(t)$. Using a linear function for $t_D(t)$ lengthening is one of the simplest time-varying extensions. A linear function of time also corresponds to the first term of a *Taylors' Series* expansion of some more general $t_D(t)$ analytic function, giving this epidemiology extension:

$$t_D(t) \equiv t_{dbl}(1 + \alpha_S t), \tag{3}$$

$$K_D(t) \equiv \frac{\ln 2}{t_D(t)} = (\ln 2)/[t_{dbl}(1 + \alpha_S t)] \equiv K_o/(1 + \alpha_S t). \tag{4}$$

The $t = 0$ initial values for $t_D(t)$ and $K_D(t)$ become the new constants t_{dbl} and K_o, which characterize the initial exponential growth phase. The α_S coefficient in Equation (3) is a new *Social Mitigation Parameter* [*SMP*] that helps quantify the effectiveness of the society-wide *Mitigation Measures* as a whole.

The α_S value expresses how well non-infected people manage to avoid the virus contagion. As a lumped parameter, it likely reflects an average value over many processes, known and unknown, which comprise *Social Mitigation*, to supplement the contact-to-contact tracking that initially sets t_{dbl} or K_o.

Substituting Equation (4) into Equation (1) gives:

$$N(t) = N_o \exp\left[+K_o t/(1+\alpha_S t)\right], \quad (5)$$

as one of the simplest models for CoVID-19 spread. A pure exponential growth (or decay) has no *memory*, while Equation (5) for $\alpha_S > 0$, has a *memory*. The $t = 0$ start time of first mitigation changes the future history. To include $t < 0$ requires replacing Equation (3) by $t_D(t) \equiv t_{dbl}\left(1+\max\left[0,\alpha_S t\right]\right)$, which has a corner at $t = 0$ that preserves the *memory* of when mitigation first started.

2. Model Features

The Equation (1) exponential growth pandemic model implicitly assumes a large uninfected population allowing the disease to easily spread ("*dilute pandemic*"). When almost everybody is infected ("*saturated pandemic*"), exponential growth shuts off, and Equation (1) no longer applies.

On 3/10/2020, *German Chancellor Angela Merkel* [1] noted that she "*estimates that 60% to 70% of the German population will contract the coronavirus*", indicating that *saturated pandemic* models were being considered as a worst-case.

Even that worst-case condition assumes: 1) recovered coronavirus patients are no longer infectious, and 2) surviving a CoVID-19 infection confers absolute immunity to re-infection. More recently, South Korea [2] found 91 cases of clinically recovered patients later testing as CoVID-19 positive. They may also shed viable coronaviruses in their phlegm and fecal matter, furthering disease spread. Although these effects are not modeled here, those additional transmission modes could turn a 60% - 70% hope into a calculated 99+% consequence.

These factors show why CoVID-19 modeling beyond Equation (1) is needed, especially to see if society-wide *Mitigation Measures* can naturally halt disease spread, without necessitating a *saturated pandemic* condition. We show next that Equation (5) allows for this pandemic shut-off, even for the *dilute pandemic* case. Since both K_o and α_S in Equation (5) have the same units, their ratio is dimensionless. The long-term limit of Equation (5) gives:

$$\lim_{t\to+\infty}\left[N(t)/N_o\right] = \exp\left[+K_o/\alpha_S\right], \quad (6)$$

setting an average value for the total number of all follow-on infections arising from a single individual. Since it depends only on the ratio of the original pandemic growth factor K_o to the α_S *SMP*, this model shows the impact of accounting for a broader environment beyond individual contact tracking.

The early spread of CoVID-19 cases outside of China, and the early USA CoVID-19 data [3] both had nearly exponential rises, as shown in **Figure 1**. A purely exponential rise gives a straight line on a *log-plot* {*log*(# *of cases*) *vs linear time*}. The initial *doubling-time* for the USA was $t_{dbl} \approx 2.02$ days, giving

$K_o \approx 0.343/\text{day}$ using Equations (1)-(2). This initial CoVID-19 data was prior to any significant *Mitigation Measures* being implemented.

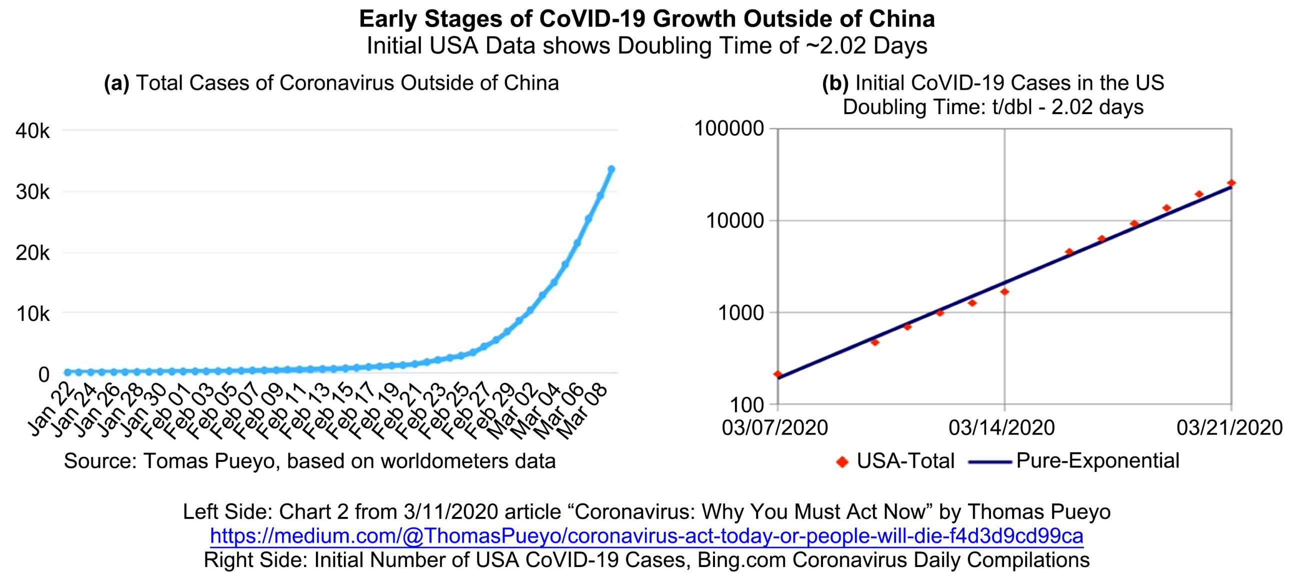

Figure 1. Early CoVID-19 Cases: (a) Outside of China, and (b) Just in the US. Both graphs show nearly exponential growth.

On March 19, 2020, Governor Gavin Newsom of California ordered a CoVID-19 "stay-at-home" lockdown of virtually all of California's nearly 40 million residents. Similar statewide CoVID-19 lockdowns were ordered by the Governors of Illinois, New York, Indiana, Michigan, Ohio, Washington, West Virginia and Wisconsin.

The slowing of CoVID-19 spread by implementing large-scale societal *Mitigation Measures* can be fairly rapid, as illustrated by the USA CoVID-19 data of Figure 2, which covers March 2020.

The impact of these multi-state *Mitigation Measures* is evident in Figure 2 as a sudden transition on the *log-plot* from a straight-line to having downward curvature, which the α_S *Social Mitigation Parameter* (*SMP*) aims to quantify. The local slope in Figure 2 also decreases right after the onset of *Mitigation Measures*, indicating further slowing of CoVID-19 spread.

A well-documented South Korean coronavirus cluster can also be used to help estimate the expected size of the α_S *SMP*. That CoVID-19 cluster determined that a single infected person at the *Shinjeongji Church* caused infection of about 4482 people within the 47-day time interval between January 20, 2020 and March 8, 2020. Using K_o from Figure 1(b) provides this α_S estimate:

$$\ln(4.482) \approx (0.343/\text{day})(47\ \text{days}) / \left[1 + \alpha_S (47\ \text{days})\right], \tag{7}$$

$$\alpha_S = 0.01952/\text{day} \approx 0.02/\text{day}, \tag{8}$$

which is indicative of minimal mitigation. If additional deliberate mitigation measures doubled α_S to $\alpha_S \approx 0.04/\text{day}$, Equation (6) would give:

$$\lim_{t\to+\infty}\left[N(t)/N_o\right]=\exp\left[+\frac{(0.343/\text{day})(47\text{ days})}{1+(0.04/\text{day})(47\text{ days})}\right]\approx 270\,, \tag{9}$$

for the number of infections per person, a $16.6X$ reduction from 4482.

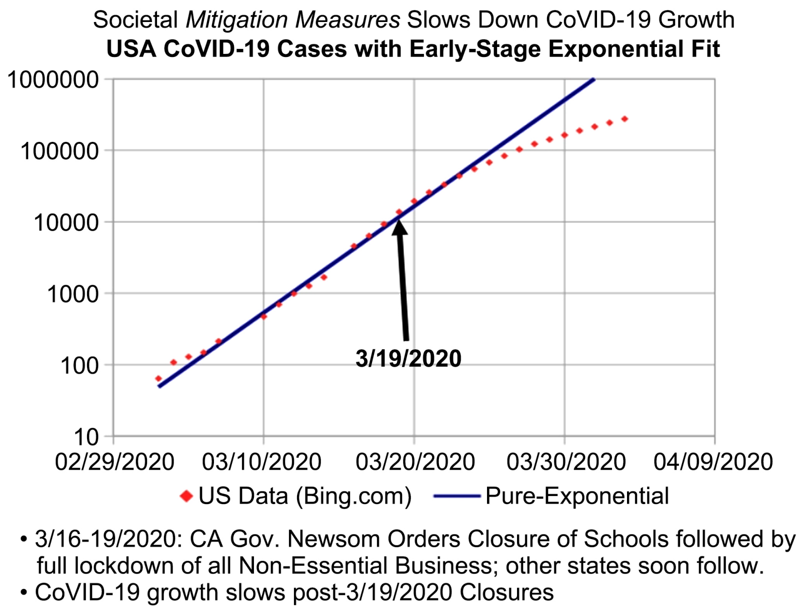

Figure 2. USA CoVID-19 data, pre-vs-post mid-March 2020. Multi-State *Mitigation Measures* slowed growth, transitioning from straight-line to downward curvature.

Since $N(t)$ in Equation (5) represents a total number of cases, it is similar to a *cumulative distribution function* (*cdf*), which is used in reliability and also has time as its fundamental variable. The derivative of Equation (5), $\mathrm{d}N(t)/\mathrm{d}t$, is analogous to an unnormalized *probability density function* (*pdf*), which can be used to predict a *pandemic peak*:

$$\begin{aligned}\{pdf\}&\equiv\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{N(t)}{N_o}\right]=\frac{\mathrm{d}}{\mathrm{d}t}\left\{\exp\left[+K_o t/(1+\alpha_S t)\right]\right\}\\&=+K_o\left(\frac{1}{1+\alpha_S t}\right)^2\exp\left[+K_o t/(1+\alpha_S t)\right]\end{aligned}, \tag{10}$$

$$\{pdf\}=+K_o\left[\frac{N(t)}{N_o}\right]\Big/(1+\alpha_S t)^2\,, \tag{11}$$

The time t_P of the *pandemic peak* is set by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\{pdf\}\equiv 0\,, \tag{12}$$

$$\alpha_S t_P=\frac{1}{2}\left[K_o/\alpha_S\right]-1\,, \tag{13}$$

where the Equation (13) simplification arises from the Equation (12) constraint. Substituting $K_o=0.343/\text{day}$ and $\alpha_S=0.02/\text{day}$ from Equation (8) into Equation (13) gives $t_P\approx 379\text{ days}$. Increasing mitigation to $\alpha_S=0.04/\text{day}$, keeping the same $K_o = 0.343/\text{day}$, now gives $t_P \approx 82$ days, which is almost a 4.6 X reduction in the *pandemic peak* timing for doubling the *Social Mitigation* effect from its original baseline value. These examples highlight the tremendous impact that even a small amount of enhanced *Social Mitigation* can have.

While the $\alpha_S = 0$ limit of Equation (11) recovers the Equation (1) standard exponential growth, both the $\{pdf\}$ and $\left[N(t)/N_o\right]$ growth are then unbounded. However, even a small $\alpha_S > 0$ value in Equation (11) will have an enormous impact on the predicted long-time behavior. Since Equation (6) showed that $\left[N(t)/N_o\right]$ now approaches a finite value for all $\alpha_S > 0$, this new $\{pdf\}$ asymptotic limit:

$$\lim_{t \to +\infty, \alpha_S > 0} \left[\{pdf\}\right] \sim \frac{\{\text{Constant}\}}{(\alpha_S t)^2}, \tag{14}$$

also results. The $\{pdf\}$ that arises from this model all have an initial exponential rise, coupled with the Equation (14) "*long tail*" at large times, which means that new CoVID-19 cases may arise for a long time, even if significant *Mitigation Measures* are in place.

The Equation (14) $\{pdf\}$ prediction also differs substantially from the widely-used *University of Washington IHME* (*Institute for Health Metrics and Evaluation*) projections, which use symmetric Gaussians for both the $\{pdf\}$ rise and fall [4]. Thus, these methods provide an alternative risk-bound for evaluating potential CoVID-19 worst-case scenarios.

3. Determining $\{t_{dbl}, \alpha_S\}$ from CoVID-19 Data

Explicit numerical values for $\{t_{dbl}, \alpha_S\}$ parameters were determined from the CoVID-19 data as follows. Rewriting Equation (5) as:

$$\ln\left[N(t)/N_o\right] = \left[+K_o t/(1+\alpha_S t)\right] \equiv (\ln 2) t / \left[t_{dbl}(1+\alpha_S t)\right]. \tag{15}$$

allowed data fitting to be done on a $Y - vs - X$ *log-plot*, using $Y = \ln\left[N(t)/N_o\right]$ as the ordinate and $X \leftrightarrow t$ as the abscissa, to calculate and minimize the *root-mean-square* (*rms*) *error*.

The $t = 0$ point in Equation (15) sets N_o. To best model *Mitigation Measures*, this point was usually chosen at the start of a downward curvature on a *log-plot*, so that $N(t=0) \equiv N_I$, where the N_I is now the first data point in the analysis. The prior $t < 0$ regime can often have nearly pure exponential growth, as in **Figure 2**, and those regions should not be part of *rms-error* minimization for evaluating *Mitigation Measures*.

The N_F final data point, measured at the most recent $t = t_F$ time:

$$N(t = t_F) \equiv N_F. \tag{16}$$

was also fixed for each dataset, so that only $\{t_{dbl}, \alpha_S\}$ value pairs that meet both $N(t=0) \equiv N_I$ and Equation (16) were used.

In practice, an α_S was chosen first. The *Excel™_Tools_Goal-Seek* function was used to adjust t_{dbl} to obey Equation (16), setting the *rms-error* between the dataset and Equation (15), with the final $\{t_{dbl}, \alpha_S\}$ having the least *rms-error*.

In the following figures, all CoVID-19 raw data came from the publicly available Microsoft "*COVID-19 Tracker*" site [5]. When no updates were available, that site repeated the prior day data, whereas we used the geometric mean of the day-prior and day-after data for interpolation.

4. Effects of Varying the Initial Zero-Time Point

Starting with:

$$N(t) = \mathbf{1}\exp\left[+K_o t/(1+\alpha_S t) + \ln(N_o)\right], \quad (17)$$

$$\lim_{t\to+\infty}\left[N(t)\right] = \mathbf{1}\exp\left[+(K_o/\alpha_S) + \ln(N_o)\right], \quad (18)$$

$$t_{dbl}^{o} \equiv (\ln 2)/K_o, \quad (19)$$

using a shifted time-scale normalization point is examined next:

$$N_S(t') = \mathbf{1}\exp\left[+K_o(t'+t_A)/\left[1+\alpha_S(t'+t_A)\right] + \ln(N_o)\right], \quad (20)$$

This $N_S(t')$ function should closely match Equation (17) with a shifted time axis: $t = t' + t_A$, but the best fit parameter numerical values change. Since:

$$\left[1+\alpha_S(t'+t_A)\right] = \left[1+\alpha_S t_A\right]\left[1+\frac{\alpha_S t'}{(1+\alpha_S t_A)}\right], \quad (21)$$

$$\frac{K_o(t'+t_A)}{\left[1+\alpha_S(t'+t_A)\right]} = \left[1+\frac{t_A}{t'}\right]\frac{(K_o t')}{\left[1+\alpha_S t_A\right]} \Bigg/ \left[1+\frac{\alpha_S t'}{(1+\alpha_S t_A)}\right], \quad (22)$$

then defining:

$$\alpha_A \equiv \alpha_S/(1+\alpha_S t_A), \quad (23)$$

$$K_A \equiv K_o/(1+\alpha_S t_A), \quad (24)$$

$$t_{dbl}^{A} \equiv (\ln 2)/K_A = (1+\alpha_S t_A)t_{dbl}^{o} = (1+\alpha_S t_A)(\ln 2)/K_o, \quad (25)$$

it gives:

$$\begin{aligned}\frac{K_o(t'+t_A)}{\left[1+\alpha_S(t'+t_A)\right]} &= \left[1+\frac{t_A}{t'}\right]K_A t' \Big/ \left[1+\alpha_A t'\right] \\ &= K_A t'/\left[1+\alpha_A t'\right] + K_A t_A/\left[1+\alpha_A t'\right]\end{aligned}. \quad (26)$$

These equations highlight the net effect of time-shifting. For $t_A > 0$, when t' begins after *Mitigation Measures* have started, the shifted-time-axis results in a larger calculated *doubling-time* and a smaller *SMP* α_A-value. For $t_A < 0$, when t' may include *Mitigation Measures* already in place at the Equations (17)-(19) $t = 0$ point, this shifted-time-axis results in a smaller calculated *doubling-time* and a larger *SMP* α_A-value.

Finally, for small t', where $\alpha_A t' < 1$, using Equations (18) and (26) gives:

$$\mathbf{1}\exp\left[\frac{+K_o t}{(1+\alpha_S t)} + \ln(N_o)\right] \approx \mathbf{1}\exp\left[\frac{+K_A t'}{(1+\alpha_A t')} + \ln(N_A)\right], \quad (27)$$

$$\ln(N_A) = \ln(N_o) + K_A t_A, \tag{28}$$

which shows that the $t' = 0$ new initial state should have an N_A starting value obeying $N_A > N_o$ for $t_A > 0$, and $N_A < N_o$ for $t_A < 0$. However, whether {N_o, K_o, α_S}, or an alternative {N_A, K_A, α_A}, are used to parameterize a given data set, the net overall function fit and predictions, as a function of calendar date, should remain fairly self-consistent, even when some ambiguity exists as to when *Mitigation Measures* first were noticeably effective.

5. USA and Selected States Model Results

The model predictions for CoVID-19 spread in the USA is shown in **Figure 3**. This analysis only included data after mid-March 2020, when several State Governors first instituted mandatory *Mitigation Measures*. Results give an *SMP* estimate of α_S ≈ 0.05945/day, a USA initial *doubling-time* of $t_{dbl}^{initial}$ ≈ 2.1758 days, which lengthens to $t_{dbl}^{\{\text{at Peak}\}}$ ≈ 5.83 days at the projected *pandemic peak* of 4/19/2020. The predicted total number of CoVID-19 cases is 5,464,000, giving a projected 1.67% final infection rate, if the present level of *Mitigation Measures* or their equivalent, are continued.

These predictions assume no "*second wave*" of infection or re-infection. They also do not include the effect of additional *Mitigation Measures*, which could further increase the {t_{dbl}, α_S} values, and significantly reduce the projected final number of CoVID-19 cases.

Figure 4 shows model predictions for CoVID-19 evolution in California. Only data after 3/21/2020 was included in the analysis, after California Governor Gavin Newsom instituted mandatory *Mitigation Measures*. It gives an *SMP* estimate of α_S ≈ 0.03546/day, with an initial *doubling-time* of $t_{dbl}^{initial}$ ≈ 2.5017 days, which lengthens to $t_{dbl}^{\{\text{at Peak}\}}$ ≈ 9.774 days at the projected *pandemic peak* of 6/07/2020. The predicted total number of CoVID-19 cases is 1,123,700, giving a projected 2.813% final infection rate, at the present level of *Mitigation Measures*.

Figure 5 shows model predictions for CoVID-19 evolution in New York. A relatively high *SMP* estimate of α_S ≈ 0.1031/day was found, coupled with a relatively short initial *doubling-time* of $t_{dbl}^{initial}$ ≈ 0.9395 days, which creates a high narrow spike in daily new cases. The present model projects a New York *pandemic peak* around 4/10/2020, with an estimated at-peak *doubling-time* of $t_{dbl}^{\{\text{at Peak}\}}$ ≈ 3.36 days. The predicted total number of cases is 1,218,000, giving a projected 6.072% final infection rate.

Figure 6 shows model predictions for CoVID-19 evolution in Washington State. An initial *doubling-time* of $t_{dbl}^{initial}$ ≈ 2.2005 days and an *SMP* value of α_S ≈ 0.0366/day were found, with a projected *pandemic peak* around 6/04/2020. The relatively low number of cases at the *Mitigation Measures* start helps to give a predicted total number of cases of 557,600, corresponding to a 7.15% final infection rate.

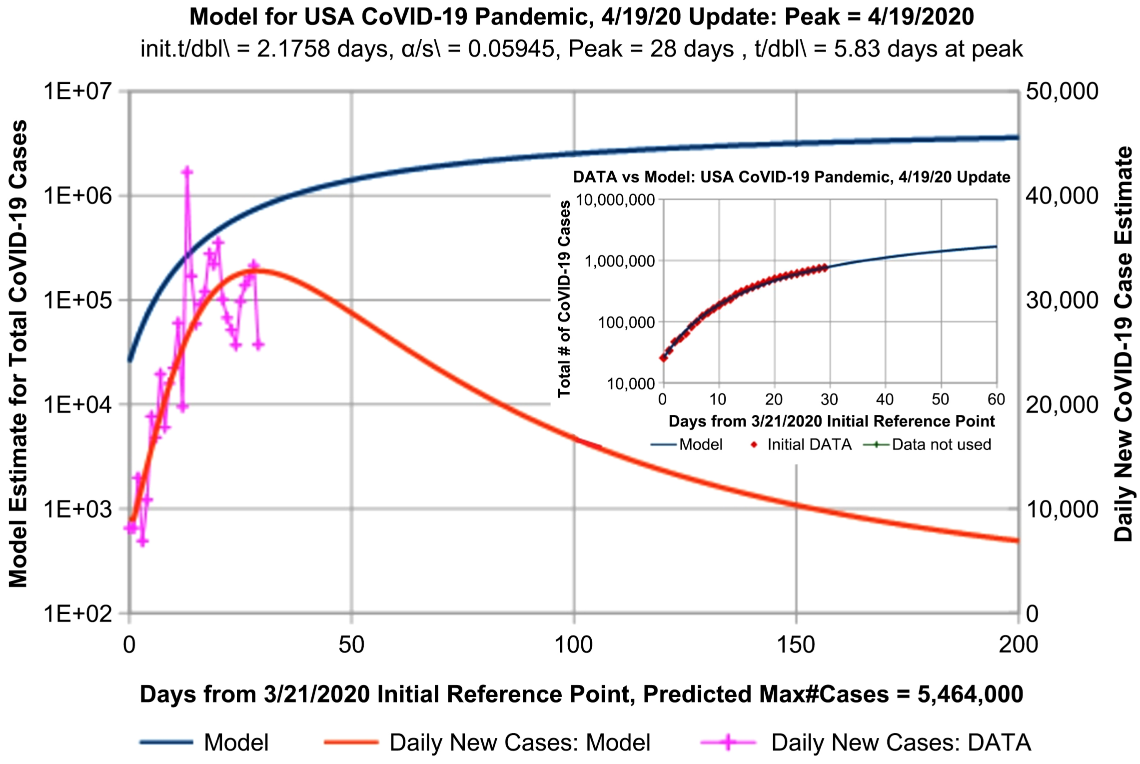

Figure 3. USA Model Predictions. To allow better mitigation predictions, only data after mid-March 2020 was included, when several Governors instituted mandatory lockdowns.

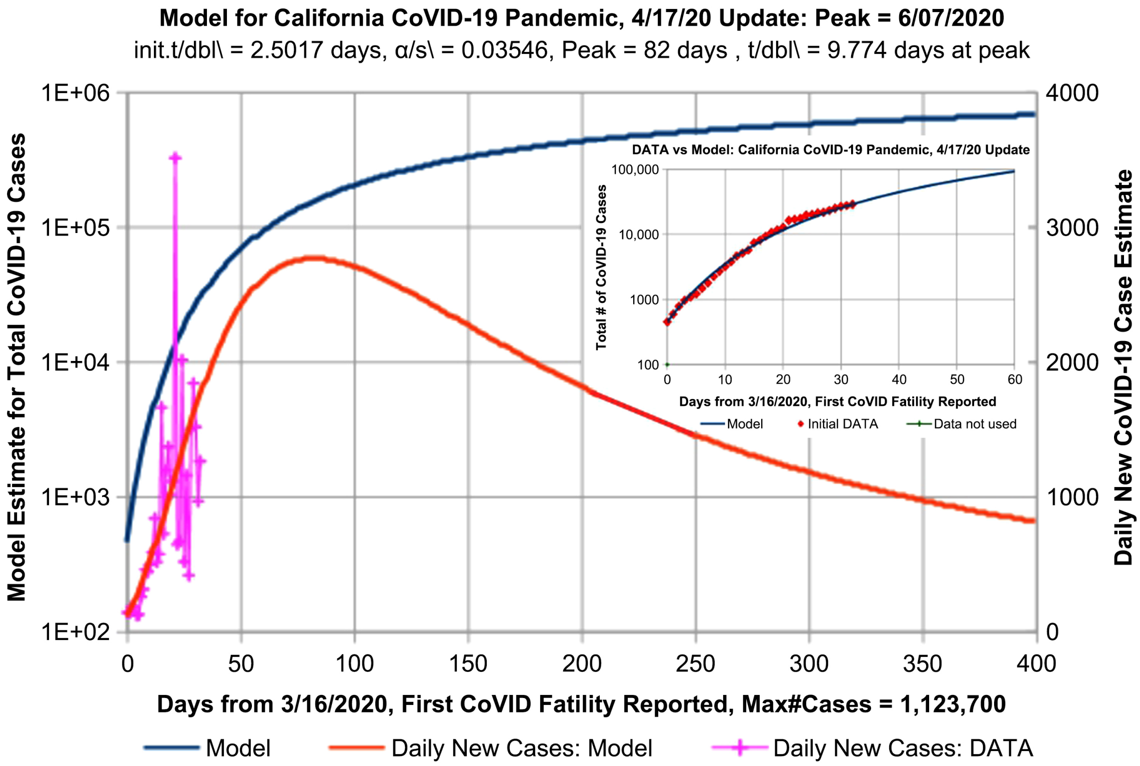

Figure 4. Predicted California CoVID-19 results. After Gov. Gavin Newsom instituted widespread *Mitigation Measures*, projections showed significant improvement.

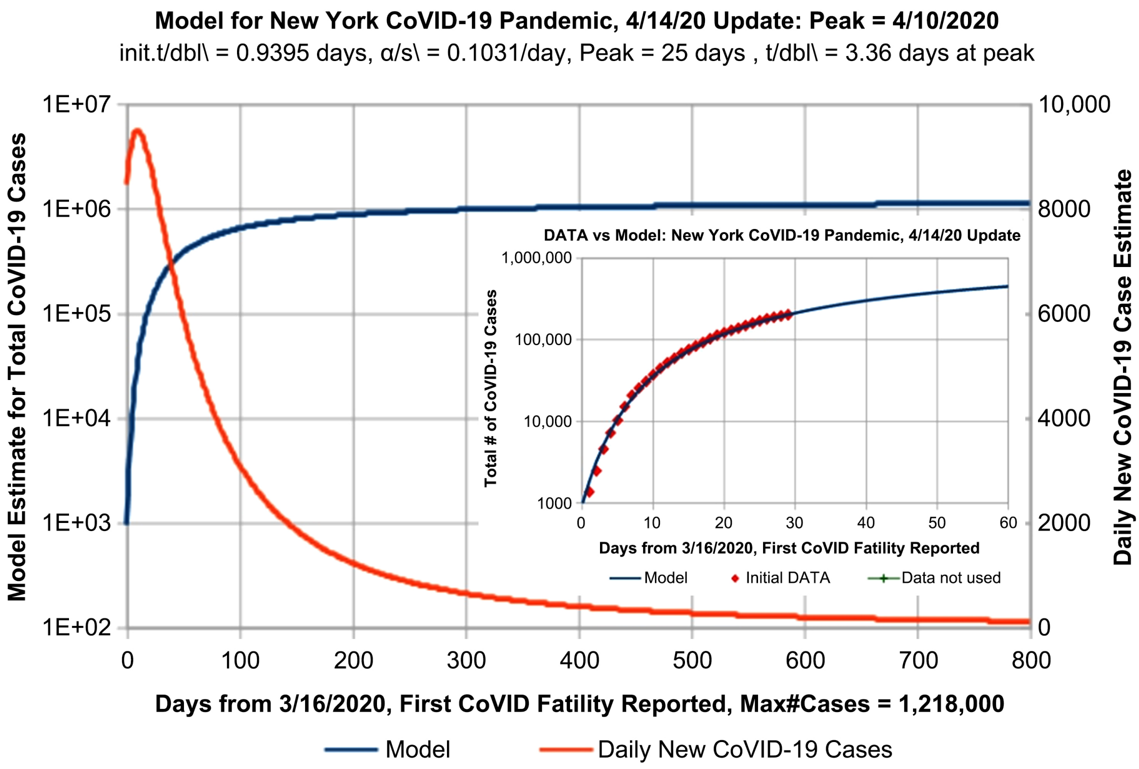

Figure 5. Predicted New York CoVID-19 results. A relatively high *Mitigation Measure* level and a short intrinsic doubling time creates a narrow spike in daily new cases.

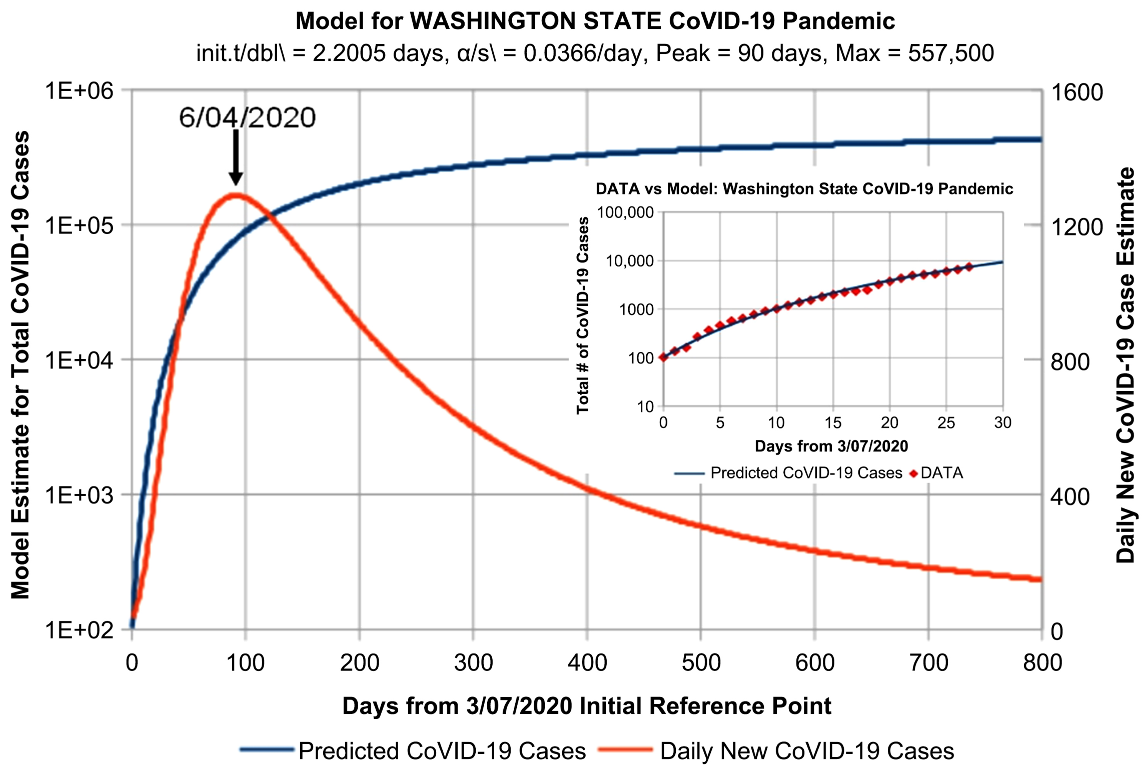

Figure 6. Predicted Washington State CoVID-19 results. The relatively low number of cases at *Mitigation Measure* start helps to give a relatively low final number of cases.

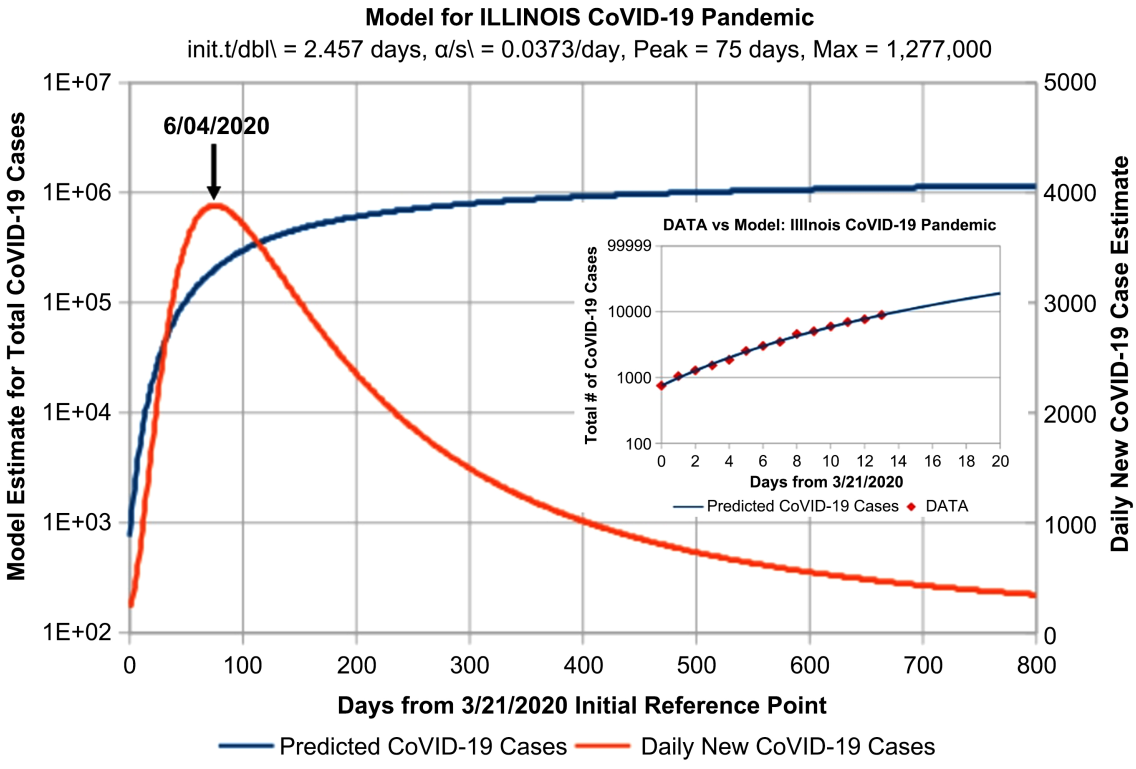

Figure 7. Predicted Illinois CoVID-19 results. The slow *doubling-time* and moderate amount of *Mitigation Measures* gives a slow increase to the predicted CoVID-19 peak.

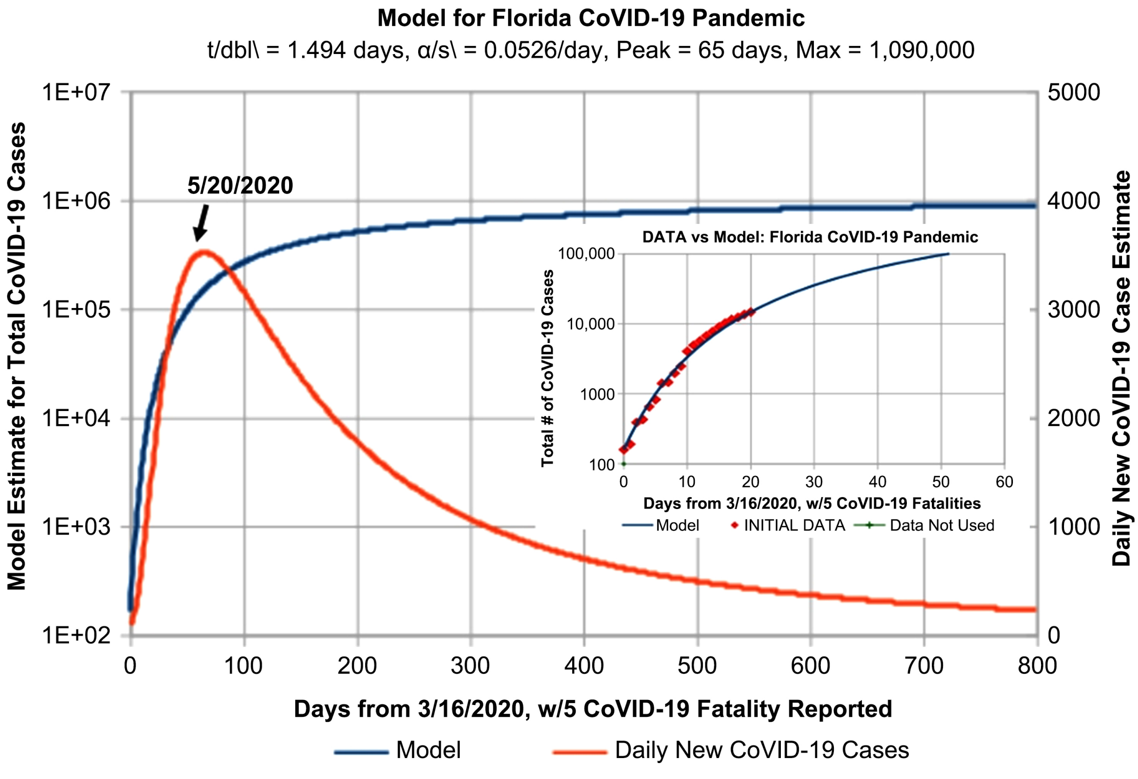

Figure 8. Predicted Florida CoVID-19 results. Many Florida counties instituted their own Mitigation Measures prior to a state-wide lockdown, substantially slowing CoVID-19 growth.

Figure 7 shows model predictions for CoVID-19 evolution in Illinois. An initial *doubling-time* of $t_{dbl}^{initial} \approx 2.457$ days and moderate *SMP* value of $\alpha_S \approx 0.0373$/day combine to give a projected *pandemic peak* around 6/04/2020, similar to Washington State, but having a higher predicted total number of cases at 1,277,000, and a projected 11.47% final infection rate.

Figure 8 shows model predictions for CoVID-19 evolution in Florida. Many Florida counties instituted their own Mitigation Measures prior to a state-wide lockdown, slowing CoVID-19 growth. A somewhat high *SMP* value of $\alpha_S \approx 0.0526$/day, and an initial *doubling-time* of $t_{dbl}^{initial} \approx 1.494$ days results. A *pandemic peak* is estimated at around 5/20/2020, with a predicted total number of cases at 1,090,000, and a projected 4.96% final infection rate.

6. World and Selected Countries Model Results

Figure 9 shows model predictions of CoVID-19 evolution for the whole World. The present-day *doubling-time* value of $t_{dbl}^{initial} \approx 5.761$ days likely represents a combination of small urban, large urban, and rural area results. However, the calculated low *SMP* estimate of $\alpha_S \approx 0.01712$/day shows that nearly 4.43% of the World's population could be at risk for eventual CoVID-19 infection. At these present levels, the projected *pandemic peak* is around 8/15/2020, with potentially hundreds of millions of people being infected.

Figure 10 shows model predictions for CoVID-19 evolution in China, covering their "*first wave*" of early exposure and early mitigation. Data were included that was prior to a "*New Reporting Method*" being used, which started off with one sudden data jump, and nearly level CoVID-19 follow-on results. The present model predicts what number of cases could have resulted, had the reporting method not changed. Draconian *Mitigation Measures* helped to contain the pandemic to *Hubei Province* and *Wuhan*. These projections show that those *Mitigation Measures* have impressively contained CoVID-19 spread.

Figure 11 shows model predictions for CoVID-19 evolution in South Korea, covering the period of their country's early exposure and initial mitigation methods. Pre-pandemic *Mitigation Measures*, including extensive contact-tracing and large-scale CoVID-19 testing, were implemented. These projections show that those *Mitigation Measures*, as an alternative to China's methods, also have impressively contained CoVID-19 spread.

Figure 12 shows model predictions for CoVID-19 evolution in Italy. An initial *doubling-time* of $t_{dbl}^{initial} \approx 1.4648$ days and *SMP* estimate of $\alpha_S \approx 0.05282$/day give a *pandemic peak* around 4/29/2020, with a predicted number of total cases at 1,764,000 and a projected 2.92% final infection rate.

Figure 13 shows model predictions for CoVID-19 evolution in Germany. The relatively high *SMP* estimate of $\alpha_S \approx 0.07614$/day with an initial *doubling-time* of $t_{dbl}^{initial} \approx 1.4177$ days combine to give a projected *pandemic peak* at around 4/08/2020, with a predicted total number of cases of 700,100 and a projected 0.84% final infection rate. These values would make Germany one of the countries with the least impact in Europe. They represent predicted final CoVID-19 infection rates that are significantly lower than the original 60% - 70% early worst-case estimates highlighted by *German Chancellor Angela Merkel*.

Figure 14 shows model predictions for CoVID-19 evolution in Spain. An *SMP* estimate of $\alpha_S \approx 0.07058/\text{day}$, which is comparable to Germany, and a smaller initial *doubling-time* of $t_{dbl}^{initial} \approx 1.1778$ days combine to give more predicted CoVID-19 cases than Germany. The estimated *pandemic peak* is around 4/21/2020, with a predicted number of total cases at 1,526,000 and a projected 3.26% final infection rate.

Figure 15 shows model predictions for CoVID-19 evolution in Ecuador. Reports of chaos in Ecuador have been alarming. Yet the present data show a significant and somewhat unexpected leveling off in the number of reported CoVID-19 cases. This result could mean that some yet unknown *Mitigation Measures* may be operating. Alternatively, the data could mean that there is a dire CoVID-19 testing and reporting shortfall operating amidst chaos.

Figure 16 shows model predictions for CoVID-19 evolution in India. These initial data show virtually no mitigation at present, having one of the lowest calculated *SMP* estimates of $\alpha_S \approx 0.0148/\text{day}$, with an initial *doubling-time* of $t_{dbl}^{initial} \approx 3.135$ days. At this rate, nearly 17.38% of the population of India could eventually become infected. The estimated *pandemic peak* is around 5/30/2021, which would be 441 days after the first CoVID-19 fatality was reported, on 3/14/2020. Additional *Mitigation Measures*, further increasing the $\{t_{dbl}, \alpha_S\}$ values, as well as using additional modeling parameters may significantly reduce these projected number of CoVID-19 cases.

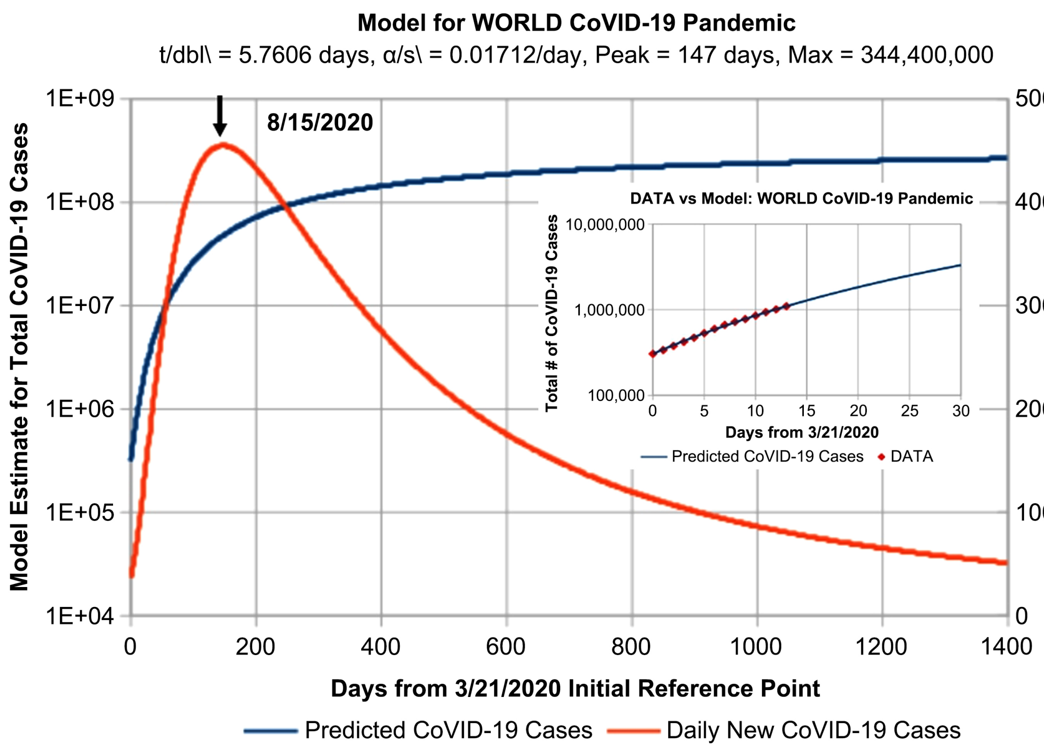

Figure 9. Model predictions for the WORLD, showing present-day low level of mitigation.

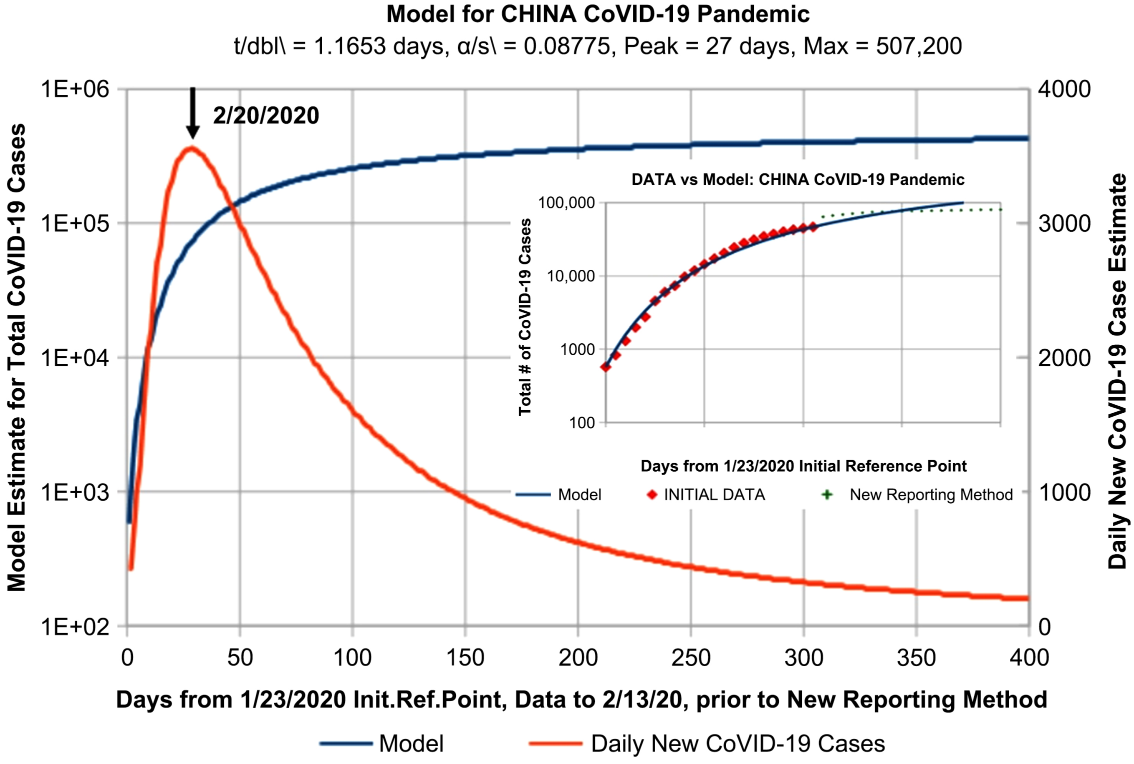

Figure 10. Predicted CHINA CoVID-19 results, using pre-"*New Reporting Method*" data. Draconian *Mitigation Measures* helped to contain pandemic to *Hubei Province* and *Wuhan*.

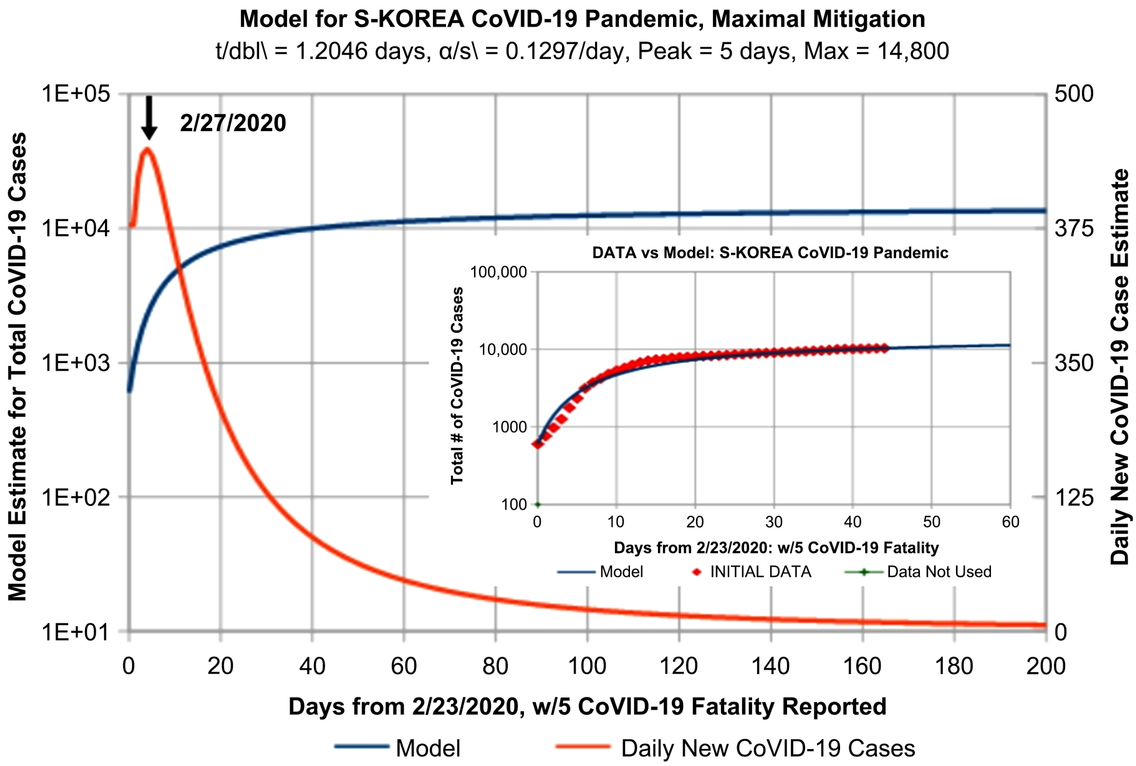

Figure 11. Predicted SOUTH KOREA CoVID-19 results. Pre-pandemic contact-tracing and large-scale CoVID-19 testing as *Mitigation Measures* have contained the pandemic.

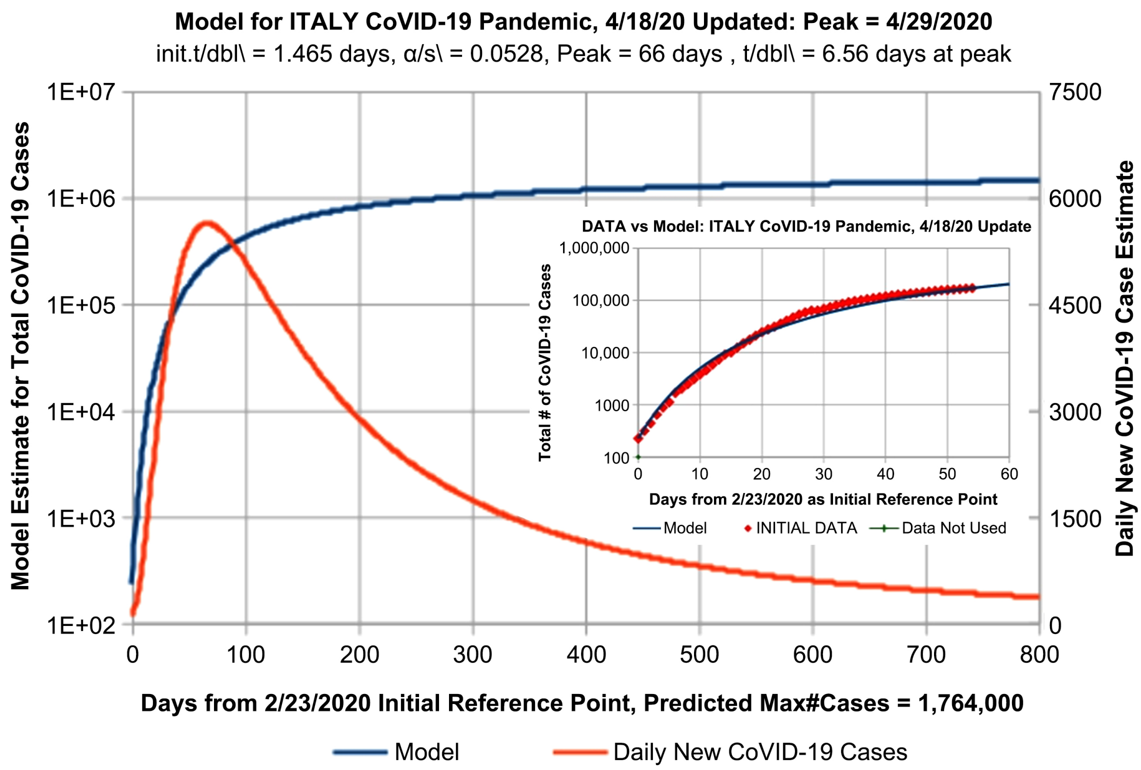

Figure 12. Predicted ITALY CoVID-19 results. Additional curvature in the actual CoVID-19 data vs Model makes these predictions a likely worst-case.

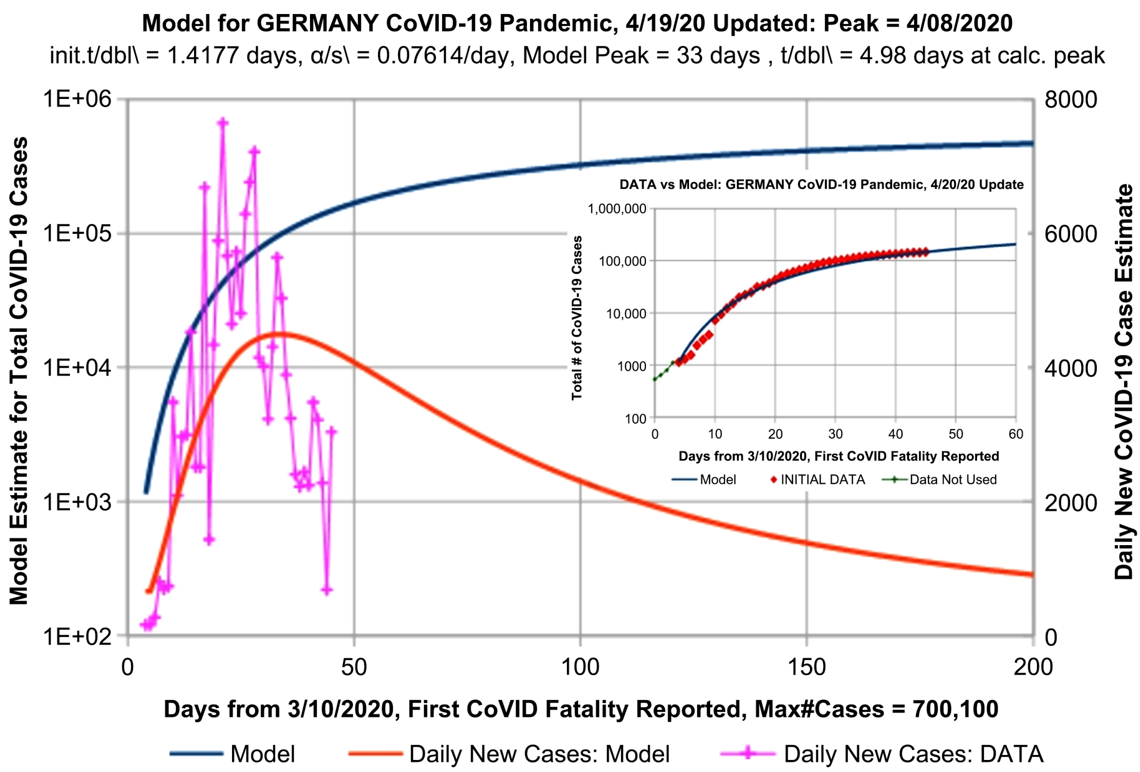

Figure 13. Predicted GERMANY CoVID-19 results. This model gives a more gradual function for the Daily New CoVID-19 cases, making these predictions a likely worst-case.

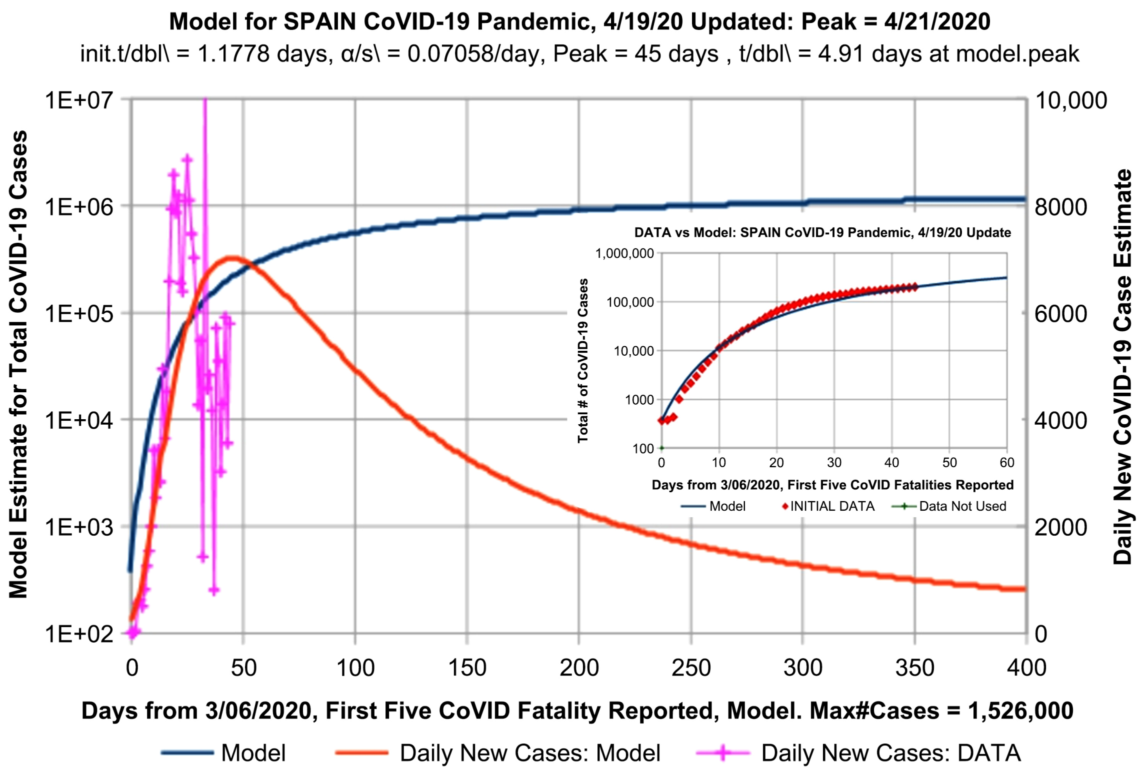

Figure 14. Predicted SPAIN CoVID-19 results. This model gives a more gradual function for Daily New CoVID-19 cases, making these predictions a likely worst-case.

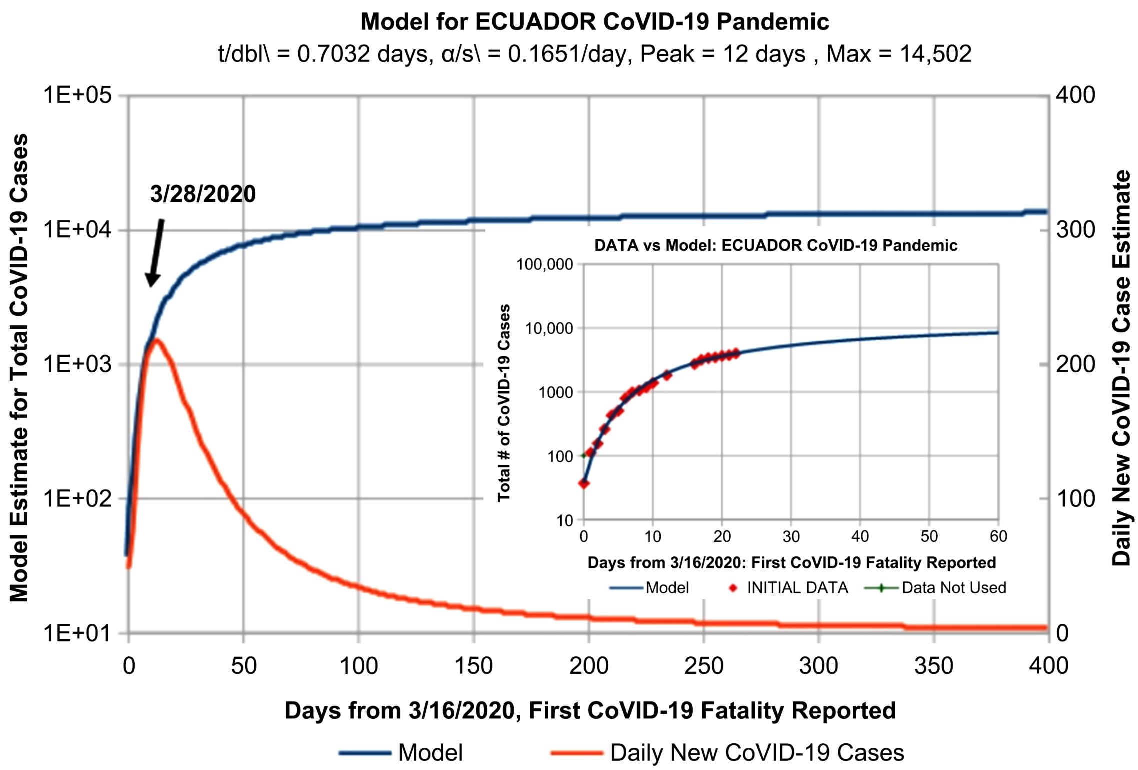

Figure 15. Predicted ECUADOR CoVID-19 results. Reports of chaos in Ecuador have been alarming. Poor CoVID-19 tracking and low testing may have skewed these results.

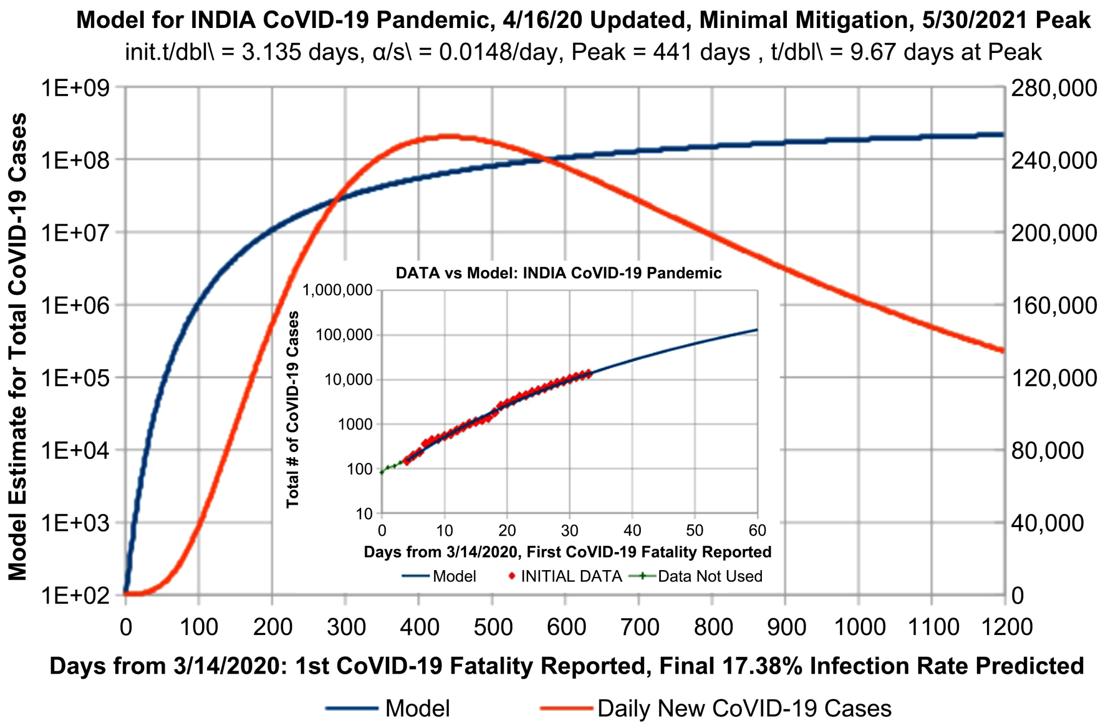

Figure 16. Predicted INDIA CoVID-19 results. Data shows only minimal mitigation at present. Further mitigations should help make these predictions a worst-case result.

7. Augmented Peak Shape Modeling

Using a new *Social Mitigation Parameter* [*SMP*] α_S, as in Equation (3), successfully models pandemic shut-off, even in the *dilute pandemic* limit. However, as the **Figures 3-16** insets show, many of the data-vs-model comparisons have the data trending above the model near the final $t = t_F$ data point.

Since Equation (3) for $t_D(t)$ is linear, using an *Additional Modeling Parameter* [*AMP*] β_S in a higher order polynomial, may fit the $\{pdf\}$ shape better. A quadratic function for t_D:

$$t_D = t_{dbl}\left(1 - \beta_S Z + \alpha_S Z^2\right), \tag{29}$$

where $N(t)/N_o$ still approaches a constant at long times, as in Equation (6), then sets $Z^2 \equiv t$, giving this extension of Equation (3):

$$t_D(t) = t_{dbl}\left(1 - \beta_S \sqrt{t} + \alpha_S t\right). \tag{30}$$

Values of $\beta_S > 0$ in Equation (30) allow the predicted $\left[N(t)/N_o\right]$ values to rise above the $\beta_S = 0$ model predictions, and to have a smaller *doubling-time*, for the same $\{t_{dbl}, \alpha_S\}$. However, the best fit $\{t_{dbl}, \alpha_S\}$ values will also differ between the $\beta_S > 0$ and $\beta \equiv 0$ cases, so these changes are relative.

The new $\{pdf\}$ function for Equation (30) is:

$$\begin{aligned}\{pdf\} &\equiv \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{N(t)}{N_o}\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left\{\exp\left[+K_o t\Big/\left(1 - \beta_S\sqrt{t} + \alpha_S t\right)\right]\right\} \\ &= K_o\left[\frac{N(t)}{N_o}\right]\left(1 - \frac{1}{4}\beta_S^2 t\right)\Big/\left[\left(1 + \alpha_S t - \beta_S\sqrt{t}\right)^2\left(1 + \frac{1}{2}\beta_S\sqrt{t}\right)\right]\end{aligned}. \tag{31}$$

When $\{pdf\}=0$ in Equation (31), it estimates an end-point for the pandemic at:

$$t_{END}=4/\beta_S^2, \tag{32}$$

while predicting this maximum number of pandemic cases at t_{END}:

$$N(t_{END})=N_o \exp\left[K_o/\left(\alpha_S-\frac{1}{4}\beta_S^2\right)\right]. \tag{33}$$

As an example, this augmented model was applied to CoVID-19 evolution in Italy. As shown in Figure 17, this Equation (31) $\{pdf\}$ function gives a better fit to the observed number of daily new CoVID-19 cases.

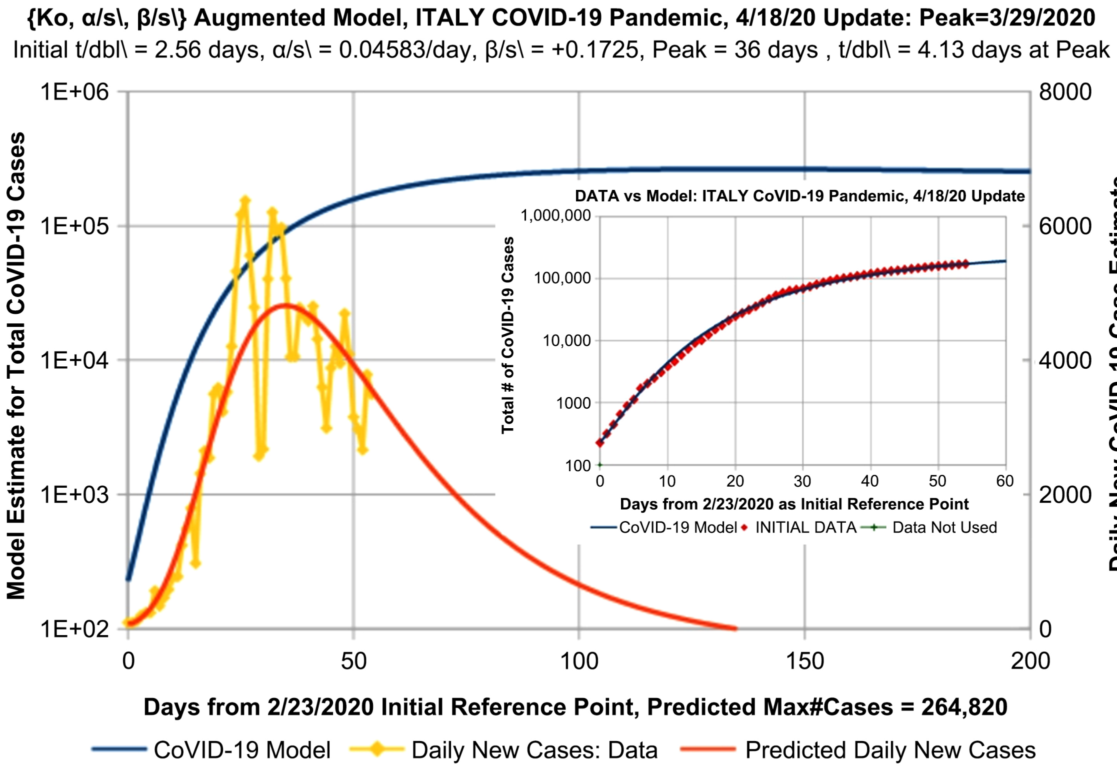

Figure 17. Predicted ITALY CoVID-19 results, using an augmented 2-parameter $\{\alpha_S, \beta_S\}$ *Social Mitigation* model. Total number of CoVID-19 cases is much less than Figure 12, but the model post-peak drop is much steeper, making this a likely best-case result.

In this augmented model, the initial *doubling-time* of $t_{dbl}^{initial} \approx 2.5566$ days, along with estimates for the *Mitigation Measure* parameters of $\alpha_S \approx 0.04583/\text{day}$ and $\beta_S \approx +0.1725$, combine to significantly reduce the projected maximum number of CoVID-19 cases down to 264,820 which is about $7X$ less compared to using α_S alone, as in Figure 12. This augmented model sets an estimated *pandemic peak* at 3/29/2020, with a projected pandemic end-point around 7/7/2020, which is also significantly more optimistic.

The true CoVID-19 pandemic progress is likely to be in between Figure 12 as a worst-case, and Figure 17 as a best-case projection. The geometric mean of the

Figure 12 and Figure 17 results set an average of 683,500 cases for Italy at the CoVID-19 pandemic end. These bounds also highlight the amount of uncertainty that is intrinsic to these empirically based methods.

8. Summary and Conclusions

The standard exponential for modeling pandemics starts with an N_o known number of initial cases at some reference time $t=0$. Epidemiologists work to determine a pandemic growth factor K_D, which sets the *doubling-time* t_D for the number of pandemic cases.

The CoVID-19 disease, caused by the SARS-CoV-2 coronavirus pathogen, initially showed both regional and global exponential growth. It resulted in a *doubling-time* of $t_D \approx 2.02$ days for the US, as highlighted in Figure 1.

An exponential growth normally only halts when it runs out of materials. In epidemiology that point often occurs when there are virtually no more uninfected people left, which we call a *saturated pandemic*. The exponential growth function is only applicable when infection rates are much lower than saturation, which we call a *dilute pandemic*.

A modification to exponential growth is developed here, which allows ratio of the number of pandemic cases, $N(t)$, compared to its N_o initial value at $t=0$:

$$\lim_{t\to+\infty}\left[N(t)/N_o\right]=\mathbf{M}, \tag{34}$$

to approach a final constant, denoted $\mathbf{M}$, while still being in a *dilute pandemic* condition. This result is attributed to the inclusion of society-wide *Mitigation Measures* to stop pandemic growth, before the value of $\mathbf{M}$ reaches the whole population value.

Society-wide *Mitigation Measures* aim to progressively lengthen the t_D *doubling-time*, essentially making $t_D(t)$. Most analyses presented here used a linear function of time as the simplest non-constant model for $t_D(t)$:

$$t_D(t)\equiv t_{dbl}\left(1+\alpha_S t\right), \tag{35}$$

$$K_D(t)\equiv\frac{\ln 2}{t_D(t)}=(\ln 2)/\left[t_{dbl}\left(1+\alpha_S t\right)\right]\equiv K_o/\left(1+\alpha_S t\right), \tag{36}$$

where $t_D(t=0)\equiv t_{dbl}$, and $K_D(t=0)\equiv K_o$. Here, α_S is a new *Social Mitigation Parameter* (*SMP*), to quantify societal *Mitigation Measures*. This Equation (35) extension of pure exponential growth gives:

$$N(t)=N_o\exp\left[+K_o t/\left(1+\alpha_S t\right)\right], \tag{37}$$

as an empirical equation for modeling CoVID-19 spread. Since both K_o and α_S in Equation (5) have the same units, their ratio is a dimensionless number. The long-term limit of Equation (37) gives:

$$\lim_{t\to+\infty}\left[N(t)/N_o\right]=\exp\left[+K_o/\alpha_S\right], \tag{38}$$

setting a final value for the Equation (34) constant $\mathbf{M}$, allowing these predictions to be applicable to the *dilute pandemic* limit. The CoVID-19 number of estimated cases per day is given by:

$$\{pdf\} = \frac{\mathrm{d}}{\mathrm{d}t}\left[N(t)/N_o\right], \tag{39}$$

$$\lim_{t\to+\infty,\alpha_S>0}\left[\{pdf\}\right] \sim \frac{\{\text{Constant}\}}{(\alpha_S t)^2}, \tag{40}$$

which combines an initial exponential rise with "*long tail*" at large times. In this model, new CoVID-19 cases can continue to arise for a long time, even with significant *Mitigation Measures* in place.

Analysis of available CoVID-19 data using this model shows that it can match observed data fairly well, both from various US states **Figures 3-8**, as well as for different global countries **Figures 9-16**. However, using a single parameter to encompass all societal *Mitigation Measures* often gives a slightly larger slope on a *log-plot*, compared to the latest measured data values, which makes this model a likely worst-case estimate.

A second data-fitting parameter β_S was also used in an augmented model, to better fit the $\{pdf\}$ data:

$$N(t) = N_o \exp\left[+K_o t\Big/\left(1-\beta_S\sqrt{t}+\alpha_S t\right)\right], \tag{41}$$

$$\max\left[N(t)/N_o\right] = \exp\left[+K_o\Big/\left(\alpha_S - \frac{1}{4}\beta_S^2\right)\right], \tag{42}$$

$$t_{END} = 4/\beta_S^2, \tag{43}$$

where t_{END} becomes an estimated pandemic end-point, where zero new CoVID-19 cases per day could occur.

As a representative example, this augmented model was applied to the CoVID-19 data from Italy in **Figure 17**. Those results show that this augmented model allows a better fit to the observed number of new daily CoVID-19 cases, but the absence of a CoVID-19 tail in its $\{pdf\}$ function makes this $\{K_o, \alpha_S, \beta_S\}$ augmented model a likely best-case result, with the original $\{K_o, \alpha_S\}$ model being a likely worst-case estimate.

This class of CoVID-19 pandemic models all enable pandemic shut-off even in the *dilute pandemic* limit, with only a small fraction of the total population being infected. These models also provide estimates for: 1) the maximum number of cases near pandemic shutoff, 2) the size and shape of the *pandemic peak* [$\mathrm{d}N(t)/\mathrm{d}t$], and 3) pandemic peak timing [t_P]. These models and analyses may help enhance planning and preparation to maximize resource use, potentially increasing individual and collective CoVID-19 pandemic survival rates.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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