Scientific Research Publishing

Open Journal of Applied Sciences, 2024, 14, 1996-2008
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Magnetic Field Curves and Magnetic Equipotential Surfaces around Crossing Electrical Wires Replacing Classical Magnetic Field Lines

Geoffroy Auvert

INPG, Grenoble Alpes University, Saint Martin d'Hères, France
Email: geoffroy.auvert@grenoble-inp.org

How to cite this paper: Auvert, G. (2024) Magnetic Field Curves and Magnetic Equipotential Surfaces around Crossing Electrical Wires Replacing Classical Magnetic Field Lines. *Open Journal of Applied Sciences*, **14**, 1996-2008.
https://doi.org/10.4236/ojapps.2024.148131

Received: May 23, 2024
Accepted: August 3, 2024
Published: August 6, 2024

Abstract

This article is based on a recent model specifically defining magnetic field values around electrical wires. With this model, calculations of field around parallel wires were obtained. Now, this model is extended with the new concept of magnetic equipotential surface to magnetic field curves around crossing wires. Cases of single, double, and triple wires are described. Subsequent article will be conducted for more general scenarios where wires are neither infinite nor parallel.

Keywords

Magnetic Field Value, Magnetic Field Vector, Magnetic Field Line, Magnetic Field Curve, Equipotential Surface, Crossing Electrical Wires, Magnetic Cross Product

1. Introduction

Electrical moving charges induce magnetic fields around them and steer a compass in a perpendicular direction [1]. The existence of this magnetic field and its orientation is well established today [2], but theoretical descriptions of this field are not so clear as it is in theoretical description of gravity. For examples, magnetic field descriptions by Biot and Savart [3] are not fully accepted by Richard Feldman, Nobel Prize in physics [4]. The first difficulty is that classical superposition process [5] is not clearly acceptable. The second difficulty is about every vector of magnetic fields which are not in the same field space at any points near electrical wires [6]. In actual scientific article, no improvement is available. To address these difficulties, a new theory using another description of magnetic fields, has been recently proposed [7]. This theory is based on a new definition of magnetic equipotential surfaces and magnetic field curves. This theory agrees with the magnetic field formula given by Biot and Savart [3], but its supporting vectors are modified, including a new definition of the magnetic scalar and vector values. This new theory is under test procedures. The first one, dealing with parallel infinite electrical wires, has been published [8]. The second test, concerning crossing infinite wires, is described in the present article. The third test, in future articles, will be performed when magnetic fields use electrical segments.

This paper briefly recalls the theorical base previously detailed [7]. It is followed by the drawing procedure of equipotential surfaces and procedure to form magnetic field curves around a single electrical wire. This procedure is then applied to draw 3D figures of single, double, and triple crossing wires.

Units used in this paper, are meters (m) for distances, Amperes (A) for electrical-current in wires, and Ampere/meter (A/m) for field values and vector lengths at a point placed everywhere out of the wires. To reduce data complexity, the value of the vacuum permittivity is chosen equal to one.

Software and websites used in this article are Mathematica [9], Gimp [10] and Wikipedia [11].

In 3D figures of this article, colorations of vectors and surfaces have some importance. For vectors which all are in the same physical space, vectors having identical physical function have the same color. For surfaces, colors are as in topographic maps depending on their altitudes (z axes).

2. Theoretical Bases around Crossing Infinite Current Wires

This chapter is devoted to the field around one infinite current wire, the study case to briefly recall theoretical bases. This field extends everywhere around a wire with a field value which is decreasing away of the wire. At a given point in this field, a scalar magnetic field value is calculated. At this location, three orthogonal vectors are described. Then they have the same specific physical units in Ampere by meter (A/m). Points and vectors with the same physical units, support every mathematical operation between them.

To resume briefly, any points around the wire have a scalar (non-vector) magnetic field value. The three vectors associated to that point are: magnetic (M), attraction (A) and current (C) field vectors [7].

2.1. One Point and Three Field Vectors

In this paragraph, one point “p” around an electrical wire with a current vector (I) can be placed anywhere around it at distance “r” from the wire.

In **Figure 1**, point “p” is associated to a magnetic field value I/r. Also at that place, three vectors M, A and C, are described.

Specificities about points and vectors are:

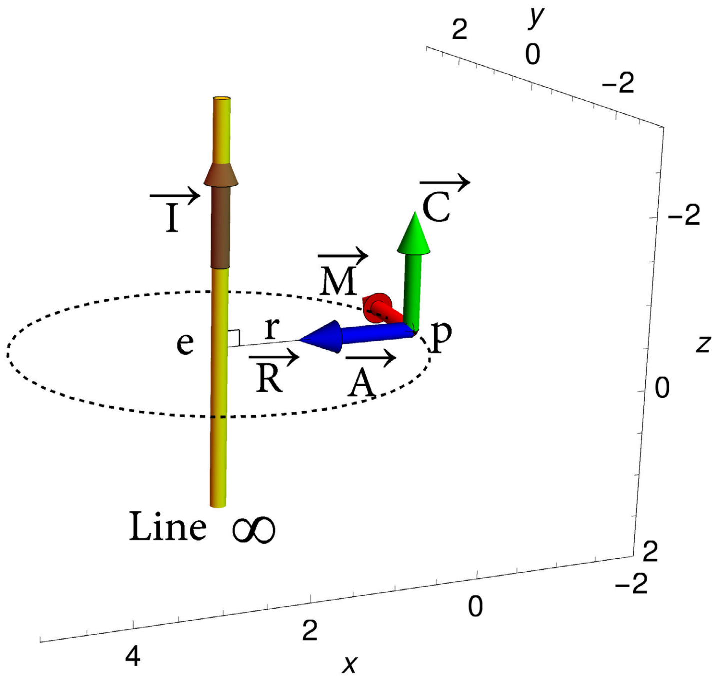

Figure 1. Classical circular field lines with one point "p" and its vectors around an infinite wire with electrical current "I", going upward. This wire imposes at point "p", a current field vector C (green) in the wire direction, an attraction vector A (blue) directed perpendicularly to the wire, and a magnetic vector M (red) in an orthogonal-direction of the wire (also named magnetic field vector). The vector R with length "r", starts at the projection of "p" at point "e" on the wire and ends at point "p". The three vectors: M, A and C, are orthogonal to each other's with the same length "I/r".

- Point "p"

In Figure 1, point "p" is at distance r of the wire which has an electrical current "I". This imposes to have a positive field value of I/r with electronic moving charges. All points and all vectors placed in the space around the wire have the same unit (A/m). This field value is not vector dependent.

- Vector C

In Figure 1, vector C starting in "p", is parallel to the wire with the same direction of electrical charges moving in the wire. Its magnitude is equal to I/r.

- Vector M

For the magnetic vector M (red) in Figure 1, its direction is defined by the classical right-hand rule around the wire [12]. This vector giving the direction of the magnetic field curves will be used to build magnetic curves.

- Vector A

The electrical current induces in point p, an attraction vector A directed toward the wire. Its magnitude is equal to I/r. Letter A stands for: Attraction.

Moreover, Figure 1 is a very general case of moving electrons around a straight wire. In the field value can be calculated and the three vectors giving directions and norms of A, C, and M, are defined. Calculations of these vectors can now be made, and these results are given in Table 1.

Table 1. Numerical relations between vectors A, C and M. At position "p" of Figure 1, far from the current wire which have a vector I direction, vectors A, C and M, have norms equal to I/r. The same physical unit, I/r, used for all three vectors, which means that they belong to is the same one physical space. This property allows various numerical operations for these vectors. These operations and their cross products are now given for the three vectors.

Field name	Attraction vector	Currentvector	Magnetic vector	Point "p"
Vectorname	$\vec{A}$ blue	$\vec{C}$ green	$\vec{M}$ red	Field value
Unityvectors	$\overrightarrow{Au}$	$\overrightarrow{Cu}$	$\overrightarrow{Mu}$	
Vector formula	$\vec{A} = \overrightarrow{-R} \cdot I/r^2$	$\vec{C} = \vec{I}/r$	$\vec{M} = \overrightarrow{-R} \wedge \vec{I}/r^2$	
Orthogonality With unit vectors	$\vec{A} = I/r\left(\overrightarrow{Cu} \wedge \overrightarrow{Mu}\right)$	$\vec{C} = I/r\left(\overrightarrow{Mu} \wedge \overrightarrow{Au}\right)$	$\vec{M} = I/r\left(\overrightarrow{Au} \wedge \overrightarrow{Cu}\right)$	Same Space Unit
Norm, value (A/m)	I/r	I/r	I/r	I/r

In Table 1, magnetic field value at point "p" is equal to the identical norms of the three vectors. The present model used in this article, is using cross product with vectors A, R, C, I and Au, Cu and Mu.

Note that formulas given in Table 1, are different from vector formulas given by Biot et Savart [3]. The main difference is that the three vectors used here are in the same physical space in (A/m).

2.1.1. Definition of Equipotential Surfaces

The concept of "equipotential surface" is frequently used in electrostatic field [13]. For example, a small electrical charge imposes, at a precise voltage, a spherical equipotential surface around itself. As proposed for the first time in our previous article [8], electrostatic and electromagnetism can be approached in the same way. For example, in magnetism with one conducting wire, points with the same I/r value build an equipotential surface in a cylinder around the wire.

On such a surface, vector C and vector M (see Figure 1) belong to a plane tangent to the equipotential surface and parallel to the wire.

As vector A is orthogonal to vectors C and M, it is directly perpendicular to the equipotential surface built around the wire. Then, this vector gives the direction needed to obtain points having any expected magnetic field value. These vectors will be largely used to determine equipotential surfaces in this article and even in the application chapter devoted to 3D drawings.

2.1.2. Drawings of Equipotential Surfaces

This chapter shows how points with the same field values around a current wire are marked to form a part of an equipotential surface.

The drawing of five points with the same field value is shown in Figure 2. The building procedure is detailed below the drawing.

The marking procedure starts at point W where the field value is infinity, then using one line in a chosen direction, the field value decreases when going away from the wire. Then, a marker is plotted where the expected field value I/r is reached. This procedure can be used for the five lines starting at point W. All points S form a circle which is named in this article as an equipotential circle.

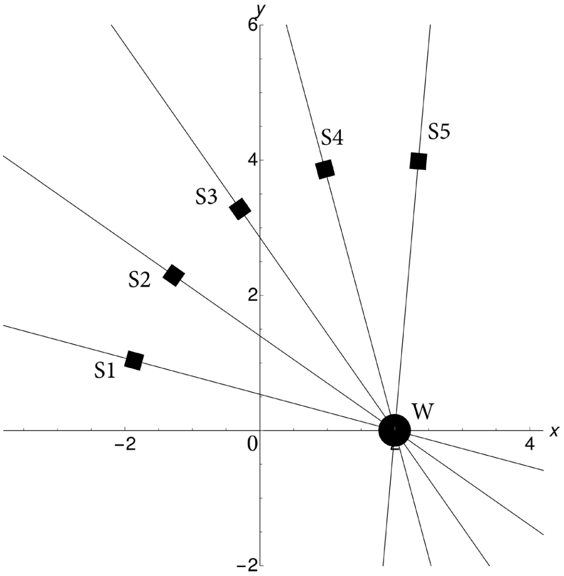

Figure 2. Marking of points with the same physical field value I/r in an x-y plane perpendicular at point "W" to an electrical wire. Several lines are drawn in the several directions. The scalar field value is evaluated all along each lines. Then, point S can be marked when specific scalar field value I/r is reached. With several lines giving points S1 to S5 in the x-y plane, a circle, centered in point W, is a part of an equipotential surface.

As the marking of equipotential circle is possible, an equipotential surface is obtained by using as many circles needed all along the wire.

Note that many other line configurations can be used to draw equipotential surfaces. When vector A is used, it is easier to calculate potential values, but it is not the simplest method.

2.2. Construction of Magnetic Field Curves in Equipotential Surfaces

Field curves will be drawn in equipotential surfaces only by using the direction of vector M, as shown below. Therefore, magnetic field curves having the same field values are always on one equipotential surface.

Figure 3 shows how one field curve is drawn around a single wire in an x-y plane perpendicular to the wire. The drawing procedure is described below the drawing.

These field curves are first, guided by vectors M and second, re-evaluated, in the direction of vector A, the scalar field value.

The drawing procedure of field curves starts a point d0 out of the wire. From d0, and guided by vector A, the field value is calculated and mark p1 is drawn when the expected scalar value I/r of the expected equipotential surface is reached. At p1 and guide by vector M1, point d1 can be marked on this vector M1 direction. As vector M1 is tangent to the field curve, d1 does not have the expected scalar field value. To obtain point p2 with the expected field value, the procedure started previously at point d0 must be repeated. By making five times this procedure, points p1 to p5 are calculated and become visible in Figure 3.

They form a circle centered at point W. These points p have the same field value and they form the same circle as in **Figure 2**. It is called “magnetic field curve”. Remind you that magnetic vector M is used as a guide for the magnetic field curve.

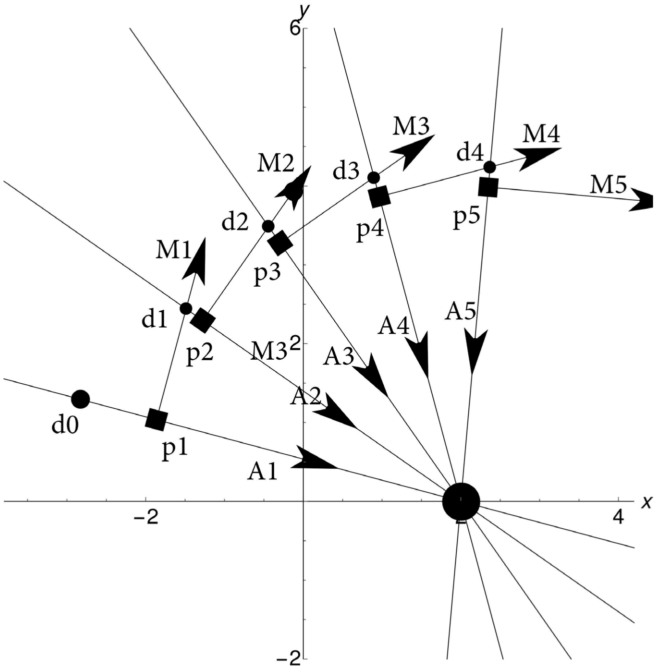

Figure 3. Drawing successive field points forming a magnetic field curve around a single electrical wire. The wire is perpendicular to the x-y plane and goes through point W. Vectors M are magnetic field vectors at points p. In the left side, d0 is the starting point to calculate successively all points p which are forming around the wire, the field curve which stays in the equipotential surface.

This procedure in 2D representations will be extended to 3D representations in the application chapter.

2.3. Two Crossing Infinite-Current Wires

This chapter describes the modifications imposed when adding a second electrical wire.

This is shown in **Figure 4**.

In **Figure 4**, vectors have different colors. Brown vectors determine electrical current in yellow wires. The three other colors are for vectors in the same physical space with field unit in (A/m). Green colored vectors are parallel to the wires, blue vectors have attractive directions to the wires, and red vectors are the cross product of other green and blue vectors.

It is possible to add vectors having the same physical units. With a larger number of wires, additions are given by the following formulas. Formula 1 is an addition of green vectors C. Formula 2 is addition of attractive blue vectors. Formula 3 is an addition of magnetic red vectors.

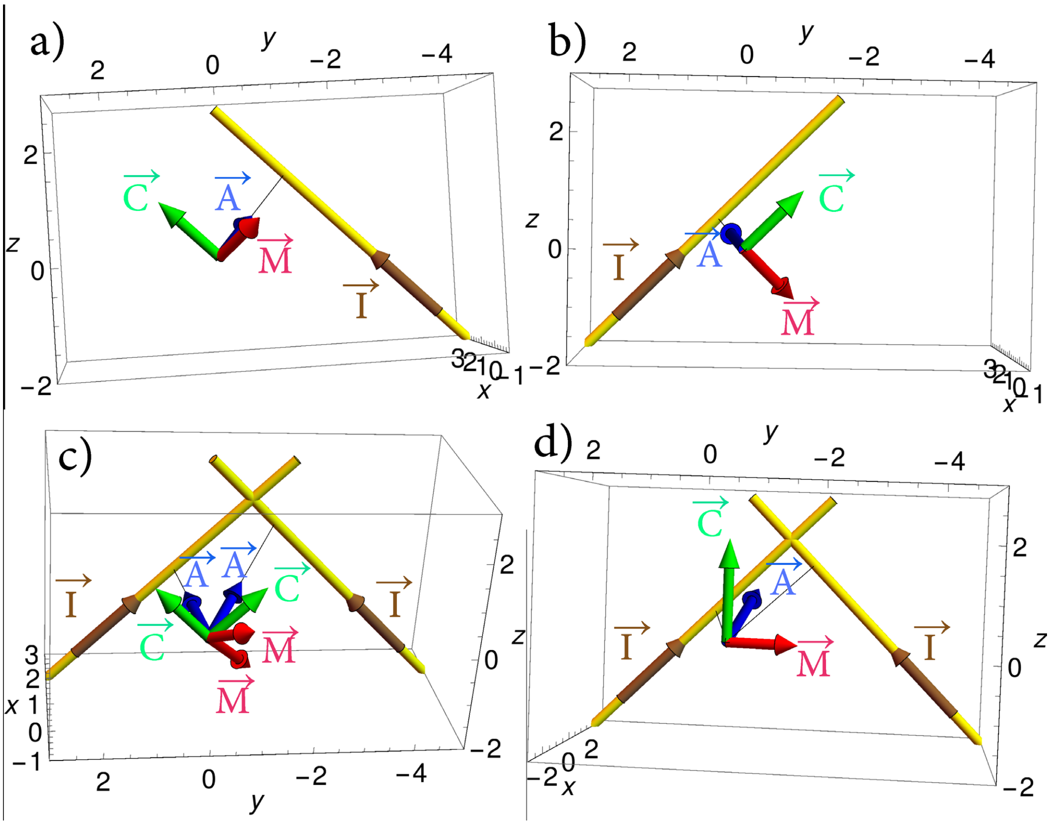

Figure 4. Vectors at point "p" with two crossing infinite wires with current "I. Each wire imposes in point "p", a scalar field value with magnetic vectors M (red), vectors A (blue) and vectors C (green). For each wire, the three vectors A, C and M are orthogonal, but after additions, all these vectors are not with the same norm which depends to the distance with the corresponding wire.

$$\vec{C} = \sum_i \left(\frac{I_i}{r_i} * \overrightarrow{Cu_i} \right) \tag{1}$$

$$\vec{A} = \sum_i \left(\frac{I_i}{r_i} * \overrightarrow{Au_i} \right) \tag{2}$$

$$\vec{M} = \sum_i \left(\frac{I_i}{r_i} * \overrightarrow{Mu_i} \right) \tag{3}$$

This particularity is applicable to the scalar field value which is the addition of every field values of each wire. This addition of scalar field values allows to calculate equipotential surface. This addition capability will be now used for 3D drawings in the following application chapter.

3. Application

The objective of this chapter is to build points of magnetic field equipotential surfaces on which magnetic field curves are also placed. This drawing procedure is not conformed to classical magnetic field vectors used to evaluate field lines [14]. In the present article, surfaces and curves are drawn according to model described above in three theoretical conditions: one, two and three electrically active wires.

3.1. One Infinite Wire with Equipotential Surfaces and Field Curves

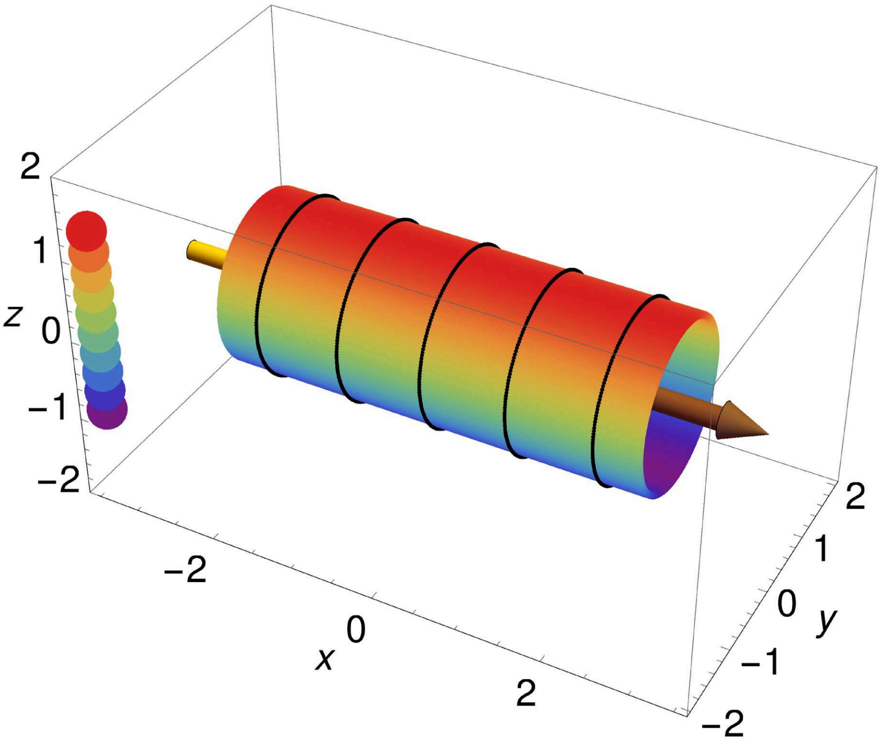

Figure 5. One wire with its electronic current I, is along the *x* axes and passes at $y = z = 0$. The cylinder is around the wire and with its field value of one (A/m). The cylinder color depends on the *z* value *i.e.*, like in a topographic map. The black magnetic field curves are drawn on the cylindric equipotential surface forming circles (as in classical drawing).

In **Figure 5**, equipotential surface value is chosen to be one (A/m) with a current of one ampere in the wire. These results in a cylindrical equipotential surface is located at one meter away and centered around the wire. On this cylinder, magnetic field curves are drawn using the previous procedure which starts in the vector M direction. Then a correction of the scalar field value is calculated in the vector A direction.

Note: In **Figure 5**, black field curves drawn on the equipotential cylinder are fully compatible with results obtain with classical field lines when only one wire is present [14].

3.1.1. Two Crossing Infinite Wires with Equipotential Magnetic Surfaces and Magnetic Field Curves

Every point on the colored equipotential surface in **Figure 6**, has the same field value of one (A/m). Consequently, none of these points depend on any vector. For black field curves, only vector directions are used.

In **Figure 6**, black points are calculated with one starting point which is followed by more than 1000 points. The calculation stops when the point becomes very near the starting point.

Any field curves in **Figure 6**, never touch another curve. This observation agrees with the proposition of Gauss which says that magnetic fields is a vector field with a zero divergence *i.e.* it has neither source nor end [15].

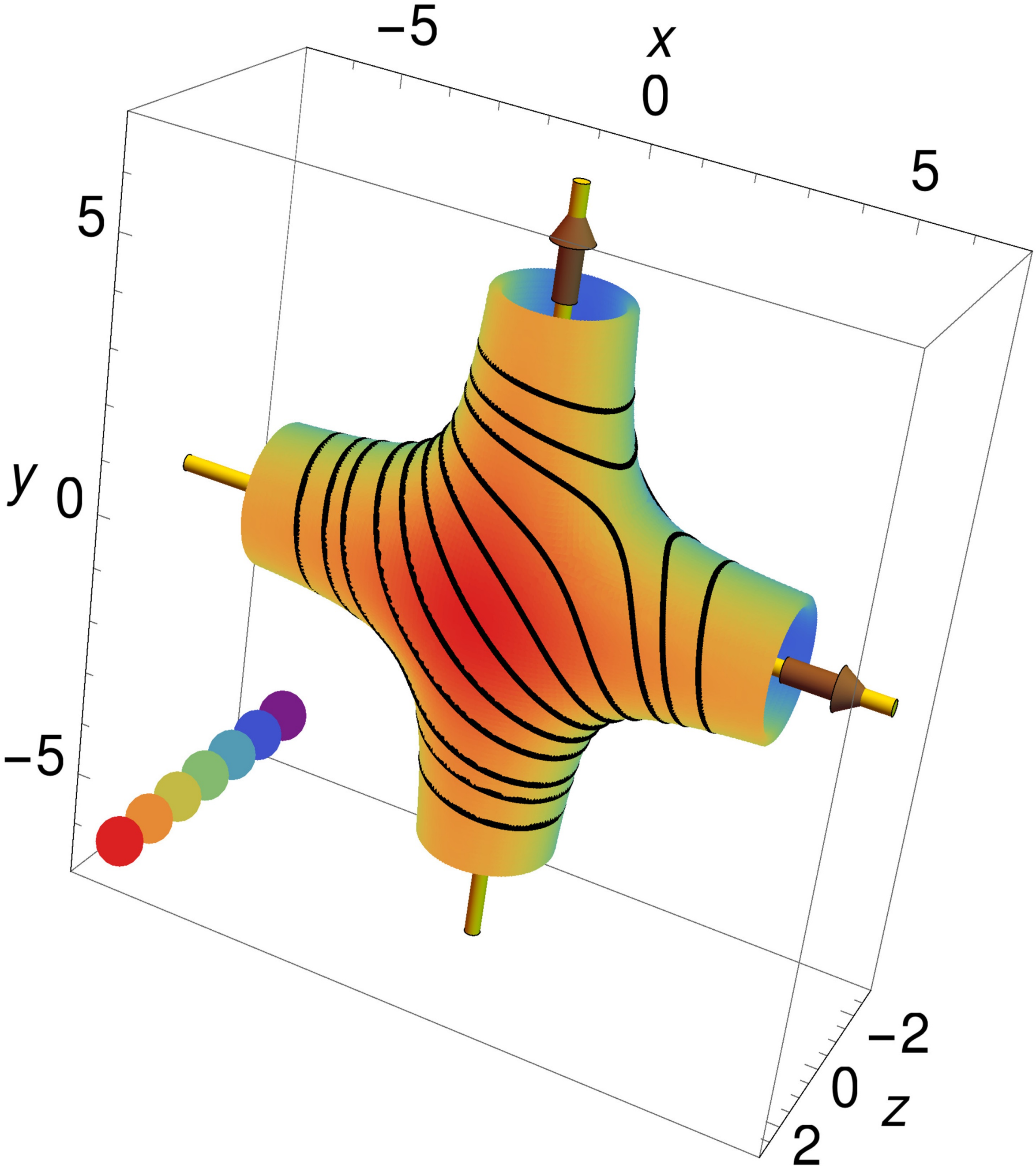

Figure 6. Equipotential surface and magnetic field curves at a field value of one (A/m) around two perpendicular infinite wires places along x and y axes. The z scalar field values at any point of the equipotential surface have the color depending on its altitude. It is red when z is the highest, but yellow or green when z decreases and becomes blue for negative z value like in point placed behind the brown vector of the current along x axes. On this surface, magnetic curves are in black color. These curves do not have end points wherever they start. Also, these curves which stay on the multi-colored surface, look non-circular but they never contact each other.

3.1.2. Three Crossing Infinite Wires with Magnetic Equipotential Surfaces and Magnetic Field Curves

In **Figure 7**, field curves seem to be nearly like in **Figure 6**, but it will be shown in **Figure 8**, that some curves have a very particular drawing.

As previously for two wires, each starting points have a return point equal to the starting point after it is drawn on the equipotential surface.

Details about crossings of black field curves are in the discussion chapter.

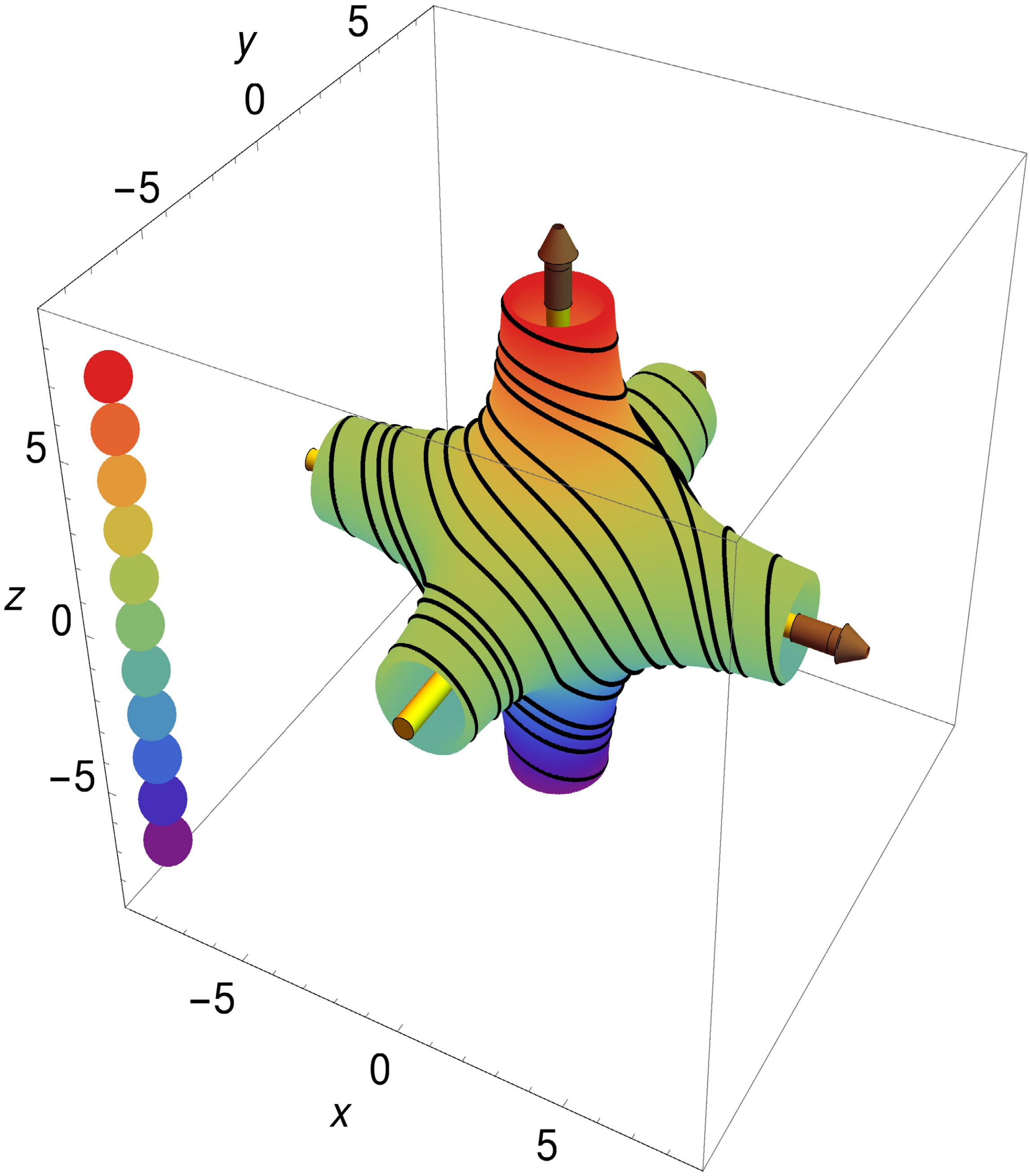

Figure 7. One equipotential surface and magnetic field curves at a field value of one (A/m) around three perpendicular infinite wires places along axes *x*, *y* and *z*. These curves are nearly circular when they are near one wires, *x* or *y* or *z*. These curves are deformed near crossing point of the wires.

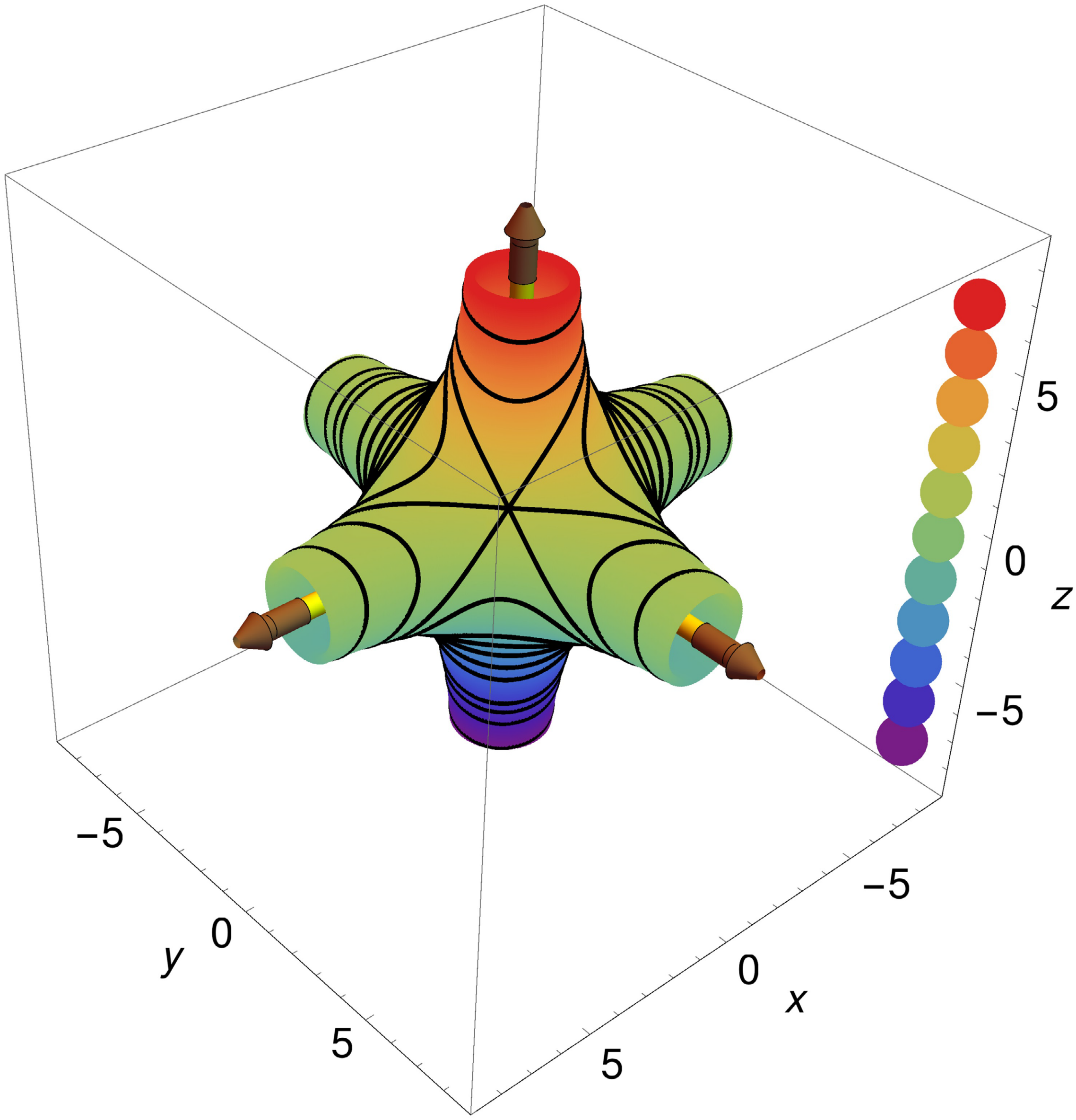

Figure 8. Top view of an equipotential surface with magnetic field curves around three infinite wires placed along x, y and z axes. The colors of the equipotential surface are determined by the z value. When $z = 9$, the color is red. When z is around zero, the color of the equipotential surface is green. When z is at $z = -5$, the color is dark blue. In the center between the axes x, y, z, one magnetic field curve is crossing itself three times. This topside line separates three domains placed above from the center domain which is just below. This drawing has a unique rotation line $x = y = z$ with 120 degree. Moreover, the same non-visible domain repartitions exist on the part place behind this drawing.

4. Discussion

4.1. Number of Domains

In our previous publication [8], definition of borders between domains has been given. To summarize, a border is a magnetic field curve which cross itself separating two domains. In **Figure 5**, as curves are circular, they never cross themselves. In this case, the physical space around the wire is made of one domain.

When three wires cross as in **Figure 8**, the field curve looks like a three-leaf clover. This is a multi-domain case: Its center seen to be the center of the drawing. In that case, each leaf depicts the clover has one domain, to which one can add the external, and main. By symmetry and on the hidden side of the equipotential surface, there are also three internal domains. The total number of domains is thus seven.

In the case of two wire crossing, as in **Figure 6**, the borders have not been drawn but one could reasonably suppose that five domains exist.

4.2. On Precision of Calculations and Results

In the present article, studied field lines are about ten meter long. Mathematically, the iterative process implies that a calculation error adds up at each step of the process. The precision of the software thus plays a crilical role in the proximity of the last point with the starting point.

For the curves drawn in the present article, the distance between successive points is chosen to be 5 * 10^(−3) m. This gives about several thousand points on each field curves. The distance between the starting point and end point is around this value. At our drawing resolution, end points are blended with starting points and positions are not visible on the figures.

For the clover of **Figure 8**, the crossing position of leaves is mathematically known to be the square root of 9/2. To reach this point, precision of the starting point must be 10^(−8) m. With it, the lower distance of point of the curve to the pass connection point is 14 * 10^(−3) m. This is small enough so they cannot be seen on curves drawn in the equipotential surfaces.

5. Conclusion

This article is based on a theoretical hypothesis that magnetic field points are first based on a scalar field value and secondarily with three vectors. The first is to draw equipotential surface without using vector but having points with a common field value. In this surface, magnetic field curves are guided by a magnetic vector further adapted with a second vector. These curves never connect each other but some very particular crossing curves exist. They are named domains borders separating areas in the equipotential surfaces. These magnetic field curves are drawn around one, two and three crossing wires. Soon, equipotential surfaces and magnetic field curves around segments of electrical wires will be studied.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Magnetic Field. https://en.wikipedia.org/wiki/Magnetic_field

[2] Orsted's Law. https://en.wikipedia.org/wiki/Oersted%27s_law

[3] Biot-Savart Law. https://en.wikipedia.org/wiki/Biot–Savart_law

[4] Feynman Lectures. https://www.feynmanlectures.caltech.edu/II_01.html#Ch1-S5

[5] Superposition Process in Physique. https://en.wikipedia.org/wiki/Superposition_principle

[6] Electric Currents (along a Wire). https://en.wikipedia.org/wiki/Biot%E2%80%93Savart_law

[7] Auvert, G. (2023) A Unified Definition of Electrostatic and Magnetic Fields. *Open Journal of Applied Sciences*, **13**, 793-780. https://doi.org/10.4236/ojapps.2023.135063

[8] Auvert, G. (2024) Utilizing Iso-Value Field Curves in Lieu of Magnetic Field Lines amid Infinite and Parallel Electrical Wires. *Open Journal of Applied Sciences*, **14**, 70-84. https://doi.org/10.4236/ojapps.2024.141006

[9] Mathematica (2024) Champaign, Illinois. https://www.wolfram.com/

[10] GIMP (2019) Development Team. https://www.gimp.org

[11] Wikipedia. Wikimedia Foundation, 28 May 2022. https://en.wikipedia.org/

[12] Right Hand Rule. https://en.wikipedia.org/wiki/Right-hand_rule

[13] Champ électrostatique: Surfaces équipotentielles. https://fr.wikipedia.org/wiki/Champ_électrique

[14] Field Line. https://en.wikipedia.org/wiki/Field_line

[15] Gauss's Law for Magnetism. https://en.wikipedia.org/wiki/Gauss%27s_law_for_magnetism