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Study and FDTD Modeling of the Influence of Direct Lightning Shock on the Power Transmitted by a High-Voltage Power Transmission Line

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Abstract

Electric towers of high voltage transmission lines are more exposed to natural lightning phenomena thanks to their high heights. These lines are crossed by powerful current sources to dissipate in the ground, which can, at one time or another, create disturbances or other phenomena can be generated. This is why we have set ourselves the objective of studying the FDTD modeling of the influence of direct lightning strikes on the power transmitted by a High-Voltage power line. To do this, we have implemented Kirchhoff's laws to model the power transmitted by a High-Voltage power line in a steady state. Calculating the electromagnetic field generated by lightning requires the lightning current along the channel and its spatiotemporal distribution, the bi-exponential models and that of engineers were chosen and used to reproduce the physical phenomena best. Several works have been published in the literature and various mathematical models are proposed, to study the filamentous nature of power lines which has led to a more flexible modelling, based on the transmission line model, associated with the field theory developed from Maxwell's equations, which explain the interaction between a lightning wave and a power transmission line. The resolution of the line equations in the lightning shock regime was the subject of the FDTD method to obtain the results in the spatiotemporal domain. Through this research, we are interested in the study of the spatiotemporal distribution of the lightning current wave to model the radiated electromagnetic field and to examine the influence of the overvoltage induced by the atmospheric discharge on the transportable power of a High Voltage AC Transmission line, for good selective protection to illuminate the parasites. 2D simulations based on proposed models were developed as well as the verification of the consistency of the different models, by comparing the fractal dimensions of the results of our program with those of the figures obtained experimentally. The aspects developed in this article could have direct implications in practical applications in the engineering and design of high-voltage transmission systems.

Keywords

Modeling, Direct lightning Strike, Lightning Current, Induced Overvoltages, Transmitted Power, HV Line, Kirchhoff Equations, Coupling Equation, Bi-Exponential Model, Agrawal Model, Engineers' Model, Transmission Line Theory, Maxwell's Equation, FDTD Method

1. Introduction

The considerable development of the electrical energy transmission network in its various components which ensure an optimal distribution of energy transits, real-time knowledge of the electrical quantities characteristic of the latter, makes it possible to ensure its control and command and gradually leads to the study of the various disturbances [1]; by their mode of transmission such as the phenomenon of wave propagation which is the basis of many cases of information transmission such as Hertzian waves, telecommunications networks, energy distribution networks, lightning, nuclear electromagnetic pulse [2]; by their form in particular, interruptions, the flicker phenomenon and high frequencies [3] as well as in aeronautics to name but a few; and by the fact that they affect electrical or human organs [4]. In order of priority and impact rate in the stress classes, lightning represents the unpredictable and most harmful natural phenomenon on all electro-energy systems [5] and also, on the environment, if we can affirm that nowadays, energy carriers adequately control the protection of the network against accidental internal faults, it is not the same for its protection against lightning, in particular during an indirect impact where it radiates significant electromagnetic fields and which will induce by electromagnetic coupling cruel overvoltages in their targets, in particular the electrical energy transmission networks [6] as well as in telecommunications.

External surges propagating in the electrical network are likely to cause dielectric breakdowns in the materials that provide insulation for the network components [7], the drop in transmissible active power can cause untimely disconnection in the case of lines operating without power reserve margin, etc. [8] These therefore require a detailed study of the propagation of these surges, supposed to be the most severe constraints on electro-energy systems, to predict the level they can reach at any point in the electrical network to recommend adequate solutions [9].

The major concern today for both operators and consumers and the energy transport whose main element is the power line is to control the transmission parameters, particularly in alternating current, to ensure stability and increase efficiency that can illuminate parasites for good selective protection [10]. Given the complexity of the study of the parameters of a line as well as the number of data entered have several unknowns; we will limit ourselves to the study of fluctuations in the transportable power for a compensated inductive line to approach reality.

2. Study Method

In this research, we were interested in the analysis of the behavior of AC HV lines during an atmospheric discharge. Because these transmission lines are the most exposed to disturbances during a lightning discharge as we mentioned previously. Their study of transient behavior is generally based on the theory of transmission lines with electrical components R, L, C and G [11]. Several equations exist for their modeling depending on the shape and characteristics of the material used in the installation. The bi-exponential models [12] and the so-called engineering models [13]-[16] were chosen and used for the analytical modeling of the spatio-temporal distribution of the lightning current. This allows to study the interaction between a lightning wave and a power transmission line, of which several types of research have been done in this field and various mathematical models have been proposed, as well as, the different studies on power lines have led to a more flexible modeling based on the different models of transmission lines, associated with the field theory developed from Maxwell's equations. The resolution of the equations of the lines in the lightning shock regime was the subject of the FDTD method to obtain the results in the spatio-temporal domain.

The choice of the proposed models and specific parameters is justified by the flexibility in the implementation of the modeling of the phenomenon studied, reduces the calculation time and makes it possible to easily simulate the effects of lightning shocks, which gives the possibility of calculating and evaluating the risks and designing more robust infrastructures.

The study required the development of a calculation code using computer tools, based on the FDTD method [17]-[20]. The developed code was then validated by comparisons of the results obtained with experimental results drawn and compared to other studies. Validation against real measurements allowed us to establish the accuracy and reliability of the proposed models. The evaluation of the spatio-temporal distribution of the lightning current wave during the direct impact and the overvoltage induced by the lightning discharge at the output of the HV line interested us a lot [21] [22]. The results obtained are consistent with the analyses carried out by other researchers in this field. Thus, we have highlighted the influence of this overvoltage on the power transportable by an HV line. Some models allowed us to carry out these studies, in particular the theoretical models and their different resolutions as well as the FDTD model of the power transmitted by an HV line in lightning regime. The study of the influence of direct lightning strike on HTAC transmission lines is essential to ensure the safety, reliability and efficiency of electrical systems, while minimizing the risks associated with extreme climatic events such as thunderstorms that can cause serious short circuits.

3. Theoretical Models

3.1. Modeling of Power Transit on HV Lines in Normal Mode

The voltage of a line must remain fairly constant because the active power consumed by customers varies. Generally, the voltage variation from zero to full load should not exceed a certain value ranging from 5% to 10% of the nominal voltage depending on the type of system. The maximum power that a line can carry also depends on the voltage variation [23]. Here, the study is done on a compensated inductive line. We consider an inductive line of negligible resistance but which has an inductive reactance X and a capacitive reactance X_c. The voltage V measured at the end of the line decreases when the load increases [23].

$$V_\infty = E - X \cdot I \tag{1}$$

with: E: the electromotive force in Volts.

The relation (1) between the voltage V at the load busbar and the load current I is called. This system which defines the equation of a straight line passing through V and having the slope as shown in **Figure 1** next.

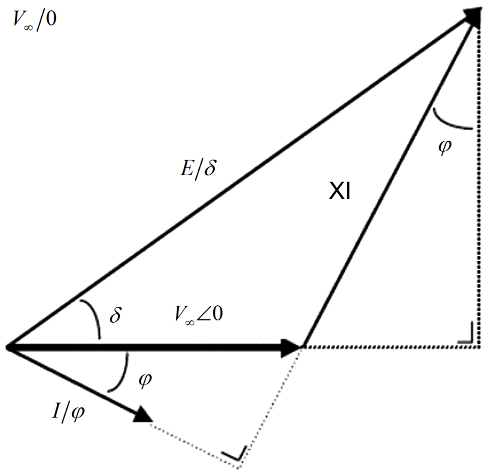


Figure 1. Vector diagram of electrical quantities.

As a reminder, this model only makes sense in the case of a steady state. We also consider that this machine is connected to a so-called "infinite" network, *i.e.* one *whose* voltages and phase remain constant regardless of the disturbances imposed [23]. The operation of this system is then described by Equation (1), written in polar notation. The network here represents the synchronous reference. This results in the vector diagram in **Figure 2** below.

$$E - jX \cdot I = V_\infty \tag{2}$$

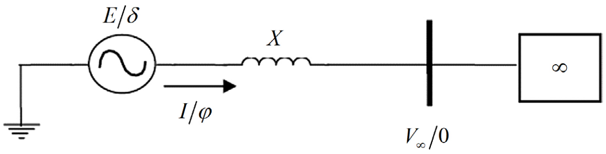


Figure 2. Diagram of an inductive line.

The regulation can be improved and the transportable power increased by adding a capacitive reactance X_c at the end of the line. The voltage V can be kept constant by adding the value of X_c so that the reactive power supplied by the capacitor see expression (3) below is equal to half the reactive power absorbed by the line see expression (4) below [23]:

$$P_{ab} = \frac{u^2}{x_c} \tag{3}$$

$$P_f = x_c \cdot I^2 \tag{4}$$

with: P_f : power supplied by the capacitor in kVAr and P_{ab} : reactive power absorbed by the line in kVAr.

However, we see that there is always an upper limit $P_{\max}$ to the active power carried by the line.

$$P_{ac} = \frac{E}{x}\left(2E - V_\infty\right) \cdot \sin\delta \tag{5}$$

with: P_{ac} : Upper limit to the active power that the line transports in MW.

Relation (5) shows that the transmissible power as a function of the phase shift angle "δ" exists between the machine and the infinite network. The curve of this power as a function of the phase shift is then half-sinusoidal see **Figure 3** below [23].

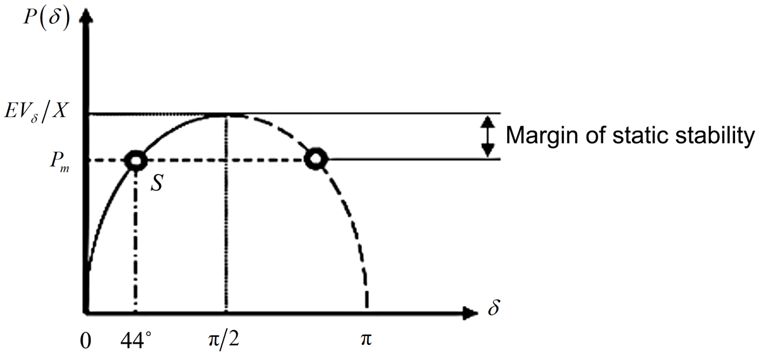


Figure 3. Transmissible power as a function of the phase shift between current and voltage.

Points "S" such that "δ" is less than π/2 are stable, while points "I" for which "δ" ∈ [π/2, π] are unstable. This can be explained by the following two remarks: Let "P_m" be the mechanical power supplied to the generator, cfr **Figure 3** [23]:

For "$\delta < \pi/2$", if a small disturbance occurs tending to increase or decrease the rotor angle, the electrical power produced becomes greater or less than the mechanical power rotor angle, the electrical power produced becomes greater or less than the mechanical power. Given the equation of motion of the synchronous machine, the angle δ tends to decrease because "$\frac{d^2\delta}{dt^2} < 0$". In both cases, there is a propensity of the system to return to equilibrium.

In both cases, the system tends to return to equilibrium.

For "$\delta > \pi/2$", the behavior is the opposite: far from its equilibrium position, the rotor tends to move further away from it! Although in theory the stability zone extends from 0 to π/2, in practice we are limited to a value of the order of $\delta_{\max} \sim$ 30° to 35°, for a heavily loaded network. The stability margin thus preserved makes it possible to maintain the system in a stable state, even in the event of a transient disturbance such as a change in a modification of the load distribution during a network reconfiguration, a modification of energy production, or in the event of a temporary fault or loss of the generator set.

Note: A distinction must be made between the angle "δ" between the voltages of two neighboring network nodes and the angle between two. The value of the latter can reach 45° [23].

In the equivalent diagram of **Figure 4** below, we neglect, as a first approximation, the effect of the capacities of the capacitors and the inherent resistance of the lines. The latter is then represented only by an inductance X as shown in the following diagram [23]:

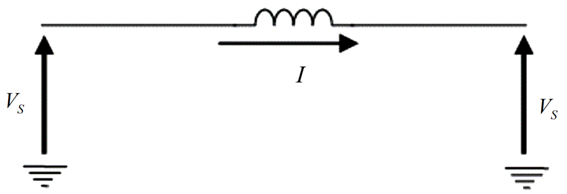


Figure 4. Simplified diagram of the line.

By a reasoning similar to the previous one, we deduce that the power transfer is guided by expression 6. These results can also be obtained from the load distribution equations.

$$P = \frac{u_s^2}{x} \cdot \sin\delta \tag{6}$$

Either:

$$P = \frac{u_s}{x}\left(2u_s - u_R\right) \cdot \sin\delta \tag{7}$$

with: P: power transmissible by the inductive line in MW; u_s: source voltage in kV and u_R: voltage at the line output or at the load terminals kV.

Disturbances in the network load pattern have a direct impact on this angle δ, which requires it to be kept, for stability reasons, below a value generally lower than 30° - 35°. For a connection loaded at its natural power, this angle limits the series reactance, and therefore sets a limit length. The maximum power $P_{\max}$ is reached if $\delta = 30^\circ$ and $u_R = u_s$.

$$P_{\max} = \frac{u_s}{2 \cdot x}\left(2u_s - u_R\right) \tag{8}$$

Legend: $P_{\max}$: Maximum power in MW; u_s : Source voltage in kV; u_R : Load voltage in kV *and* x : Line reactance in Ohm .

3.2. Modeling of There Lightning as a Source of Radiation

The lightning channel and the elements carrying the lightning current to the ground create an electromagnetic field. Induced currents and voltages will then appear in nearby conductors. The resulting potential differences can in turn cause breakdowns in the electronic or electrical elements connected to these conductors. These breakdowns can also be of high intensity and create a risk of ignition or destruction of the same type as that created by the direct strike. Some equipment sensitive to electromagnetic disturbances can be disturbed or destroyed by the field created by a nearby lightning strike. In the presence of a thundercloud, a cloud-to-ground discharge can be initiated. This is due to a deformation of the electric field [17] [18] [21].

3.2.1. Temporal Models of the Current Wave the Base of the Lightning Channel

To model lightning, it is considered as a time-varying current wave [23]. The oldest and simplest analytical model is a difference of two decaying exponentials. It was proposed by Bruce and Golde in 1941 [Cooray V., 2003] [24]-[27]. It is represented by the following relation:

$$i(t) = I_0\left(e^{-At} - e^{-Bt}\right) \tag{9}$$

I_0 is the amplitude of the base current, A and B are time constants.

The analytical model usually adopted is that proposed by Heidler, frequently known as the "Heidler function", where the lightning wave is represented by the base channel current by the following expression [Heidler F., 1985] [28]:

$$i(0,t) = \frac{I_0}{\eta} \frac{\left(\frac{t}{\tau_1}\right)^n}{1 + \left(\frac{t}{\tau_1}\right)^n} e^{\frac{-t}{\tau_2}} \tag{10}$$

Legend:

I_0 : amplitude of the base channel current in kA;

τ_1 : Rise time constant

τ_2 : Decay time constant;

η : Correction factor for wave amplitude and

n : is an exponent varying between 2 and 10.

The Heidler function is preferred over the classical (double exponential) form for the following reasons:

- The Heidler function has a derivative equal to 0 at $t = 0$, which fits well with experimentally measured lightning strikes;
- It allows precise adjustment of the current amplitude, the maximum current derivative and the transferred electric charge independently by varying I_0, τ_1 and τ_2.

In order to reproduce a particular lightning waveform, a combination of two Heidler functions can be used. Relation (2) then becomes:

$$i(0,t)=\frac{I_{01}}{\eta_1}\cdot\frac{\left(\frac{t}{\tau_{11}}\right)^{n_1}}{1+\left(\frac{t}{\tau_{21}}\right)^{n_1}}\mathrm{e}^{\frac{-t}{\tau_{22}}}+\frac{I_{02}}{\eta_2}\cdot\frac{\left(\frac{t}{\tau_{11}}\right)^{n_2}}{1+\left(\frac{t}{\tau_{21}}\right)^{n_2}}\mathrm{e}^{\frac{-t}{\tau_{22}}} \tag{11}$$

A combination of the Heidler function and the double exponential (relations 1 and 3) was used by Nucci to reproduce a typical lightning wave obtained by measurements [Nucci, CA, Diendorfer G., Uman M., Rachidi F., Ianoz M., and Mazzetti C., 1990] [29].

3.2.2. Model of the Spatio-Temporal Distribution of Lightning Current in the Channel

In recent years, several models of the return arc, with different degrees of complexity, have been developed to assess its electromagnetic radiation (e.g. [30]-[36]). One of the major difficulties in modeling the lightning channel is that the current can only be measured at the base of the channel; however, to determine the radiated electric and magnetic fields, it is necessary to know the current distribution along the channel. The proposed return arc models differ essentially from each other in the description of the spatial and temporal distributions of the current along the lightning channel $i(z',t)$. In [33], the back-arc models are classified into four categories: the gas dynamic models, the electromagnetic models [37] [38], the distributed circuit models [39] [40] and the engineering models [30] [32] [34]. A general description of the four models is given in [33] [34]. In this paper, we will consider only the five models presenting respectively the spatial and temporal distribution of the back-arc current, namely, the BG, TL, TCS, MTLL and MTLE models. which, Rakov [41] (and recently) [33] [42], expressed the technology models by the following generalized equation:

$$i(z',t)=u\left(t-\frac{z'}{v_f}\right)\cdot P(z')\cdot i\left(0,t+\frac{z'}{c}\right) \tag{12}$$

where: $u(t)$: is the Heaviside function $\begin{cases} t\geq\frac{z'}{v_f}\rightarrow u(t)=1 \\ aileurs\rightarrow u(t)=0 \end{cases}$

$P(z')$: is the height-dependent attenuation factor introduced by (Rakov and Dulzon) [41], and v_f: is the upward return arc propagation speed, v: is the propagation speed of the current wave. Table 1 below summarizes the parameters $P(z')$, and v_f for different models mentioned above:

Table 1. $P(z')$ and v_f in Equation (12) for the 5 models described above [41].

Models	$P(z')$	v_f
BG	1	∞
TL	1	v
TCS	1	$-C$
MTLL	$1 - z'/h$	v
MTLE	$\mathrm{Exp}(-z'/\lambda)$	v

Based on expressions (9) and (4) below, we implemented the five spatio-temporal distribution models of the lightning current wave:

$$\begin{cases} i(t) = I_0\left(\mathrm{e}^{-At} - \mathrm{e}^{-Bt}\right) \\ i(z',t) = u\left(t - \dfrac{z'}{v_f}\right)\cdot P(z')\cdot i\left(0, t + \dfrac{z'}{v_f}\right) \end{cases}$$

3.2.3. Bruce-Golde Model (BG)

Bruce and Golde [23] considered that the return arc propagates vertically from the ground with a velocity “*v*” which is less than the speed of light “*c*”. The current at each point of the channel can be expressed, mathematically as follows:

$$i(z',t) = i\left(0, t - \frac{z'}{v}\right) \tag{13}$$

By combining equation (9) and (13), we obtain below, the BG model reproducing the spatio-temporal distribution of the lightning current wave.

$$i(z',t) = I_0\left(\mathrm{e}^{-A\left(t-\frac{z'}{v}\right)} - \mathrm{e}^{-B\left(t-\frac{z'}{v}\right)}\right) \tag{14}$$

3.2.4. “Transmission Line” (TL) Model

This model equates the lightning channel to a lossless transmission line where a current pulse propagates from the ground at a constant return arc velocity “*v*”. This model was proposed by “Uman” and “McLain” in 1969 [42], and is widely used to this day. The current distribution is defined by:

$$i(z',t) = i\left(0, t - \frac{z'}{v}\right) \tag{14a}$$

By staging expression (9) and (14a), we easily find the TL model corresponding to the lightning current distribution.

$$i(z',t) = I_0\left(\mathrm{e}^{-A\left(t-\frac{z'}{v}\right)} - \mathrm{e}^{-B\left(t-\frac{z'}{v}\right)}\right) \tag{15}$$

3.2.5. Modified Transmission Line Model (MTL Models)

In order to overcome the shortcomings of the TL model while maintaining its simplicity which allows easy use in coupling calculations, a modification to the TL model has been proposed by (Nucci *et al.*), (Rachidi and Nucci), and (Rakov and Dulzon) [43]-[45]. These two models are described below:

$$i(z',t)=\left(1-\frac{z'}{h}\right)\cdot i\left(0,t-\frac{z'}{v}\right) \tag{16}$$

Combining Equation (9) and (16), we find the "modified transmission line" (MTL) model that can reproduce the spatio-temporal distribution of lightning current.

$$i(z',t)=\left(1-\frac{z'}{h}\right)\cdot I_0\cdot\left(\mathrm{e}^{-A\left(t-\frac{z'}{v}\right)}-\mathrm{e}^{-B\left(t-\frac{z'}{v}\right)}\right) \tag{17}$$

3.2.6. Modified Exponential Decay Transmission Line Model (MTLE)

In this model the spatio-temporal distribution, the parameter λ represents the decay rate of the current intensity along the channel; its value, was determined by (Nucci and Rachidi) (λ = about 2 km) [46], and by (Lin *et al.*) using simultaneous recordings of electromagnetic fields at several distances published in [47] [48]. The MTLE model is the most used, the compact MTLE model of the lightning current distribution $i(z',t)$ is defined by:

$$i(z',t)=\mathrm{e}^{\frac{-z'}{\lambda}}\cdot i\left(0,t-\frac{z'}{v}\right) \tag{18}$$

The staging of Equations (9) and (18) gives the generic MTLE model below:

$$i(z',t)=I_0\cdot\left(\mathrm{e}^{-A\left(t-\frac{z'}{v}\right)}-\mathrm{e}^{-B\left(t-\frac{z'}{v}\right)}\right)\cdot\mathrm{e}^{\frac{-z'}{\lambda}} \tag{19}$$

3.3. Formalism of the Calculation of Electromagnetic Field Radiated by Lightning

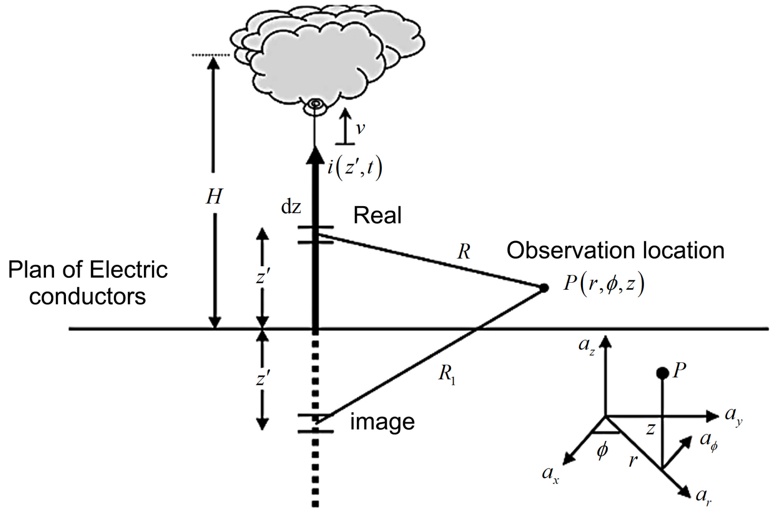


Figure 5. Representation of the lightning channel in the Hertzian dipole method for calculating emitted electromagnetic fields (perfectly conductive ground).

For the calculation of the electromagnetic field radiated by a ground-cloud lightning discharge, the commonly adopted geometry is that shown in **Figure 5** above. The Lightning channel is considered as a one-dimensional vertical antenna of height H placed above a conductive plane. The return arc propagates vertically from the ground with a speed v. It is traversed by a current whose spatio-temporal distribution $i\left(z',t\right)$ determines the electromagnetic field at any point in space.

For the case of a perfectly conductive ground, the dipole formalism offers the advantage of a mathematical model which can be written in both frequency and time, which avoids mathematical transformations (FFT or Laplace) [48].

$$\begin{cases}
\mathrm{d}E_r\left(r,z,t\right)=\dfrac{\mathrm{d}z'}{4\pi\varepsilon_0}\left[\dfrac{3r\left(z-z'\right)}{R^5}\displaystyle\int_0^t i\left(z',\tau-\dfrac{R}{c}\right)\mathrm{d}\tau+\dfrac{3r\left(z-z'\right)}{cR^4}i\left(z',t-\dfrac{R}{c}\right)\right. \\
\qquad\qquad\left.-\dfrac{r\left(z-z'\right)}{c^2R^3}\dfrac{\partial i\left(z',t-\dfrac{R}{c}\right)}{\partial t}\right] \\
\mathrm{d}E_z\left(r,z,t\right)=\dfrac{\mathrm{d}z'}{4\pi\varepsilon_0}\left[\dfrac{2\left(z-z'\right)^2-r^2}{R^5}\displaystyle\int_0^t i\left(z',\tau-\dfrac{R}{c}\right)\mathrm{d}\tau\right. \\
\qquad\qquad\left.+\dfrac{2\left(z-z'\right)^2-r^2}{cR^4}i\left(z',t-\dfrac{R}{c}\right)-\dfrac{r^2}{c^2R^3}\dfrac{\partial i\left(z',t-\dfrac{R}{c}\right)}{\partial t}\right] \\
\mathrm{d}H_\varphi\left(r,z,t\right)=\dfrac{\mathrm{d}z'}{4\pi}\left[\dfrac{r}{R^3}i\left(z',t-\dfrac{R}{c}\right)+\dfrac{r}{cR^2}\dfrac{\partial i\left(z',t-\dfrac{R}{c}\right)}{\partial t}\right] \\
R=\sqrt{r^2-\left(z-z'\right)^2}
\end{cases} \qquad (20)$$

where: $i\left(z',t\right)$: the current flowing through the dipole $\mathrm{d}z'$ at time t; ε_0: the permittivity of the vacuum; c: the speed of light; R: the distance between the dipole and the observation point; r: the horizontal distance between the channel and the observation point.

3.4. FDTD Modeling of the Interaction Between a Lightning Wave and a Power Transmission Line

In order to study the interaction between a lightning wave and a wire structure (power transmission line, cable, pylon, etc.), several works have been published in the literature and various mathematical models are proposed [49] [50].

The most rigorous modeling is that based on antenna theory which consists of solving an integral equation in electric or magnetic field by the numerical method called moments [51]. However, the threadlike nature of power lines has led to a more flexible modeling, it is the modeling by transmission line theory which is established under certain conditions [52]. In this case it is a question of solving the equations of the lines with or without second member in time or frequency.

For this purpose, we present the modeling based on the resolution of the equations of the lines with or without second member by FDTD of the interaction between a lightning wave (direct and indirect impact) and a power transmission line. The field theory developed from Maxwell's equations [53], is a rigorous modeling but not always easy to apply and allows to establish the second theory that of electric circuits. At any point in space, which is not located on a separation surface between two media, that is to say, in a linear, homogeneous, and isotopic medium (LHI), the general equations of Maxwell specify that [54]:

$$\nabla \times \boldsymbol{E}(t,r) = -\frac{\partial \boldsymbol{B}(t,r)}{\partial t} \tag{21}$$

$$\nabla \times \boldsymbol{H}(t,r) = \boldsymbol{J}(t,r) + \varepsilon_0 \frac{\partial \boldsymbol{E}(t,r)}{\partial t} \tag{22}$$

$$\nabla \cdot \boldsymbol{D}(t,r) = \rho(t,r) \tag{23}$$

$$\nabla \cdot \boldsymbol{B}(t,r) = 0 \tag{24}$$

The basic variables of these equations are:

$\boldsymbol{B}$: Magnetic induction [Tesla: T]

$\boldsymbol{H}$: Magnetic field intensity [Ampere/meter^2: Am^{-2}]

$\boldsymbol{D}$: Electric flux density [Dove/meter: Cm^{-2}]

$\boldsymbol{E}$: Electric field density [Volt/meter: Vm^{-1}]

$\boldsymbol{J}$: Electric current density [ampere/meter^2: Am^{-2}]

ρ : Electric charge density [Coulomb/meter^3: Cm^{-3}]

With:

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{25}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{26}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{27}$$

And:

μ : Magnetic permeability

ε : Electrical permeability

σ : Electrical conductivity

3.4.1. Coupling Equations Between an External Electromagnetic Field and Transmission Line

Consider a transmission line with n conductors parallel to the x-axis and located at a height h above a ground of finite conductivity and subjected to an electromagnetic disturbance and $\left[\boldsymbol{E}^e\right]$ as $\left[\boldsymbol{B}^e\right]$ shown in **Figure 6**.

From equations (22) and (23) [54], we can model the coupling between an external electromagnetic field and a transmission line via the following system of equations [55]:

$$\begin{cases} \dfrac{\mathrm{d}\left[U(x)\right]}{\mathrm{d}x}+[Z]\left[I(x)\right]=j\omega\left[\int_0^h B_y^e(x,z)\,\mathrm{d}z\right]+\left[E_x^e(x,0)\right] \\ \dfrac{\mathrm{d}\left[I(x)\right]}{\mathrm{d}x}+[Y]\left[U(x)\right]=-[Y]\left[\int_0^h E_z^e(x,z)\,\mathrm{d}z\right] \end{cases} \tag{28}$$

with: $[Z]$ and $[Y]$: respectively denote the longitudinal linear impedance and the transverse admittance of the line.

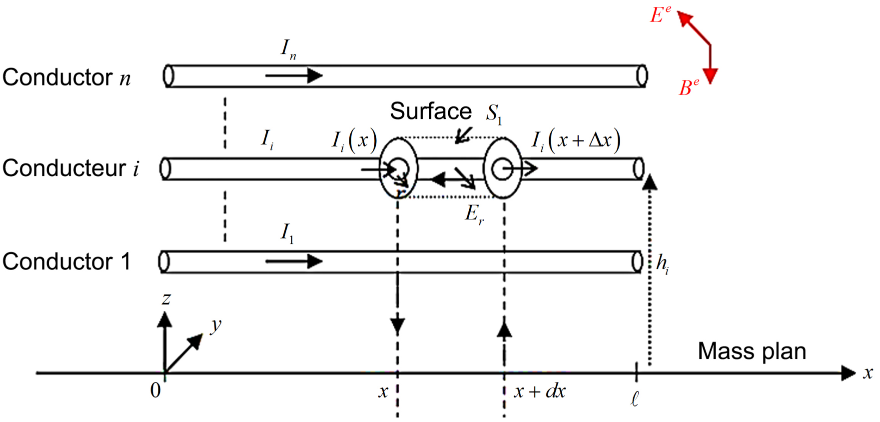


Figure 6. Geometry of an n-conductor line illuminated by an external electromagnetic field.

The coupling equations of the system (28), Taylor model, can be expressed in the form of three equivalent formulations, in which the source terms are functions:

- Electric and magnetic components of the exciting field "Taylor model" [56];
- Components of the exciting electric field "Agrawal model" [43];
- Components of the exciting magnetic field "Rachidi model" [57].

Remaining within the fundamental assumptions of transmission lines [52], for this article, we treat the coupling of the lightning wave with a wire structure from the model established by Agrawal [57]. This model is numerically more interesting than the other two, because it only involves a single source term in one of the two equations of the system (28); this source term does not contain any differentiation with respect to time and space. In the formulation established by Agrawal, the coupling equations are in terms of so-called "Scattered Voltage" voltages and total currents. For the case of a line with linear parameters independent of frequency, the coupling equations are:

$$\begin{cases} \dfrac{\partial\left[u^s(x,t)\right]}{\partial x}+[R]\left[i(x,t)\right]+[L]\dfrac{\partial\left[i(x,t)\right]}{\partial t}=\left[E_x^e(x,h,t)\right] \\ \dfrac{\partial\left[i(x,t)\right]}{\partial x}+[G]\left[u^s(x,t)\right]+[C]\dfrac{\partial\left[u^s(x,t)\right]}{\partial t}=[0] \end{cases} \tag{29}$$

where: $\left[E_x^s(x,h,t)\right]$: is the vector containing the tangential component of the exciting electric field on each conductor; $\left[u^s(x,t)\right]$ is the vector of diffracted voltages in the time domain; $\left[i(x,t)\right]$: is the vector of line currents in the time domain; $[R]$, $[L]$, $[G]$ and $[C]$: are respectively the matrices of the resistances, inductances, conductances and linear capacitances of the line.

In the case of a direct lightning strike on an overhead line, injection of a quantity of energy into the line. This is a localized injection at a point, which is the case considered by our article (**Figure 7** below).

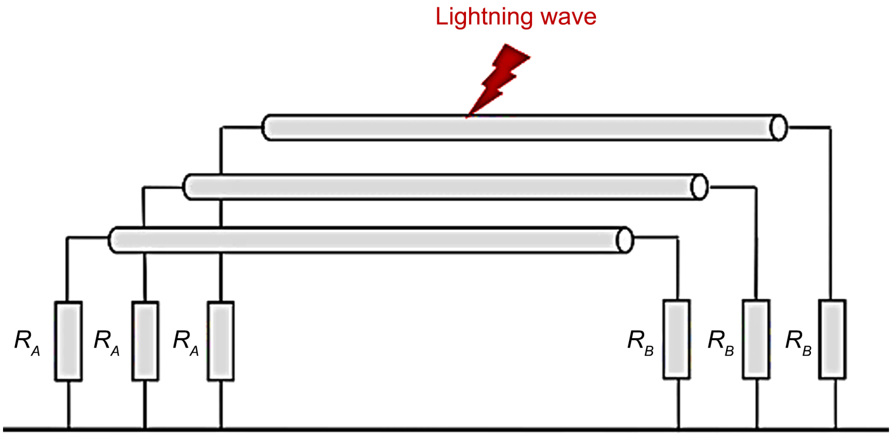


Figure 7. Direct impact of a lightning wave on an overhead line.

Then, the system of equations (28) is written as follows:

$$\begin{cases} \dfrac{\partial\left[u(x,t)\right]}{\partial x}+[R]\left[i(x,t)\right]+[L]\dfrac{\partial\left[i(x,t)\right]}{\partial t}=[0] \\ \dfrac{\partial\left[i(x,t)\right]}{\partial x}+[G]\left[u(x,t)\right]+[C]\dfrac{\partial\left[u(x,t)\right]}{\partial t}=[0] \end{cases} \tag{30}$$

Equation (1) of system (30) can be written in the following form:

$$\frac{\partial t}{\partial t}\frac{\partial\left[u(x,t)\right]}{\partial x}+[R]\left[i(x,t)\right]+[R]\frac{\partial\left[i(x,t)\right]}{\partial x}=0 \tag{31}$$

$$\frac{\partial t}{\partial x}\frac{\partial\left[u(x,t)\right]}{\partial t}+[R]\left[i(x,t)\right]+[R]\frac{\partial\left[i(x,t)\right]}{\partial x}=0 \tag{32}$$

Or: $\dfrac{\partial t}{\partial x}=v$ propagation speed of the lightning shock wave in m/sec. For this purpose, let us inject this expression into (32). We therefore find:

$$\frac{1}{v}\frac{\partial\left[u(x,t)\right]}{\partial t}+[R]\left[i(x,t)\right]+[R]\frac{\partial\left[i(x,t)\right]}{\partial x}=0 \tag{33}$$

where: $\left[u(x,t)\right]$ and $\left[i(x,t)\right]$: vectors of nodal voltages and currents in transient lightning regime.

3.4.2. FDTD Resolution of Lightning-HT Transmission Line Coupling Equations

For the coupling analysis by solving the line equations by FDTD, the conductor is subdivided alternately into current and voltage nodes (**Figure 8**).

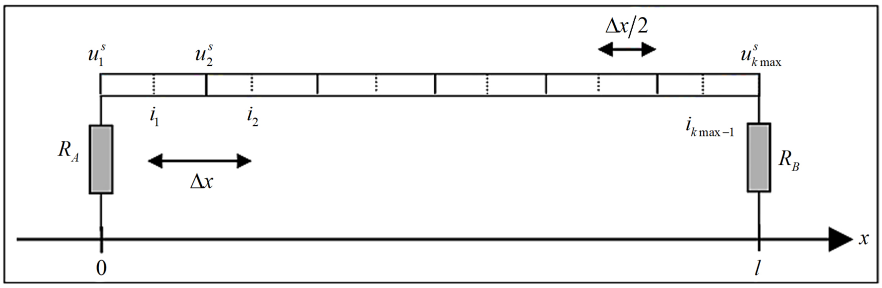


Figure 8. Spatial discretization of a conductor [55].

Two consecutive nodes of the same type are separated by an interval Δx. Note that the end nodes are voltage nodes and that the exciting electric field tangential to the conductor is calculated on the current nodes [55].

As in space, the current and voltage are offset by half a time step. More precisely, the current samples are $\Delta t/2$ *ahead* of the voltage. Figure 9 and Equation (34) illustrate the interleaving of voltages and currents in space and time.

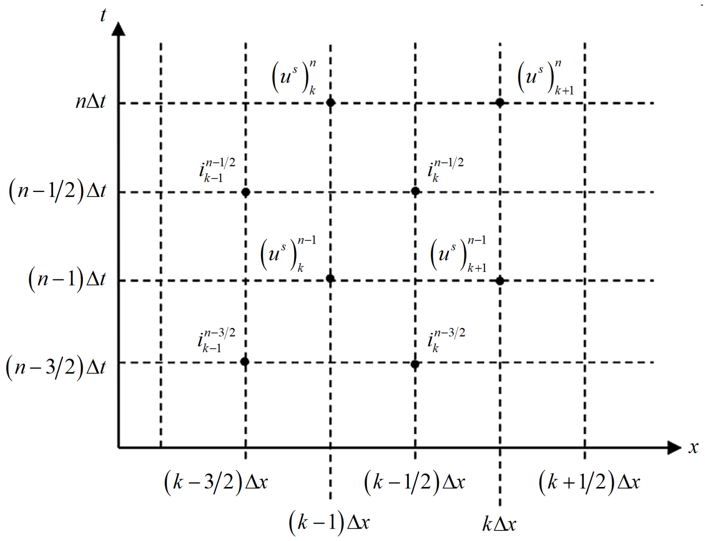


Figure 9. Spatial and temporal interleaving.

$$\begin{cases} (u^s)_k^n = u^s((k-1)\Delta x, n\Delta t) \\ (i)_k^{n+1/2} = i((k-1/2)\Delta x, (n+1/2)\Delta t) \\ (E_x^e)_k^{n+1/2} = E_x^e((k-1/2)\Delta x, (n+1/2)\Delta t) \end{cases} \tag{34}$$

With:

$$\begin{cases} t_{\max} = n_{\max}\Delta t \\ l = (k_{\max}-1)\Delta x \end{cases} \tag{35}$$

More precise approximations known as centered differences are found by expanding Equation (30) in Taylor series. In the case of a direct lightning strike on an overhead line, the FDTD discretization of Equations (30) allows us to write the following system of recurrence equations, respectively for $k = 1, \cdots, k_{\max} - 1$ and $k = 2, \cdots, k_{\max} - 1$:

$$\begin{cases} \left[i_k^{n+3/2} \right] = \left[[L] \frac{\Delta x}{\Delta t} + \frac{[R]}{2} \Delta x \right]^{-1} \left\{ \left[[L] \frac{\Delta x}{\Delta t} - \frac{[R]}{2} \Delta x \right] \left[i_k^{n+1/2} \right] - \left(\left[(u)_{k+1}^{n+1} \right] - \left[(u)_k^{n+1} \right] \right) \right\} \\ \left[(u)_k^{n+1} \right] = \left[[C] \frac{\Delta x}{\Delta t} + \frac{[G]}{2} \Delta x \right]^{-1} \left\{ \left[[C] \frac{\Delta x}{\Delta t} - \frac{[G]}{2} \Delta x \right] \left[(u)_k^{n} \right] - \left(\left[i_k^{n+1/2} \right] - \left[i_{k-1}^{n+1/2} \right] \right) \right\} \end{cases} \quad (36)$$

Since we consider the case of direct lightning strike, we assume that the propagation of the induced voltage and current occurs through the longitudinal elements (R and L). For this purpose, the equation considered for the simulation is the first of the system (36). We can therefore derive the following recurrence equation:

$$u_k^{n+1} = u_k^{n-1} + \left[L \frac{\Delta x}{\Delta t} + R \frac{\Delta x}{2} \right] i_k^{n+3/2} + \left[R \frac{\Delta x}{2} - L \frac{\Delta x}{\Delta t} \right] i_k^{n+1/2} \quad (37)$$

3.4.3. FDTD Model of the Power Transmitted by a HV Line in Lightning Regime

We directly obtain the FDTD model of the power at any point along each line conductor of the HV electrical network in lightning regime, by replacing $u_R = u_k^{n+1}$ in Equation (8). We therefore obtain:

$$P_k^{n+1} = \frac{u_s}{2X} \left[2u_s - \left\{ u_k^{n-1} + \left[L \frac{\Delta x}{\Delta t} + R \frac{\Delta x}{2} \right] i_k^{n+3/2} + \left[R \frac{\Delta x}{2} - L \frac{\Delta x}{\Delta t} \right] i_k^{n+1/2} \right\} \right] \quad (38)$$

with: P_k^{n+1}: FDTD model of power in lightning surge regime.

4. Simulation

4.1. Declaration of the Parameters of the Proposed Models

4.1.1. Channel Base Current Parameters

The parameters of the current at the base of the channel for the different models are presented in Table 2 below [40]: *Propagation speed*: $v = 1.5 \times 10^8$ m/s; *Lightning impact assessment point*: $z' = 0$ km, 2 km and 4 km, vertical axis $z = 7$ km; distance from the impact observation point $r = 500$ m or 0.5 km; the height of the lightning channel was set to a value of 7.5 km; the vacuum magnetic permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m and the vacuum electrical permittivity $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m.

Table 2. Parameters of the bi-exponential lightning current model [40].

1st arc in return	Subsequent return arc
$I_0 = 30$ kA	$I_0 = 10$ kA
$A = 2 \times 10^4$ 1/s	$A = 1.4 \times 10^4$ 1/s
$B = 2 \times 10^5$ 1/s	$B = 6 \times 10^5$ 1/s

4.1.2. HT Line Settings

The numerical values of $R = 0.07$ Ω/km, $L = 0.011$ mH/km and the length of the line 262 km are given in the table of characteristics of the 220 kV line of the SNEL/DRC West interconnected network, Inga-kimwenza section [58].

4.2. Simulation Results

See **Figures 10-15** below:

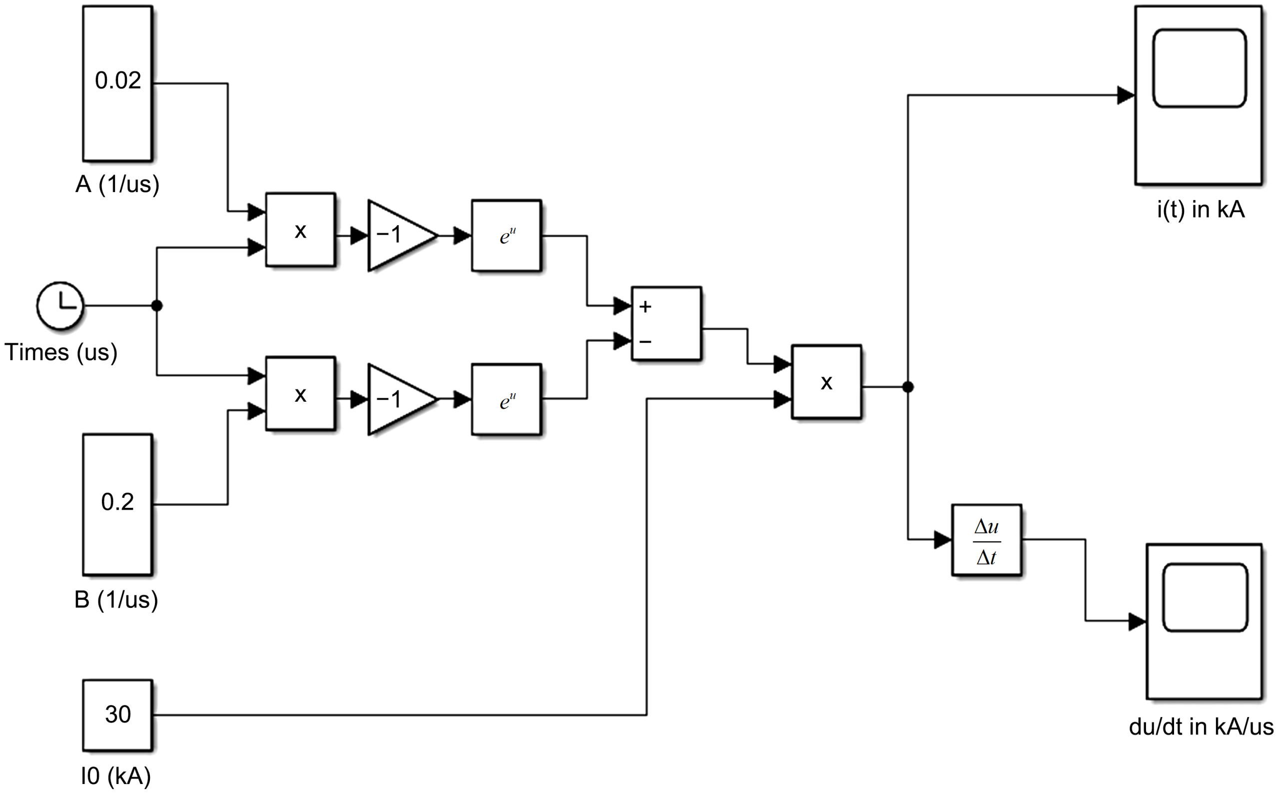
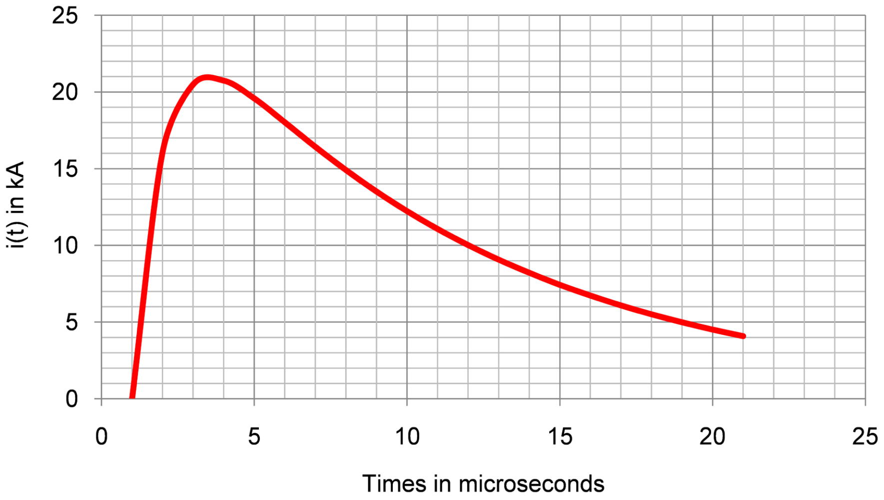
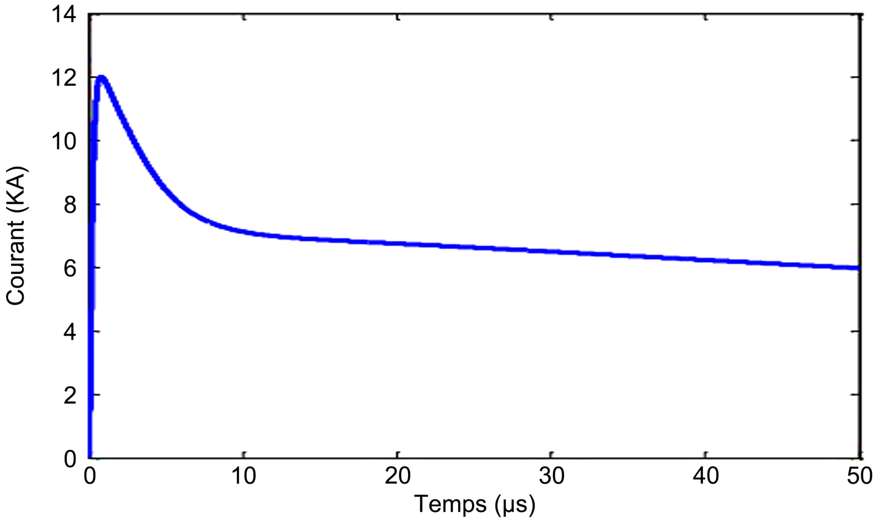


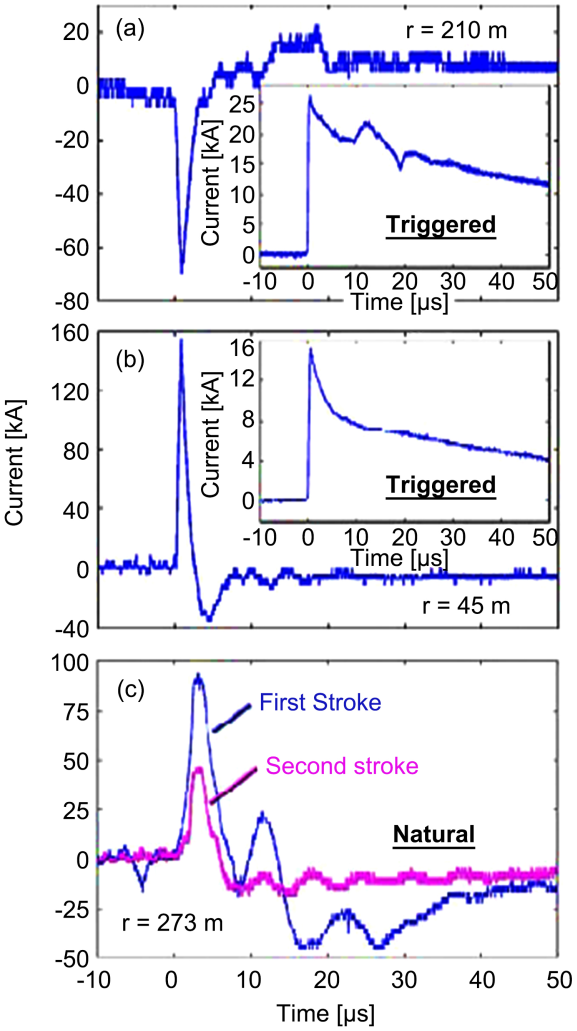
Figure 10. Numerical model of the wave $i(t)$ and the gradient $di(t)/dt$ of *the* lightning current.



(a) Calculation results

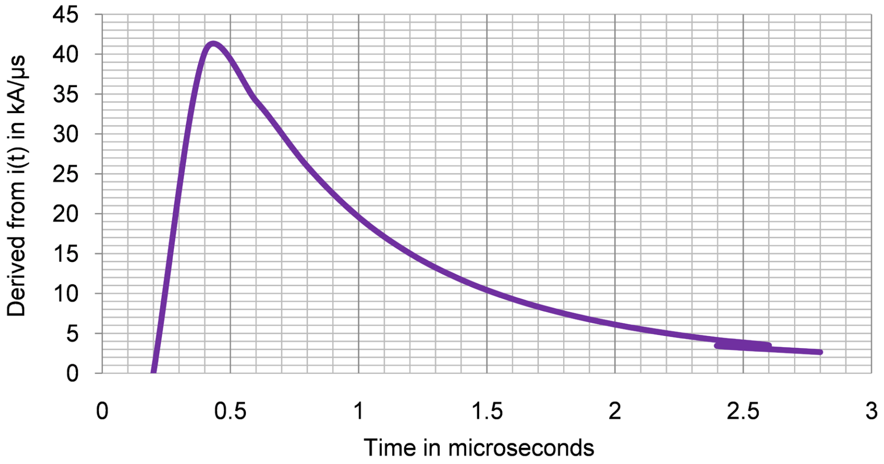


(b) Results published [Boumaiza Mostefa] [59]

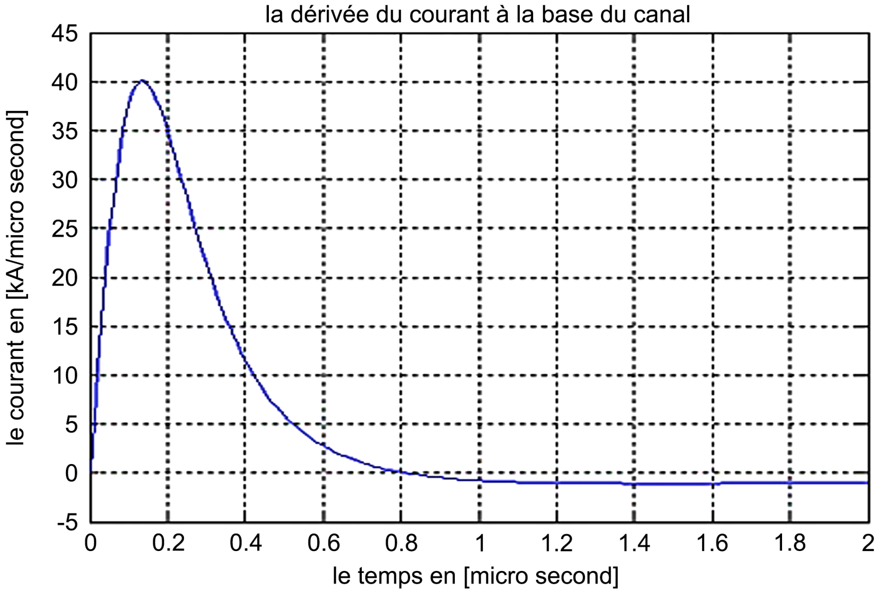


(c) Real measurement of the current at the base of the channel for lightning strikes 0503-2 and 0517-2. The lightning arc starts at *t* = 0. Shoene *et al.* [60]

Figure 11. Current at the base of the lightning channel.

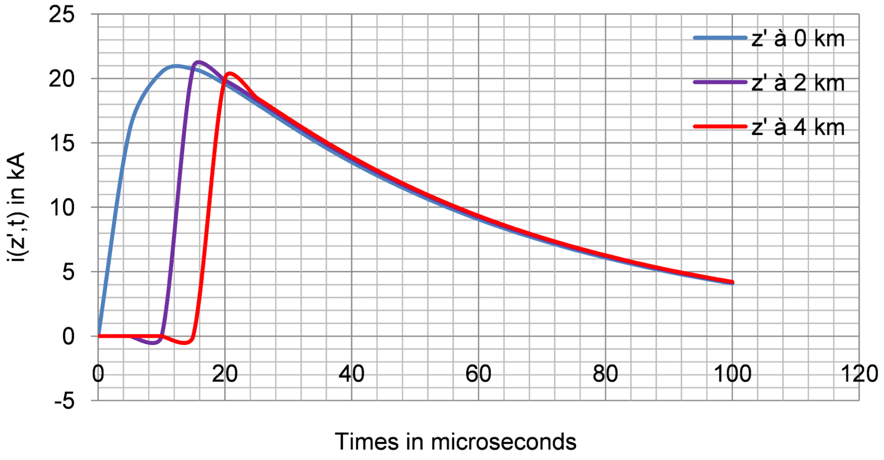


(a) Calculation results

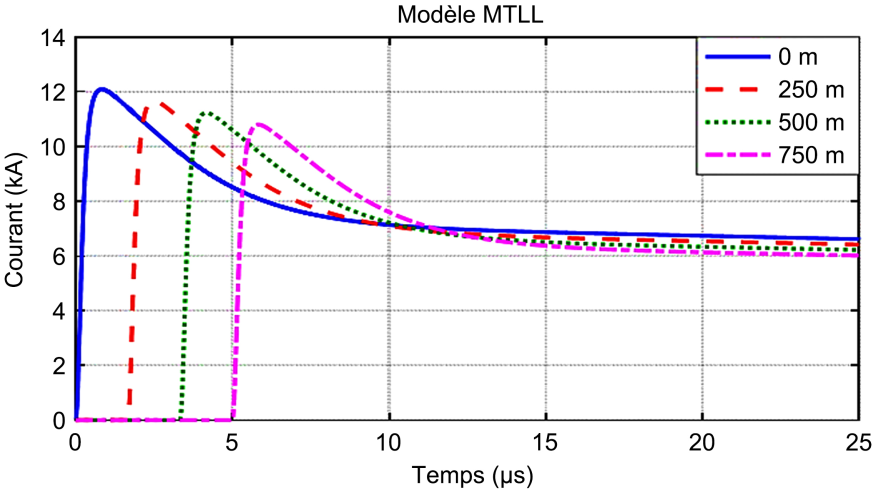


(b) Published results [Manel BIDI and Med. H. LATRECHE] [61]

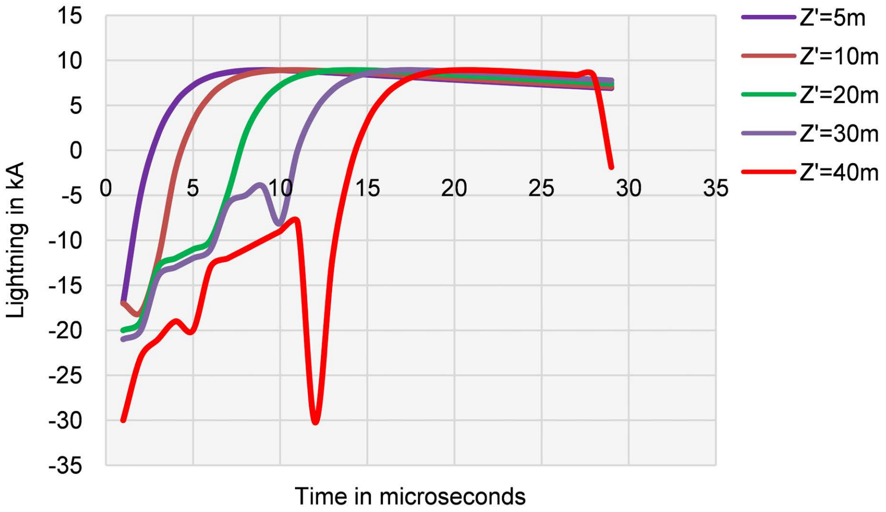
Figure 12. Temporal variation of the current at the base of the lightning channel.



(a) Calculation results

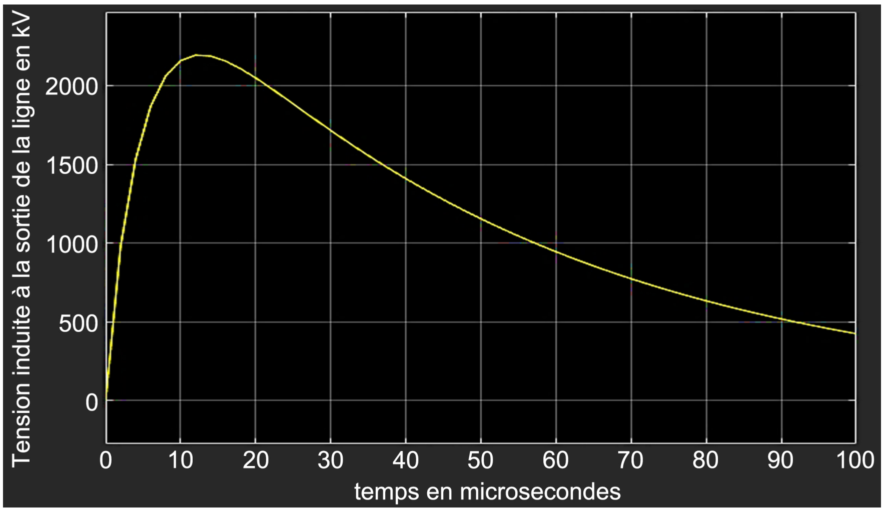


(b) Results published [HENNI Khadidja and NEDJADI Amel] [2021] [62]

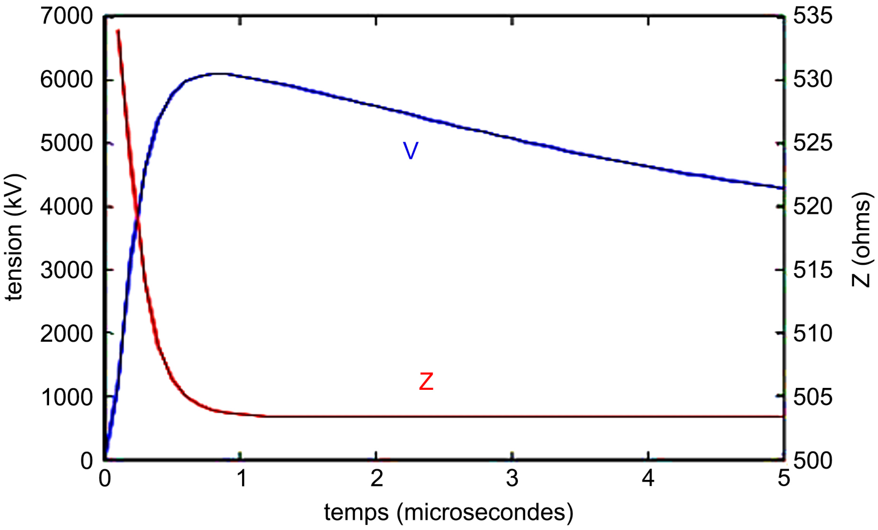


(c) Results published by [A. Bassesuka Sandoka Nzao] [2022] [63]

Figure 13. Spatio-temporal distribution of lightning current along the channel.



(a) Calculation results



(b) Results published by [JP NZURU NSEKERE in 2009] [64]

Figure 14. FDTD model of lightning induced voltage at the output of a HV line.

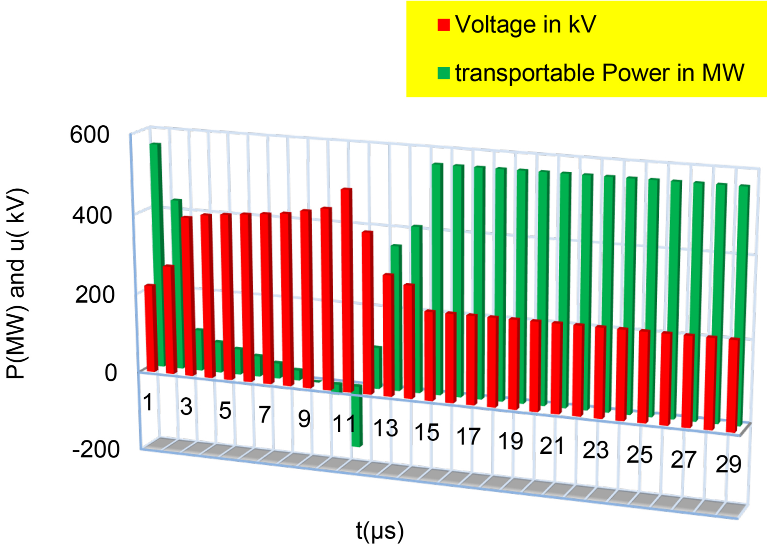


Figure 15. Calculation result: influence of the overvoltage induced by the lightning discharge on the power transportable by the HV line.

5. Discussions

HV electrical power transmission lines are the most exposed to disturbances during a lightning discharge. The study of their transient behaviour is generally based on Maxwell's field theories and transmission lines with electrical components such as *R*, *L*, *C* and *G*. Several equations exist for their modeling depending on the shape and characteristics of the material used in the installation. In order to obtain the results of different simulations, the bi-exponential, engineering, Agrawal and FDTD models have been chosen and used for the analytical and numerical modeling of the problem that is the subject of this article. For this purpose, the following results were obtained:

Figure 10 gives the numerical model reproducing the typical wave of lightning current $i(t)$ and its temporal variation $di(t)/dt$. **Figure 11(a)** shows that the passage of current at the base of the lightning channel is of an impulsive nature, that is to say, it is the cause of violent illumination of the lightning channel, responsible for the thunder, but especially for the damage produced and is characterized by the rise and fall time, this result is close to the experiments proposed in the work of Boumaiza Mostefa [59] to reproduce a typical lightning wave obtained by measurements (see **Figure 11(b)**). **Figure 11(c)** shows the results of actual channel base current measurements for lightning strikes 0503-2 and 0517-2. The lightning arc starts at $t = 0$. Shoene *et al.* [60]. And, this helps to strengthen the validity of the proposed models.

Figure 12(a) shows the temporal variation of the current at the base of the lightning channel. It is observed that the temporal variation of the lightning current can reach very high values, in this case, 40kA/mSec of amplitude. This result is confirmed by that proposed in the work by [Manel BIDI and Med. H. LATRECHE] [61] demonstrated in **Figure 12(b)**.

Figure 13(a) gives the spatio-temporal distribution of the current along the lightning channel. These results show that, for heights below the front of the return arc, *i.e.* $z' \leq vt$, this current is equal to the current at the base of the lightning channel, and in the case where $z' > vt$, this current is negative or even zero. This result brings together the experiments proposed by [HENNI Khadîdja and NEDJADI Amel, 2021] [62] presented in **Figure 13(b)** and by [A. Bassesuka Sandoka, 2022] [63] see **Figure 13(c)**.

Figure 14(a) gives the simulation result of the FDTD model of the voltage induced by lightning at the output of an HV line during the direct impact. It is noted that this induced voltage reaches an amplitude of 2200 kV under the direct impact and is directly proportional to the temporal variation of the disturbance current. This result is close to those presented in the work proposed by [J. P NZURU NSEKERE 2009] [64] see **Figure 14(b)**.

Finally, **Figure 15** shows that, during the period from 0 to 2 microseconds, the power at the output of the HV line is 560 MW if the load voltage is equal to the source voltage, *i.e.* $u_s = u_c = 220$ kV. In the event of a lightning strike, during the period from 2 to 15 microseconds, a transient condition appears, thus causing an overvoltage of amplitude 510 kV likely to cause a power drop at the output of the line. The power drop only occurs in transient conditions or when the overvoltage reaches 24% of the nominal voltage. This is only transient but can be inconvenient for the proper operation of the HV line. This situation leads to an increase in the frequency of outages in the case of lines operating without a power reserve margin. These outages are responsible for losses of undistributed kilowatt hours, that is, loss of revenue for the electricity transmission and distribution company.

6. Conclusions

At the end of our work, we studied the FDTD modeling of the transmissible power of an HV transmission line in lightning impulse regime. The focus was on a compensated HV inductive line because most of the lines operate in a compensated manner. We were interested in the study of the spatio-temporal distribution of the lightning current wave to model the radiated electromagnetic field and to examine the influence of the overvoltage induced by the atmospheric discharge on the transmittable power of an AC high voltage transmission line. The original contribution of this article is part of the implementation of an FDTD model of the power transmitted by an AC HV power line in direct lightning overvoltage regime. The aspects developed in this article could have direct implications in the practical applications of engineering and design of high voltage transmission systems. After theoretical considerations where we established some mathematical models that helped us define the problem, we took a practical case to do the simulation, the line being powered by a 220 kV source, it results that, the lightning current being of an impulse nature is characterized by the rise and fall time likely to cause an overvoltage on the HV power line. The power drop only occurs in transient mode or when the overvoltage reaches 24% of the nominal voltage. This is only transient but can be annoying for the proper operation of the HV line.

The study presented in this paper is crucial for several reasons, such as infrastructure safety because HV transmission lines are often subjected to extreme weather conditions, including lightning. This allows to simulate the effects of direct lightning strikes, assess risks and design more robust infrastructures.

In this type of research field, and due to the complexity of the lightning phenomenon and its accompanying effects, the situation remains fertile. Sensitivity analysis of the results to the variation of model parameters could be considered in the future to assess the robustness of the results. Our efforts remain motivated to achieve a comprehensive and general development of new electromagnetic field formulations that reflect the reality of the physical phenomenon.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Aguet, M. and Morf, J.J. (1981) Electricity, Treaty of Electricity, Volume XII. Polytechnic Press of Lausanne.

[2] Aguet, M. and Ianovici, M. (1982) High Voltage, Treaty of Electricity, Volume XXII. Polytechnic Press of Lausanne.

[3] Gardiol, F. (1996) L'Electromagnétisme, Treaty of Electricity, Volume III. Polytechnic Press of Lausanne.

[4] Haddad, A. and Warne, D. (1996) Advances in High Voltage Engineering. The Institution of Engineering and Technology.

[5] Begamudre, R.D. (1986) Extra High Voltage AC Transmission Engineering. New Age International Publisher.

[6] Denno, K. (1922) High Voltage AC Transmission Engineering. CRC Press.

[7] Roy, G., Gary, C. and Hutzler, B. (1988) The Dielectric Properties of Air and Very

High Voltages. Eyrolles.

[8] Gary, C. (1995) Crown Effect in Alternative Tension.

[9] Pelissier, R. (1975) Electric Power Networks, Volume 4, Propagation of Electric Waves on Power Lines. Dunod Technical.

[10] Ianovici, M. and Morf, J.J. (1983) Electromagnetic Compatibility. Romandie Polytechnic Press.

[11] Aguet, M. and Morf, J.J. (1987) Electrical Energy. EPFL Electricity, Electronics and Electrical Engineering Treatise. Romandie Polytechnic Press, 335 p.

[12] Guerrieri, S., Nucci, C.A., Rachidi, F. and Rubinstein, M. (1998) On the Influence of Elevated Strike Objects on Directly Measured and Indirectly Estimated Lightning Currents. *IEEE Transactions on Power Delivery*, **13**, 1543-1555. https://doi.org/10.1109/61.714865

[13] Yang, G., Yu, Z., Zhang, B., Gu, S. and He, J. (2018) Evaluation of Lightning Current from Magnetic Field Based on Deconvolution Method. *IEEE Transactions on Electromagnetic Compatibility*, **60**, 679-684. https://doi.org/10.1109/temc.2017.2747514

[14] Uman, M.A. and McLain, D.K. (1970) Lightning Return Stroke Current from Magnetic and Radiation Field Measurements. *Journal of Geophysical Research*, **75**, 5143-5147. https://doi.org/10.1029/jc075i027p05143

[15] Affolter, J.F. (2000) Energy and Electrical Networks IESE, EIVD Handout of Advanced Courses.

[16] Allab, K. (1984) Elements of Mathematical Analysis-Function of a Real Variable. Scientific Schools.

[17] Spiegel, M.R. (1975) Schaum's Outline of Theories and Problems of Advanced Calculus. McGraw-Hill, 160-165.

[18] Rachidi, F. (2002) Electromagnetic Effects of Lightning on Electric Lines, Electromagnetic Compatibility EMC, Course Handout, LRE, EPFL.

[19] de Coulon, F. and Jufer, M. (1984) Introduction to Electrical Engineering, Treatise on Electricity, Electronics and Electrical Engineering from EPFL. Presses Polytechniques Romandes, 347 p.

[20] Lasne, L. (2004) Electrical Engineering: Basics and Electrical Network. University of Bordeaux, Department of Electronics-Electrical Engineering-Automation.

[21] Van Cutsem, T. (1998) Electrical Power Systems. University of Liège, Department of Electricity, Electronics and Computer Science (Institut Montefiore).

[22] Cooray, V. (2003) The Lightning Flash. IEE Press. https://doi.org/10.1049/PBPO034E

[23] Guemri, B. (2004) Study and Analysis of the Electromagnetic Field-Transmission Line Coupling. Magister's Thesis, USTO-MB.

[24] Bermudez Arboleda, J.L. (2003) Lightning Currents and Electromagnetic Fields Associated with Return Strokes to Elevated Strike Objects. These Doctorate es Science, EPFL.

[25] Nucci, C.A., Diendorfer, G., Uman, M.A., Rachidi, F., Ianoz, M. and Mazzetti, C. (1990) Lightning Return Stroke Current Models with Specified Channel-Base Current: A Review and Comparison. *Journal of Geophysical Research*: *Atmospheres*, **95**, 20395-20408. https://doi.org/10.1029/jd095id12p20395

[26] Ianovici, M. and Morf, J.J. (1985) Electromagnetic Compatibility. 2nd Edition, Romandie Polytechnic Press.

[27] Cocquerelle, J.L. (1999) C.E.M. and Power Electronics. Editions Technip.

[28] Degauque, P. and Hamelin, J. (1990) Electromagnetic Compatibility: Radioelectric

Noise and Disturbance. Technical and Scientific Collection of Telecommunications. Éditions Dunod.

[29] Orzan, D. (1998) External and Internal Coupling between an Electromagnetic Field and a Network of Multi-Wire Lines. Doctoral Thesis, Swiss Federal Institute of Technology in Lausanne (EPFL).

[30] Sommerfeld, A. (1909) Über die Ausbreitung der Wellen in der drahtlosen Telegraphie. *Annalen der Physik*, **333**, 665-736. https://doi.org/10.1002/andp.19093330402

[31] Rachidi, F. (1991) Electromagnetic Effects of Lightning on Aerial Transmission Lines, Modeling and Simulation. Doctoral Thesis, Swiss Federal Institute of Technology in Lausanne.

[32] Dib, D. (1997) Study and Analysis of the Propagation of Atmospheric Overvoltages in HV Lines. Thesis of Magister, University of Annaba.

[33] Gary, C., Dragan, G. and Critescu, D. (1978) Attenuation of Propagation of Surges under the Influence of the Corona Effect, Extract from CIGRE, Group 33-13.

[34] Kiessling, F., Nefzger, P., Nolasco, J.F. and Kaintzyk, U. (2003) Overhead Power Lines: Planning, Design, Construction. Springer-Verlag. https://doi.org/10.1007/978-3-642-97879-1

[35] Gary, C. (2014) The Dielectric Properties of Air and Very High Voltages. Editions Eyrolles.

[36] Haddouche, A., Dib, D. and Benrettem, A. (2007) Three Phase Line Model with Corona Effect. *International Journal of Electric and Power Engineering*, **1**, 47-52.

[37] Djalel, D., Ali, H., *et al.* (2007) The Return-Stroke of Lightning Current, Source of Electromagnetic Fields (Study, Analysis and Modelling). *American Journal of Applied Sciences*, **4**, 42-48. https://doi.org/10.3844/ajassp.2007.42.48

[38] Dib, D., Haddocuhe, A. and Chemam, F. (2006) The Agression on the Electric Networks by the External Electromagnetic Fields. *International Journal of Electric and Power Engineering*, **1**, 114-121.

[39] Djalel, D. (2007) The Impact of Lightning on Electrical Networks Study, Analysis and Modeling. PhD Thesis, Badji Mokhtar Annaba University. https://biblio.univ-annaba.dz/wp-content/uploads/2014/05/THESE-DETAT-DIB.pdf

[40] Price, G.H. and Pierce, E.T. (1972) The Modeling of Channel Current in the Lightning Return Stroke. *Radio Science*, **12**, 381-388.

[41] CIGRE (2013) Guideline for Numerical Electromagnetic Analysis Method and Its Application to Surge Phenomena. Working Group C4.501.

[42] Gillard, R. (2005) Propagation of Electromagnetic Waves. INSA Rennes-France.

[43] Nekhoul, B., Harrat, B., Boufenneche, L., Chouki, M., Poljak, D. and Kerroum, K. (2014) A Simplified Approach to the Study of Electromagnetic Transients Generated by Lightning Stroke in Power Network. 2014 *International Symposium on Electromagnetic Compatibility*, Gothenburg, 1-4 September 2014, 595-600. https://doi.org/10.1109/emceurope.2014.6930975

[44] Kaouche, S., Nekhoul, B., Kerroum, K., Drissi, K.E. and Paladian, F. (2008) Induced Lightning Disturbances in a Network of Aerial Shielded Cables. *Annals of Telecommunications*, **62**, 894-924.

[45] Mezoued, S., Kaouche, S., Nekhoul, B., Kerroum, K., Khamlichi Drissi, K. and Paladian, F. (2006) Illumination of a Network of Lines or Cables by a Lightning Wave Modeling by Current Generators. *Annals of Telecommunications*, **61**, 721-747. https://doi.org/10.1007/bf03219931

[46] Paolone, M., Petrache, E., Rachidi, F., Nucci, C.A., Rakov, V., Uman, M., *et al.* (2005) Lightning Induced Disturbances in Buried Cables—Part II: Experiment and Model Validation. *IEEE Transactions on Electromagnetic Compatibility*, **47**, 509-520. https://doi.org/10.1109/temc.2005.853163

[47] Ametani, A., *et al.* (2013) Guideline for Numerical Electromagnetic Analysis Method and Its Application to Surge Phenomena.

[48] Nucci, C.A. (1995) Lightning-Induced over Voltages on Overhead Power Lines. Part 2: Coupling Models for the Evaluation of the Induced Voltages. Cigré Paper Prepared within the Framework of Task Force 33.01.01 of the CIGRE Working Group 33.01, 1994. Electra No. 162, October.

[49] Degauque, P. and Hamelin, J. (1990) Electromagnetic Compatibility, Noise and Radioelectric Disturbances. Technical and Scientific Collection of Telecommunications. Dun Edition.

[50] Harrington, R.F. (1968) Field Computation by Moment Method. Macmillan.

[51] lanoz, M., Nucci, C.A. and Tesche, F.M. (1988) Transmission Line Theory for Field-to-Transmission Line Coupling Calculations. *Electromagnetics*, **8**, 171-211. https://doi.org/10.1080/02726348808908214

[52] Taylor, C.D., Satterwhite, R.S. and Jr. Harrison, W. (1965) The Response of Terminated Two-Wire Transmission Line Excited by a Nonuniform Electromagnetic Field. *IEEE Transactions on Antennas and Propagation*, **13**, 987-989. https://doi.org/10.1109/TAP.1965.1138574

[53] Nekhoul, B., Labie, P., Zgainski, F.X., Meunier, G., Morillon, F. and Bourg, S. (1996) Calculating the Impedance of a Grounding System. *IEEE Transactions on Magnetics*, **32**, 1509-1512. https://doi.org/10.1109/20.497536

[54] Bouche, D., Ghidaglia, J.-M. and Pascal, F. (2005) Error Estimate and the Geometric Corrector for the Upwind Finite Volume Method Applied to the Linear Advection Equation. *SIAM Journal on Numerical Analysis*, **43**, 578-603.

[55] Sullivan, D.M. (1996) A Simplified PML for Use with the FDTD Method. *IEEE Microwave and Guided Wave Letters*, **6**, 97-99. https://doi.org/10.1109/75.482001

[56] Hoer, C. and Love, C. (1965) Exact Inductance Equations for Rectangular Conductors with Applications to More Complicated Geometries. *Journal of Research of the National Bureau of Standards, Section C: Engineering and Instrumentation*, **69**, 127-137. https://doi.org/10.6028/jres.069c.016

[57] Agrawal, A., Price, H. and Gurbaxani, S. (1980) Transient Response of Multiconductor Transmission Lines Excited by a Nonuniform Electromagnetic Field. *IEEE Transactions on Electromagnetic Compatibility*, **22**, 119-129. https://doi.org/10.1109/temc.1980.303824

[58] Rachidi, F. (1993) Formulation of the Field to Transmission Line Coupling Equations in Terms of Magnetic Excitation Field. *IEEE Transactions on Electromagnetic Compatibility*, **35**, 404-407. https://doi.org/10.1109/15.277316

[59] Boumaiza, M. and Labed, D. (2015) Lightning Electromagnetic Field Coupling to Overhead Power Distribution Lines. *International Journal of Applied Electromagnetics and Mechanics*, **47**, 1093-1106. https://doi.org/10.3233/jae-140144

[60] Schoene, J., Uman, M.A., Rakov, V.A., Kodali, V., Rambo, K.J. and Schnetzer, G.H. (2003) Statistical Characteristics of the Electric and Magnetic Fields and Their Time Derivatives 15 M and 30 M from Triggered Lightning. *Journal of Geophysical Research: Atmospheres*, **108**, Article No. 4192. https://doi.org/10.1029/2002jd002698

[61] Bidi, M. and Latreche, M.H. (2011) Modeling and Calculation of the Electromagnetic

Field Radiated by the Arc in Return of a Lightning Strike. CNI'2-2011, A. MIRA-Béjaia University. https://www.yumpu.com/fr/document/read/11713111/modelisation-et-calcul-du-champ-electromagnetique-rayonne

[62] Henni, K. and Nedjadi, A. (2021) Study of the Coupling of the Electro-Magnetic Field Generated by a Lightning Strike with Overhead Lines Using the 3D-FDTD Method. Master's Thesis, University of Saida Dr. Moulay Tahar.

[63] Nzao, A.B.S. (2022) Analysis of the Behavior of a HV AC Line during an Atmospheric Discharge and Evaluation of the Spatio-Temporal Distribution of the Lightning Current Wave. *Open Journal of Applied Sciences*, **12**, 175-195. https://doi.org/10.4236/ojapps.2022.122013

[64] Nsekere, J.P.N. (2009) Contribution to the Analysis and Realization of the Earthing of Electrical Installations in Tropical Regions. Doctoral Thesis, University of Liège.