

Building upon Biot-Savart Law: Behavior of Surfaces and Curves of Magnetic Field around Electrical Segments

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Abstract

The Biot-Savart law defines the magnetic field vector around an electrical segment. Its calculation involves a cross product, but certain physical properties of this operation are not fully satisfactory. This article proposes a modification to the Biot-Savart formula by replacing the vector field with a scalar field. The revised magnetic field concept is then applied to both short and long electrical segments.

Keywords

Electrical Segment with Magnetic Field, Biot-Savart Law, Magnetic Field Value, Scalar Field Value, Equipotential Surface, Equipotential Curve, Field around Electrical Segment

1. Introduction

In 1820, physicist Hans Christian Ørsted discovered that an electric current in a wire affects a nearby compass [1] [2]. This phenomenon is now understood as the result of an electromagnetic field. The work of Jean-Baptiste Biot and Félix Savart led to the well-known Biot-Savart law, which provides a formula for calculating the electromagnetic field at any point around a current-carrying wire [3] [4] and established the concept of the electromagnetic field as a vector field [5].

The Biot-Savart law is a powerful tool for understanding magnetic fields around conductors. However, Feynman criticized its incompatibility with the principle of additivity [6]. Another issue, discussed in this article, is the law's discontinuity in the immediate vicinity of the conductor.

An improvement to the Biot-Savart law was proposed in a previous article and successfully applied by the author to several parallel wires [7]. A second test was

conducted for crossing wires [8].

The key idea behind this work was to establish a correspondence between the electromagnetic field and other field descriptions, such as the electrostatic field. The proposed approach involves handling the electromagnetic field first and foremost as a scalar field. However, vectors remain useful in the description, when applied to crossing wires, leading to a hybrid approach that combines elements of both models [8].

This article presents the third application of the scalar model, this time focusing on electrical segments. The theoretical model is introduced at the beginning and compared to the classical Ørsted-Biot-Savart law. In the same section, the field model is applied to a short electrical segment.

The second section explores applications to an infinite line, half-lines, and short segments, along with equipotential surfaces and magnetic field curves.

The drawings and calculations for the scalar model were carried out using three software programs: Mathematica [9], GIMP [10], and Wikipedia [11].

2. Theoretical Bases

In the following sections, the electromagnetic field is defined as a scalar field value in which every point is combined with three orthogonal, dimensionless vectors. This definition offers two critical advantages: first, the unified nature of this space ensures the physical additivity of the magnetic field; second, the use of dimensionless vectors allows the result of cross-products to exist within the same geometrical space as the initial vectors.

Once formally described, this model will be compared to the Biot-Savart model.

2.1. Magnetic Field around a Short Elementary Segment

2.1.1. Scalar Field with Dimensionless Vectors

The scalar magnetic field model assigns a field value to each point while incorporating three orthogonal, dimensionless vectors.

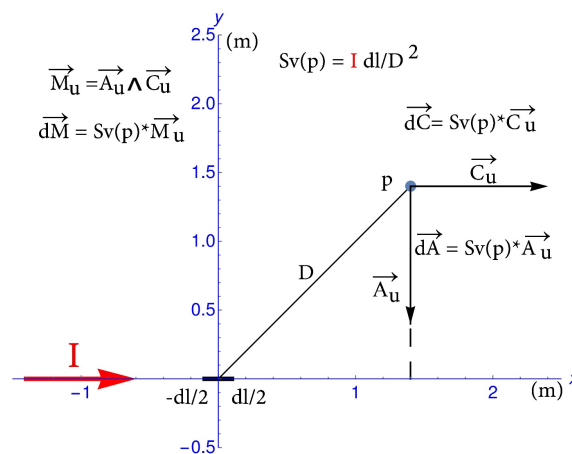


Figure 1. The magnetic field at any point p, located at a distance D from the midpoint of a short segment on the x-axis (centered at x = 0), is described by a scalar value Sv and three orthogonal, dimensionless vectors. The x, y, and z axes are in meters.

Figure 1 focuses on the field induced by a short elementary electrical segment, denoted as dl .

In **Figure 1**, three dimensionless vectors originate from p :

- Cu : aligned with the current I ,
- Au : directed from p toward the wire,
- Mu : the cross product of Au and Cu , oriented along the z -axis (not shown in the xy plane).

These vectors (Au , Cu , and Mu) are mutually perpendicular and dimensionless. By multiplying each of them by the scalar field value Sv at p , the physical vectors dA , dB , and dM are obtained, all with the same physical length and dimension.

The physical field value Sv at p is in A/m and given by product I and dl , divided by the square of the distance between p and the midpoint of the elementary electrical segment dl .

The scalar value Sv at point p always corresponds to a non-vector physical quantity. For example, in **Figure 1**, this value is given by $I dl/D^2$ (A/m).

2.1.2. Classical Magnetic Field Vectors around a Short Elementary Segment

This paragraph critically examines the Biot-Savart magnetic field model.

In this classical vector model of the magnetic field, a point p —located outside a short elementary segment dl —is considered. At this point as illustrated in **Figure 2**, three vectors with different physical units are associated.

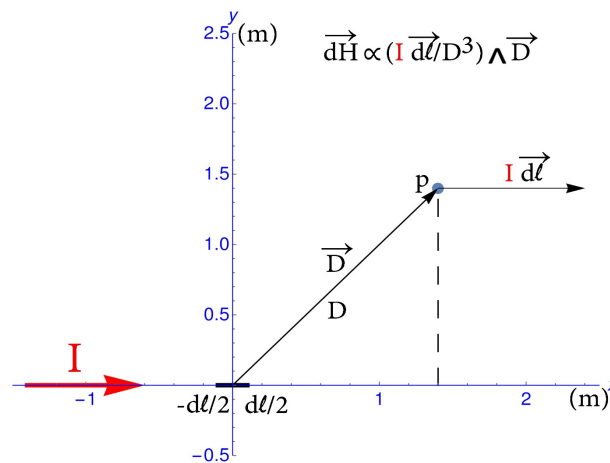


Figure 2. The Ørsted-Biot-Savart law defines three vectors depending on point p :

- $I dl$: Aligned with the current I ,
- D : Extends from the midpoint of the short elementary segment (positioned along the x -axis and centered at $x = 0$) to point p ,
- dH : The classical magnetic field vector, directed along the z -axis (not shown in the diagram), with its magnitude displayed at the top of the figure.

The value of dH is determined by the cross product of vectors $I dl$ and D .

In contrast to the previously described model, the three vectors in this case have different physical units, making them incompatible for direct comparison and

preventing them from being understood within the same geometrical space.

Let's now attempt to calculate the field values:

When point \mathbf{p} is located on the \mathbf{x} -axis, the cross product of \mathbf{D} and $\mathbf{I dl}$ becomes zero, which physically means that the small segment has no magnetic influence along this axis. Similarly, if \mathbf{p} lies directly on the segment \mathbf{dl} , \mathbf{dH} is also zero.

When point \mathbf{p} is located on the \mathbf{y} -axis, the cross product of \mathbf{D} and $\mathbf{I dl}$ evolves from zero when \mathbf{p} is very far away and becomes infinite as \mathbf{p} approaches the segment \mathbf{dl} .

This leads to a clear contradiction: at the same point \mathbf{p} (located at (0, 0)), \mathbf{dH} cannot simultaneously be zero and infinite. This discontinuity cannot be physically explained by the current model. In contrast, such a phenomenon is absent in the model described in the previous paragraph, which does not involve a cross product given the field value.

With this observation, the author proposes to work with the specific model outlined earlier, which assigns a scalar value to each point and is associated with three dimensionless orthogonal vectors.

2.1.3. Scalar Field Model with Dimensionless Vectors around an Electrically Active Segment

Here, the first model described in **Figure 1** is applied to determine the iso-magnetic surfaces and field curves around an elementary segment carrying a current, and to represent them in a 3D drawing.

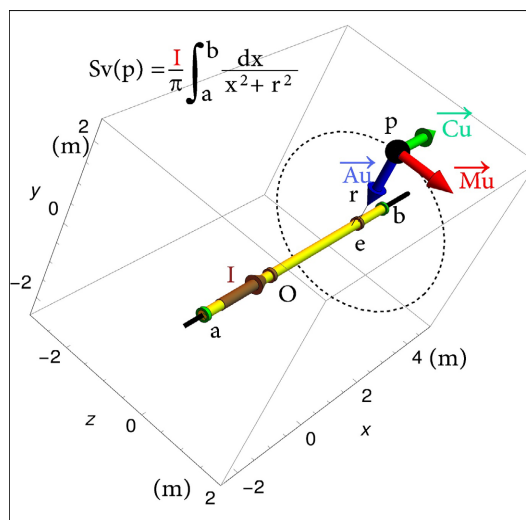


Figure 3. An electrical segment induces a scalar magnetic field value \mathbf{Sv} for a point \mathbf{p} placed at distance \mathbf{r} in a perpendicular direction of the \mathbf{x} axis which is supporting the segment \mathbf{ab} in yellow color. At all points around the segment at a distance \mathbf{r} , the magnetic field value has the same scalar value. All points compose a circle centered in \mathbf{e} , located on axis \mathbf{x} , and having the same equipotential magnetic value. \mathbf{Au} , \mathbf{Cu} and \mathbf{Mu} are dimensionless unit vectors as defined in **Figure 1**. Dimensionless vector \mathbf{Mu} is the cross-product of \mathbf{Au} and \mathbf{Cu} .

The magnetic field value is the same at all points on the circle centered on the \mathbf{x} -axis, where the segment is located as in **Figure 3**. This circle represents a mag-

netic field curve, with the vector \mathbf{Mu} being tangent to it.

By extending this concept to the entire segment, it is possible to represent an iso-magnetic field surface, which is associated with circular magnetic field curves. An example of this is shown in **Figure 4**.

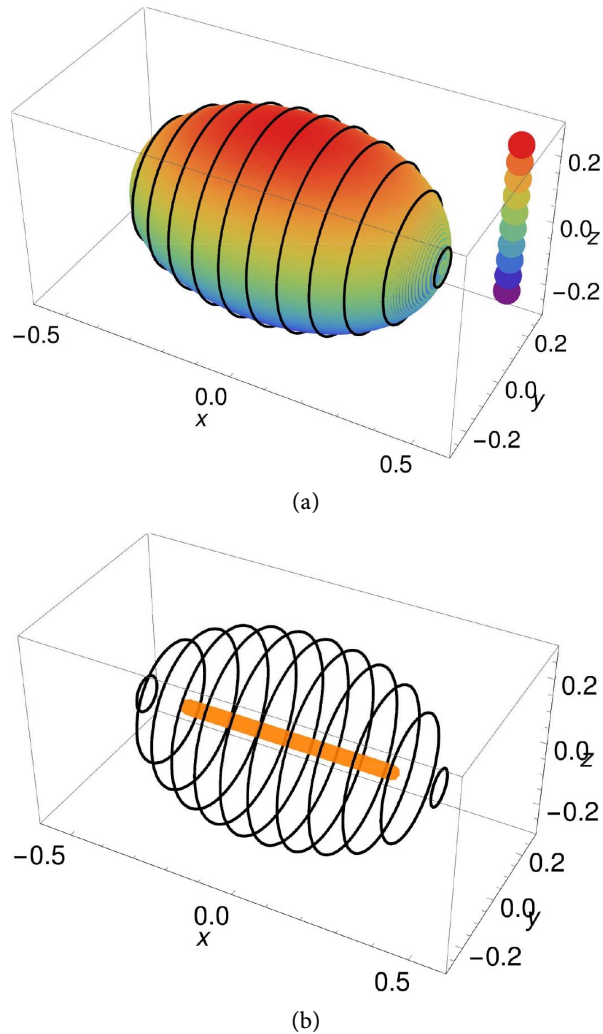


Figure 4. Iso-magnetic surfaces and magnetic field curves at 2 (A/m) around an electrically active segment (orange), located on the x-axis from positions -0.3 for point **a** to $+0.3$ meters for point **b**.

In **Figure 4(a)**, the colored surface represents points with the same magnetic field values. In **Figure 4(b)**, the circles represent magnetic field curves around the segment, each having the same physical value as the iso-magnetic surface in **Figure 4(a)**.

The scalar magnetic field value induced by a linear, electrically active segment from **a** to **b** on the **x**-axis can be determined by integrating the elementary value $I dl \mathcal{D}^2$, as described in **Figure 3**. The result is given by Equation (1)

$$Sv(x, r) = \frac{1}{\pi} \int_a^b \frac{dx}{x^2 + r^2} \tag{1}$$

Equation (1) provides the scalar magnetic field value $\mathcal{S}v$ at point p , which is located a distance r from the wire along the x-axis, with current I flow through the segment $a b$. For example, when point p is on the segment between a and b , the magnetic field scalar value becomes infinite.

If point p is on the x-axis (with $r = 0$) but outside the segment, the magnetic field decays according to a hyperbolic function, $1/x$, and approaches zero as the distance x becomes infinite.

In other words, this model ensures that the field outside the segment remains continuous.

3. Applications

3.1. Infinite Line

An infinite line is described by setting $a = -\infty$ and $b = +\infty$. In this case, Equation (1) gives $\mathcal{S}v = I/r$. This expression has been extensively used in our previous articles on infinite straight wires [7] [8] [12].

3.2. Half-Line

A half-line on the x axis of **Figure 5** in a x y plane can be described with endpoints extending from $a = -\infty$ and $b = 0$.

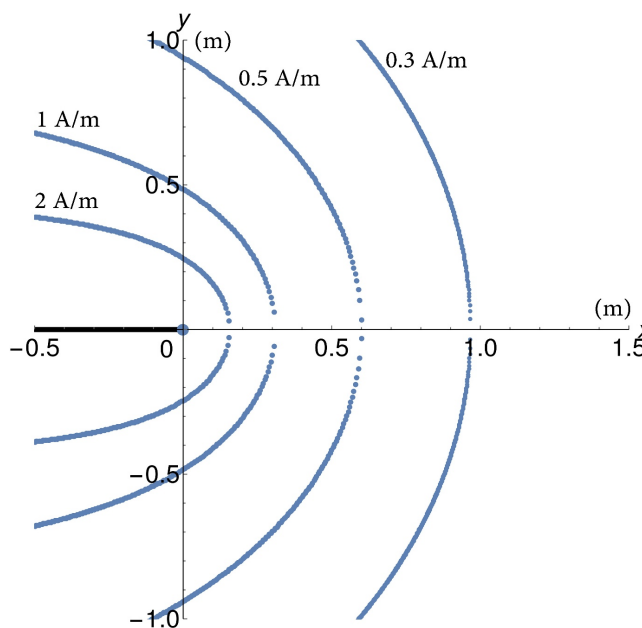


Figure 5. A half-line on the x-axis extends from $a = -\infty$ to $b = 0$. The field value increases from zero to $+\infty$ as point p approaches the origin of the half-line. The field value remains non-zero along the positive x-axis.

One can observe in **Figure 5** that there is no discontinuity in the field value, and that the field becomes infinite everywhere near the half-line.

3.3. Two Half-Lines on x Axis

In **Figure 6**, an infinitely long electrical wire is segmented into two half-lines, with a gap between $x = 0$ and $x = 0.6$ m.

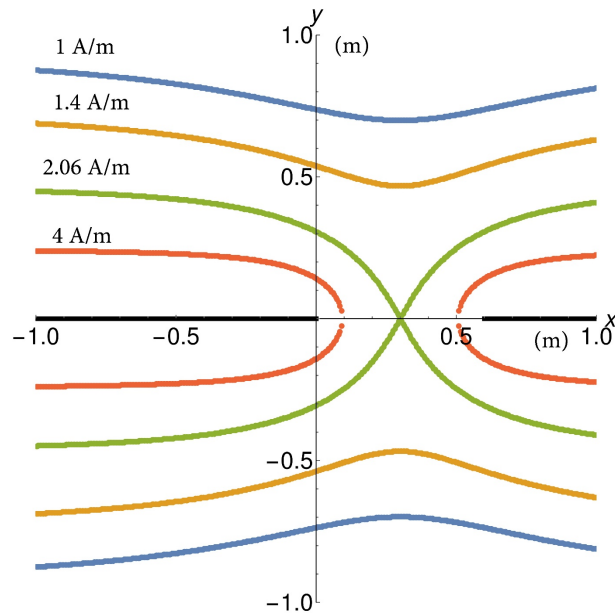


Figure 6. Two aligned half-lines, each carrying an electrical current of 1 A. The additivity of the scalar field enables the construction of magnetic equipotential surfaces intersecting the xy -plane. These surfaces become parallel to both half-lines when the distance from the segments is greater than the separation between the two half-lines. When the scalar field value exceeds 2.06 A/m, the magnetic field surfaces separate, each enclosing one of the half-lines. The boundary at 2.06 A/m is depicted by the green line.

As shown in **Figure 6**, a scalar field value of 2.06 A/m defines a boundary that differentiates between distinct types of equipotential surfaces, separating those near the half-lines from those farther away. This boundary divides the equipotential surfaces into three distinct groups. Close to the electrical half-lines, the surfaces form two separate groups, each associated with one of the half-lines. Beyond the boundary, one equipotential surface encloses both half-lines, as illustrated by the blue surface in **Figure 6**.

3.4. Two Half-Lines with Opposite Current

In the present paragraph and in **Figure 7**, however, the currents flow in opposite directions: +1 A on the left side and -1 A on the right side.

In **Figure 7**, the magnetic field curves closely resemble the surfaces depicted in **Figure 4** when they are near one of the half-lines. However, a key difference arises at the vertical line, which appears as a planar surface in a 3D view and is positioned midway between the two half-lines. On this plane, the magnetic field value is exactly 0 A/m, acting as a natural boundary that separates the regions influenced by each half-line's current.

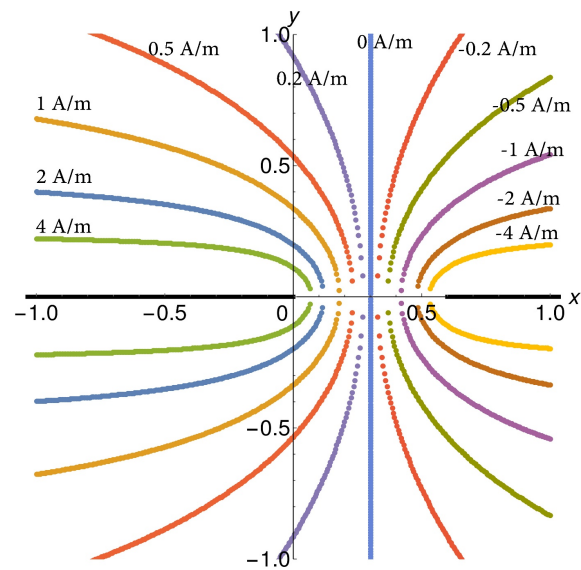


Figure 7. Two infinite half-lines, separated by 0.6 m, carrying opposite electrical currents (+1 A on the left and -1 A on the right). Due to this opposing current flow, no single group of magnetic surfaces can encompass both half-lines. Instead, magnetic surfaces with negative field values are concentrated around the right-side half-line, while those with positive field values surround the left-side half-line. This configuration creates a distinct separation between the magnetic field regions associated with each half-line.

3.5. Cross Sections of Iso-Magnetic Surfaces around One Segment

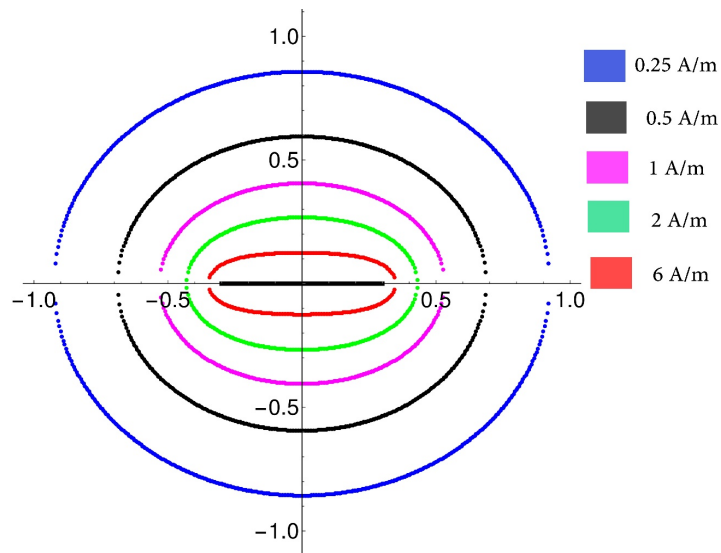


Figure 8. A segment extending from $a = -0.3$ m to $b = +0.3$ m induces magnetic field surfaces around it. At low magnetic field values, the blue surface is nearly spherical, appearing as circular shape in the xy -plane. As the magnetic field value increases, the surfaces become progressively elongated, eventually adopting a shape that closely resembles the segment itself.

The significance of this diagram of **Figure 8**, lies in its ability to incorporate additional segments, enabling the representation of more complex geometrical

configurations. By extending this concept, one can analyze the interaction of multiple current-carrying segments and their corresponding magnetic field surfaces, providing deeper insights into more intricate electromagnetic structures.

4. Discussion

4.1. Dimensionless Vectors

In this article, three dimensionless vectors are used:

- The first induced by the electrical current is named vector C_u and with the same direction as the current flow.
- The second, an attraction vector, is named A_u , perpendicular to the wire, indicating the direction to go to the moving electric current.
- The third, a magnetic field vector, is the dimensionless cross product of the first two vectors, indicating the direction of the magnetic field, much like a compass.

The cross product with these three vectors can be applied by using A_u and C_u to have M_u , or by using C_u and M_u to have A_u and for M_u and A_u to have C_u . This is a very important property available in this model.

4.2. Continuity of the Magnetic Field Value

In this article and in **Figure 8**, the magnetic field value at any point around a segment is defined by three orthogonal vectors with the same physical value (not one vector as in the classical model). The key consequence of this approach is that the field values induced by multiple segments can be added together to obtain the total field at any point. This additivity is a fundamental property shared by all physical processes, such as gravitational attraction.

4.3. Comments about this Article

This article is a small part of a long series. It has the number 4 and it uses a field value and three vectors. These three vectors were previously defined and published in 2022 [12]. This first article has been defined like to electrostatic field. Then, this definition was applied to two articles composed of long wire [7] [8]. After these three articles, it was necessary to apply this definition to short segments. During this work, the author found no experimental data about field around electrical segments.

Note that this article is devoted to university students and a future article will be written to be understandable by specialists.

5. Conclusion

Modifications to the Biot-Savart Law were necessary to enhance the physical definition of the magnetic field around short segments. The proposed model enables the evaluation of the magnetic field at any point around an electrical current, without exhibiting physical contradictions or discontinuities. It also aligns with the additivity principles commonly used in physics. Using this model, equipotential

magnetic surfaces have been successfully represented. Future applications of this model to more complex geometries, such as a circular or solenoid configuration, will be described in future works.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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